Segmentation of texts in Old Babylonian mathematics

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Chapter 3
Segmentation of texts in Old Babylonian mathematics

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Abstract[2]: This chapter offers an analysis of the segmentation of texts into lists of mathematical problems written on clay tablets during the Old Babylonian period (early second millennium BCE). The study focuses on mathematical series texts, that is, long lists of statements written on several numbered tablets. Two aspects are considered: material segmentation (sections, columns, tablets...) and textual segmentation (statements, groups of statements), as well as the relationship between these two aspects. It is shown that the analysis of parts of text may be a powerful tool for the reconstruction of the entire series and for detecting the operations on texts which produced the series.

Introduction

The bulk of known Mesopotamian mathematical texts, written on clay tablets in cuneiform script, dates from the Old Babylonian period (ca. 2000–1600 BCE). Among these texts, a large majority are school exercises written by young students in the framework of the elementary education of scribes. By contrast with 'school texts', the other mathematical texts can be termed as 'advanced texts', even if the boundary between these two categories is not always clear. The context of production of advanced mathematical texts is uncertain but is probably linked to the activities of scribal schools. The geographical provenance of mathematical tablets is generally unknown because almost all of the tablets housed today in European and American Museums and private collections come from illegal excavations, and were bought by curators and collectors from antiquarians or dealers. As a consequence, many tablets found in the same *locus*

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have been scattered along the obscure pathways of the antiquities market. The discussion on parts of texts is affected by this history. Indeed, in most cases, we can access only isolated individual tablets, where, often, only a fragment of the text is preserved. Yet, if we examine segments of text at the scale of individual tablets, most of the relevant information is lost. The present discussion on parts of text is conducted from coherent lots, in which the nexus of texts can be grasped. For this reason, I focus on a tablet which belongs to a group that can be identified with a relatively high degree of probability. This tablet is preserved at the Louvre Museum under number AO 9071 and contains a list of problem statements.[3]

AO 9071 belongs to a series, that is, a group of several numbered tablets. The serial organization of AO 9071 is visible at the end of the tablet, where a colophon indicates that the tablet is the seventh in a series. With this minimal information, we can already distinguish two embedded parts of text: a portion of series text noted within a single tablet, which belongs to an entire series text noted on several numbered tablets. Other kinds of segmentation appear in this first glance when looking at the material aspects of the tablet (see the copy in the Appendix): the obverse and the reverse; the columns; in each column, the sections, that is, small boxes in which a text of one or several lines is noted; and finally, the lines. Zooming in more closely, many other levels of segmentation appear in the text. Zooming out to the series the tablet belongs to, higher levels of segmentation can also be detected.

In this chapter, I first describe the material and textual segmentation on AO 9071, and the relationship between these two kinds of segmentation. On this basis, I try to detect the textual operations which produced the text noted on this tablet, and more broadly in the series it belongs to. Specificities of practices with parts of text in series texts are highlighted. For this purpose, I compare the textual operations which produced the series texts to other practices developed in scholarly milieus in the Old Babylonian period, for example, the production of super-series (concatenations of series) and catalogues (lists of problem statements noted on a single tablet without serial numbers). It is shown that the examination of parts of text may be a powerful tool for the reconstruction of the entire series and for the detection of the operations on texts which produced the series.

3 I published this text in Proust (2009).

1. Tablet AO 9071

1.1 General Description

Tablet AO 9071 was bought by the Louvre Museum from a dealer, Elias Géjou, before 1924, the year of its entry in the inventory (Proust 2009: 169). The provenience and date are unknown, but paleographic evidence shows that it is most likely that the tablet comes from a city in Southern or central Mesopotamia and dates from the Old Babylonian period. Closer examination of AO 9071 suggests that the tablet comes from the same context as other similar mathematical series texts.

The text covers both sides of the tablet, and, on each side, is organized into three columns. As usual on cuneiform tablets, the columns run from left to right on the obverse, and from right to left on the reverse, while, inside a column, the text is written from left to right and from top to bottom. The text on the reverse is the continuation of the text on the obverse when the tablet is rotated around the lower edge. The language is uncertain: the writing uses only Sumerograms (cuneiform signs representing Sumerian words or grammatical particles), but it is not clear if the text was supposed to be uttered in Sumerian or in Akkadian, or if it is a purely graphic artifact which does not represent any spoken language (see discussion in Proust (2009: 229–230)). The text contains a list of 95 problem statements dealing with the length, the width and the surface of a rectangle. The solution of the problems is always the same: the length is 30 *ninda* and the width is 20 *ninda*. Most of the known examples of series text are based on rectangles with these same dimensions.

Each statement is inscribed in a box delimited by horizontal lines and by the vertical lines which also mark the columns (see photo and copy in Fig. 1).

