Piecewise Polynomial Model of the Aerodynamic Coefficients of the Generic Transport Model and its Equations of Motion

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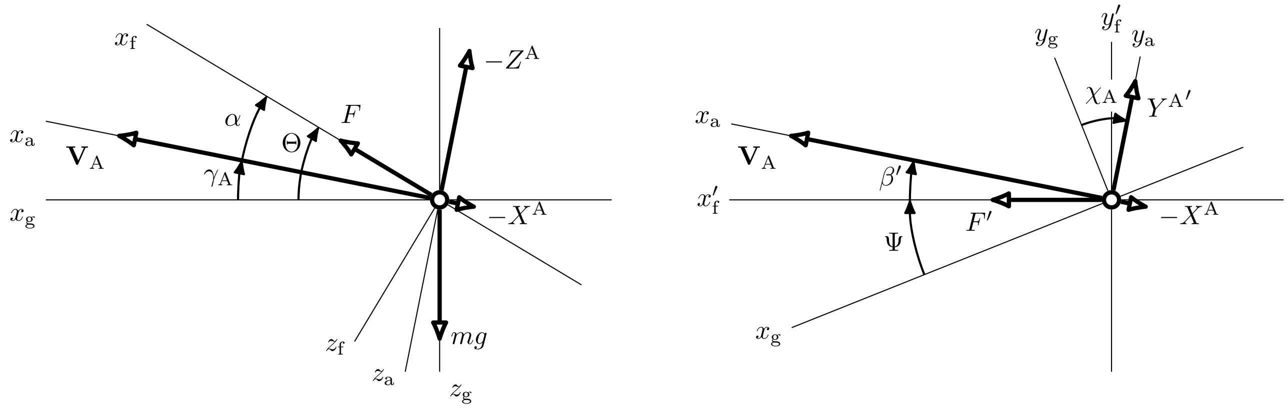
Abstract

The purpose of this document is to illustrate the piecewise polynomial model which has been derived from wind-tunnel measurement data of the Generic Transport Model (GTM) using the `pwpfit` toolbox. For implementation details and use in MATLAB, please refer to the source code at https://github.com/pwpfit/GTMpw.

Preliminaries

If not stated otherwise, all variables are in SI units. We will refer to the following axis systems of ISO 1151-1: the *body axis system* $(x_\mathrm{f}, y_\mathrm{f}, z_\mathrm{f})$ aligned with the aircraft's fuselage; the *air-path axis system* $(x_\mathrm{a}, y_\mathrm{a}, z_\mathrm{a})$ defined by the velocity vector $\mathbf{V}_\mathrm{A}$; and the *normal earth-fixed axis system* $(x_\mathrm{g}, y_\mathrm{g}, z_\mathrm{g})$. The orientation of the body axes with respect to the normal earth-fixed system is given by the attitude angles Φ, Θ, Ψ and to the air-path system by angle of attack α and side-slip β; the orientation of the air-path axes to the normal earth-fixed system is given by azimuth χ_A, inclination γ_A, and bank-angle μ_A. (Fig. 1.)



(a) Longitudinal axes ($\beta = \mu_\mathrm{A} = 0$).

(b) Horizontal axes ($\gamma_\mathrm{A} = 0$).

Fig. 1: Axis systems with angles and vectors. Projections into the plane are marked by $'$.

I. Aerodynamic Coefficients

The piecewise polynomial models of the aerodynamic coefficients are given

$$C_\odot(\alpha, \beta, \dots) = \begin{cases} C_\odot^{pre}(\alpha, \beta, \dots) & \text{if } \alpha \leq \alpha_0, \\ C_\odot^{post}(\alpha, \beta, \dots) & \text{else,} \end{cases} \tag{1}$$

where $C_\odot \in \{C_\mathrm{X}, C_\mathrm{Y}, C_\mathrm{Z}, C_\mathrm{l}, C_\mathrm{m}, C_\mathrm{n}\}$ are polynomials in angle of attack, side-slip, surface deflections, and normalized body rates; and the boundary is found at

$$\alpha_0 = 16.111°. \tag{2}$$

The polynomials in low and high angle of attack, $C_\odot^{pre}$, $C_\odot^{post}$, are sums

$$C_\odot^{pre} = C_{\odot\alpha}^{pre}(\alpha) + C_{\odot\beta}^{pre}(\alpha, \beta) + C_{\odot\xi}^{pre}(\alpha, \beta, \xi) + C_{\odot\eta}^{pre}(\alpha, \beta, \eta) + C_{\odot\zeta}^{pre}(\alpha, \beta, \zeta) + C_{\odot p}^{pre}(\alpha, \hat{p}) + C_{\odot q}^{pre}(\alpha, \hat{q}) + C_{\odot r}^{pre}(\alpha, \hat{r}); \tag{3}$$

$$C_\odot^{post} = C_{\odot\alpha}^{post}(\alpha) + C_{\odot\beta}^{post}(\alpha, \beta) + C_{\odot\xi}^{post}(\alpha, \beta, \xi) + C_{\odot\eta}^{post}(\alpha, \beta, \eta) + C_{\odot\zeta}^{post}(\alpha, \beta, \zeta) + C_{\odot p}^{post}(\alpha, \hat{p}) + C_{\odot q}^{post}(\alpha, \hat{q}) + C_{\odot r}^{post}(\alpha, \hat{r}). \tag{4}$$

[1]ONERA – The French Aerospace Lab, Department of Information Processing and Systems, Centre Midi-Pyrénées, Toulouse, 31055, Franceand ENAC, Université de Toulouse, Drones Research Group; now with the University of Michigan, e-mail: tcunis@umich.edu.

[2]Rutgers, The State University of New Jersey, Department of Mechanical & Aerospace Engineering, Piscataway, NJ 08854, USA, e-mail: laurent.burlion@rutgers.edu.

[3]ENAC, Université de Toulouse, Drones Research Group, Toulouse, 31055, France, e-mail: jean-philippe.condomines@enac.fr.

In the following subsections, we present the polynomial terms obtained using the `pwpfit` toolbox. Coefficients of absolute value lower than 10^{-2} have been omitted for readability.

A. Domain of low angle of attack

Polynomials in angle of attack:

$$C_{\mathrm{X}\alpha}^{pre} = -0.039 + 0.244\alpha + 4.452\alpha^2 - 17.394\alpha^3; \tag{5}$$

$$C_{\mathrm{Z}\alpha}^{pre} = -0.017 - 5.241\alpha - 1.866\alpha^2 + 28.466\alpha^3; \tag{6}$$

$$C_{\mathrm{m}\alpha}^{pre} = 0.119 - 1.465\alpha + 8.129\alpha^2 - 31.983\alpha^3; \tag{7}$$

$C_{\mathrm{Y}\alpha}^{pre}$, $C_{\mathrm{l}\alpha}^{pre}$, $C_{\mathrm{n}\alpha}^{pre}$ are zero by definition.

