Reasons to believe in a social environment

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Abstract

We present a logic which supports reasoning about an agent's belief formation and belief change due to evidence provided by other agents in the society. We call this logic DEL-ES which stands for "Dynamic Epistemic Logic of Evidence Sources". The term 'evidence source' refers to an agent in the society who provides evidence to believe something to another agent. According to DEL-ES, if an agent has gathered a sufficient amount of evidence in support a given fact φ then, as a consequence, she should start to believe that φ is true. A sound and complete axiomatization for DEL-ES is given. We discuss some of its interesting properties and illustrate it in a concrete example from legal contexts.

Keywords: Modal logic, Epistemic Logic, Evidences, Reasons.

1 Introduction

A. J. Ayer put in his book [2, p. 3] :

> A rational man is one who makes a proper use of reason: and this implies, among other things, that he correctly estimates the strength of evidence.

In the same vain, this paper attempts to look into the ways that a rational agent handles evidence, as *reasons*, to support or reject her beliefs. Notions of evidence and justification pervade in legal contexts, in that various parties that are involved would collect evidence, and then use them to support their own claims or beliefs, or reject those of the opposing parties. The following three aspects suddenly become relevant, namely, evidence collection, belief formation

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and belief revision. We will pursue the problems that arise when we pay more attention to the social enviroment in which evidence is not simply given, but provided by certain sources through communication.

Logical investigation of evidence and justification is not something new to us. Here we only mention two research areas that have inspired our work. [1] proposed so-called justication logic which explicitly expresses evidence as a term and possible manipulations of evidence as operations over terms. This framework has been further connected to the notion of beliefs and belief revision in [5]. The notion of evidence, understood as a proposition, [2] and its relationships with belief, are studied in recent papers [7,8]. The latter work has a single-agent perspective and clearly states that it left open the issues of evidence sources (i.e., where does the evidence come from?). This is where we started our journey.

Motivated differently, social influence in terms of individual's belief change has caught a lot of attention in recent years. [16] presented a finite state automata model with a threshold to deal with social influence. As a simple case, agent i would change her belief from p to $\neg p$ if all her neighbors believe $\neg p$. This model can successfully explain social phenomena, like peer pressure, and behavior adoption. [3] further enriches this model by introducing quantitative measurement on trust between agents and strength of evidence, and stipulates how these parameters influence one's valuation of evidence. [18] studies the phenonemon of trust-based belief change, that is, belief change that depends on the degree of trust the receiver has in the source of information. Viewed in line of social choice theory, one can also think of belief formation or change as a process of aggregating opinions from different reliable information sources, as e.g. [13].

In this paper we would like to combine ideas from the above two research areas, taking evidence source into account, and consider its roles in formation and dynamics of beliefs. A rational agent is someone who is aware of reasons for her beliefs, and who is willing to change her beliefs when facing new evidence.

The contribution of the paper is a new logic which supports reasoning about an agent's belief formation and belief change due to evidence provided by other agents in the society. We call this logic DEL-ES which stands for "Dynamic Epistemic Logic of Evidence Sources". It is assumed that the evidence source is social, that is, it is an agent in the society who provides evidence to believe something to another agent. The central idea of the logic DEL-ES is that an agent accumulates evidence in support of a given proposition φ from other agents in the society and the body of evidence in support of φ can become a reason to believe φ. This is reminiscient of Keynes's well-known concept of "weight of evidence" (or "weight of argument"), as clearly defined in the following paragraph from the famous treatise on probability [14, p. 77]:

> As the relevant evidence at our disposal increases, the magnitude of the probability of the argument may either decrease or increase, according as the

[2] Semantically, as a set of possible worlds.

> new knowledge strengthens the unfavourable or the favourable evidence; but something seems to have increased in either case, - we have more substantial basis upon which to rest our conclusion. I express this by saying that an accession of new evidence increases the weight of argument.

Using Keynes' terminology, we assume that an agent has a reason to believe that φ is true if the weight of evidence supporting φ is considered by her *sufficient* to believe φ. In this paper we take the notion of weight in a qualitative sense, namely, the amount of evidence matters. We will leave the quantitative reading for future investigation.

The paper is organized as follows. In Section 2 we present the syntax and semantics of the logic DEL-ES, while in Section 3 we discuss some of its general properties. The semantics of DEL-ES combines a relational semantics for the concepts of knowledge and belief and a neighbourhood semantics for the concept of evidence. A sound and complete axiomatization for the logic is given in Section 4. The completeness proof is non-standard, given the interrelation between the concepts of belief and knowledge, on the one hand, and the concept of evidence, on the other hand. In Section 5 we illustrate the logic DEL-ES in a concrete example. Finally, in Section 6 we conclude.

2 Dynamic epistemic logic of evidence sources

In this section, we present the syntax and the semantics of the logic DEL-ES. DEL-ES has a static component, called EL-ES, which includes modal operators for beliefs, knowledge and evidence sources *plus* special atomic formulas that allow us to represent an agent's disposition to form beliefs based on evidence, namely, how much evidence the agent needs to collect in support of a fact before starting to believe that the fact is true. DEL-ES extends EL-ES with two kinds of dynamic operators: (i) operators for describing the consequences on an agent's epistemic state of the operation of receiving some new evidence, and (ii) operators for describing the consequences on an agent's epistemic state of the operation of restoring belief consistency.

On the technical level, DEL-ES combine methods and techniques from Dynamic Epistemic Logic (DEL), that has been developed in the past decades (cf. [4,11,6]), with methods and techniques from neighbourhood semantics for modal logic (cf. [10]).

2.1 Syntax

Assume a countable set of atomic propositions $Atm = \{p, q, \ldots\}$ and a finite set of agents $Agt = \{1, \ldots, n\}$. The set of groups (or coalitions) is defined to be $2^{Agt*} = 2^{Agt} \setminus \{\emptyset\}$. Elements of 2^{Agt*} are denoted by $J, J', \ldots$ For every $J \in 2^{Agt*}$, $card(J)$ denotes the cardinality of J.

The language of DEL-ES, denoted by $\mathcal{L}_{\textsf{DEL-ES}}$, is defined by the following grammar in Backus-Naur Form (BNF):

$$\begin{array}{lcl} \alpha & ::= & \varphi!_{i \hookleftarrow j} \mid \circ_i \\ \varphi & ::= & p \mid \mathsf{trs}(i,x) \mid \neg\varphi \mid \varphi \wedge \psi \mid \mathsf{K}_i\varphi \mid \mathsf{B}_i\varphi \mid \mathsf{E}_{i,j}\varphi \mid [\alpha]\varphi \end{array}$$

where p ranges over Atm, i, j over Agt and $1 \leq x \leq card(Agt)$. The other Boolean constructions $\top$, $\bot$, $\vee$, $\rightarrow$ and $\leftrightarrow$ are defined from p, $\neg$ and $\wedge$ in the standard way.