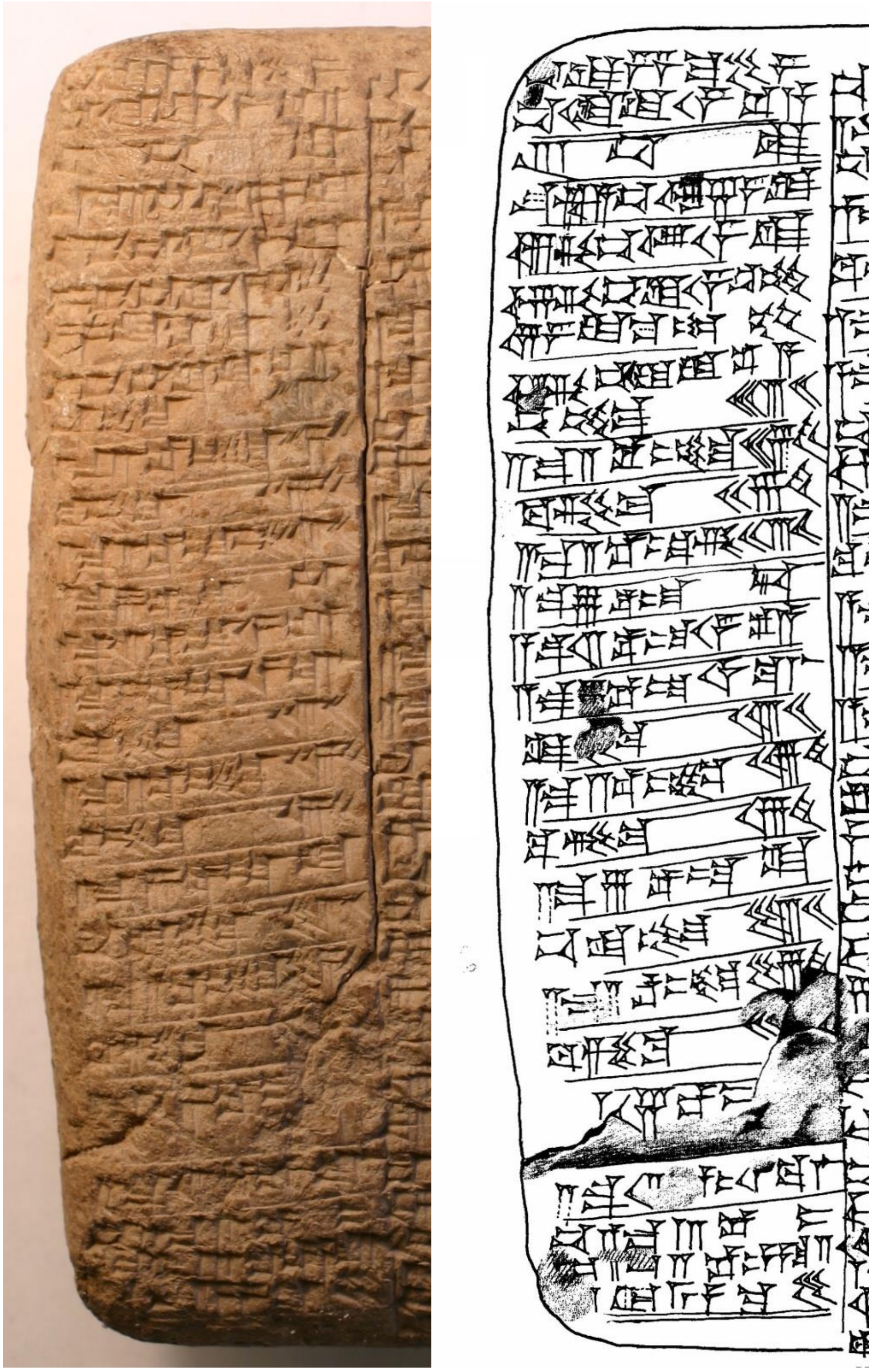

Figure 1: AO 9071, obverse, Column i (photo and copy C. Proust, courtesy of the Louvre Museum)

The text ends at the bottom of the left column on the reverse with a colophon stating: “95 sections, this is the 7th tablet” (1(geš$_2$) 3(u) 5(diš) im-šu / dub 7-kam-ma)—see Fig. 2. Here, specific technical terminology relating to parts of texts can be recognized: the section (im-šu), and the tablet (dub).

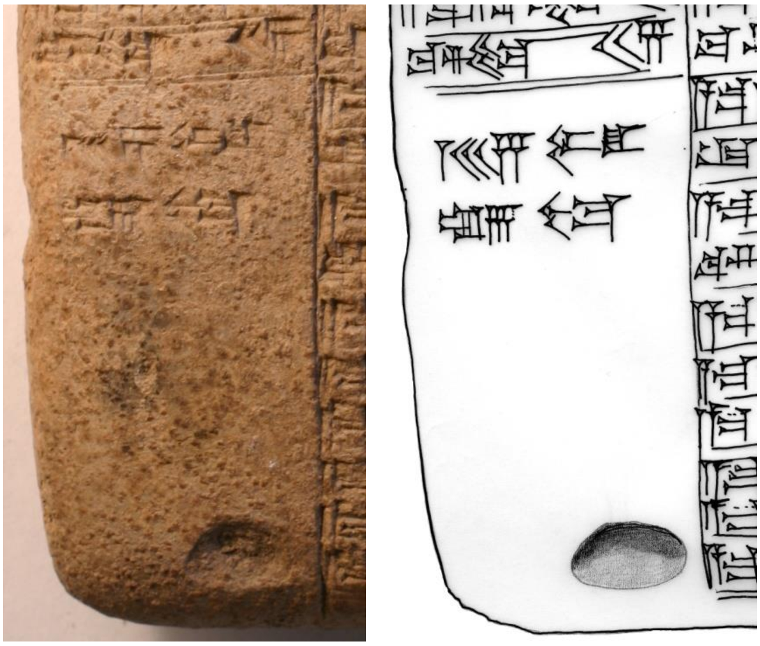

Figure 2: AO 9071, reverse, bottom of Column iii (photo and copy C. Proust, courtesy of the Louvre Museum)

1.2 The Sections: Boxes or Statements?

What is the entity named 'im-šu' in the colophon? Is it a box or a textual segment? At first glance, a section is a box which contains one statement. However, if we count the boxes on AO 9071, only 93 boxes are visible, while, according to the colophon, 95 'im-šu' were counted by the scribe who wrote the tablet. A close reading of the statements makes it clear that horizontal lines were omitted inside the 37th and 50th boxes, and that both boxes actually contain two statements (Proust 2009: 194). Thus, the 95 entities counted in the colophon are the statements, not the boxes. Other examples show similar discrepancies between the number of statements and the number of boxes. For instance, YBC 4713 is the 10th tablet in a series and contains 34 problem statements dealing with the same rectangle as AO 9071. However, according to the colophon, the tablet contains '37 sections' (3(u) 7(diš) im-šu). This discrepancy can be explained by the fact that problem statements #9, #15 and #31 straddle two columns; thus, these statements cover two boxes. The scribe counted the boxes (37), which are more numerous than the statements (34). Another example, tablet YBC 4712 is the 13th in a series and contains 48 problem statements, again dealing with the same rectangle as on AO 9071. However, if we count the boxes, we find 49 boxes, because one of the statements (#8) straddles two columns. The colophon states that the tablet contains '48 sections' (4(u) 8(diš) im-šu). In this case, the scribe counted the statements and not the boxes.[4] As we see, in many cases, there is no biunivocal relation between statements and boxes in series texts, and the items counted by the scribes are sometimes the statements and sometimes the boxes.[5]

These observations raise the problem of the meaning of the Sumerogram 'im-šu', which I translate as 'section'. Word for word, 'im-šu' means 'hand tablet' (im = clay or tablet; šu = hand). In some literary texts, this term is used to designate a small round or square tablet containing a calculation or a literary school exercise, perhaps to be taken home as homework.[6] In mathematical texts, the term 'im-šu' occurs only in relation to counted sections in the colophons of catalogues and series texts. The examples above show that at one and the same time, 'im-šu' means: the container (a box delimited by horizontal and vertical lines) and the content (the statement inscribed inside it). Sometimes, the first meaning dominates, as in YBC 4713, where the boxes are counted, and sometimes the second meaning dominates, as in YBC 4712, where the statements are counted. My translation as 'section' [7] designates both the textual segment (statement) and the material segment (box), which generally coincide, even if some exceptions appear on series tablets, as shown above. The word 'section' conveys some ambiguity, as does the Sumerian word 'im-šu'.

