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► **To cite this version:**

D. Dochain, A. Rapaport. An asymptotic observer for batch processes with single biogas measurement. 2nd IFAC Conference on Modelling, Identification and Control of Nonlinear Systems (MICNON 2018), Jun 2018, Guadalajara, Mexico. pp.420-424, 10.1016/j.ifacol.2018.07.315 . hal-01826938

HAL Id: hal-01826938

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Submitted on 30 Jun 2018

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An asymptotic observer for batch processes with single biogas measurement

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Abstract: In this paper we propose an observer for an example of systems for which the boundary of its domain is unobservable, and all trajectories converge to this boundary. The proposed case study, a bioreactor in batch operating conditions with one simple microbial growth reaction and gas production is standard and largely encountered in practical situations. Comparison of the proposed observer with a classical Luenberger observer has been performed: it shows that this later does not guarantee a convergence with no estimation error.

Keywords: State observer, observability, biological systems, biosystem.

1. INTRODUCTION

The state observation of nonlinear systems is a wide and active research area resulting in a large scientific publications on the subject. Let us point out a few (e.g. Gauthier & Kupka [1998] Kazantzis & Kravaris [1998], Krener & Isidori [1983]) as well as the interesting and nicely survey paper by Krener (Krener [2004]). In the present paper, we would like to address a state observation issue that, as far as we know, has yet been addressed, i.e. the problem of the on-line reconstruction of the variables of nonlinear systems for which there is a loss of observability on the boundary of its domain. we shall indeed concentrate on a biological system example with one biomass x , one substrate s and gaseous outflow rate y measured on-line when the initial condition (x_0, s_0) is unknown. Incidentally the design of the state estimate takes advantage of the fundamental reaction invariant property of reaction systems (Gavalas [1968]).

The paper is organized as follows. Section 2 introduces the dynamical model of the biological system. Section 3 provides an analysis of the observability properties of the system under study. An asymptotic observer is derived in Section 4 while Section 5 gives a Luenberger observer for the purpose of comparison. Section 6 provides numerical simulation results to illustrate the performance of the proposed observer as well as its comparison with the Luenberger observer designed in Section 5. Section 7 studies the behavior of our proposed observer for specific growth rate models (Hill, Haldane) that do not satisfy Hypothesis 2.

2. DYNAMICAL MODEL

Let us consider the dynamical model of a simple microbial growth reaction in a batch reactor (see e.g. Bastin & Dochain [1990]):

$$\begin{cases} \dot{x} = \mu(s)x \\ \dot{s} = -\mu(s)x \end{cases} \quad (1)$$

where x and s stand for the concentrations in biomass and substrate, respectively. Without loss of generality, we assume that the yield coefficient of the transformation of the substrate into biomass is equal to 1. The specific growth rate function $\mu(\cdot)$ satisfies the usual assumption:

Hypothesis 1. The function $\mu(\cdot)$ is Lipschitz continuous on \mathbb{R}_+ , positive on $(0, +\infty)$ with $\mu(0) = 0$.

For convenience, we denote the number

$$\bar{\mu} = \max_{s>0} \mu(s)$$

This number could be finite or not. In the present work, we consider that a gaseous by-product flow rate, such as biogas (e.g. Bernard et al. [2001]), is measured on-line as a quantity proportional to the output variable

$$y = \mu(s)x.$$

It often happens in batch bioprocesses that the initial quantities of reactants (x_0, s_0) are not well known while the biogas production is the only available measurement during the operation of the process. As the solutions of (1) clearly satisfy $\lim_{s \rightarrow +\infty} s(t) = 0$, the initial quantity of substrate can be recovered as

$$s_0 = \lim_{t \rightarrow +\infty} \int_0^t y(\tau) d\tau$$

However the main interest is usually to estimate the total production of biomass, that is $\lim_{t \rightarrow +\infty} x(t)$. The purpose of the present work is the present a simple and reliable methodology to estimate the x concentration.

3. OBSERVABILITY ANALYSIS

Equivalently to the set of differential equations (1), one can consider the dynamics in the (z, s) coordinates with $z = x + s$

$$\frac{d}{dt} \begin{bmatrix} z \\ s \end{bmatrix} = f(z, s) = \begin{bmatrix} 0 \\ -\mu(s)(z - s) \end{bmatrix} \quad (2)$$

$$y = h(z, s) = \mu(s)(z - s) \quad (3)$$

on the positive cone

$$\mathcal{C} := \{(z, s) \in \mathbb{R}^2 \mid z > s > 0\}$$

Here we require a stronger assumption on the function μ .