The language of $\mathsf{EL\text{-}ES}$ (Epistemic Logic of Evidence Sources), denoted by $\mathcal{L}_{\mathsf{EL\text{-}ES}}$, the fragment of $\mathsf{DEL\text{-}ES}$ without dynamic operators, is defined by the following:

$$\varphi \quad ::= \quad p \mid \mathsf{trs}(i, x) \mid \neg\varphi \mid \varphi \wedge \psi \mid \mathsf{K}_i\varphi \mid \mathsf{B}_i\varphi \mid \mathsf{E}_{i,j}\varphi$$

K_i is the standard modal operator of knowledge and $\mathsf{K}_i\varphi$ has to be read "agent i knows that φ is true". B_i is an operator for belief and $\mathsf{B}_i\varphi$ has to be read "agent i believes that φ is true". The dual of the knowledge operator is defined as follows:

$$\widehat{\mathsf{K}}_i\varphi \stackrel{\mathtt{def}}{=} \neg\mathsf{K}_i\neg\varphi$$

while the dual of the belief operator is defined as follows:

$$\widehat{\mathsf{B}}_i\varphi \stackrel{\mathtt{def}}{=} \neg\mathsf{B}_i\neg\varphi$$

$\mathsf{E}_{i,j}\varphi$ has to be read "agent i has evidence in support of φ based on the information provided by agent j".

$\mathsf{trs}(i, x)$ is a constant that has to be read "agent i has a level of epistemic cautiousness equal to x". Agent i's level of epistemic cautiousness corresponds to the amount of evidence in support of a given fact that agent i needs to collect before forming the belief that the fact is true.

We distinguish two types of events denoted by α: $\varphi!_{i \hookleftarrow j}$ and $\circ_i$. The symbol $\varphi!_{i \hookleftarrow j}$ denotes the event which consists in agent j providing evidence to agent i in support of φ, whereas the symbol $\circ_i$ denotes the event which consists in agent i restoring the consistency of her beliefs. The formula $[\alpha]\varphi$ has to be read "φ will hold after the occurrence of the event α". The dual of the dynamic operator $[\alpha]$ is defined as follows:

$$\langle\alpha\rangle\varphi \stackrel{\mathtt{def}}{=} \neg[\alpha]\neg\varphi$$

where $\langle\alpha\rangle\varphi$ has to be read "it is possible that the event α occurs and, if it occurs, φ will hold afterwards". Clearly, $\langle\alpha\rangle\top$ and $[\alpha]\bot$ have to read, respectively, "it is possible that the event α occurs" (or α is executable) and "it is impossible that the event α occurs" (or α is inexecutable).

The reason why we introduce events of the form $\circ_i$ and corresponding dynamic operators $[\circ_i]$ is that the process of accumulating new evidence may lead to inconsistency of beliefs. In such a situation, an agent may want to restore consistency of her beliefs and start the accumulation of new evidence to discover new truths. This issue will be clearly illustrated in Section 3.

Let us immediately define the following abbreviations for every $i \in Agt$ and

$1 \leq x \leq card(Agt)$:

$$\mathsf{E}_i^{\geq x}\varphi \stackrel{\mathtt{def}}{=} \bigvee_{J \in 2^{Agt*}: card(J)=x} \ \bigwedge_{j \in J} \mathsf{E}_{i,j}\varphi$$

$$\mathsf{E}_i\varphi \stackrel{\mathtt{def}}{=} \bigvee_{1 \leq x \leq card(Agt)} \big(\mathsf{E}_i^{\geq x}\varphi \wedge \neg \mathsf{E}_i^{\geq x}\neg\varphi\big)$$

$$\mathsf{R}_i\varphi \stackrel{\mathtt{def}}{=} \mathsf{E}_i\varphi \wedge \bigvee_{1 \leq x \leq card(Agt)} \big(\mathsf{E}_i^{\geq x}\varphi \wedge \mathsf{trs}(i,x)\big)$$

$$\mathsf{QR}_i\varphi \stackrel{\mathtt{def}}{=} \neg\mathsf{E}_i\neg\varphi \wedge \bigvee_{1 \leq x \leq card(Agt)} \big(\mathsf{E}_i^{\geq x-1}\varphi \wedge \mathsf{trs}(i,x)\big)$$

We use the convention $\mathsf{E}_i^{\geq 0}\varphi \stackrel{\mathtt{def}}{=} \top$ and $\mathsf{E}_i^{\geq card(Agt)+1}\varphi \stackrel{\mathtt{def}}{=} \bot$.

$\mathsf{E}_i^{\geq x}\varphi$ has to be read "agent i has at least x evidence in support of φ". $\mathsf{E}_i\varphi$ has to be read "agent i has a *decisive* evidence for φ" in the sense that she has more evidence in support of φ than evidence in support of $\neg\varphi$. $\mathsf{R}_i\varphi$ has to be read "agent i has a *sufficient* reason to believe that φ is true". According to our definition, an agent has a sufficient reason to believe that φ is true if and only if:

(i) she has a decisive evidence for φ,

(ii) the amount of evidence in support of φ is equal to or above her threshold of epistemic cautiousness.

As we will highlight in Section 4, a sufficient reason to believe that φ is true ensures that the agent will form the corresponding belief that φ is true. The last abbreviation QR_i defines the concept of quasi-reason: an agent has a *quasi-sufficient reason* to believe that φ is true, denoted by $\mathsf{QR}_i\varphi$, if and only if an additional evidence in support of φ will provide to the agent a sufficient reason to believe that φ is true.

2.2 Semantics

The main notion in semantics is given by the following definition of evidence source model which provides the basic components for the interpretation of the logic $\mathsf{DEL\text{-}ES}$:

Definition 2.1 [Evidence Source Model] An evidence source model (ESM) is a tuple $M = (W, E, D, S, T, V)$ where:

- W is a set of worlds or situations;
- $E : Agt \longrightarrow 2^{W \times W}$ s.t. for all $i \in Agt$, $E(i)$ is an epistemic relation on W;
- $D : Agt \longrightarrow 2^{W \times W}$ s.t. for all $i \in Agt$, $D(i)$ is a doxastic relation on W;
- $S : Agt \times Agt \times W \longrightarrow 2^{2^W}$ is an evidence source function;
- $T : Agt \times W \longrightarrow \{k \in \mathbb{N} : 0 \leq k \leq card(Agt)\}$ is an epistemic threshold function;
- $V : W \longrightarrow 2^{Atm}$ is a valuation function;

and which satisfies the following conditions for all $i, j \in Agt$ and $w, v \in W$:

(**C1**) every $E(i)$ is an equivalence relation;

(**C2**) $D(i) \subseteq E(i)$;

(**C3**) if $wE(i)v$ then $D(i)(w) = D(i)(v)$;

(**C4**) if $wE(i)v$ then $S(i, j, w) = S(i, j, v)$;

(**C5**) if $X \in S(i, j, v)$ then $X \subseteq E(i)(w)$;

(**C6**) $\emptyset \notin S(i, j, v)$;

(**C7**) if $wE(i)v$ then $T(i, w) = T(i, v)$;

(**C8**) if $card(Agt_{i,X,M,w}) > card(Agt_{i,W \setminus X,M,w})$ and $card(Agt_{i,X,M,w}) \geq T(i, w)$
then $D(i)(w) \subseteq X$;

where, for any binary relation R on W, $R(w) = \{v \in W : wRv\}$ and for all $X \subseteq W$:

$$Agt_{i,X,M,w} = \{j \in Agt : X \in S(i, j, w)\}.$$

For notational convenience, we write E_i instead of $E(i)$ and D_i instead of $D(i)$. For every $w \in W$, $E_i(w)$ and $D_i(w)$ are called, respectively, agent i's information set and belief set at w. Agent i's information set at w is the set of worlds that agent i envisages at world w, while agent i's belief set at w is the set of worlds that agent i thinks to be possible at world w.

Constraint C1 ensures that the epistemic relation $E(i)$ is nothing but the indistinguishability relation traditionally used to model a fully introspective and truthful notion of knowledge.

Constraint C2 ensures that the set of possible worlds is included in the set of envisaged worlds. Indeed, following [15], a ESM requires that an agent is capable of assessing whether an envisaged situation is *possible* or not. [3]

Constraint C3 just means that if two worlds are in the same information set of agent i, then agent i has the same belief set at these two worlds. In other words, an agent knows her beliefs.

$S(i, j, w)$ is the set of evidence that agent j has provided to agent i where, following [7], a piece of evidence is identified with a set of worlds. Constraint C4 means that if two worlds are in the same information set of agent i, then agent i has the same evidence at these two worlds. In other words, an agent knows her evidence. Constraint C5 just means that an agent can have evidence only about facts which are compatible with her current information set. Constraint C6 means that an agent cannot have evidence about inconsistent facts.

[3] Here we take the term "envisaged" to be synonymous of the term "imagined". Clearly, there are situations that one can imagine that she considers impossible. For example, a person can imagine a situation in which she is the president of French republic and, at the same time, considers this situation impossible.

$T(i, w)$ corresponds to agent i's level of epistemic cautiousness at world w. Constraint C7 just means that if two worlds are in the same information set of agent i, then agent i has the same level of epistemic cautiousness at these two worlds. In other words, an agent knows her level of epistemic cautiousness.

Constraint C8 relates evidence with belief. Suppose that the amount of evidence in support of a given fact is: (i) equal or higher than my level of epistemic cautiousness and (ii) is higher than the amount of evidence in support of its negation. Then, I should start to believe this fact. Specifically, conditions (i) and (ii) together provide a sufficient reason to believe the fact in question.

Truth conditions of $\mathsf{DEL\text{-}ES}$ formulas are inductively defined as follows.

Definition 2.2 [Truth conditions] Let $M = (W, E, D, S, T, V)$ be a ESM and let $w \in W$. Then:

$$
\begin{aligned}
M, w \models p &\Longleftrightarrow p \in V(w) \\
M, w \models \mathsf{trs}(i, x) &\Longleftrightarrow T(i, w) = x \\
M, w \models \neg\varphi &\Longleftrightarrow M, w \not\models \varphi \\
M, w \models \varphi \wedge \psi &\Longleftrightarrow M, w \models \varphi \text{ and } M, w \models \psi \\
M, w \models \mathsf{K}_i\varphi &\Longleftrightarrow \forall v \in E_i(w) : M, v \models \varphi \\
M, w \models \mathsf{B}_i\varphi &\Longleftrightarrow \forall v \in D_i(w) : M, v \models \varphi \\
M, w \models \mathsf{E}_{i,j}\varphi &\Longleftrightarrow ||\varphi||^M_{i,w} \in S(i, j, w) \\
M, w \models [\varphi!_{i\hookleftarrow j}]\psi &\Longleftrightarrow \text{ if } M, w \models \widehat{\mathsf{K}}_i\varphi \text{ then } M^{\varphi!_{i\hookleftarrow j}}, w \models \psi \\
M, w \models [\circ_i]\psi &\Longleftrightarrow \text{ if } M, w \models \mathsf{B}_i\bot \text{ then } M^{\circ_i}, w \models \psi
\end{aligned}
$$

where

$$||\varphi||^M_{i,w} = \{v \in W : M, v \models \varphi\} \cap E_i(w),$$

$M^{\varphi!_{i\hookleftarrow j}}$ and M°_i} are updated models defined according to the following Definitions 2.3 and 2.4.

According to the truth conditions: agent i knows that φ at world w if and only if φ is true in all worlds that at w agent i envisages, and agent i believes that φ at world w if and only if φ is true in all worlds that at w agent i considers possible. Moreover, at world w agent j has provided evidence in support of φ to agent i if and only if, at w, agent i has the fact corresponding to the formula φ (i.e., $||\varphi||^M_{i,w}$) included in her evidence set $S(i, j, w)$. In what follows, we define the updated models triggered by the two kinds of events:

Definition 2.3 [Update via $\varphi!_{i\hookleftarrow j}$] Let $M = (W, E, D, S, T, V)$ be a ESM. Then, $M^{\varphi!_{i\hookleftarrow j}}$ is the tuple $(W, E, D^{\varphi!_{i\hookleftarrow j}}, S^{\varphi!_{i\hookleftarrow j}}, T, V)$ such that, for all $k, l \in$

Agt and $w \in W$:

$$D_k^{\varphi!_{i\hookleftarrow j}}(w) = \begin{cases} D_k(w) \cap ||\varphi||_{k,w}^M & \text{if } k = i \text{ and } M, w \models \mathsf{QR}_i\varphi \\ D_k(w) & \text{otherwise} \end{cases}$$

$$S^{\varphi!_{i\hookleftarrow j}}(k, l, w) = \begin{cases} S(k, l, w) \cup \{||\varphi||_{k,w}^M\} & \text{if } k = i \text{ and } l = j \\ S(k, l, w) & \text{otherwise} \end{cases}$$

Definition 2.4 [Update via $\circ_i$] Let $M = (W, E, D, S, T, V)$ be a SSM. Then, M°_i} is the tuple $(W, E, D^{\circ_i}, S^{\circ_i}, T, V)$ such that, for all $j, k \in Agt$ and $w \in W$:

$$D_j^{\circ_i}(w) = \begin{cases} E_j(w) & \text{if } j = i \\ D_j(w) & \text{otherwise} \end{cases}$$

$$S^{\circ_i}(j, k, w) = \begin{cases} \emptyset & \text{if } j = i \\ S(j, k, w) & \text{otherwise} \end{cases}$$

As highlighted in Definition 2.3, the event consisting in agent j providing evidence to agent i in support of φ, has two consequences: (i) a new evidence in support of φ is added to the set of evidence provided by agent j to agent i, and (ii) if before getting the new information agent i has a quasi-sufficient reason to believe φ then, after getting it, the agent will start to believe φ.