Polynomials in angle of attack and side-slip:

$$C_{\mathrm{X}\beta}^{pre} = 0.012 + 0.013\alpha - 2.049\alpha^2 + 0.027\alpha\beta + 0.066\beta^2 + 9.808\alpha^3 + 0.249\alpha^2\beta - 0.572\alpha\beta^2 - 10.651\alpha^4 - 1.178\alpha^3\beta + 1.936\alpha^2\beta^2 - 0.043\beta^4; \tag{8}$$

$$C_{\mathrm{Y}\beta}^{pre} = -1.024\beta + 0.219\alpha\beta - 0.307\alpha^2\beta + 0.070\beta^3 + 4.667\alpha^3\beta - 1.062\alpha\beta^3; \tag{9}$$

$$C_{\mathrm{Z}\beta}^{pre} = -0.035 - 0.018\alpha + 2.911\alpha^2 + 0.024\beta^2 - 6.328\alpha^3 - 0.098\alpha^2\beta + 4.018\alpha\beta^2 - 8.805\alpha^4 + 0.328\alpha^3\beta - 8.609\alpha^2\beta^2 + 0.165\beta^4; \tag{10}$$

$$C_{\mathrm{l}\beta}^{pre} = -0.144\beta + 0.177\alpha\beta + 0.052\alpha^2\beta + 0.203\beta^3 + 3.495\alpha^3\beta - 1.113\alpha\beta^3; \tag{11}$$

$$C_{\mathrm{m}\beta}^{pre} = 0.068 - 0.205\alpha - 9.113\alpha^2 + 0.021\alpha\beta - 1.396\beta^2 + 54.917\alpha^3 + 0.170\alpha^2\beta - 1.633\alpha\beta^2 - 83.622\alpha^4 - 0.990\alpha^3\beta + 11.760\alpha^2\beta^2 - 0.027\alpha\beta^3 + 1.164\beta^4; \tag{12}$$

$$C_{\mathrm{n}\beta}^{pre} = 0.228\beta - 0.228\alpha\beta - 0.230\beta^3 - 1.697\alpha^3\beta + 0.539\alpha\beta^3. \tag{13}$$

Polynomials in angle of attack, side-slip, and aileron deflections:

$$C_{\mathrm{X}\xi}^{pre} = 0.033\alpha - 0.262\alpha^2 + 0.042\beta^2 + 0.014\beta\xi + 0.135\xi^2 + 1.195\alpha^3 + 0.094\alpha^2\beta - 0.147\alpha\beta^2 + 0.016\alpha\beta\xi - 0.129\alpha\xi^2 - 2.732\alpha^4 - 0.308\alpha^3\beta + 0.993\alpha^2\beta^2 - 0.575\alpha^2\beta\xi + 0.161\alpha^2\xi^2 - 0.111\beta^4 - 0.048\beta^2\xi^2 - 0.012\beta\xi^3 - 0.499\xi^4; \tag{14}$$

$$C_{\mathrm{Y}\xi}^{pre} = 0.080\beta - 0.022\xi - 1.925\alpha\beta - 0.036\alpha\xi + 0.501\alpha^2\beta + 0.229\alpha^2\xi - 0.021\beta^3 - 0.244\beta\xi^2 - 1.469\alpha^3\beta - 0.751\alpha^3\xi + 3.189\alpha\beta^3 + 5.135\alpha\beta\xi^2 + 0.227\alpha\xi^3; \tag{15}$$

$$C_{\mathrm{Z}\xi}^{pre} = -0.030\alpha - 0.322\alpha^2 - 0.033\beta^2 - 0.140\beta\xi + 0.029\xi^2 + 3.097\alpha^3 + 0.094\alpha^2\beta - 0.328\alpha\beta^2 + 0.385\alpha\beta\xi - 0.247\alpha\xi^2 - 5.289\alpha^4 - 0.308\alpha^3\beta + 0.142\alpha^2\beta^2 + 0.169\alpha^2\beta\xi - 0.795\alpha^2\xi^2 + 0.109\beta^4 + 0.027\beta^3\xi - 0.058\beta^2\xi^2 + 0.197\beta\xi^3 + 0.118\xi^4; \tag{16}$$

$$C_{\mathrm{l}\xi}^{pre} = 0.021\beta - 0.079\xi - 0.158\alpha\beta + 0.046\alpha\xi + 0.149\alpha^2\beta + 0.698\alpha^2\xi - 0.066\beta^3 + 0.027\beta^2\xi - 0.036\beta\xi^2 + 0.069\xi^3 - 0.508\alpha^3\beta - 1.316\alpha^3\xi + 0.338\alpha\beta^3 + 0.016\alpha\beta^2\xi + 0.453\alpha\beta\xi^2 - 0.139\alpha\xi^3; \tag{17}$$

$$C_{\mathrm{m}\xi}^{pre} = -0.039 + 0.059\alpha + 0.163\alpha^2 + 0.401\beta^2 - 0.013\beta\xi - 0.564\xi^2 + 2.568\alpha^3 + 0.094\alpha^2\beta - 1.548\alpha\beta^2 + 0.068\alpha\beta\xi - 0.439\alpha\xi^2 - 4.594\alpha^4 - 0.308\alpha^3\beta - 1.295\alpha^2\beta^2 - 0.048\alpha^2\beta\xi + 0.356\alpha^2\xi^2 + 0.085\beta^4 + 0.082\beta^3\xi + 0.084\beta^2\xi^2 + 0.038\beta\xi^3 + 2.201\xi^4; \tag{18}$$

$$C_{\mathrm{n}\xi}^{pre} = -0.249\alpha\beta + 0.014\alpha\xi + 0.041\alpha^2\beta + 0.053\alpha^2\xi + 0.029\beta^3 - 0.017\beta\xi^2 + 0.016\xi^3 + 0.337\alpha^3\beta + 0.048\alpha^3\xi + 0.337\alpha\beta^3 + 0.049\alpha\beta^2\xi + 0.690\alpha\beta\xi^2 - 0.122\alpha\xi^3. \tag{19}$$

Polynomials in angle of attack, side-slip, and elevator deflections:

$$C_{\mathrm{X}\eta}^{pre} = -0.036\alpha - 0.292\alpha^2 + 0.148\alpha\eta - 0.102\eta^2 + 1.173\alpha^3 - 0.411\alpha^2\eta + 0.033\alpha\beta^2 - 0.011\alpha\beta\eta + 0.105\alpha\eta^2 - 0.036\eta^3; \quad (20)$$

$$C_{\mathrm{Y}\eta}^{pre} = -0.267\beta + 1.036\alpha\beta - 3.141\alpha^2\beta + 0.532\beta^3; \quad (21)$$

$$C_{\mathrm{Z}\eta}^{pre} = 0.041\alpha - 0.016\beta - 0.526\eta + 0.205\alpha^2 + 0.074\alpha\beta + 0.050\alpha\eta - 0.012\beta^2 - 1.770\alpha^3 - 0.060\alpha^2\beta + 0.815\alpha^2\eta + 0.306\alpha\beta^2 - 0.028\alpha\beta\eta + 0.181\alpha\eta^2 + 0.297\beta^2\eta + 0.012\beta\eta^2 + 0.645\eta^3; \quad (22)$$

$$C_{\mathrm{l}\eta}^{pre} = -0.012\beta + 0.057\alpha\beta - 0.286\alpha^2\beta + 0.047\beta^3; \quad (23)$$

$$C_{\mathrm{m}\eta}^{pre} = 0.028 + 0.032\alpha - 0.054\beta - 1.851\eta - 0.172\alpha^2 + 0.206\alpha\beta - 0.172\alpha\eta - 0.199\beta^2 + 0.026\beta\eta - 0.173\eta^2 - 0.300\alpha^3 - 0.068\alpha^2\beta + 5.136\alpha^2\eta + 0.816\alpha\beta^2 - 0.023\alpha\beta\eta + 0.897\alpha\eta^2 + 0.693\beta^2\eta + 0.111\beta\eta^2 + 1.324\eta^3; \quad (24)$$

$$C_{\mathrm{n}\eta}^{pre} = -0.057\beta + 0.142\alpha\beta - 0.344\alpha^2\beta + 0.110\beta^3. \quad (25)$$