Anyway, the facts that the sections are marked by visual elements—horizontal and vertical lines—and that they are counted in the colophon, suggest that for the ancient scribes, the

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4 Neugebauer already observed this phenomenon (Neugebauer 1935-1937: I, 433, Note 12a).

5 As observed by Julie Lefebvre in her commentaries on the present paper (SAW seminar July 2013), 'It seems that "im-šu" have somewhat the same status as "*alinéa*" in French: sometimes it only refers to a material phenomenon, a blank space generated by an indentation; sometimes it means the textual content (in legal texts, "*alinéa* 15", for example).'

6 It seems that this is the meaning of 'im-šu' in Lines 7 and 10 of the literary text published by Kramer under the title *Schooldays* (Kramer 1949). Kramer translates 'im-šu' as 'hand copy': '(In) the afternoon, my hand copies were prepared for me […] I spoke to my father of my hand copies' (kin-sig im-šu-mu ma-an-gub-bu-uš […] ad-da-mu im-šu-mu KA in-an-dug$_4$-ma) (Kramer 1949: 201, 205). However, in his commentary, Kramer refers to the translation 'section' adopted by Neugebauer and Sachs: 'In Line 7, 'im-šu' is to be rendered as "section" or "paragraph" according to Neugebauer and Sachs [*Mathematical Cuneiform Text*] 125.' (Kramer 1949: 214).

7 I follow Neugebauer (1935–1937) in *Mathematische Keilschrifttexte I–III* (in German: *Abschnitte*), and Neugebauer and Sachs in *Mathematical Cuneiform Text*. Thureau-Dangin tentatively translated 'im-šu' as '*case*' (Thureau-Dangin 1938: 148, Note 1). However, the French word '*case*' (box in English) refers to material segmentation only.

sections are the basic units of text. In the following, other kinds of parts of text are detected through the analysis of how the textual sections are arranged.

1.3 Textual Segmentation

Do the sections form groups with some consistency? In other genres of mathematical cuneiform texts, for example, in catalogue texts, the list of statements is clearly organized in several groups, each group presenting high internal consistency. A good example of such coherent groups can be found on catalogue YBC 4657, in which 31 statements deal with the dimensions and the cost of an excavation. Three coherent groups can be detected by analyzing the statements. Moreover, two of the groups correspond to other tablets containing the procedure to be followed for solving the problems of the group: procedure text YBC 4663 contains the solutions of the first group of catalogue YBC 4657, and procedure text YBC 4662 contains the solutions of the third group of catalogue YBC 4657.[8] (see Table 1).

Catalogue text YBC 4657	Procedure texts
Group I (#1–8)	YBC 4663
Group II (#9–18)	
Group III (#19–28)	YBC 4662
#29–31	

Table 1: Groups in catalogue text YBC 4657

The second group probably corresponds to a procedure text which has not been discovered, or identified. The last three statements do not really form a group and seem to have been added for another and as yet unclear reason. In each group, a given statement is not an isolated item, but shares common features with the other statements in the group, and the meaning of one individual problem comes from the meaning of the other problems in the group. It appears that, in catalogue texts, the groups of problems are delimited according to the nature of the procedures to be used to solve them. The procedures command the consistency of the groups.[9]

8 Tablets YBC 4657, YBC 4663 and YBC 4662 are Old Babylonian mathematical texts from southern Mesopotamia published in Neugebauer and Sachs (1945). For more details on the relationship between these texts, see Neugebauer and Sachs (1945: 73) and Proust (2012: 138).

9 For a detailed analysis of the procedures in sets of problems, see Proust (forthcoming).

Can we detect such groups on tablet AO 9071? The textual segmentation on AO 9071 can be apprehended by examining the beginning of the text, for example the text noted in the first column of the obverse (see Fig. 1). The translation of the obverse, Column i, is the following (the section numbers headed by '#' and the line numbers headed by '*l.*' are mine).

#	*l.*	Translation
1	1.	The length and the width I added: 50 *ninda*
	2.	The length exceeds the width by 10 *ninda*.
2	3.	2/3 of the length: the width.
3	4.	Half of the length and 5 *ninda*: the width.
4	5.	1/3 of the length and 10 *ninda*: the width.
5	6.	1/5 of the length and the width to 10 *ninda* and the length I added,
	7.	15 *ninda* and the length I added < : 1.25>.
6	8.	1/3 of that by which the length exceeds the width
	9.	to the length I added: 33.20.
7	10.	2 times I repeated, I added : 36.40.
8	11.	I subtracted: 26.40.
9	12.	2 times I repeated, I subtracted: 23.20.
10	13.	9 times I repeated: the length.
11	14.	12 times I repeated: excess by 10 *ninda*.
12	15.	6 times I repeated: less by 10 *ninda*.
13	16.	To the width
		I added: 23.20.
14	17.	2 times I repeated, I added: 26.40.
15	18.	I subtracted: 16.40.
16	19.	6 times I repeated : the width
17	20.	To the length and the width
		I added: 53.20.
18	21.	2 times I repeated, I added: 56.40.
19	22.	I subtracted: 46.40.
20	23.	15 times I repeated:
	24.	I equalized.
21	25.	12 times I repeated: less by 10 *ninda*.

22 26. The length 3 times I repeated,
27. the width 2 times I repeated, I added: 2.10.
28. [The length] and the width I added: 50.