Again in Definition 2.4, the event consisting in restoring the consistency of agent i's beliefs has two consequences: (i) all agent i's sets of evidence become empty, and (ii) agent i starts to consider possible all situations that she envisages. More concisely, the operation of restoring belief consistency makes an agent to forget everything she has in her mind except her knowledge. This includes the agent's evidence as well as her beliefs. In other words, by restoring belief consistency, an agent "cleans up" her mind in order to start the accumulation of new evidence and the discovery of new truths.

Notice that the event $\varphi!_{i\hookleftarrow j}$ is executable, denoted by $\langle\varphi!_{i\hookleftarrow j}\rangle\top$, if and only if $\widehat{\mathsf{K}}_i\varphi$ holds and the event $\circ_i$ is executable, denoted by $\langle\circ_i\rangle\top$, if and only if $\mathsf{B}_i\bot$ holds. This means that an agent cannot provide to another agent evidence in support of φ if this evidence conflicts with her knowledge, and an agent will not restore consistency of her beliefs unless her beliefs are inconsistent.

For every $\varphi \in \mathcal{L}_{\mathsf{DEL\text{-}ES}}$, we write $\models \varphi$ to mean that φ is valid w.r.t. the class of ESMs, that is, for every $M = (W, E, D, S, T, V)$ and for every $w \in W$ we have $M, w \models \varphi$. We say that φ is satisfiable w.r.t. the class of ESMs if and only if $\neg\varphi$ is not valid w.r.t. the class of ESMs.

3 Properties

In this section we want to focus on some interesting properties of the logic DEL-ES. We start with the following static properties about the relationship between sufficient reason and quasi-sufficient reason:

$$\models \mathsf{R}_i\varphi \rightarrow \mathsf{QR}_i\varphi \tag{1}$$

$$\models (\mathsf{R}_i\varphi \wedge \mathsf{R}_i\neg\varphi) \rightarrow \mathsf{B}_i\bot \tag{2}$$

The validity (1) highlights that sufficient reason is stronger than quasi-sufficient reason, while, according to the validity (2), two conflicting reasons lead to belief inconsistency.

Let us now consider some properties that only apply to the propositional fragment of the logic $\mathsf{DEL\text{-}ES}$. Let $\mathcal{L}_{Atm}$ be the propositional language build out of the set of atoms Atm. Then, we have the following validities for $\varphi, \psi \in \mathcal{L}_{Atm}$:

$$\models (\neg\mathsf{E}_{i,j}\varphi \wedge \mathsf{QR}_i\varphi) \rightarrow [\varphi!_{i\hookleftarrow j}]\mathsf{R}_i\varphi \tag{3}$$

$$\models (\neg\mathsf{E}_{i,j}\varphi \wedge \mathsf{QR}_i\varphi) \rightarrow [\varphi!_{i\hookleftarrow j}]\mathsf{B}_i\varphi \tag{4}$$

$$\models (\neg\mathsf{E}_{i,j_1}\varphi \wedge \ldots \wedge \neg\mathsf{E}_{i,j_x}\varphi \wedge \mathsf{trs}(i,x)) \rightarrow [\varphi!_{i\hookleftarrow j_1}]\ldots[\varphi!_{i\hookleftarrow j_x}]\mathsf{R}_i\varphi \tag{5}$$

$$\models (\neg\mathsf{E}_{i,j_1}\varphi \wedge \ldots \wedge \neg\mathsf{E}_{i,j_x}\varphi \wedge \mathsf{trs}(i,x)) \rightarrow [\varphi!_{i\hookleftarrow j_1}]\ldots[\varphi!_{i\hookleftarrow j_x}]\mathsf{B}_i\varphi \tag{6}$$

$$[\chi!_{i\hookleftarrow j}]B_i\varphi \rightarrow [\chi!_{i\hookleftarrow j}][\theta!_{i\hookleftarrow k}]B_i\varphi \tag{7}$$

$$\models (\neg\mathsf{E}_{i,j}\varphi \wedge \mathsf{QR}_i\varphi \wedge \mathsf{K}_i(\varphi \rightarrow \psi)) \rightarrow [\varphi!_{i\hookleftarrow j}]\mathsf{B}_i\psi \tag{8}$$

According to (3), if agent i has a quasi-sufficient reason to believe that the propositional formula φ is true and agent j has not provided to agent i evidence in support of φ then, after j does that, i will have a sufficient reason to believe φ. According to (4), if agent i has a quasi-sufficient reason to believe that the propositional formula φ is true and agent j has not provided to agent i evidence in support of φ then, after j does that, i will start to believe φ. The validities (5) and (6) are similar properties for sequences of informative events: if agent i has a level of epistemic cautiousness equal to x and there are x agents who have not provided to agent i evidence in support of the propositional formula φ then, after they do that, i will have a sufficient reason to believe φ and, as a consequence, i will start to believe φ. The validity (7) highlights that, by getting more evidence, an agent decreases her uncertainty about the truth of propositional formulas. The validity (8) highlights the relationship between knowledge and belief from a dynamic point of view.

The reason why we need to impose that φ and ψ are propositional formulas is that there are $\mathsf{DEL\text{-}ES}$-formulas such as the Moore-like formula $p \wedge \neg\mathsf{B}_i p$ for which the previous validities (3)-(8) do not hold. For instance, the following formula is not valid:

$$(\neg\mathsf{E}_{i,j}(p \wedge \neg\mathsf{B}_i p) \wedge \mathsf{QR}_i(p \wedge \neg\mathsf{B}_i p)) \rightarrow [(p \wedge \neg\mathsf{B}_i p)!_{i\hookleftarrow j}]\mathsf{B}_i(p \wedge \neg\mathsf{B}_i p)$$

This is intuitive since if I think that my uncertainty about p could be unjustified since p is possibly true and someone gives me a decisive evidence in support of this fact then, as a consequence, I should start to believe that p is true and that I believe this (since I have introspection over my beliefs).