Polynomials in angle of attack, side-slip, and rudder deflections:

$$C_{\mathrm{X}\zeta}^{pre} = -0.011 + 0.026\alpha + 0.351\alpha^2 + 0.022\beta^2 + 0.091\beta\zeta - 2.731\alpha^3 - 0.042\alpha\beta^2 - 0.040\alpha\beta\zeta + 0.013\alpha\zeta^2 + 6.059\alpha^4 + 0.053\alpha^2\beta^2 + 0.245\alpha^2\beta\zeta - 0.211\alpha^2\zeta^2 - 0.011\beta^4 - 0.042\beta^2\zeta^2 - 0.130\beta\zeta^3 - 0.056\zeta^4; \quad (26)$$

$$C_{\mathrm{Y}\zeta}^{pre} = 0.187\beta + 0.308\zeta + 0.820\alpha\beta - 0.501\alpha\zeta + 0.160\alpha^2\beta + 1.251\alpha^2\zeta - 0.455\beta^3 - 0.224\beta^2\zeta + 0.168\beta\zeta^2 - 0.235\zeta^3 + 14.007\alpha^3\beta - 6.802\alpha^3\zeta - 2.757\alpha\beta^3 + 0.307\alpha\beta^2\zeta - 0.678\alpha\beta\zeta^2 + 1.093\alpha\zeta^3; \quad (27)$$

$$C_{\mathrm{Z}\zeta}^{pre} = -0.106\alpha + 0.016\beta + 0.254\alpha^2 - 0.074\alpha\beta + 0.055\beta^2 + 0.054\beta\zeta + 0.117\zeta^2 + 3.459\alpha^3 + 0.060\alpha^2\beta + 0.089\alpha\beta^2 - 0.367\alpha\beta\zeta + 0.063\alpha\zeta^2 - 6.397\alpha^4 - 0.984\alpha^2\beta^2 + 1.934\alpha^2\beta\zeta - 1.049\alpha^2\zeta^2 - 0.099\beta^4 - 0.140\beta^3\zeta - 0.047\beta^2\zeta^2 - 0.043\beta\zeta^3 - 0.122\zeta^4; \quad (28)$$

$$C_{\mathrm{l}\zeta}^{pre} = 0.024\zeta + 0.027\alpha\beta - 0.027\alpha\zeta + 0.150\alpha^2\beta + 0.068\alpha^2\zeta - 0.013\beta^3 - 0.026\beta^2\zeta - 0.017\zeta^3 + 0.600\alpha^3\beta - 0.251\alpha^3\zeta - 0.172\alpha\beta^3 + 0.046\alpha\beta^2\zeta + 0.020\alpha\beta\zeta^2 + 0.040\alpha\zeta^3; \quad (29)$$

$$C_{\mathrm{m}\zeta}^{pre} = -0.016 - 0.039\alpha + 0.054\beta + 0.335\alpha^2 - 0.206\alpha\beta + 0.032\beta^2 - 0.340\beta\zeta + 0.298\zeta^2 - 7.653\alpha^3 + 0.068\alpha^2\beta + 0.723\alpha\beta^2 - 0.354\alpha\beta\zeta + 0.021\alpha\zeta^2 + 23.281\alpha^4 - 1.037\alpha^2\beta^2 + 0.588\alpha^2\beta\zeta - 1.013\alpha^2\zeta^2 - 0.311\beta^4 - 0.304\beta^3\zeta + 0.213\beta^2\zeta^2 + 0.680\beta\zeta^3 - 0.247\zeta^4; \quad (30)$$

$$C_{\mathrm{n}\zeta}^{pre} = 0.040\beta - 0.145\zeta + 0.109\alpha\beta + 0.033\alpha\zeta + 0.606\alpha^2\beta - 0.264\alpha^2\zeta - 0.108\beta^3 + 0.114\beta^2\zeta - 0.052\beta\zeta^2 + 0.088\zeta^3 - 1.391\alpha^3\beta + 1.698\alpha^3\zeta - 0.387\alpha\beta^3 - 0.098\alpha\beta^2\zeta + 0.077\alpha\beta\zeta^2 - 0.090\alpha\zeta^3. \quad (31)$$

Polynomials in angle of attack and normalized body p-rate:

$$C_{\mathrm{Y}\hat{p}}^{pre} = -0.071\hat{p} - 0.138\alpha\hat{p} - 0.013\hat{p}^2 - 0.862\alpha^2\hat{p} + 0.383\alpha\hat{p}^2 + 14.142\hat{p}^3; \quad (32)$$

$$C_{\mathrm{l}\hat{p}}^{pre} = -0.266\hat{p} - 0.247\alpha\hat{p} - 0.015\hat{p}^2 + 3.159\alpha^2\hat{p} + 0.540\alpha\hat{p}^2 - 2.843\hat{p}^3; \quad (33)$$

$$C_{\mathrm{n}\hat{p}}^{pre} = -0.083\hat{p} - 0.044\alpha\hat{p} - 0.065\hat{p}^2 + 1.221\alpha^2\hat{p} + 0.073\alpha\hat{p}^2 + 5.811\hat{p}^3; \quad (34)$$

$C_{\mathrm{X}\hat{p}}^{pre}$, $C_{\mathrm{Z}\hat{p}}^{pre}$, $C_{\mathrm{m}\hat{p}}^{pre}$ are zero due to lack of GTM measurement data.

Polynomials in angle of attack and normalized body q-rate:

$$C_{\mathrm{X}\hat{q}}^{pre} = -0.030\alpha + 0.851\hat{q} + 0.076\alpha^2 + 12.434\alpha\hat{q} + 571.330\hat{q}^2 + 0.127\alpha^3 + 23.482\alpha^2\hat{q} + 2196.700\alpha\hat{q}^2 + 2529.200\hat{q}^3; \quad (35)$$

$$C_{\mathrm{Z}\hat{q}}^{pre} = -32.946\hat{q} - 0.203\alpha^2 - 32.151\alpha\hat{q} + 1401.800\hat{q}^2 - 0.282\alpha^3 - 80.578\alpha^2\hat{q} + 1239.200\alpha\hat{q}^2 + 2345.900\hat{q}^3; \quad (36)$$

$$C_{\mathrm{m}\hat{q}}^{pre} = -0.024 - 0.033\alpha - 42.378\hat{q} + 0.496\alpha^2 - 2.991\alpha\hat{q} + 765.020\hat{q}^2 + 0.909\alpha^3 + 29.585\alpha^2\hat{q} + 2214.800\alpha\hat{q}^2 + 2370.400\hat{q}^3; \quad (37)$$

$C_{\mathrm{Y}\hat{q}}^{pre}$, $C_{\mathrm{l}\hat{q}}^{pre}$, $C_{\mathrm{n}\hat{q}}^{pre}$ are zero due to lack of GTM measurement data.

Polynomials in angle of attack and normalized body r-rate:

$$C_{\mathrm{Y}\hat{r}}^{pre} = 0.748\hat{r} + 0.865\alpha\hat{r} + 0.881\hat{r}^2 + 1.625\alpha^2\hat{r} - 1.485\alpha\hat{r}^2 + 2.662\hat{r}^3; \tag{38}$$
$$C_{\mathrm{l}\hat{r}}^{pre} = 0.147\hat{r} + 0.755\alpha\hat{r} + 0.862\alpha^2\hat{r} + 0.125\alpha\hat{r}^2 - 5.986\hat{r}^3; \tag{39}$$
$$C_{\mathrm{n}\hat{r}}^{pre} = -0.337\hat{r} - 0.249\alpha\hat{r} - 0.471\hat{r}^2 - 0.912\alpha^2\hat{r} + 1.028\alpha\hat{r}^2 + 19.902\hat{r}^3; \tag{40}$$

$C_{\mathrm{X}\hat{r}}^{pre}$, $C_{\mathrm{Z}\hat{r}}^{pre}$, $C_{\mathrm{m}\hat{r}}^{pre}$ are zero due to lack of GTM measurement data.

B. Domain of high angle of attack

Polynomials in angle of attack:

$$C_{\mathrm{X}\alpha}^{post} = 0.019 - 0.130\alpha + 0.169\alpha^2 - 0.022\alpha^3; \tag{41}$$
$$C_{\mathrm{Z}\alpha}^{post} = -0.365 - 2.712\alpha + 1.647\alpha^2 - 0.369\alpha^3; \tag{42}$$
$$C_{\mathrm{m}\alpha}^{post} = 0.247 - 2.847\alpha + 2.748\alpha^2 - 1.105\alpha^3; \tag{43}$$

$C_{\mathrm{Y}\alpha}^{post}$, $C_{\mathrm{l}\alpha}^{post}$, $C_{\mathrm{n}\alpha}^{post}$ are zero by definition.