Hypothesis 2. The function μ is C^2 , concave and increasing on \mathbb{R}_+ , with $\mu(0) = 0$.

Such an assumption is fulfilled for the well-known Monod function (see Figure 1)

$$\mu(s) = \frac{\mu_{\max} s}{K_s + s}$$

the most largely specific growth rate model used in biotechnology and biological systems.

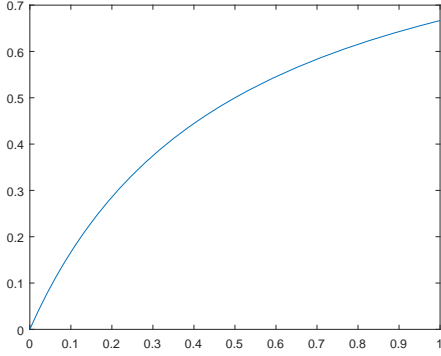


Fig. 1. The Monod model

Lemma 3. Under Hypothesis 2, the system (2)(3) is differentially observable on \mathcal{C} , but not on its boundary.

Proof. On the open cone \mathcal{C} , one has $\mu(s) > 0$ and can write

$$\begin{aligned} L_f h(z, s) &= -[\mu'(s)(z - s) - \mu(s)]\mu(s)(z - s) \\ &= -\left(\frac{\mu'(s)}{\mu(s)}h(z, s) - \mu(s)\right)h(z, s). \end{aligned} \quad (4)$$

For a given non-negative number y , we define the function

$$\varphi_y(s) = \mu(s) - \frac{\mu'(s)}{\mu(s)}y$$

whose derivative is

$$\varphi'_y(s) = \mu'(s) - \left(\frac{\mu''(s)}{\mu(s)} - \frac{\mu'(s)^2}{\mu(s)^2}\right)y.$$

Under Hypothesis 2, $\varphi'_y(s)$ is positive for any s , and therefore the inverse φ_y^{-1} is well defined from $\varphi(\mathbb{R}_+)$ to \mathbb{R}_+ . As $h(z, s) > 0$ on \mathcal{C} , one can write

$$\begin{cases} s = \varphi_{h(z,s)}^{-1}\left(\frac{L_f h(z,s)}{h(z,s)}\right) \\ z = \frac{h(z,s)}{\mu(s)} + s \end{cases}$$

and conclude that the map

$$\begin{bmatrix} z \\ s \end{bmatrix} \mapsto \begin{bmatrix} h(z,s) \\ L_f h(z,s) \end{bmatrix}$$

is injective on \mathcal{C} , that is the system is differentially observable on \mathcal{C} .

On the boundary of \mathcal{C} , one has $L_{f^i} h \equiv 0$ for any integer i and therefore the system is not differentially observable on $\partial\mathcal{C}$.

One can note that the boundary $z = s$ (that is $x = 0$) of the cone \mathcal{C} is invariant but repulsive for the dynamics, while the boundary $s = 0$ is attractive. Therefore for any initial condition with $z_0 > s_0$ (that is $x_0 > 0$), the solution converges to the boundary $s = 0$ where the system is no longer observable. This feature prevents classical constructions of nonlinear observers, such as the high-gain observer, that requires the (differential) observability on a compact invariant set (which here has to contain $s = 0 \dots$).

4. AN ASYMPTOTIC OBSERVER

In this section, we first show that the system is detectable from any positive initial condition, without requiring Hypothesis 2.

Proposition 4. Assume that one as $y(0) > 0$, then one has

$$\lim_{t \rightarrow +\infty} s(t) = 0, \quad \lim_{t \rightarrow +\infty} x(t) = z$$

where

$$z = \frac{y(0)}{\mu\left(\int_0^{+\infty} y(\tau) d\tau\right)} + \int_0^{+\infty} y(\tau) d\tau \quad (5)$$

for any $t \geq 0$.