In the first statement (#1), the sum and the difference of the length and the width (implicitly of a rectangle) are given as, respectively, 50 *ninda* and 10 *ninda*.[10] Even if no question appears, one guesses that tacitly the length and the width of the rectangle are being sought. This is a quite common problem, well attested in other Old Babylonian sources. A simple calculation leads to the solution, namely, the length is 30 *ninda* and the width is 20 *ninda*.

In the second statement (#2), the relation '2/3 of the length is the width' is given. This information is not sufficient to find the length and the width. Another relation between the length and the width is expected. Where is this other relation? Here, a general principle adopted in series texts is applied: when several successive statements use the same piece of information, this information itself only appears in the first statement; it is implicit in those that follow. According to this principle, the missing relation was already stated in the previous section, namely in the first line: 'The length and the width I added: 50 *ninda*.' Similarly, in #2–6, only one relation between the length and the width are given; the other relation necessary to find the length and the width is also that stated in the first line of #1. In short, statements #2–6 cannot be read independently: they depend on the first line of the first statement.

In Statement #7, we read '2 times I repeated, I added: 36.40.' For a modern reader who seeks to solve the problem, something seems to be missing from this statement. What is repeated? What is added to what? Again, the missing information is to be looked for in the previous statement: what is repeated (i.e. doubled) is the expression given in #6, Line 8 '1/3 of that by which the length exceeds the width' (say, P). The result of this doubling of P is added to something given in #6, Line 9: the length (say, S). Thus, the relation given in #7 cannot be apprehended without the information given in #6. But another relation is necessary to find the length and the width: this missing relation is again that given in #1, Line 1. Finally, the statement in #7 depend on Statements #6 and #1. In the same way, Statements #8–12 depend on #6 and #1.

[10] The *ninda* is a unit of length of about 6 m.

Statement #13 is similar to the previous ones, except for the expression *S*, which is no longer the length, but the width. It is the same for the following statements, #14–16, which thus depend on #13, #6 and #1.

In Statement #17, the expression *S* changes again: it is now the length plus the width. This is the same for the following statements, #18–21, which depend on #17, #6 and #1.

In Statement #22, a new complete statement is given, and the statements that follow, enumerated in Column ii, are generated with the same process as the one just described.

No marks indicate in the cuneiform text the way in which each statement depends on the previous ones, which make the reading of the statements quite difficult. For the modern reader, I introduced indentations which make these dependencies clear. This modern layout indicates the tree structure of the list of statements. The text displayed with such indentations, which does not exist in the original text, is reproduced below. This layout allows the modern reader to reconstruct the full statement represented by each actual elliptic statement noted in the sections.

#	*l.*	
1	1.	The length and the width I added: 50 *ninda*
	2.	The length exceeds the width by 10 *ninda*.
2	3.	2/3 of the length: the width.
3	4.	Half of the length and 5 *ninda*: the width.
4	5.	1/3 of the length and 10 *ninda*: the width.
5	6.	1/5 of the length and the width to 10 *ninda* and the length I
	7.	added,
		15 *ninda* and the length I added < : 1.25>.
6	8.	1/3 of that by which the length exceeds the width
	9.	to the length I added: 33.20.
7	10.	2 times I repeated, I added : 36.40.
8	11.	I subtracted: 26.40.
9	12.	2 times I repeated, I subtracted: 23.20.
10	13.	9 times I repeated: the length.

11	14.	12 times I repeated: excess by 10 *ninda*.
12	15.	6 times I repeated: less by 10 *ninda*.
13	16.	To the width I added: 23.20.
14	17.	2 times I repeated, I added: 26.40.
15	18.	I subtracted: 16.40.
16	19.	6 times I repeated : the width
17	20.	To the length and the width I added: 53.20.
18	21.	2 times I repeated, I added: 56.40.
19	22.	I subtracted: 46.40.
20	23.	15 times I repeated:
	24.	I equalized.
21	25.	12 times I repeated: less by 10 *ninda*.
22	26.	The length 3 times I repeated,
	27.	the width 2 times I repeated, I added: 2.10.
	28.	[The length] and the width I added: 50.

Which parts of text can we detect in this extract of AO 9071? Statements #1–21 depend on the first line and thus form an indivisible group. Inside this group, two other sub-groups appear: the first includes Statements #2–6, which depend only on #1, and the second includes Statements #7–21 which depend on #1 and #6. But this latter sub-group can, in turn, be decomposed into three sub-sub-groups: #7–12, #13–16 and #17–21. A new cycle of interdependent statements begins at the bottom of Column i of the obverse and continues in Column ii (#22–34).

With this first analysis of the structure of the list of statements on tablet AO 9071, we have detected different levels of groups of sections. Considering the entire text noted on AO 9071, five independent groups (or cycles) can be recognized (see Table 2). Each of the five independent groups can be decomposed into sub-groups, each of which can be decomposed in turn into sub-sub-groups.

Group I	#1–21	obverse, Column i
Group II	#22–34	obverse, Column i–ii
Group III	# 35–60	obverse, Column ii–iii – reverse Column iv
Group IV	#61–71	reverse Column iv–v
Group V	#72–93	reverse Column v–vi

Table 2: Independent groups (or cycles) on AO 9071

Table 2 shows that the textual segmentation into groups, sub-groups and sub-sub-groups on AO 9071 does not always adhere to the typographical segmentation into columns and sides. The second group (or cycle) begins at the bottom of the first column and continues on the second, and, in the same way, the other groups straddle columns. We have seen above (Sect. 1.2) that textual segmentation into statements does not always adhere to the material segmentation into boxes: for example, a statement may begin at the bottom of one column and continue at the top of the following column. As observed above, no visual marks such as double lines, a blank space or an indentation, nor typographical marks such as columns could help the ancient user of the tablet to localize the different parts of a full statement, scattered in several dependent sections. The groups, sub-groups and sub-sub-groups can only be identified from textual and mathematical considerations.

Contrasts between catalogues and series texts can be underlined. In series texts, the groups of problems are formed by systematic variations of the statements. Moreover, while the groups in catalogue texts are juxtaposed, the structure is more complex on tablet AO 9071, as the five groups contain nested sub- and sub-sub-groups. Are these five apparently independent groups (or cycles) juxtaposed, or are they themselves embedded in a larger structure? The entire text must be apprehended at the scale of the series that the tablet belongs to.