The following two validities apply to any formula of the language $\mathcal{L}_{\mathsf{DEL\text{-}ES}}$:

$$\models (\mathsf{B}_i\varphi \wedge \neg\mathsf{E}_{i,j}\neg\varphi \wedge \mathsf{QR}_i\neg\varphi) \rightarrow [\neg\varphi!_{i\hookleftarrow j}]\mathsf{B}_i\bot \tag{9}$$

$$\models [\circ_i]\neg\mathsf{B}_i\bot \tag{10}$$

According to the validity (9) if agent i believes that φ is true, has a quasi-sufficient reason to believe that φ is false and agent j has not provided to agent i evidence in support of the fact that φ is false then, after j does that, i's beliefs will become inconsistent. Validity (10) highlights the role of the event $\circ_i$ in restoring consistency of i's beliefs.

4 Axiomatization

Let us now present sound and complete axiomatizations for the logic EL-ES and its dynamic extension DEL-ES. As we will show, the completeness proof of the logic EL-ES is non-standard, given the interrelation between the concepts of belief and knowledge, on the one hand, and the concept of evidence, on the other hand. The completeness proof of EL-ES is based on a canonical model construnction. All axioms of EL-ES, except one, are used in the usual way to prove that the canonical model so constructed is a ESM. There is a special axiom of the logic EL-ES, about the interrelation between knowledge and evidence, that is used in an unusual way to prove the truth lemma.

Definition 4.1 [EL-ES] We define EL-ES to be the extension of classical propositional logic given by the following rules and axioms:

$$(\mathsf{K}_i\varphi \wedge \mathsf{K}_i(\varphi \rightarrow \psi)) \rightarrow \mathsf{K}_i\psi \qquad (\mathbf{K}_{\mathsf{K}_i})$$

$$\mathsf{K}_i\varphi \rightarrow \varphi \qquad (\mathbf{T}_{\mathsf{K}_i})$$

$$\mathsf{K}_i\varphi \rightarrow \mathsf{K}_i\mathsf{K}_i\varphi \qquad (\mathbf{4}_{\mathsf{K}_i})$$

$$\neg\mathsf{K}_i\varphi \rightarrow \mathsf{K}_i\neg\mathsf{K}_i\varphi \qquad (\mathbf{5}_{\mathsf{K}_i})$$

$$(\mathsf{B}_i\varphi \wedge \mathsf{B}_i(\varphi \rightarrow \psi)) \rightarrow \mathsf{B}_i\psi \qquad (\mathbf{K}_{\mathsf{B}_i})$$

$$\bigvee_{0 \leq x \leq card(Agt)} \mathsf{trs}(i,x) \qquad (\mathbf{AtLeast}_{\mathsf{trs}(i,x)})$$

$$\mathsf{trs}(i,x) \rightarrow \neg\mathsf{trs}(i,y) \text{ if } x \neq y \qquad (\mathbf{AtMost}_{\mathsf{trs}(i,x)})$$

$$\mathsf{K}_i\varphi \rightarrow \mathsf{B}_i\varphi \qquad (\mathbf{Mix1}_{\mathsf{K}_i,\mathsf{B}_i})$$

$$\mathsf{B}_i\varphi \rightarrow \mathsf{K}_i\mathsf{B}_i\varphi \qquad (\mathbf{Mix2}_{\mathsf{K}_i,\mathsf{B}_i})$$

$$\mathsf{trs}(i,x) \rightarrow \mathsf{K}_i\mathsf{trs}(i,x) \qquad (\mathbf{Mix}_{\mathsf{K}_i,\mathsf{trs}(i,x)})$$

$$\mathsf{E}_{i,j}\varphi \rightarrow \mathsf{K}_i\mathsf{E}_{i,j}\varphi \qquad (\mathbf{Mix1}_{\mathsf{K}_i,\mathsf{E}_{i,j}})$$

$$\neg\mathsf{E}_{i,j}\bot \qquad (\mathbf{Cons}_{\mathsf{E}_{i,j}})$$

$$(\mathsf{E}_{i,j}\varphi \wedge \mathsf{K}_i(\varphi \leftrightarrow \psi)) \rightarrow \mathsf{E}_{i,j}\psi \qquad (\mathbf{Mix2}_{\mathsf{K}_i,\mathsf{E}_{i,j}})$$

$$\mathsf{R}_i\varphi \rightarrow \mathsf{B}_i\varphi \qquad (\mathbf{SuffReas})$$

$$\frac{\varphi}{\mathsf{K}_i\varphi} \qquad (\mathbf{Nec}_{\mathsf{K}_i})$$

Notice that the rule of necessitation for the belief operator is provable by means of the rule of inference $(\mathbf{Nec}_{\mathsf{K}_i})$ and Axiom $(\mathbf{Mix1}_{\mathsf{K}_i,\mathsf{B}_i})$. Moreover, Axiom 4 for the belief operator is provable by means of Axioms $(\mathbf{Mix1}_{\mathsf{K}_i,\mathsf{B}_i})$ and $(\mathbf{Mix2}_{\mathsf{K}_i,\mathsf{B}_i})$. Axiom 5 for the belief operator is provable by means of Axioms $(\mathbf{Mix1}_{\mathsf{K}_i,\mathsf{B}_i})$, $(\mathbf{Mix2}_{\mathsf{K}_i,\mathsf{B}_i})$, $\mathbf{K}_{\mathsf{K}_i}$, $\mathbf{T}_{\mathsf{K}_i}$, $\mathbf{4}_{\mathsf{K}_i}$ and $\mathbf{5}_{\mathsf{K}_i}$. A syntactic proof can be found in [17].

Theorem 4.2 *The logic EL-ES is sound and complete for the class of ESMs.*

Proof. It is routine to check that the axioms of EL-ES are all valid w.r.t. the class of ESMs and that the rule of inference ($\mathbf{Nec}_{\mathsf{K}_i}$) preserves validity.

To prove completeness, we use a canonical model argument.

We consider maximally consistent sets of formulas in $\mathcal{L}_{\textsf{EL-ES}}$ (MCSs). The following proposition specifies some usual properties of MCSs.

Proposition 4.3 *Let Γ be a MCS and let $\varphi, \psi \in \mathcal{L}_{\textsf{EL-ES}}$. Then:*

- *if $\varphi, \varphi \to \psi \in \Gamma$ then $\psi \in \Gamma$;*
- *$\varphi \in \Gamma$ or $\neg\varphi \in \Gamma$;*
- *$\varphi \vee \psi \in \Gamma$ iff $\varphi \in \Gamma$ or $\psi \in \Gamma$.*

The following is the Lindenbaum's lemma for our logic. As the proof is standard (cf. [9, Lemma 4.17]) we omit it here.