Proof. When $y(0) > 0$, one has $x_0 > 0$ and $s_0 > 0$. Clearly, the solution of (1) fulfills $(x(t), s(t)) \rightarrow (z, 0)$ when t tends to $+\infty$, where $z = x_0 + s_0$. Therefore, one can write

$$s(t) = \int_t^{+\infty} y(\tau) d\tau$$

at any time $t \geq 0$, and then one also

$$x(t) = \frac{y(0)}{\mu\left(\int_0^{+\infty} y(\tau) d\tau\right)}.$$

Finally, one obtains

$$z = x(t) + s(t) = \frac{y(0)}{\mu\left(\int_0^{+\infty} y(\tau) d\tau\right)} + \int_t^{+\infty} y(\tau) d\tau$$

One can then consider the following asymptotic observer.

Proposition 5. For any initial condition (x_0, s_0) with $x_0 > 0$ and $s_0 > 0$, the following observer

$$\begin{cases} \dot{v}(t) = y(t), & v(0) = 0 \\ \hat{x}(t) = \frac{y(0)}{\mu(v(t))} + v(t), & (t > 0) \end{cases} \quad (6)$$

fulfills

$$\lim_{t \rightarrow +\infty} \hat{x}(t) - x(t) = 0$$

Moreover, the error $\hat{x} - x$ is decreasing with time.

Proof. Note that for $t > 0$, the solution of (6) is given by the expression

$$\hat{x}(t) = \frac{y(0)}{\mu\left(\int_0^t y(\tau) d\tau\right)} + \int_0^t y(\tau) d\tau$$

The convergence of the observer is then a simple consequence of Proposition 4. The time derivative of the x -error is determined straightforwardly as

$$\frac{d}{dt}(\hat{x} - x)(t) = -y(t_0) \frac{\mu' \left(\int_{t_0}^t y(\tau) d\tau \right)}{\left[\mu \left(\int_{t_0}^t y(\tau) d\tau \right) \right]^2} y(t) < 0$$

which shows the monotonic behavior of the error.

Remark 6. The internal variable $v(\cdot)$ of the observer (6) is well defined for any $t \geq 0$, while the estimation $\hat{x}(\cdot)$ as “output” of this system is defined only for $t > 0$, which is quite unusual in the observer constructions. More precisely, one has $\lim_{t \rightarrow 0^-} \hat{x}(t) = +\infty$, but in practice the estimation $\hat{x}(\cdot)$ drops down very quickly from large values at small times $t > 0$, as it can be seen on numerical simulations in Section 6. This is not related to a “peaking phenomenon” in the dynamics (there is no “high-gain”), but is simply due to the fact that \hat{x} is not defined at $t = 0$.

Remark 7. The fact that the error $\hat{x} - x$ is decreasing with time guarantees that $\hat{x}(t)$ is an upper estimation of $x(t)$ at any time t and that the estimator does not oscillate as it could happen with high-gain observers.

5. A LUENBERGER OBSERVER

In this section, we consider functions $\mu(\cdot)$ that satisfy Hypothesis 2, so that the differential observability is fulfilled during the transient. We study the behavior of an Luenberger observer when time tends towards infinity. As we aim at reconstructing the biomass x , and the output y is not a state variable of the original dynamics, we write the system (1) in (x, y) coordinates as follows

$$\begin{cases} \dot{\hat{x}} = y \\ \dot{\hat{y}} = \phi(x, y)y \end{cases} \quad (7)$$

with

$$\phi(x, y) = -\mu' \circ \mu^{-1} \left(\frac{y}{x} \right) x + \frac{y}{x}$$

Note that μ^{-1} is well defined on $[0, \bar{\mu}]$ under the Hypothesis 2. However, the map ϕ has a singularity at $x = 0$, but solutions of (1) satisfy $x(t) \geq x_0$ at any $t > 0$ and therefore avoid this singularity for any initial condition with $x_0 > 0$. For the derivation of an observer, one has to extend this dynamics for values of x that are non positive or such that y/x is larger than $\bar{\mu}$. Let ϵ be a positive number, and consider the map

$$\tilde{\phi}(x, y) = \phi(\max(x, \epsilon, y/\bar{\mu}), y)$$

which coincides with ϕ along any solution of (7) with $x_0 \geq \epsilon$. Moreover $\tilde{\phi}$ is Lipschitz continuous on $\mathbb{R} \times \mathbb{R}_+$. We can then consider a Luenberger observer in the (x, y) coordinates for the dynamics with ϕ replaced by $\tilde{\phi}$:

$$\begin{cases} \dot{\hat{x}} = y + G_1(\hat{y} - y(t)) \\ \dot{\hat{y}} = \tilde{\phi}(\hat{x}, y(t))y(t) + G_2(\hat{y} - y(t)) \end{cases} \quad (8)$$