Polynomials in angle of attack and side-slip:

$$C_{\mathrm{X}\beta}^{post} = 0.101\alpha - 0.239\alpha^2 + 0.025\beta^2 + 0.199\alpha^3 + 0.160\alpha\beta^2 - 0.011\beta^3 - 0.055\alpha^4 - 0.148\alpha^2\beta^2 + 0.010\alpha\beta^3 - 0.043\beta^4; \tag{44}$$
$$C_{\mathrm{Y}\beta}^{post} = -0.393\beta - 2.181\alpha\beta + 1.680\alpha^2\beta - 0.591\beta^3 - 0.402\alpha^3\beta + 1.287\alpha\beta^3; \tag{45}$$
$$C_{\mathrm{Z}\beta}^{post} = -0.040 + 0.183\alpha - 0.268\alpha^2 + 0.039\alpha\beta - 0.293\beta^2 + 0.168\alpha^3 - 0.088\alpha^2\beta + 3.329\alpha\beta^2 - 0.039\alpha^4 + 0.056\alpha^3\beta - 2.148\alpha^2\beta^2 + 0.165\beta^4; \tag{46}$$
$$C_{\mathrm{l}\beta}^{post} = 0.074\beta - 0.339\alpha\beta + 0.111\alpha^2\beta - 0.142\beta^3 + 0.113\alpha\beta^3; \tag{47}$$
$$C_{\mathrm{m}\beta}^{post} = 0.357 - 2.954\alpha - 0.096\beta + 7.802\alpha^2 + 0.510\alpha\beta - 0.589\beta^2 - 7.667\alpha^3 - 0.736\alpha^2\beta - 1.740\alpha\beta^2 + 2.469\alpha^4 + 0.296\alpha^3\beta + 1.931\alpha^2\beta^2 - 0.040\alpha\beta^3 + 1.164\beta^4; \tag{48}$$
$$C_{\mathrm{n}\beta}^{post} = 0.197\beta - 0.186\alpha\beta - 0.286\alpha^2\beta - 0.238\beta^3 + 0.160\alpha^3\beta + 0.567\alpha\beta^3. \tag{49}$$

Polynomials in angle of attack, side-slip, and aileron deflections:

$$C_{\mathrm{X}\xi}^{post} = 0.079 - 0.611\alpha + 1.476\alpha^2 + 0.135\beta^2 - 0.025\beta\xi - 0.083\xi^2 - 1.489\alpha^3 - 0.253\alpha\beta^2 - 0.013\alpha\beta\xi + 0.897\alpha\xi^2 + 0.531\alpha^4 + 0.194\alpha^2\beta^2 + 0.026\alpha^2\beta\xi - 0.717\alpha^2\xi^2 - 0.111\beta^4 - 0.048\beta^2\xi^2 - 0.012\beta\xi^3 - 0.499\xi^4; \tag{50}$$
$$C_{\mathrm{Y}\xi}^{post} = -1.486\beta - 0.059\xi + 4.744\alpha\beta + 0.127\alpha\xi - 4.548\alpha^2\beta - 0.097\alpha^2\xi + 2.041\beta^3 + 2.727\beta\xi^2 + 0.121\xi^3 + 2.605\alpha^3\beta + 0.031\alpha^3\xi - 4.143\alpha\beta^3 + 0.013\alpha\beta^2\xi - 5.429\alpha\beta\xi^2 - 0.185\alpha\xi^3; \tag{51}$$
$$C_{\mathrm{Z}\xi}^{post} = 0.038 - 0.332\alpha + 0.762\alpha^2 - 0.043\beta^2 - 0.277\xi^2 - 0.480\alpha^3 - 0.326\alpha\beta^2 - 0.034\alpha\beta\xi + 0.676\alpha\xi^2 + 0.054\alpha^4 + 0.262\alpha^2\beta^2 - 0.033\alpha^2\beta\xi - 0.206\alpha^2\xi^2 + 0.109\beta^4 + 0.027\beta^3\xi - 0.058\beta^2\xi^2 + 0.197\beta\xi^3 + 0.118\xi^4; \tag{52}$$
$$C_{\mathrm{l}\xi}^{post} = -0.107\beta - 0.035\xi + 0.419\alpha\beta - 0.042\alpha\xi - 0.500\alpha^2\beta + 0.084\alpha^2\xi + 0.082\beta^3 + 0.035\beta^2\xi + 0.194\beta\xi^2 + 0.030\xi^3 + 0.253\alpha^3\beta - 0.032\alpha^3\xi - 0.188\alpha\beta^3 - 0.013\alpha\beta^2\xi - 0.365\alpha\beta\xi^2; \tag{53}$$
$$C_{\mathrm{m}\xi}^{post} = -0.062 + 0.636\alpha - 1.619\alpha^2 - 0.520\beta^2 + 0.051\beta\xi - 0.061\xi^2 + 1.436\alpha^3 + 1.727\alpha\beta^2 - 0.202\alpha\beta\xi - 2.612\alpha\xi^2 - 0.434\alpha^4 - 1.297\alpha^2\beta^2 + 0.105\alpha^2\beta\xi + 1.727\alpha^2\xi^2 + 0.085\beta^4 + 0.082\beta^3\xi + 0.084\beta^2\xi^2 + 0.038\beta\xi^3 + 2.201\xi^4; \tag{54}$$
$$C_{\mathrm{n}\xi}^{post} = -0.170\beta + 0.013\xi + 0.458\alpha\beta - 0.034\alpha\xi - 0.227\alpha^2\beta + 0.037\alpha^2\xi + 0.328\beta^3 + 0.017\beta^2\xi + 0.397\beta\xi^2 - 0.045\xi^3 + 0.161\alpha^3\beta - 0.021\alpha^3\xi - 0.726\alpha\beta^3 - 0.017\alpha\beta^2\xi - 0.781\alpha\beta\xi^2 + 0.095\alpha\xi^3. \tag{55}$$

Polynomials in angle of attack, side-slip, and elevator deflections:

$$C_{\mathrm{X}\eta}^{post} = 0.042 - 0.276\alpha - 0.011\eta + 0.546\alpha^2 + 0.061\alpha\eta + 0.039\beta^2 - 0.063\eta^2 - 0.293\alpha^3 - 0.063\alpha^2\eta - 0.088\alpha\beta^2 - 0.032\alpha\eta^2 - 0.036\eta^3; \tag{56}$$

$$C_{\mathrm{Y}\eta}^{post} = -0.608\beta + 1.973\alpha\beta - 2.152\alpha^2\beta + 0.532\beta^3; \tag{57}$$

$$C_{\mathrm{Z}\eta}^{post} = -0.100 + 0.559\alpha - 0.594\eta - 0.926\alpha^2 + 0.029\alpha\beta + 0.585\alpha\eta + 0.110\beta^2 - 0.035\eta^2 + 0.452\alpha^3 - 0.024\alpha^2\beta - 0.230\alpha^2\eta - 0.127\alpha\beta^2 + 0.319\alpha\eta^2 + 0.297\beta^2\eta + 0.012\beta\eta^2 + 0.645\eta^3; \tag{58}$$

$$C_{\mathrm{l}\eta}^{post} = -0.049\beta + 0.150\alpha\beta - 0.152\alpha^2\beta + 0.047\beta^3; \tag{59}$$

$$C_{\mathrm{m}\eta}^{post} = 0.090 - 0.372\alpha - 1.880\eta + 0.436\alpha^2 + 1.470\alpha\eta + 0.019\beta\eta - 0.087\eta^2 - 0.139\alpha^3 - 0.341\alpha^2\eta + 0.076\alpha\beta^2 + 0.589\alpha\eta^2 + 0.693\beta^2\eta + 0.111\beta\eta^2 + 1.324\eta^3; \tag{60}$$

$$C_{\mathrm{n}\eta}^{post} = -0.070\beta + 0.142\alpha\beta - 0.187\alpha^2\beta + 0.110\beta^3. \tag{61}$$