2. The Series of AO 9071

2.1 General Description

Which tablets belonged to the same series as AO 9071? This is difficult to identify because, as pointed out in the introduction, the series tablets were excavated by illegal diggers and were dispersed through the antiquities market to various European and American museums without any archaeological information. Twenty tablets containing mathematical series text are known to date.[11] Most of these are now housed at the Yale Babylonian Collection and were published by Neugebauer (1935–1937) in a special chapter in *Mathematische Keilschrifttexte I–III* (MKT) (which also includes two samples from the Vorderasiatische Museum, Berlin); two other tablets containing series text are kept at the Oriental Institute, Chicago (A 24194 and A 24195) and were published by Neugebauer and Sachs (1945) in *Mathematical Cuneiform Text* (MCT), ten years after MKT; and, finally, I recently discovered two new samples at the Louvre Museum, AO 9071 and AO 9072 (Neugebauer 1935–1937: Chap. VII; Neugebauer and Sachs 1945: texts T and U; Proust 2009. Tablets containing series text, or at least the majority of those identified to date, probably come from the same provenience. They seem to reflect a new mathematical culture that developed toward the end of the Old Babylonian period in 'peripheral' regions, to use Høyrup's expression (Høyrup 2000: 50; Høyrup 2001: 199; Høyrup 2002: 351).[12] This new mathematical culture was nevertheless steeped in ancient Mesopotamian mathematical traditions, as pointed out by Høyrup, and may have emerged in communities of scholars who settled in northern Babylonia, perhaps in Kiš or Sippar, after the southern cities were destroyed and abandoned around 1740 BCE, and after people of Ur and Uruk, including groups of scholars, fled to the north.[13]

The known series texts shed some light on the composition of the original entire series that AO 9071 belongs to. Neugebauer suggested that the tablets containing series text published in MKT belong to at least three different series, which he labeled A, B and C (Neugebauer 1935–1937: I, Chap. VII; see last column of Table 6 below). The first task is to identify the common features

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11 The complete list is provided in Proust (2012: 150–151, Table C). In this publication, I labeled the tablets containing mathematical series text as S1, S2, … S20.

12 . Neugebauer, in his chapter on '*serientexte*' (Neugebauer 1935–1937: I, 387, Chap. VII), supposed that series tablets come from Kiš: 'All texts discussed in this chapter come from the antiquities market. The two Berlin tablets (VAT 7528 and 7537) were purchased in Paris from Géjou and were probably inventoried between 1911 and 1912. In this purchase were also texts that, according to indications given by the merchant, came from Kish or localities in the immediate vicinity of Kish.' (Alle in diesem Kapitel behandelten Texte stammen aus dem Antikenhandel. Die beiden Berliner Tafeln (VAT 7528 und 7537) sind von Gejou in Paris gekauft und vermutlich zwischen 1911 und 1912 inventarisiert worden. In diesem Kauf befinden sich auch Texte, die nach der Angabe des Händlers aus Kiš oder Orten unmittelbar bei Kiš stammen). Friberg considered that the provenience of tablets containing series text is more probably to be located in southern Mesopotamia, more precisely in Ur. He distinguished two different sub-groups among tablets containing series text: a group Sa and a group Sb, the latter only including the two tablets kept at the Oriental Institute of Chicago A 24194 and A 24195 (Friberg 2000: 172)

13 See the bibliographies in Proust (2009: 169–170; 229) and Proust (2012: 14).

of tablets belonging to the same series in order to select the tablets which share their features with AO 9071.

2.2 Features of an Entire Series Text

Are the set of tablets belonging to the same series homogeneous from different points of view, whether thematic, mathematical or textual? Some light on the answers to these questions is provided by known tablets containing series text which belong with certainty to the same series. Two of them have been identified: YBC 4713 (which bears the serial number 10) and YBC 4712 (which bears the serial number 13). Indeed, the content of these two tablets has been found again in a third, YBC 4668, which contains 288 statements and bears the serial number 3. Neugebauer has shown that YBC 4668 belonged to a gigantic series made up of several regular series, each of which included a dozen tablets Neugebauer (1935–1937: I, 385). 'Super-series' tablet YBC 4668 contains not only the content of YBC 4713 and 4712, but also three other portions which probably belong to the same series. These five portions of probably the same series are described in Table 3.

Portions of series text	**Tablet**	**Serial number of the tablet**
First	YBC 4713	10
Second	Unknown	(11)
Third	Unknown	(12)
Fourth	YBC 4712	13
Fifth	Unknown	(14)

Table 3: The five portions of series on the super-series tablet YBC 4668

Considerations on parts of text, whether material or textual, will be essential in the following identification of features of an entire series text. We can already see, at this stage, that we now have a higher level part of text: the super-series, which contains several series, which themselves contain the content of several tablets.

Considering the five portions of probably the same series text noted on YBC 4668, we can observe that they deal with the same topic, namely, our immutable rectangle whose dimensions are 30 *ninda* by 20 *ninda*. Thus, it seems that one entire series text is thematically homogeneous.

Are these five portions of series text homogeneous from a mathematical point of view? The features of the five texts are summarized in Table 4. All of the statements on super-series tablet YBC 4668 either begin with the data of the surface of the rectangle or depend on statements which provide this surface. This surface is always the same and is invariably stated as follows: 'The surface (measures) 1 *eše GAN*' (in Sumerian*:* a-ša$_3$ 1(eše$_3$) GAN$_2$).[14] Such an area corresponds to the number 10 in sexagesimal place value notation.[15] This relation is represented in Table 4 below by the abbreviation 'E1: $u \times s = 10$', where E1 represents the first relation, or 'first equation', u represents the length (uš in Sumerian) and s represents the width (sag in Sumerian).