Lemma 4.4 *Let Δ be a EL-ES-consistent set of formulas. Then, there exists a MCS Γ such that $\Delta \subseteq \Gamma$.*

Let the canonical ESM model be the tuple $M^c = (W^c, E^c, D^c, S^c, T^c, V^c)$ such that:

- W^c is set of all MCSs;
- for all $w, v \in W^c$ and $i \in Agt$, $wE_i^c v$ iff, for all $\varphi \in \mathcal{L}_{\textsf{EL-ES}}$, if $\mathsf{K}_i\varphi \in w$ then $\varphi \in v$;
- for all $w, v \in W^c$ and $i \in Agt$, $wD_i^c v$ iff, for all $\varphi \in \mathcal{L}_{\textsf{EL-ES}}$, if $\mathsf{B}_i\varphi \in w$ then $\varphi \in v$;
- for all $w \in W^c$ and $i, j \in Agt$, $S^c(i,j,w) = \{A_\varphi(i,j,w) : \mathsf{E}_{i,j}\varphi \in w\}$;
- for all $w \in W^c$ and $i \in Agt$, $T^c(i,w) = x$ iff $\mathsf{trs}(i,x) \in w$;
- for all $w \in W^c$ and $p \in Atm$, $p \in V^c(w)$ iff $p \in w$;

where $A_\varphi(i,j,w) = \{v \in E_i^c(w) : \varphi \in v\}$.

Thanks to Axioms $\mathbf{AtLeast}_{\mathsf{trs}(i,x)}$ and $\mathbf{AtMost}_{\mathsf{trs}(i,x)}$, it is easy to check that the model M^c is well-defined as the function T^c exists.

We have to prove that M^c is a ESM by showing that it satisfies conditions C1-C8 in Definition 2.1. The proof is a routine exercise and uses of Proposition 4.3: Condition C1 is satisfied because of Axioms $\mathbf{T}_{\mathsf{K}_i}$, $\mathbf{4}_{\mathsf{K}_i}$ and $\mathbf{5}_{\mathsf{K}_i}$; Condition C2 is satisfied because of Axiom $\mathbf{Mix1}_{\mathsf{K}_i,\mathsf{B}_i}$; Condition C3 is satisfied because of Axiom $\mathbf{Mix2}_{\mathsf{K}_i,\mathsf{B}_i}$; Condition C4 is satisfied because of Axiom $\mathbf{Mix1}_{\mathsf{K}_i,\mathsf{E}_{i,j}}$; Condition C6 is satisfied because of Axiom $\mathbf{Cons}_{\mathsf{E}_{i,j}}$; Condition C7 is satisfied because of Axiom $\mathbf{Mix}_{\mathsf{K}_i,\mathsf{trs}(i,x)}$; Condition C8 is satisfied because of Axiom **SuffReas**; Condition C5 is satisfied by construction of the model M^c and, in particular, by definition of $A_\varphi(i,j,w)$. Here we only show that M^c satisfies Conditions C4 and C5.

As for C4, suppose that $wE_i^c v$ and $X \in S^c(i,j,w)$. The latter means that $X = \{u \in E_i^c(w) : \varphi \in u\}$ and $\mathsf{E}_{i,j}\varphi \in w$ for some φ. Hence, by Proposition 4.3 and Axiom $\mathbf{Mix1}_{\mathsf{K}_i,\mathsf{E}_{i,j}}$, we have $\mathsf{K}_i\mathsf{E}_{i,j}\varphi \in w$. By $wE_i^c v$ and the definition of

E_i^c, from the latter it follows that $\mathsf{E}_{i,j}\varphi \in v$. Hence, by the definition of S^c, we have $Y = \{u \in E_i^c(v) : \varphi \in u\} \in S^c(i, j, v)$. Since E_i^c is an equivalence relation and $wE_i^c v$, we have $E_i^c(w) = E_i^c(v)$. Thus, $X = Y$. Hence, $X \in S^c(i, j, v)$.

As for C5, suppose that $X \in S^c(i, j, w)$. The latter means that $X = \{u \in E_i^c(w) : \varphi \in u\}$ and $\mathsf{E}_{i,j}\varphi \in w$ for some φ. Thus, clearly, $X \subseteq E_i^c(w)$.

The next step in the proof consists in stating the following existence lemma. The proof is again standard (cf. [9, Lemma 4.20]) and we omit it.

Lemma 4.5 *Let $\varphi \in \mathcal{L}_{\textsf{EL-ES}}$ and $w \in W^c$. Then:*

- *if $\widehat{\mathsf{K}}_i\varphi \in w$ then there exists $v \in W^c$ such that $wE_i^c v$ and $\varphi \in v$;*
- *if $\widehat{\mathsf{B}}_i\varphi \in w$ then there exists $v \in W^c$ such that $wD_i^c v$ and $\varphi \in v$.*

Finally, we can prove the following truth lemma.

Lemma 4.6 *Let $\varphi \in \mathcal{L}_{\textsf{EL-ES}}$ and $w \in W^c$. Then, $M^c, w \models \varphi$ iff $\varphi \in w$.*

Proof. The proof is by induction on the structure of the formula. Here we only prove the case $\varphi = \mathsf{E}_{i,j}\psi$ which is the most interesting one as it uses a non-standard technique. The other cases are provable in the standard way (cf. [9, Lemma 4.21]).

($\Rightarrow$) Suppose $M^c, w \models \mathsf{E}_{i,j}\psi$. Thus, $\{u \in E_i^c(w) : M^c, u \models \psi\} \in S^c(i, j, w)$. Hence, by definition of S^c, there exists χ such that $\mathsf{E}_{i,j}\chi \in w$ and $\{u \in E_i^c(w) : \chi \in u\} = \{u \in E_i^c(w) : M^c, u \models \psi\}$. Thus, by induction hypothesis, $\{u \in E_i^c(w) : \chi \in u\} = \{u \in E_i^c(w) : \psi \in u\}$. Now, suppose that $\mathsf{K}_i(\chi \leftrightarrow \psi) \notin w$. By Proposition 4.3, it follows that $\neg\mathsf{K}_i(\chi \leftrightarrow \psi) \in w$. This means that $\widehat{\mathsf{K}}_i((\chi \wedge \neg\psi) \vee (\neg\chi \wedge \psi)) \in w$. By Lemma 4.5, the latter implies that there exists $v \in W^c$ such that $wE_i^c v$ and $((\chi \wedge \neg\psi) \vee (\neg\chi \wedge \psi)) \in v$ which is in contradiction with $\{u \in E_i^c(w) : \chi \in u\} = \{u \in E_i^c(w) : \psi \in u\}$. Thus, we have $\mathsf{K}_i(\chi \leftrightarrow \psi) \in w$. From $\mathsf{E}_{i,j}\chi \in w$ and $\mathsf{K}_i(\chi \leftrightarrow \psi) \in w$, by Proposition 4.3 and Axiom $\mathbf{Mix2}_{\mathsf{K}_i,\mathsf{E}_{i,j}}$, it follows that $\mathsf{E}_{i,j}\psi \in w$.