Polynomials in angle of attack, side-slip, and rudder deflections:

$$C_{\mathrm{X}\zeta}^{post} = 0.016 - 0.140\alpha + 0.478\alpha^2 + 0.082\beta^2 + 0.203\beta\zeta - 0.122\zeta^2 - 0.669\alpha^3 - 0.316\alpha\beta^2 - 0.463\alpha\beta\zeta + 0.456\alpha\zeta^2 + 0.298\alpha^4 + 0.274\alpha^2\beta^2 + 0.325\alpha^2\beta\zeta - 0.307\alpha^2\zeta^2 - 0.011\beta^4 - 0.042\beta^2\zeta^2 - 0.130\beta\zeta^3 - 0.056\zeta^4; \tag{62}$$

$$C_{\mathrm{Y}\zeta}^{post} = 3.051\beta - 0.205\zeta - 11.790\alpha\beta + 1.702\alpha\zeta + 14.450\alpha^2\beta - 2.320\alpha^2\zeta - 2.302\beta^3 - 0.260\beta^2\zeta - 0.182\beta\zeta^2 + 0.353\zeta^3 - 6.130\alpha^3\beta + 1.104\alpha^3\zeta + 3.811\alpha\beta^3 + 0.433\alpha\beta^2\zeta + 0.566\alpha\beta\zeta^2 - 0.996\alpha\zeta^3; \tag{63}$$

$$C_{\mathrm{Z}\zeta}^{post} = 0.220 - 1.248\alpha + 2.437\alpha^2 - 0.029\alpha\beta - 0.083\beta^2 + 0.142\beta\zeta - 0.081\zeta^2 - 2.074\alpha^3 + 0.024\alpha^2\beta + 0.345\alpha\beta^2 - 0.163\alpha\beta\zeta + 0.591\alpha\zeta^2 + 0.654\alpha^4 - 0.149\alpha^2\beta^2 + 0.107\alpha^2\beta\zeta - 0.422\alpha^2\zeta^2 - 0.099\beta^4 - 0.140\beta^3\zeta - 0.047\beta^2\zeta^2 - 0.043\beta\zeta^3 - 0.122\zeta^4; \tag{64}$$

$$C_{\mathrm{l}\zeta}^{post} = 0.129\beta - 0.471\alpha\beta + 0.047\alpha\zeta + 0.546\alpha^2\beta - 0.070\alpha^2\zeta - 0.091\beta^3 - 0.019\beta^2\zeta + 0.012\beta\zeta^2 - 0.203\alpha^3\beta + 0.031\alpha^3\zeta + 0.106\alpha\beta^3 + 0.021\alpha\beta^2\zeta - 0.010\alpha\beta\zeta^2 - 0.033\alpha\zeta^3; \tag{65}$$

$$C_{\mathrm{m}\zeta}^{post} = 0.429 - 3.325\alpha + 8.084\alpha^2 + 0.245\beta^2 - 0.974\beta\zeta + 0.199\zeta^2 - 7.822\alpha^3 - 0.363\alpha\beta^2 + 2.617\alpha\beta\zeta + 0.150\alpha\zeta^2 + 2.615\alpha^4 + 0.142\alpha^2\beta^2 - 1.957\alpha^2\beta\zeta - 0.220\alpha^2\zeta^2 - 0.311\beta^4 - 0.304\beta^3\zeta + 0.213\beta^2\zeta^2 + 0.680\beta\zeta^3 - 0.247\zeta^4; \tag{66}$$

$$C_{\mathrm{n}\zeta}^{post} = 0.149\beta - 0.146\zeta - 0.223\alpha\beta + 0.055\alpha\zeta + 0.015\alpha^2\beta + 0.184\alpha^2\zeta - 0.377\beta^3 + 0.176\beta^2\zeta - 0.036\beta\zeta^2 + 0.067\zeta^3 - 0.013\alpha^3\beta - 0.106\alpha^3\zeta + 0.568\alpha\beta^3 - 0.318\alpha\beta^2\zeta + 0.020\alpha\beta\zeta^2 - 0.015\alpha\zeta^3. \tag{67}$$

Polynomials in angle of attack and normalized body p-rate:

$$C_{\mathrm{Y}\hat{p}}^{post} = -0.058\hat{p} - 0.590\alpha\hat{p} + 0.970\hat{p}^2 + 0.582\alpha^2\hat{p} - 3.112\alpha\hat{p}^2 + 14.142\hat{p}^3; \tag{68}$$

$$C_{\mathrm{l}\hat{p}}^{post} = 0.044\hat{p} - 0.589\alpha\hat{p} + 0.133\hat{p}^2 + 0.464\alpha^2\hat{p} + 0.013\alpha\hat{p}^2 - 2.843\hat{p}^3; \tag{69}$$

$$C_{\mathrm{n}\hat{p}}^{post} = 0.113\hat{p} - 0.504\alpha\hat{p} - 0.240\hat{p}^2 + 0.369\alpha^2\hat{p} + 0.695\alpha\hat{p}^2 + 5.811\hat{p}^3; \tag{70}$$

$C_{\mathrm{X}\hat{p}}^{post}$, $C_{\mathrm{Z}\hat{p}}^{post}$, $C_{\mathrm{m}\hat{p}}^{post}$ are zero due to lack of GTM measurement data.

Polynomials in angle of attack and normalized body q-rate:

$$C_{\mathrm{X}\hat{q}}^{post} = 0.034 - 0.209\alpha + 22.961\hat{q} + 0.246\alpha^2 - 76.364\alpha\hat{q} + 821.140\hat{q}^2 - 0.097\alpha^3 + 59.647\alpha^2\hat{q} + 1308.300\alpha\hat{q}^2 + 2529.200\hat{q}^3; \tag{71}$$

$$C_{\mathrm{Z}\hat{q}}^{post} = 0.015 - 0.385\alpha - 91.233\hat{q} + 0.978\alpha^2 + 199.050\alpha\hat{q} + 1312.200\hat{q}^2 - 0.677\alpha^3 - 165.610\alpha^2\hat{q} + 1557.900\alpha\hat{q}^2 + 2345.900\hat{q}^3; \tag{72}$$

$$C_{\mathrm{m}\hat{q}}^{post} = 0.128 - 0.470\alpha - 11.370\hat{q} + 0.432\alpha^2 - 145.970\alpha\hat{q} + 1029.300\hat{q}^2 - 0.173\alpha^3 + 145.910\alpha^2\hat{q} + 1275.000\alpha\hat{q}^2 + 2370.400\hat{q}^3; \tag{73}$$

$C_{\mathrm{Y}\hat{q}}^{post}$, $C_{\mathrm{l}\hat{q}}^{post}$, $C_{\mathrm{n}\hat{q}}^{post}$ are zero due to lack of GTM measurement data.

Polynomials in angle of attack and normalized body r-rate:

$$C_{\mathrm{Y}\hat{r}}^{post} = 3.868\hat{r} - 11.818\alpha\hat{r} + 1.677\hat{r}^2 + 7.267\alpha^2\hat{r} - 4.315\alpha\hat{r}^2 + 2.662\hat{r}^3; \tag{74}$$

$$C_{\mathrm{l}\hat{r}}^{post} = -0.154\hat{r} + 3.156\alpha\hat{r} + 0.019\hat{r}^2 - 3.867\alpha^2\hat{r} + 0.042\alpha\hat{r}^2 - 5.986\hat{r}^3; \tag{75}$$

$$C_{\mathrm{n}\hat{r}}^{post} = -0.639\hat{r} + 0.714\alpha\hat{r} - 0.239\hat{r}^2 - 0.510\alpha^2\hat{r} + 0.203\alpha\hat{r}^2 + 19.902\hat{r}^3; \tag{76}$$

$C_{\mathrm{X}\hat{r}}^{post}$, $C_{\mathrm{Z}\hat{r}}^{post}$, $C_{\mathrm{m}\hat{r}}^{post}$ are zero due to lack of GTM measurement data.