Portion of series text	**First equation E1**	**Other equations**	**Variations on relations between**
First	$u \times s = 10$	E2, E3, E4	u, s, coefficients, auxiliary unknowns
Second	$u \times s = 10$	E2	u, s, $u + s$, $u - s$, u^2, s^2, $(u + s)^2$, $(u - s)^2$
Third	$u \times s = 10$	E2	Damaged; seems similar to the second e
Fourth	$u \times s = 10$	E2	u, s, u/s, s/u, auxiliary unknowns
Fifth	$u \times s = 10$	E2, E3, E4	u, s, coefficients, auxiliary unknowns, pairs of reciprocals

Table 4: Mathematical patterns in YBC 4668

The mathematical features of the five portions of series text included in YBC 4668 are not the same:

- In the first portion (which corresponds to the content of series tablet YBC 4713), two unknown coefficients are introduced in addition to the length and the width, and the problems can be described in modern terms as systems of four equations with four unknowns. The list of statements is generated by variations which affect, in the 'equations' E2, E3 and E4, the relations between the length, the width, the coefficients and some auxiliary unknowns.
- In the second portion (which does not correspond to the content of a known series tablet), the statements provide only a second equation, E2, and the variations affect the

[14] 1 *eše GAN* is 6 *GAN*, that is, 6 times the area of a 10 *ninda*-side square, about 3600 m².

[15] Indeed, 10 is the product of 30, the length, by 20, the width, in floating sexagesimal place value notation. See Proust (2013) for more explanation on sexagesimal place value notation and floating multiplication.

relationships between the following parameters: the length, the width, the difference and the sum of these dimensions, and their squares (these parameters are represented, in Table 4, by u, s, $u + s$, $u - s$, u^2, s^2, $(u + s)^2$, $(u - s)^2$).

- The third portion is too damaged to be described in detail but seems to be close to the second. The demarcation between the second and the third single series texts is hypothetical and grounded on the structure of the list.
- In the fourth of the five portions (which corresponds to the content of series tablet YBC 4712), the mathematical structure is also similar to the second, but the quotients u/s and s/u are among the parameters of the E2 equation.
- The fifth portion (which does not correspond to the content of a known series tablet) is similar to the first, with additional sophistications such as unknown pairs of reciprocals.

To sum up, the first equation, which evaluates the surface, is the same for all the statements on super-series tablet YBC 4668, and the variations affect the other equations. The statements on YBC 4668 can be described in modern terms as quadratic systems of two equations with two unknowns (in the second, third and fourth portions of series text) or of four equations with four unknowns (in the first and the fifth portions). The mathematical pattern of the statements differs throughout super-series tablet YBC 4668, as two different models are found, whereas only one mathematical pattern is found in a single portion of series text.

Which parts of text can we recognize on super-series tablet YBC 4668? The portion of super-series it contains includes five portions of probably the same series text. Each of the portions of series text is generated in a similar way to that described for AO 9071 (Sect. 1), and thus includes groups (cycles), sub-groups and sub-sub-groups. Does this textual segmentation coincide with material segmentation? First, we observe that no cycle seems to be interrupted at the end of a single portion of series text, which means that each portion of series text contains only complete cycles. Second, if we observe how the text on YBC 4668 is divided into columns, we see that the end of a portion of series text coincides with the end of a column on super-series tablet YBC 4668 (see Table 5). In particular, the change of side (from obverse to reverse) seems to occur between the end of the second portion of series text and the beginning of the third portion, insofar as the demarcation between these portions can be guessed. We can conclude that, in the case of tablet YBC 4668, the textual segmentation coincides quite well with the material segmentation.[16]

Portions of series text on YBC 4668	**Tablet**	**Side of YBC 4668**	**Columns of YBC 4668**
First	YBC 4713	Obverse	i–ii
Second			iii–iv
Third		Reverse	i
Fourth	YBC 4713		ii–iii
Fifth			iv

Table 5: Textual and material parts of text in YBC 4668

This quick trip into the world of super-series, that is, series of series, provides some light on the possible features of series texts. First, we have seen that the entire series texts are thematically homogeneous. This conclusion is confirmed by the fact that the thematic homogeneity is always respected in the scale of a portion of series text, written on one tablet: some series tablets contain only problems on fields, others only on bricks, others only on canals, another only on economics and so on. Second, several mathematical patterns can be found in the same series, but these mathematical patterns seem to exhibit some common features. Third, at least on super-series tablet YBC 4668, the textual segmentation seems to adhere to the material segmentation, but this is not always the case. Keeping in mind these results concerning the portion of the series observed on super-series tablet YBC 4668, we can attempt to partially reconstruct the series AO 9071 belongs to.

2.3 The Tablets which may Belong to the Same Series as AO 9071

The most conclusive criterion is thematic. As it is highly probable that an entire series consisted of problems dealing with the same topic, we can select the tablets dealing with the rectangle whose dimensions are 30 *ninda* by 20 *ninda* as possible candidates for belonging to the same

16 However, the scribe(s) who wrote the other known tablets which probably contain super-series, A 24194 and A 24195, seem not to have attempted to end each single series text exactly at the end of a column, or at the end of the obverse or at the end of the tablet (Proust 2015: 310–311).

series as AO 9071. The eleven known series tablets that meet this criterion are listed in Table 6.[17]

Museum number	Label	Number of sections	Serial number	Pattern of statements	MKT group
YBC 4710	S2	35	4	Quadratic systems E1: $u \times s = 10$ E2, E3, E4: variations on relations between u, s and other parameters.	A
YBC 4713	S5	37	10 (part of super-series tablet YBC 4668)	Quadratic system E1: $u \times s = 10$ E2, E3, E4: variations on relations between u, s, coefficients, and auxiliary unknowns	A
YBC 4712	S6	48	13 (part of super-series tablet YBC 4668)	Quadratic system E1: $u \times s = 10$ E2: variations on u, s, u/s, s/u, and auxiliary unknowns	A
YBC 4715	S8	ca. 60	destroyed	Quadratic systems E1: $u \times s = 10$ E2: variations on relations between u, s, and auxiliary unknowns	A
VAT 7537	S7	ca. 45	destroyed	Quadratic systems E1: $u \times s = 10$ E2: variations on relations between u, s, $u + s$, $u - s$, u^2, s^2, $(u + s)^2$, $(u – s)^2$	A
YBC 4697	S9	25	3	Quadratic systems E1: $u \times s = 10$ E2: variations on relations between u, s, $u + s$, $u - s$, u^2, s^2, $(u + s)^2$, $(u – s)^2$	A
YBC 4709	S3	56	5, 6, 8 or 9 (9 is the most probable)	Quadratic systems E1: $u \times s = 10$	A

17 From the twenty known series texts, I excluded the three super-series (see Sect. 2.3); YBC 4714, because the type of tablet and the structure of the text are different; YBC 4696, whose colophon does not contain a serial number; YBC 4708 and YBC 4673, which deal with bricks, VAT 7528, which deals with canals; and YBC 4698, which deals with economic topics (see Middeke-Conlin and Proust 2014). Note that in the following, to facilitate the use of Table 6, I designate the tablets by the publication numbers S1, … S14 that I have used in previous articles.

