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# Maximum thermal emission of subwavelength spherical objects

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## Abstract

We calculate in this note the maximum thermal emission of a spherical object which size is much smaller than the thermal wavelength. Using reciprocity theorem under the form of the Kirchhoff law and using the recent results of Grigoriev et al. [1], we show that such a spherical object has its thermal emission which is bound to a certain value scaling as  $T^2$ .

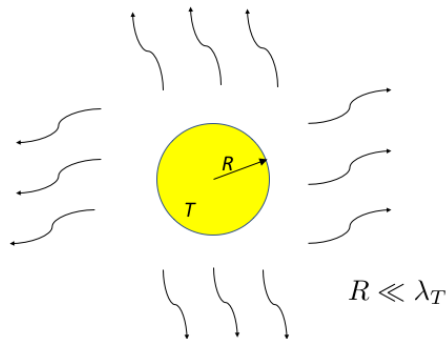


FIG. 1: Nanoparticle at temperature  $T$  with a radius much smaller than the thermal wavelength  $\lambda_T$ .

## I. SYSTEM CONSIDERED

We consider a spherical object filled with a material which permittivity is given by  $\epsilon$  and temperature is given by  $T$ . The system is depicted in Fig. 1. The simple question which is addressed in this note is to know what is the maximum thermal emission of such a spherical object when its size is much smaller than the thermal wavelength  $\lambda_T$  given by the Wien law and corresponding to the wavelength at which Planck emission is maximum ( $\lambda_T T = 2898 \mu\text{m K}$ ).

## II. MAXIMUM THERMAL EMISSION OF A DIPOLAR NANOPARTICLE

Let us now calculate the thermal emission of a spherical object. A consequence of the reciprocity theorem in electromagnetism is the Kirchhoff law stating that the absorptivity of any object is equal to its emissivity. Thermal emission  $\phi_\omega$  in the most general case is thus the product of the absorption cross section  $\sigma_{abs}$  and of the so called thermal emittance  $H_\omega^0 = \hbar\omega/[4\pi^2c^2(\exp[\hbar\omega/k_bT])]$

$$\phi_\omega = \sigma_{abs} H_\omega^0 = \frac{\hbar\omega^3}{4\pi^2c^2(\exp[\hbar\omega/k_bT] - 1)} \sigma_{abs} \quad (1)$$

where,  $\omega$  is the angular frequency considered,  $\hbar$  is the reduced Planck constant,  $c$  the velocity of light and  $k_b$  the Boltzmann constant. In the case of an isolated sphere, the absorption cross section can be obtained exactly with Mie theory. When this sphere has a radius which

is much smaller than the wavelength considered, one is in the dipolar approximation and the absorption cross section is completely dominated by the first term given by the Mie theory. Grigoriev et al. [1] have shown that it is possible to achieve an ideal absorption if the sphere permittivity follows a very specific function on  $\rho = \omega R/c$  where  $R$  is the sphere radius. When this condition is fulfilled, the absorption is maximized and the maximum absorption cross section is given by

$$\sigma_{abs}^{max} = \frac{5\pi}{k_0^2} \quad (2)$$

where  $k_0 = \omega/c$ . Note that under these conditions, the scattering cross section is equal to the absorption cross section. Reporting the maximum scattering cross section in the expression of the flux and integrating over all the frequencies, the maximum flux emitted by a spherical object in the dipolar approximation is

$$\phi^{max} = \frac{5\pi k_b^2 T^2}{24\hbar} = 1.18 \times 10^{-12} \text{ W K}^{-2} T^2 \quad (3)$$

which is apart to a numerical number the product of the quantum of thermal conductance with the temperature. Note that this flux does not depend on the sphere size (although the optimized dielectric function maximizing absorption cross-section  $\sigma_{abs}$  depends on it) and that it scales as  $T^2$ .

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[1] V. Grigoriev, N. Bonod, J. Wenger, and B. Stout, ACS Photonics **2**, 263 (2015).