II. Equations of Motion

We have the aerodynamic forces and moments in body axis system

$$\begin{bmatrix} X^{\mathrm{A}} \\ Y^{\mathrm{A}} \\ Z^{\mathrm{A}} \end{bmatrix}_{\mathrm{f}} = \frac{1}{2}\varrho S V_{\mathrm{A}}^2 \begin{bmatrix} C_{\mathrm{X}} \\ C_{\mathrm{Y}} \\ C_{\mathrm{Z}} \end{bmatrix}; \qquad \begin{bmatrix} L^{\mathrm{A}} \\ M^{\mathrm{A}} \\ N^{\mathrm{A}} \end{bmatrix}_{\mathrm{f}} = \frac{1}{2}\varrho S V_{\mathrm{A}}^2 \begin{bmatrix} b\, C_{\mathrm{l}} \\ c_{\mathrm{a}} C_{\mathrm{m}} \\ b\, C_{\mathrm{n}} \end{bmatrix} + \begin{bmatrix} X^{\mathrm{A}} \\ Y^{\mathrm{A}} \\ Z^{\mathrm{A}} \end{bmatrix}_{\mathrm{f}} \times \left(\mathbf{x}_{\mathrm{cg}} - \mathbf{x}_{\mathrm{cg}}^{\mathrm{ref}}\right); \tag{77}$$

and the weight force by rotation into the body axis system

$$X_{\mathrm{g}}^{\mathrm{G}} = -g\sin\Theta; \tag{78}$$

$$Y_{\mathrm{g}}^{\mathrm{G}} = -g\sin\Phi\cos\Theta; \tag{79}$$

$$Z_{\mathrm{g}}^{\mathrm{G}} = -g\cos\Phi\cos\Theta; \tag{80}$$

The resulting forces lead to changes in the velocity vector, in body axis system, by

$$\dot{\mathbf{V}}_{\mathrm{A\,f}} = \frac{1}{m}\begin{bmatrix} X^{\mathrm{A}} + X^{\mathrm{G}} + X^{\mathrm{F}} \\ Y^{\mathrm{A}} + Y^{\mathrm{G}} \\ Z^{\mathrm{A}} + Z^{\mathrm{G}} \end{bmatrix}_{\mathrm{f}} + \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times \mathbf{V}_{\mathrm{A\,f}}. \tag{81}$$

with the thrust $X_{\mathrm{f}}^{\mathrm{F}} = F$ (engines aligned with the x_{f}-axis). For a symmetric plane ($I_{xy} = I_{yz} = 0$), the resulting moments in body axis are given as

$$L_{\mathrm{f}} = L_{\mathrm{f}}^{\mathrm{A}} - q\,r\,(I_z - I_y) + p\,q\,I_{zx}; \tag{82}$$

$$M_{\mathrm{f}} = M_{\mathrm{f}}^{\mathrm{A}} + M_{\mathrm{f}}^{\mathrm{F}} - p\,r\,(I_x - I_z) - \left(p^2 - r^2\right) I_{zx}; \tag{83}$$

$$N_{\mathrm{f}} = N_{\mathrm{f}}^{\mathrm{A}} - p\,q\,(I_y - I_x) - q\,r\,I_{zx}; \tag{84}$$

with $M_{\mathrm{f}}^{\mathrm{F}} = l_{\mathrm{t}} F$ (engines symmetric to the x_{f}-y_{f}-plane and shifted vertically from the origin by l_{t}) and the inertias I_x, I_y, I_z, I_{zx}. The changes of angular body rates are then given as

$$\dot{p} = \frac{1}{I_x I_z - I_{zx}^2}\left(I_z L_{\mathrm{f}} + I_{zx} N_{\mathrm{f}}\right); \tag{85}$$

$$\dot{q} = \frac{1}{I_y} M_{\mathrm{f}}; \tag{86}$$

$$\dot{r} = \frac{1}{I_x I_z - I_{zx}^2}\left(I_{zx} L_{\mathrm{f}} + I_x N_{\mathrm{f}}\right). \tag{87}$$

Here, the normalized body rates have been used with

$$\begin{bmatrix} \hat{p} \\ \hat{q} \\ \hat{r} \end{bmatrix} = \frac{1}{2V_{\mathrm{A}}}\begin{bmatrix} b\,p \\ c_{\mathrm{a}}\,q \\ b\,r \end{bmatrix} \iff \begin{bmatrix} p \\ q \\ r \end{bmatrix} = 2V_{\mathrm{A}}\begin{bmatrix} b^{-1}\,\hat{p} \\ c_{\mathrm{a}}^{-1}\,\hat{q} \\ b^{-1}\,\hat{r} \end{bmatrix} \tag{88}$$

and

$$\begin{bmatrix} \dot{\hat{p}} \\ \dot{\hat{q}} \\ \dot{\hat{r}} \end{bmatrix} = \frac{1}{V_\mathrm{A}} \left(\frac{1}{2} \begin{bmatrix} b\,\dot{p} \\ c_\mathrm{a}\,\dot{q} \\ b\,\dot{r} \end{bmatrix} - \dot{V}_\mathrm{A} \begin{bmatrix} \hat{p} \\ \hat{q} \\ \hat{r} \end{bmatrix} \right). \tag{89}$$

The change of attitude is finally obtained by rotation into normal earth-fixed axis system:

$$\dot{\Phi} = p + q \sin\Phi \tan\Theta + r \cos\Phi \tan\Theta; \tag{90}$$

$$\dot{\Theta} = q \cos\Phi - r \sin\Phi; \tag{91}$$

$$\dot{\Psi} = q \sin\Phi \cos^{-1}\Theta + r \cos\Phi \cos^{-1}\Theta. \tag{92}$$

III. Longitudinal Model

The longitudinal model is restricted to the x_a-z_a-plane, assuming $\beta = \mu_\mathrm{A} = \chi_\mathrm{A} = 0$.

A. *Aerodynamic Coefficients*

The longitudinal aerodynamic coefficients, $C_\mathrm{L}, C_\mathrm{D}, C_\mathrm{m}$, are obtained as

$$C_\mathrm{L}(\alpha,\eta) = C_{\mathrm{L}\alpha}(\alpha) + C_{\mathrm{L}\eta}(\alpha,\eta) \quad \text{with}$$

$$C_{\mathrm{L}\alpha}(\alpha) = \begin{cases} 0.017 + 5.234\alpha + 1.985\alpha^2 - 30.060\alpha^3 & \text{if } \alpha \leq 16.634^\circ \\ 0.279 + 3.251\alpha - 3.235\alpha^2 + 0.708\alpha^3 & \text{else} \end{cases}, \tag{93}$$

$$\begin{aligned} C_{\mathrm{L}\eta}(\alpha,\eta) = & -0.000 + 0.003\alpha + 0.521\eta - 0.072\alpha^2 - 0.416\alpha\eta \\ & + 0.089\eta^2 + 0.051\alpha^3 + 0.039\alpha^2\eta - 0.293\alpha\eta^2 - 0.479\eta^3; \end{aligned} \tag{94}$$

$$C_\mathrm{D}(\alpha,\eta) = C_{\mathrm{D}\alpha}(\alpha) + C_{\mathrm{D}\eta}(\alpha,\eta) \quad \text{with}$$