				E2: variations on relations between u, s, $u + s$, $u - s$, u^2, s^2, $(u + s)^2$, $(u - s)^2$	

YBC 4711	S12	131	4, 5 or 6	Quadratic systems E1: $u \times s = 10$ E2: variations on relations between u and s	B
YBC 4695	S11	97	5	Linear and quadratic systems E1: variations on relations between u and s E2: variations on relations between u and s	B
AO 9071	S13	93	7	Linear systems E1: variations on relations between u and s E2: variations on relations between u and s	
AO 9072	S14	ca. 170	destroyed	Quadratic systems? E1 not stated (probably implicitly $u \times s = 10$) E2: variations on relations between u and s	

Table 6: Series texts dealing with rectangular fields

As another criterion, the number of statements can be taken into consideration. From this point of view, two groups can be recognized, as shown in Table 6. The tablets containing less than 60 statements, which belong to Neugebauer's Group A, and the tablets containing between 90 and 170 statements, which include two tablets belonging to Neugebauer's Group B and the two tablets from the Louvre.[18]

Let us consider the mathematical criteria in turn. The most common mathematical pattern found in series texts is a quadratic system of two relations between the length and the width of a rectangle. The first relation (E1) is the surface of the rectangle, as in super-series tablet YBC 4668. The second equation (E2) is a linear relation between the length and the width. This E2 linear relation describes systematic cycles of variations implemented on the expressions *P* and *S* in the same process as we have seen for AO 9071. This pattern can be found in texts S3, S7, S9, S12 and probably S14 (see Table 6). Among them, tablets S3, S7 and S9 were classified in

[18] The tablets containing more than 200 statements are the three known 'super-series' tablets (YBC 4668, A 24194 and A 24195), which do not appear in Table 6, even if they deal with fields.

Group A by Neugebauer, and indeed, present a specific feature: the variable expressions are relations between the square of the length and the width, or a simple combination of them (u^2, s^2, $(u + s)^2$, $(u - s)^2$). Another pattern is close to this one, with the difference being that the first equations describe linear or quadratic variations; this is the pattern we have found on AO 9071 and also on YBC 4695 (S11). In both patterns, the solution is always the same: the length is 30 *ninda* and the width is 20 *ninda*.

The patterns we have found in the texts which make up super-series tablet YBC 4668 are different and much more complex: sometimes four unknown values are to be sought; the relations between the unknown parameters include quotients, reciprocals, coefficients and auxiliary lengths. The solutions are nevertheless the immutable 30 *ninda* and 20 *ninda* for the dimensions of the squares, and additional values for other parameters introduced in the statements (coefficients and auxiliary lengths). These complex mathematical patterns are found in series texts classified as Group A by Neugebauer (S2, S5, S6 and S8, which include the two series texts included in super-series tablet YBC 4668).

Tablet YBC 4695 (S11) is similar to AO 9071 according to all of the criteria examined above: theme, number of statements and mathematical pattern. Its colophon indicates that YBC 4695 (S11) is the fifth of a series, while AO 9071 is the seventh tablet of a series. It is highly probable that both belong to the same series. If we consider the other pattern close to that found in the two tablets of this possible series, two other tablets are good candidates to be part of the same series: YBC 4711 (S12) and AO 9072.[19] The fact that tablet AO 9072 was bought from Géjou by the Louvre Museum together with AO 9071 is additional evidence that both may come from the same place and may have belonged to the same series. The serial number of YBC 4711 (S12) is damaged, but we can distinguish two rows of wedges and thus the possible numbers may be 4, 5 or 6, with a slightly better probability for 4. The serial number of AO 9072 is destroyed, but I have shown elsewhere that this tablet probably came after AO 9071 in the series and was not the direct continuation of AO 9071 because the first statement of AO 9072 relies on missing pieces of information which are not provided on AO 9071.

[19] Neugebauer stresses that YBC 4695 and YBC 4711 probably belong to the same series. (Neugebauer 1935–1937: I, 385)

If we now consider the number of sections as a non-significant criterion, tablets VAT 7537 (S7), YBC 4697 (S9) and YBC 4709 (S3) are also possible candidates to be part of the same series, because the mathematical pattern of the statements they contain are close to that found in YBC 4711 (S12).

Finally, we obtain a set of seven tablets which may have belonged to the same series as AO 9071, with various degrees of probability: high for YBC 4695 (S11), good for YBC 4711 (S12) and AO 9072 (S14), and acceptable for VAT 7537 (S7), YBC 4715 (S8) and YBC 4697 (S9). A possible composition and arrangement of this hypothetical series is illustrated in Table 7.

Museum number	Label	Type	Number of sections	Serial number in colophon	Possible serial number	Degree of probability
YBC 4697	S9	M(3,3)	25	3	3	acceptable
YBC 4711	S12	M(3,3)	131	4, 5 or 6	4	good
YBC 4695	S11	M(3,3)	97	5	5	high
AO 9071	S13	M(3,3)	93	7	7	
YBC 4709	S3	M(3,3)	56	5, 6, 8 or 9	8	acceptable
AO 9072	S14	M(4,5)	ca. 170	destroyed	(9?)	good
VAT 7537	S7	M(3,3)	ca. 45	destroyed		acceptable

Table 7: Tablets which may belong to the same series as AO 9071

As the series reconstructed in Table 7 is lacunar and very hypothetical, we can hardly rely on it to analyze the parts of text at the scale of an entire series. However, some observations on the main groups (or cycles) can be made. The case of tablet AO 9072 is interesting from this point of view. The tablet contains statements providing only one relation between the length and the width, namely, only equation E2. The other equation, E1, is not explicitly stated. This means that all the statements on AO 9072 depend on information given in a previous tablet. Thus, it seems that the list of statements on AO 9072 is part of a cycle which covers more than a single tablet. If we consider the parts of text at the scale of an entire series, it appears that the main groups (cycles) are not always juxtaposed parts but can themselves be embedded in a larger group.