Let us write the error equation as follows:

$$\frac{d}{dt} \begin{bmatrix} e_x \\ e_y \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & G_1 \\ \delta(t)y(t) & G_2 \end{bmatrix}}_{A(t)} \begin{bmatrix} e_x \\ e_y \end{bmatrix}$$

where

$$\delta(t) = \begin{cases} \frac{\tilde{\phi}(\hat{x}(t), y(t)) - \phi(x(t), y(t))}{\hat{x}(t) - x(t)} & \text{if } \hat{x}(t) \neq x(t) \\ \partial_x \phi(x(t), y(t)) & \text{if } \hat{x}(t) = x(t) \end{cases}$$

Under Hypothesis 2, the derivative μ' is bounded on \mathbb{R}_+ and then the map $x \mapsto \tilde{\phi}(x, y)$ has linear growth for any

$y \in \mathbb{R}_+$. As $y(\cdot)$ is bounded, $\delta(\cdot)$ is thus bounded whatever the solution $\hat{x}(\cdot)$ is.

Therefore, as one has $\lim_{t \rightarrow +\infty} y(t) = 0$, we obtain

$$\lim_{t \rightarrow +\infty} A(t) = \bar{A} = \begin{bmatrix} 0 & G_1 \\ 0 & G_2 \end{bmatrix}$$

which is not a Hurwitz matrix. Therefore, the asymptotic convergence of the error towards 0 is not guaranteed and the choice the gains G_1, G_2 does not allow to assign the speed of convergence.

6. NUMERICAL SIMULATIONS

For the Monod function

$$\mu(s) = \frac{s}{1+s}$$

with the initial condition $(s_0, b_0) = (1.1, 1.5)$ we have compared the asymptotic observer with the Luenberger one initialized with $(\hat{x}, \hat{y}) = (1, y(0))$ and various gains G_1, G_2 . We found systematically a biased asymptotic error of the Luenberger observer, as depicted on Fig. 2 (for $G_1 = G_2 = -20$ that gave the best result). These simulations show that the innovation $\hat{y} - y$ of the Luenberger observer reaches zero while the x error has not yet converged to zero, which explains its non null asymptotic error. We have

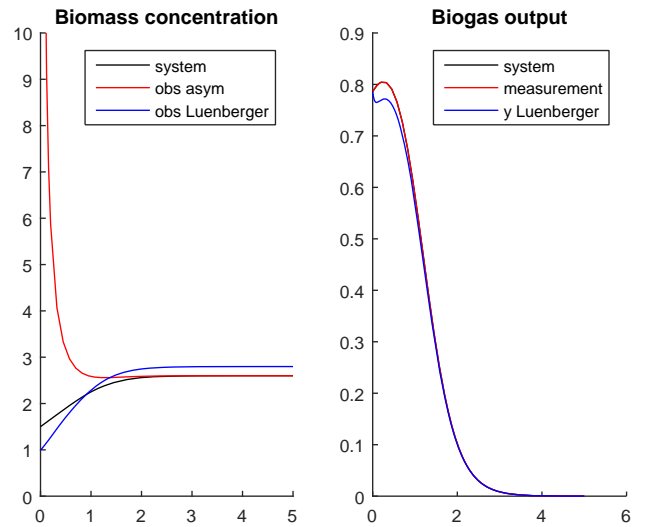


Fig. 2. Comparison of our proposed observer with the Luenberger observer in absence of noise

also performed simulations with measurements randomly disturbed by a white noise proportionate up to 10% of the signal (see Fig. 3). It shows the good behavior of the asymptotic observer with respect to measurement noise. Indeed, the dynamics of \hat{x} in equations (6) use integrals of the output, and not directly the output (apart the value of y at time 0) as classical observers, which filters the noise. The asymptotic observer is mainly affected by the error on the initial measurement. We have also tested the observer with a louder noise (up to 25% of the signal). Then, a large error on the initial measurement leads, as expected, to an asymptotic bias, as for the Luenberger observer (see Fig. 4).