$$C_{\mathrm{D}\alpha}(\alpha) = \begin{cases} 0.029 - 0.110\alpha + 2.364\alpha^2 + 3.948\alpha^3 & \text{if } \alpha \leq 16.634^\circ \\ -0.170 + 1.427\alpha + 0.719\alpha^2 - 0.486\alpha^3 & \text{else} \end{cases}, \tag{95}$$

$$\begin{aligned} C_{\mathrm{D}\eta}(\alpha,\eta) = & +0.008 - 0.012\alpha + 0.112\eta + 0.040\alpha^2 + 0.183\alpha\eta \\ & - 0.069\eta^2 - 0.053\alpha^3 - 0.043\alpha^2\eta - 0.070\alpha\eta^2 - 0.628\eta^3; \end{aligned} \tag{96}$$

$$C_\mathrm{m}(\alpha,\eta) = C_{\mathrm{m}\alpha}(\alpha) + C_{\mathrm{m}\eta}(\alpha,\eta) \quad \text{with}$$

$$C_{\mathrm{m}\alpha}(\alpha) = \begin{cases} 0.117 - 1.475\alpha + 8.475\alpha^2 - 32.729\alpha^3 & \text{if } \alpha \leq 16.634^\circ \\ 0.144 - 2.456\alpha + 2.304\alpha^2 - 0.950\alpha^3 & \text{else} \end{cases}, \tag{97}$$

$$\begin{aligned} C_{\mathrm{m}\eta}(\alpha,\eta) = & +0.014 + 0.165\alpha - 1.968\eta - 0.410\alpha^2 + 1.365\alpha\eta \\ & - 0.415\eta^2 + 0.186\alpha^3 - 0.144\alpha^2\eta + 0.948\alpha\eta^2 + 1.356\eta^3. \end{aligned} \tag{98}$$

B. *Equations of motion*

The longitudinal equations of motion are given as

$$\dot{V}_\mathrm{A} = \frac{1}{m}\left(F\cos\alpha - \frac{1}{2}\rho S V_\mathrm{A}^2 C_\mathrm{D}(\alpha,\eta,q) - mg\sin\gamma_\mathrm{A}\right), \tag{99}$$

$$\dot{\gamma}_\mathrm{A} = \frac{1}{mV_\mathrm{A}}\left(F\sin\alpha + \frac{1}{2}\rho S V_\mathrm{A}^2 C_\mathrm{L}(\alpha,\eta,q) - mg\cos\gamma_\mathrm{A}\right), \tag{100}$$

$$\dot{q} = \frac{1}{I_y}\left(l_\mathrm{t}F + \frac{1}{2}\rho S c_\mathrm{a} V_\mathrm{A}^2 C_\mathrm{m}(\alpha,\eta,q) - \frac{1}{2}\rho S V_\mathrm{A}^2 C_\mathrm{Z}(\alpha,\eta,q)\left(x_\mathrm{cg}^\mathrm{ref} - x_\mathrm{cg}\right) + \frac{1}{2}\rho S V_\mathrm{A}^2 C_\mathrm{X}(\alpha,\eta,q)\left(z_\mathrm{cg}^\mathrm{ref} - z_\mathrm{cg}\right)\right); \tag{101}$$

with

$$\Theta = \alpha + \gamma. \tag{102}$$

MATLAB Source Code

The source code for the aerodynamic coefficients and the equations of motion can be found at:

https://github.com/pwpfit/GTMpw

Appendix

A. Spline-based Longitudinal Coefficients

The purpose of this model is to provide an application example of a spline-based longitudinal aircraft model. It has neither been designed nor evaluated for engineering purposes. For the longitudinal equations of motion, refer to Section III-B and note

$$C_{\mathrm{D}}(\alpha,\ldots) = -C_{\mathrm{X}}(\alpha,\ldots)\cos\alpha - C_{\mathrm{Z}}(\alpha,\ldots)\sin\alpha \tag{103}$$

$$C_{\mathrm{L}}(\alpha,\ldots) = \quad C_{\mathrm{X}}(\alpha,\ldots)\sin\alpha - C_{\mathrm{Z}}(\alpha,\ldots)\cos\alpha \tag{104}$$

The longitudinal aerodynamic coefficients are obtained as

$$C_{\odot}(\alpha,\eta,\hat{q}) = C_{\odot\alpha}(\alpha) + C_{\odot\eta}(\alpha,\eta) + C_{\odot\hat{q}}(\alpha,\hat{q})\,, \tag{105}$$

where $C_{\odot} \in \{C_{\mathrm{X}}, C_{\mathrm{Z}}, C_{\mathrm{m}}\}$, $\hat{q} = c_{\mathrm{a}} q / (2V_{\mathrm{A}})$, and

$$C_{\odot\alpha}(\alpha,\eta) = \begin{cases} C_{\odot\alpha}^{(1)} & \text{if } \alpha \in (-\infty;\alpha_1), \\ C_{\odot\alpha}^{(2)} & \text{if } \alpha \in [\alpha_1;\alpha_2), \\ C_{\odot\alpha}^{(3)} & \text{if } \alpha \in [\alpha_2;\alpha_3), \\ C_{\odot\alpha}^{(4)} & \text{if } \alpha \in [\alpha_3;\alpha_3), \\ C_{\odot\alpha}^{(5)} & \text{if } \alpha \in [\alpha_4;\infty); \end{cases} \tag{106}$$

$$C_{\odot\eta}(\alpha,\eta) = \begin{cases} C_{\odot\eta}^{(1)} & \text{if } \alpha \in (-\infty;\alpha_1), \\ C_{\odot\eta}^{(2)} & \text{if } \alpha \in [\alpha_1;\alpha_2), \\ C_{\odot\eta}^{(3)} & \text{if } \alpha \in [\alpha_2;\alpha_3), \\ C_{\odot\eta}^{(4)} & \text{if } \alpha \in [\alpha_3;\alpha_3), \\ C_{\odot\eta}^{(5)} & \text{if } \alpha \in [\alpha_4;\infty); \end{cases} \tag{107}$$

$$C_{\odot\hat{q}}(\alpha,\hat{q}) = \begin{cases} C_{\odot\hat{q}}^{(1)} & \text{if } \hat{q} \in (-\infty;\hat{q}_1), \\ C_{\odot\hat{q}}^{(2)} & \text{if } \hat{q} \in [\hat{q}_1;\hat{q}_2), \\ C_{\odot\hat{q}}^{(3)} & \text{if } \hat{q} \in [\hat{q}_2;\hat{q}_3), \\ C_{\odot\hat{q}}^{(4)} & \text{if } \hat{q} \in [\hat{q}_3;\hat{q}_3), \\ C_{\odot\hat{q}}^{(5)} & \text{if } \hat{q} \in [\hat{q}_4;\infty); \end{cases} \tag{108}$$

with

$$\alpha_1 = 5°, \qquad \alpha_2 = 15°, \qquad \alpha_3 = 25°, \qquad \alpha_4 = 45°; \tag{109}$$

$$\hat{q}_1 = -0.200°, \qquad \hat{q}_2 = -0.075°, \qquad \hat{q}_3 = 0.075°, \qquad \hat{q}_4 = 0.200°; \tag{110}$$

and

$$C_{\mathrm{X}\alpha}^{(1)}(\alpha) = -0.025 - 0.002\alpha + 0.827\alpha^2; \tag{111}$$

$$C_{\mathrm{X}\alpha}^{(2)}(\alpha) = -0.204 + 2.778\alpha - 7.493\alpha^2; \tag{112}$$

$$C_{\mathrm{X}\alpha}^{(3)}(\alpha) = 0.217 - 1.218\alpha + 1.628\alpha^2; \tag{113}$$

$$C_{\mathrm{X}\alpha}^{(4)}(\alpha) = 0.022 - 0.110\alpha + 0.114\alpha^2; \tag{114}$$