($\Leftarrow$) Suppose $\mathsf{E}_{i,j}\psi \in w$. Thus, by the definition of S^c, $A_\psi(i, j, w) = \{v \in E_i^c(w) : \psi \in v\} \in S^c(i, j, w)$. Hence, by induction hypothesis, $\{v \in E_i^c(w) : M^c, v \models \psi\} \in S^c(i, j, w)$. The latter means that $M^c, w \models \mathsf{E}_{i,j}\psi$. □

To conclude the proof, suppose that φ is a EL-ES-consistent formula in $\mathcal{L}_{\textsf{EL-ES}}$. By Lemma 4.4, there exists $w \in W^c$ such that $\varphi \in w$. Hence, by Lemma 4.6, there exists $w \in W^c$ such that $M^c, w \models \varphi$.

□

The axiomatics of the logic DEL-ES includes all principles of the logic EL-ES *plus* a set of reduction axioms and the rule of replacement of equivalents.

Definition 4.7 We define DEL-ES to be the extension of EL-ES generated by the following reduction axioms for the dynamic operators $[\varphi!_{i\hookleftarrow j}]$:

$$[\varphi!_{i\hookleftarrow j}]p \leftrightarrow (\widehat{\mathsf{K}}_i\varphi \to p) \qquad (\mathbf{Red}_{\varphi!_{i\hookleftarrow j},p})$$

$$[\varphi!_{i\hookleftarrow j}]\mathsf{trs}(k,x) \leftrightarrow (\widehat{\mathsf{K}}_i\varphi \to \mathsf{trs}(k,x)) \qquad (\mathbf{Red}_{\varphi!_{i\hookleftarrow j},\mathsf{trs}(l,x)})$$

$$[\varphi!_{i\hookleftarrow j}]\neg\psi \leftrightarrow (\widehat{\mathsf{K}}_i\varphi \to \neg[\varphi!_{i\hookleftarrow j}]\psi) \qquad (\mathbf{Red}_{\varphi!_{i\hookleftarrow j},\neg})$$

$$[\varphi!_{i\hookleftarrow j}](\psi\wedge\chi) \leftrightarrow ([\varphi!_{i\hookleftarrow j}]\psi \wedge [\varphi!_{i\hookleftarrow j}]\chi) \qquad (\mathbf{Red}_{\varphi!_{i\hookleftarrow j},\wedge})$$

$$[\varphi!_{i\hookleftarrow j}]\mathsf{K}_k\varphi \leftrightarrow (\widehat{\mathsf{K}}_i\varphi \to \mathsf{K}_k[\varphi!_{i\hookleftarrow j}]\varphi) \qquad (\mathbf{Red}_{\varphi!_{i\hookleftarrow j},\mathsf{K}_k})$$

$$[\varphi!_{i\hookleftarrow j}]\mathsf{B}_k\psi \leftrightarrow (\widehat{\mathsf{K}}_i\varphi \to \mathsf{B}_k[\varphi!_{i\hookleftarrow j}]\varphi) \text{ if } k\neq i \qquad (\mathbf{Red}_{\varphi!_{i\hookleftarrow j},\mathsf{B}_k})$$

$$\begin{aligned}[\varphi!_{i\hookleftarrow j}]\mathsf{B}_i\psi \leftrightarrow \Big(\widehat{\mathsf{K}}_i\varphi \to \big((\neg\mathsf{QR}_i\varphi \wedge \mathsf{B}_i[\varphi!_{i\hookleftarrow j}]\psi)\vee \\ (\mathsf{QR}_i\varphi \wedge \mathsf{B}_i(\varphi \to [\varphi!_{i\hookleftarrow j}]\psi))\big)\Big)\end{aligned} \qquad (\mathbf{Red}_{\varphi!_{i\hookleftarrow j},\mathsf{B}_i})$$

$$[\varphi!_{i\hookleftarrow j}]\mathsf{E}_{k,l}\psi \leftrightarrow (\widehat{\mathsf{K}}_i\varphi \to \mathsf{E}_{k,l}[\varphi!_{i\hookleftarrow j}]\psi) \text{ if } k\neq i \text{ or } l\neq j \qquad (\mathbf{Red}_{\varphi!_{i\hookleftarrow j},\mathsf{E}_{k,l}})$$

$$\begin{aligned}[\varphi!_{i\hookleftarrow j}]\mathsf{E}_{i,j}\psi \leftrightarrow \Big(\widehat{\mathsf{K}}_i\varphi \to \big((\mathsf{E}_{i,j}[\varphi!_{i\hookleftarrow j}]\psi\vee \\ \mathsf{K}_i([\varphi!_{i\hookleftarrow j}]\psi \leftrightarrow \varphi)\big)\Big)\end{aligned} \qquad (\mathbf{Red}_{\varphi!_{i\hookleftarrow j},\mathsf{E}_{i,j}})$$

the following ones for the dynamic operators $[\circ_i]$:

$$[\circ_i]p \leftrightarrow (\mathsf{B}_i\bot \to p) \qquad (\mathbf{Red}_{\circ_i,p})$$

$$[\circ_i]\mathsf{trs}(k,x) \leftrightarrow (\mathsf{B}_i\bot \to \mathsf{trs}(k,x)) \qquad (\mathbf{Red}_{\circ_i,\mathsf{trs}(k,x)})$$

$$[\circ_i]\neg\varphi \leftrightarrow (\mathsf{B}_i\bot \to \neg[\circ_i]\varphi) \qquad (\mathbf{Red}_{\circ_i,\neg})$$

$$[\circ_i](\varphi\wedge\psi) \leftrightarrow ([\circ_i]\varphi \wedge [\circ_i]\psi) \qquad (\mathbf{Red}_{\circ_i,\wedge})$$

$$[\circ_i]\mathsf{K}_j\varphi \leftrightarrow (\mathsf{B}_i\bot \to \mathsf{K}_j[\circ_i]\varphi) \qquad (\mathbf{Red}_{\circ_i,\mathsf{K}_j})$$

$$[\circ_i]\mathsf{B}_j\varphi \leftrightarrow (\mathsf{B}_i\bot \to \mathsf{B}_j[\circ_i]\varphi) \text{ if } j\neq i \qquad (\mathbf{Red}_{\circ_i,\mathsf{B}_j})$$

$$[\circ_i]\mathsf{B}_i\varphi \leftrightarrow (\mathsf{B}_i\bot \to \mathsf{K}_i[\circ_i]\varphi) \qquad (\mathbf{Red}_{\circ_i,\mathsf{B}_i})$$

$$[\circ_i]\mathsf{E}_{j,k}\varphi \leftrightarrow (\mathsf{B}_i\bot \to \mathsf{E}_{j,k}[\circ_i]\psi) \text{ if } j\neq i \qquad (\mathbf{Red}_{\circ_i,\mathsf{E}_{j,k}})$$

$$[\circ_i]\mathsf{E}_{i,j}\varphi \leftrightarrow \neg\mathsf{B}_i\bot \qquad (\mathbf{Red}_{\circ_i,\mathsf{E}_{i,j}})$$

and the following rule of inference:

$$\frac{\psi_1 \leftrightarrow \psi_2}{\varphi \leftrightarrow \varphi[\psi_1/\psi_2]} \qquad (\mathbf{RRE})$$

We write $\vdash_{\mathsf{DEL\text{-}ES}} \varphi$ to denote the fact that φ is a theorem of DEL-ES.