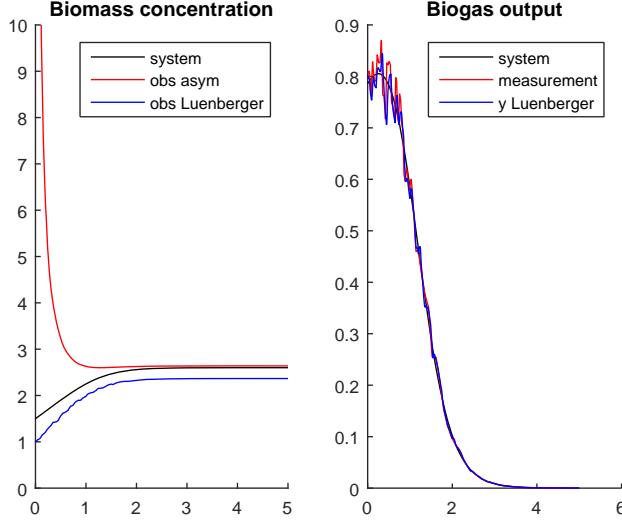


Fig. 3. Comparison of our proposed observer with the Luenberger observer with noise

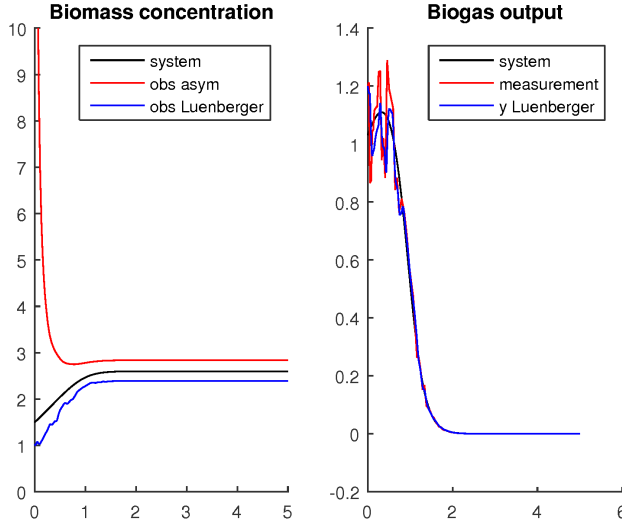


Fig. 4. Simulations with a strong noise

7. IN ABSENCE OF HYPOTHESIS 2

In this Section we consider two different growth functions $\mu(\cdot)$ that do not satisfy Hypothesis 2. This does not prevent the convergence of the asymptotic observer of Proposition 5, which requires Hypothesis 1 only to be fulfilled. The system remains detectable in the sense of Proposition 4, but the observability analysis of Section 3 can no longer be conducted. As a matter of comparison, we have simulated the Luenberger observer anyway.

7.1 Hill function

The Hill function for the specific growth rate writes as follows (see Figure 5):

$$\mu(s) = \frac{\mu_{max} s^\alpha}{K_s^\alpha + s^\alpha}$$

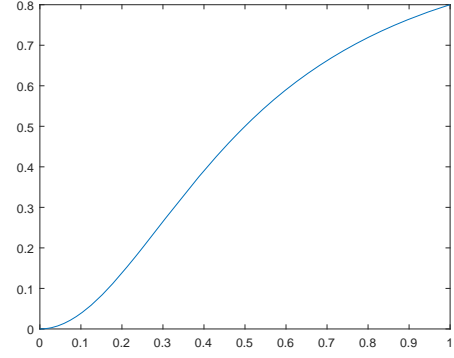


Fig. 5. The Hill model

This function is increasing but not concave (see Fig. 5). Simulations of Fig. 6 have been ran for $\mu_{max} = 1$, $K_s = 0.5$ and $\alpha = 2$.