The texture of an entire series text is thus a complex tree structure. The tiniest branches, the sections, are immediately recognizable by the reader. These fundamental units of text are parts of several nested groups. Recognizing these different nested groups is not an easy task, but this textual analysis is necessary for the reader in order to reconstruct the full statement that the content of a box refers to or, in other words, in order to simply understand the mathematical meaning of the problems. Moreover, it is not always possible for us to go back to the tree roots at the scale of a single series tablet because, as shown with the example of tablet AO 9072, the root is sometimes to be found in a previous tablet.

Only portions of series texts are available to us, so we have no vision of the entire structure of any one series text. This partial vision nevertheless allows some considerations on the way in which series texts were composed.

3. Operations on Texts

Which kind of operations on texts produced series texts? I have addressed this issue in previous publications, for example in Proust (2012); here, I just wish to underline the importance of the analysis of parts of text in such discussions.

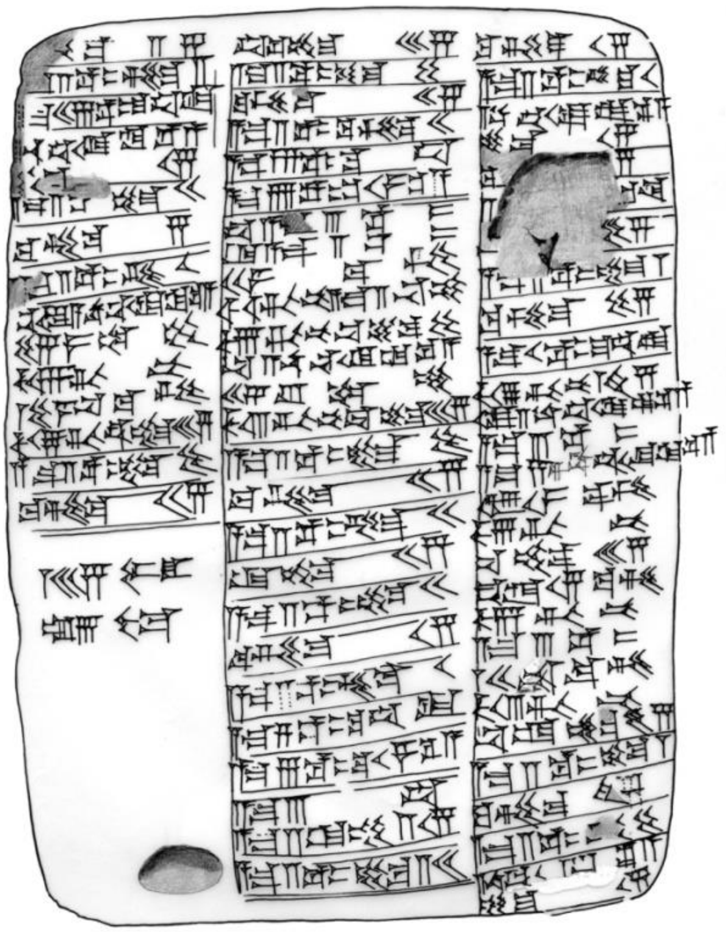
The specificity of the process which produced series texts appears more clearly when we compare it with the process which produced catalogue texts. As pointed out above (Sect. 1.3), catalogue texts are lists of problem statements similar to series texts but, unlike the series texts, do not bear serial numbers. Moreover, they are written on single-column tablets, a type usually used in the advanced level of education in southern scribal schools. The process of producing catalogue texts can be reconstructed thanks to two kinds of information: the groups of statements inside a single tablet and comparison with related procedure texts. This information suggests that catalogues are compilations, that is, the juxtaposition of groups of statements collected from different sources. The purpose of such an operation could be explained by the context of teaching, and the function of catalogues may be a kind of inventory of a possible library or the fixation of a curriculum, or both (Proust 2012).

The fact that in series texts the groups are not juxtaposed, but instead are nested, suggests that the process of producing them is different. With the example of tablet AO 9071 (Sect. 1), we have observed that each group results from systematic variations of segment of an initial statement (in Sect. 1, I labeled these segments E1, *P* and *S*). Thus the process is the reverse of a compilation: from an initial statement, which is quite simple and was well known to the scholars of the time, the statements are generated by combinatorial operations on segments of texts. It can be described as a generative process, like a seed that grows. The super-series seems to be a compilation of series texts and thus to be a combination of generative processes, which produce series texts, and a concatenation of several entire series texts. The generative process which produced series texts acts on statements, while the process which produced catalogue texts deals with procedures. Both processes probably had different goals. The generative process can be viewed as a method for exploring all of the possible statements corresponding to a given pattern, and its goal may be heuristic. With these examples, we see how different ways of segmenting texts, juxtaposed groups or nested groups reflect different operations on texts: compilation or generation. In all of the cases, however, the arrangement of groups reflects a concern for classification, whose goals are varied: archival, curricular or heuristic.

As a conclusion, the texture we have observed for series texts is different from what we are used to observing in modern books. The parts of text are not well-identified entities such as volume, chapter or paragraph, but a continuum of nested groups.

Appendix

AO 9071, copy published in Proust (2009: 177, 179)



Primary Sources

Museum number	CDLI number	Publication
A 24194	P254383	MCT: 107
A 24195	P254384	MCT: 119
AO 9071	P254392	Proust 2009
AO 9072	P416818	Proust 2009
VAT 7537	P254933	MKT: I, 466; TMB: 207
YBC 4657	P254982	MCT: 66
YBC 4662	P254983	MCT: 66
YBC 4663	P254984	MCT: 66
YBC 4668	P254986	MKT: I, 420; TMB: 162
YBC 4695	P255007	MKT: I, 501; TMB: 214
YBC 4697	P255009	MKT: I, 485; TMB: 214
YBC 4709	P255016	MKT: I, 412; TMB: 155
YBC 4710	P255017	MKT: I, 402; TMB: 148
YBC 4711	P255018	MKT: I, 503; TMB: 214
YBC 4712	P255019	MKT: I, 420; TMB: 176
YBC 4713	P255020	MKT: I, 421; TMB: 162
YBC 4715	P255022	MKT: I, 478; TMB: 190

Abbreviations

MCT: Neugebauer, et al. 1945.

MKT: Neugebauer 1935-1937.

TMB: Thureau-Dangin 1938.

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