The completeness of DEL-ES follows from Theorem 4.2, in view of the fact that the reduction axioms may be used to find, for any DEL-ES formula, a provably equivalent EL-ES formula.

Lemma 4.8 *If φ is any formula of $\mathcal{L}_{\mathsf{DEL\text{-}ES}}$, there is a formula $red(\varphi)$ in $\mathcal{L}_{\mathsf{EL\text{-}ES}}$ such that $\vdash_{\mathsf{DEL\text{-}ES}} \varphi \leftrightarrow red(\varphi)$.*

Proof. This follows by a routine induction on φ using the reduction axioms and the rule of replacement of equivalents (**RRE**) from Definition 4.7. □

As a corollary, we get the following:

Theorem 4.9 *DEL-ES is sound and complete complete for the class of ESMs.*

Proof. It is a routine exercise to check that all principles in Definition 4.7 are valid and that the rule of inference (**RRE**) preserves validity. As for completeness, if Γ is a consistent set of $\mathcal{L}_{\mathsf{DEL\text{-}ES}}$ formulas, then $red(\Gamma) = \bigwedge\{red(\varphi) : \varphi \in \Gamma\}$ is a consistent set of $\mathcal{L}_{\mathsf{EL\text{-}ES}}$ formulas (since DEL-ES is an extension of EL-ES), and hence by Theorem 4.2 there is a model M with a world w such that $M, w \models red(\Gamma)$. But, since DEL-ES is sound and for each $\varphi \in \Gamma$, $\vdash_{\mathsf{DEL\text{-}ES}} \varphi \leftrightarrow red(\varphi)$, it follows that $M, w \models red(\Gamma)$.

□

5 Illustration: is Peterson guilty?

In this section we want to illustrate how the concepts and framework we proposed in the paper can be used in understanding issues from our real life. Again, take the legal case, a judge is someone whom we trust as a rational agent. Her decision has to be made on the basis of reasons, or rather evidence. Let us consider a recent case in the US, and a small part of the timeline from online Fox News ([12]), with our notes italic in brackets. We use g to denote the proposition that "Scott Peterson is guilty" and we single out some events along the timeline that provide evidence for g or $\neg g$.

`Dec.` 24, 2002: Laci Peterson, while 8-months pregnant, is reported missing from her home in Modesto, Calif., by husband Scott Peterson. He claimed to have returned from a fishing trip and was unable to find his wife.
`Jan.` 24, 2003: Amber Frey, a massage therapist from Fresno, confirms she had a romantic relationship with Scott Peterson. [*evidence, at least, in favor of g*]
`Aug.` 22, 2003: ... Later that day, sources tell Fox News that Scott Peterson had admitted – then denied – involvement in his wife's disappearance in a wiretapped telephone conversation with his then-girlfriend Amber Frey. [*evidence supporting g*]
`Oct.` 15, 2003: Sources tell Fox News that telephone logs show that Scott Peterson called Frey hundreds of times after his wife's disappearance, contradicting prior claims that Frey pursued him. [*evidence supporting g*]
`Nov.` 3, 2003: A defense expert testifies that mitochondrial DNA tests, which cannot link evidence to a specific individual, are scientifically flawed. [evidence supporting $\neg g$]
`Nov.` 6, 2003: A police detective testifies that Scott Peterson told Frey he was a recent widower on Dec. 9, 2002, two weeks before his wife disappeared. [*evidence supporting g*]
`March` 16, 2005: Judge Alfred Delucchi formally sentences Peterson to death, calling the murder of his wife "cruel, uncaring, heartless, and callous." [*final decision made*]

As we can see, while time goes, the evidence is accumulated. Some are supporting g, some are not. This dynamic process leads the judge to form the belief that Scott Peterson is guilty ($\mathsf{B}_{judge}g$). Let us assume that the judge's level of cautiousness is equal to 4. Then, the above example can be expressed by the following formula:

$$\mathsf{trs}(judge, 4) \wedge [g!_{judge \hookleftarrow source1}][g!_{judge \hookleftarrow source2}]$$
$$[g!_{judge \hookleftarrow source3}][\neg g!_{judge \hookleftarrow source4}][g!_{judge \hookleftarrow source5}]\mathsf{B}_{judge}\, g$$

Here the judge's epistemic cautiousness, as well as the amount of evidence that have been collected determines the final decision. This might look too simple. However, we hope to have shown the potential of connecting our work to real legal practice. We believe that evidence-based analysis of legal texts can facilitate the justice system.

6 Conclusion and future directions

In this paper we have proposed a new logic, called "Dynamic Epistemic Logic of Evidence Sources", which enables us to reason about an agent's evidence-based belief formation and belief change, triggered by social communication. We have provided a complete axiomatization for both the static Epistemic Logic of Evidence Sources and its dynamic extension. We have discussed several interesting concepts that we can use in talking about evidence or reasons. For instance, having decisive evidence, and having sufficient reasons to believe. In particular, we have explicitly introduced evidence sources into our language. The new logic can be adopted to analyze. issues in legal contexts. For future directions, we identify a few. (i) We want to further study the relation between the evidence sources and the sources themselves. The same evidence provided by different sources who are situated in various communities should carry different strength. For instance, evidence from independent sources may be treated heavily than that from an internally closed community. (ii) We have emphasized that the accumulation of evidence leads to an agent's belief change and that the amount of evidence plays a role in relation with the level of epistemic cautiousness. However, it is sometimes the case that one piece of evidence counts much more than many other pieces all together. We would like to deal with such situations. (iii) Finally, an agent obtains information by social communication, and forms her beliefs on the basis of reasons. In this paper, we have investigated epistemic reasons. We plan to extend our logical framework with agents' preferences and choices in order to incorporate practical reasons in our analysis and to study their connection with epistemic reasons.

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