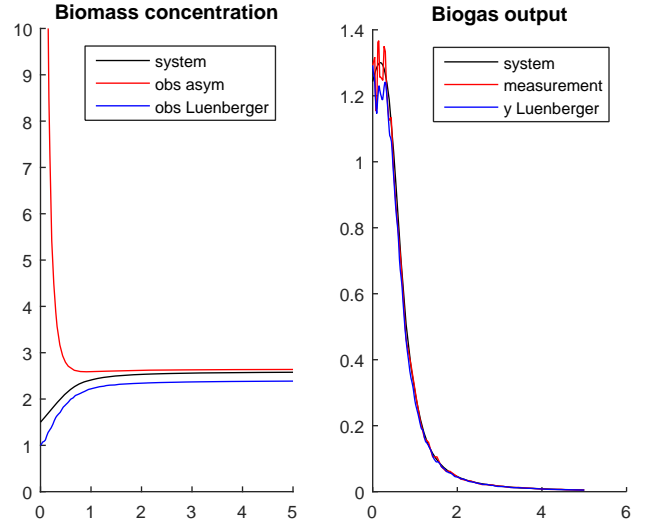


Fig. 6. Comparison of our proposed observer with the Luenberger observer with Hill function

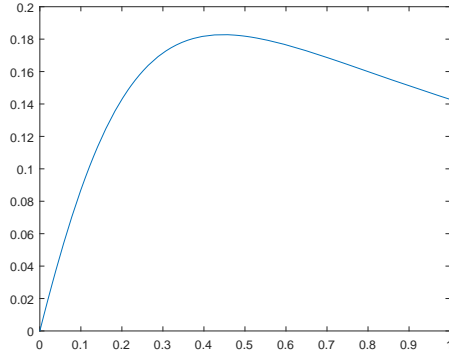
7.2 Haldane function

The Haldane model for the specific growth rate (see Figure 7) is given by the following expression:

$$\mu(s) = \frac{\mu_0 s}{K_s + s + \frac{s^2}{K_i}}$$

This function is non monotonic (see Fig. 7). Therefore the inverse of μ is not uniquely defined for the construction of the Luenberger observer in (x, y) coordinates. However, for each positive value m of the function μ , one has $\mu^{-1}(m) = \{s_-(m), s_+(m)\}$ with $0 < s_-(m) \leq \sqrt{K_s K_i} \leq s_+(m)$, and as solutions of system (1) converge to $s = 0$, we have considered $\mu^{-1}(m) = s_-(m)$ in the Luenberger observer (8) (which gives the right inverse a soon as t satisfies $s(t) \leq \sqrt{K_s K_i}$). Simulations of Fig. 8 have been ran for $\mu_0 = 1$, $K_s = 1$ and $K_i = 0.2$.

These simulations show that the proposed observer works satisfactorily in simulation for a large variety of growth



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Fig. 7. The Haldane model

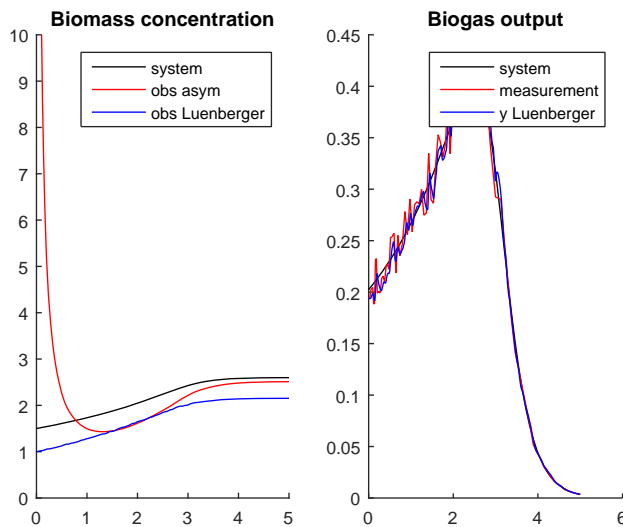


Fig. 8. Comparison of our proposed observer with the Luenberger observer with Haldane function

functions. One can note that for the Haldane function, the property of having an upper estimation of the biomass is no longer satisfied, in accordance with Proposition 5.

8. CONCLUSIONS

We have proposed a new observer for systems that are not observable on a subset that attracts the dynamics. This observer guarantees an asymptotic convergence, while a classical Luenberger observer produces an asymptotic bias. Usually, one requires from a smooth observer to have tuning parameters for assigning an exponential speed of convergence. Here, as the trajectories of the system converge to a subset of non observability, one cannot expect to obtain simultaneously an exact convergence and an assignable speed of convergence of an observer. The observer we propose does not have a adjustable speed of convergence but it guarantees an exact convergence, and numerical simulations show its good robustness with respect to measurement noise. However, the asymptotic error of the proposed observer relies mainly on the initial measurement of the output. Robust extensions of this observer will be the matter of a future work.