$$C_{\mathrm{X}\alpha}^{(5)}(\alpha) = -0.052 + 0.024\alpha + 0.063\alpha^2; \tag{115}$$

$$C_{\mathrm{Z}\alpha}^{(1)}(\alpha) = -0.028 - 4.949\alpha + 0.837\alpha^2; \tag{116}$$

$$C_{\mathrm{Z}\alpha}^{(2)}(\alpha) = 0.155 - 8.239\alpha + 14.537\alpha^2; \tag{117}$$

$$C_{\mathrm{Z}\alpha}^{(3)}(\alpha) = -0.815 - 0.298\alpha - 1.648\alpha^2; \tag{118}$$

$$C_{\mathrm{Z}\alpha}^{(4)}(\alpha) = -0.467 - 2.158\alpha + 0.786\alpha^2; \tag{119}$$

$$C_{\mathrm{Z}\alpha}^{(5)}(\alpha) = -1.052 - 0.996\alpha + 0.254\alpha^2; \tag{120}$$

$$C^{(1)}_{\mathrm{m}\alpha}(\alpha) = 0.157 - 1.724\alpha + 1.806\alpha^2 + 11.969\alpha^3; \tag{121}$$
$$C^{(2)}_{\mathrm{m}\alpha}(\alpha) = 0.565 - 10.419\alpha + 59.277\alpha^2 - 118.320\alpha^3; \tag{122}$$
$$C^{(3)}_{\mathrm{m}\alpha}(\alpha) = 7.543 - 64.304\alpha + 174.300\alpha^2 - 160.370\alpha^3; \tag{123}$$
$$C^{(4)}_{\mathrm{m}\alpha}(\alpha) = -0.866 + 1.176\alpha - 1.875\alpha^2 + 0.681\alpha^3; \tag{124}$$
$$C^{(5)}_{\mathrm{m}\alpha}(\alpha) = -1.053 + 1.752\alpha - 2.210\alpha^2 + 0.561\alpha^3; \tag{125}$$

$$C^{(1)}_{\mathrm{X}\eta}(\alpha,\eta) = -0.010\eta + 0.195\alpha\eta - 0.082\eta^2 + 0.628\alpha^2\eta + 0.213\alpha\eta^2 - 0.036\eta^3; \tag{126}$$
$$C^{(2)}_{\mathrm{X}\eta}(\alpha,\eta) = 0.011\eta - 0.009\alpha\eta - 0.060\eta^2 + 0.165\alpha^2\eta - 0.043\alpha\eta^2 - 0.036\eta^3; \tag{127}$$
$$C^{(3)}_{\mathrm{X}\eta}(\alpha,\eta) = 0.060\eta - 0.175\alpha\eta - 0.043\eta^2 + 0.087\alpha^2\eta - 0.106\alpha\eta^2 - 0.036\eta^3; \tag{128}$$
$$C^{(4)}_{\mathrm{X}\eta}(\alpha,\eta) = 0.027\eta - 0.079\alpha\eta - 0.056\eta^2 + 0.042\alpha^2\eta - 0.076\alpha\eta^2 - 0.036\eta^3; \tag{129}$$
$$C^{(5)}_{\mathrm{X}\eta}(\alpha,\eta) = -0.281\eta + 0.586\alpha\eta - 0.133\eta^2 - 0.305\alpha^2\eta + 0.021\alpha\eta^2 - 0.036\eta^3; \tag{130}$$

$$C^{(1)}_{\mathrm{Z}\eta}(\alpha,\eta) = -0.595\eta + 0.058\alpha\eta + 0.039\eta^2 + 0.045\alpha^2\eta + 0.023\alpha\eta^2 + 1.142\eta^3; \tag{131}$$
$$C^{(2)}_{\mathrm{Z}\eta}(\alpha,\eta) = -0.626\eta + 0.461\alpha\eta + 0.012\eta^2 - 0.452\alpha^2\eta + 0.332\alpha\eta^2 + 1.142\eta^3; \tag{132}$$
$$C^{(3)}_{\mathrm{Z}\eta}(\alpha,\eta) = -0.631\eta + 0.299\alpha\eta + 0.051\eta^2 + 0.224\alpha^2\eta + 0.183\alpha\eta^2 + 1.142\eta^3; \tag{133}$$
$$C^{(4)}_{\mathrm{Z}\eta}(\alpha,\eta) = -0.666\eta + 0.516\alpha\eta - 0.337\eta^2 - 0.084\alpha^2\eta + 1.074\alpha\eta^2 + 1.142\eta^3; \tag{134}$$
$$C^{(5)}_{\mathrm{Z}\eta}(\alpha,\eta) = -1.116\eta + 1.625\alpha\eta + 0.312\eta^2 - 0.768\alpha^2\eta + 0.247\alpha\eta^2 + 1.142\eta^3; \tag{135}$$

$$C^{(1)}_{\mathrm{m}\eta}(\alpha,\eta) = -1.879\eta - 0.047\alpha\eta - 0.227\eta^2 - 0.845\alpha^2\eta + 0.378\alpha\eta^2 + 1.409\eta^3; \tag{136}$$
$$C^{(2)}_{\mathrm{m}\eta}(\alpha,\eta) = -2.006\eta + 1.410\alpha\eta - 0.260\eta^2 - 0.807\alpha^2\eta + 0.756\alpha\eta^2 + 1.409\eta^3; \tag{137}$$
$$C^{(3)}_{\mathrm{m}\eta}(\alpha,\eta) = -1.916\eta - 0.071\alpha\eta - 0.704\eta^2 + 3.530\alpha^2\eta + 2.449\alpha\eta^2 + 1.409\eta^3; \tag{138}$$
$$C^{(4)}_{\mathrm{m}\eta}(\alpha,\eta) = -1.612\eta + 0.765\alpha\eta + 0.802\eta^2 + 0.017\alpha^2\eta - 1.002\alpha\eta^2 + 1.409\eta^3; \tag{139}$$
$$C^{(5)}_{\mathrm{m}\eta}(\alpha,\eta) = -1.555\eta + 0.535\alpha\eta - 0.777\eta^2 + 0.219\alpha^2\eta + 1.009\alpha\eta^2 + 1.409\eta^3; \tag{140}$$

as well as

$$C^{(1)}_{\mathrm{X}\hat{q}}(\alpha,\hat{q}) = -0.003 - 1.776\hat{q} - 0.010\alpha - 276.480\hat{q}^{(2)} - 2.319\hat{q}\alpha - 0.005\alpha^2; \tag{141}$$
$$C^{(2)}_{\mathrm{X}\hat{q}}(\alpha,\hat{q}) = 0.001 + 0.689\hat{q} + 0.004\alpha + 120.080\hat{q}^{(2)} + 1.492\hat{q}\alpha - 0.005\alpha^2; \tag{142}$$
$$C^{(3)}_{\mathrm{X}\hat{q}}(\alpha,\hat{q}) = 0.001 + 1.338\hat{q} + 0.003\alpha + 232.350\hat{q}^{(2)} + 0.558\hat{q}\alpha - 0.005\alpha^2; \tag{143}$$
$$C^{(4)}_{\mathrm{X}\hat{q}}(\alpha,\hat{q}) = 0.001 + 1.897\hat{q} + 0.004\alpha + 144.900\hat{q}^{(2)} - 0.699\hat{q}\alpha - 0.005\alpha^2; \tag{144}$$
$$C^{(5)}_{\mathrm{X}\hat{q}}(\alpha,\hat{q}) = 0.001 + 2.271\hat{q} - 0.009\alpha - 7.040\hat{q}^{(2)} + 2.981\hat{q}\alpha - 0.005\alpha^2; \tag{145}$$

$$C^{(1)}_{\mathrm{Z}\hat{q}}(\alpha,\hat{q}) = -0.008 - 25.814\hat{q} + 0.138\alpha + 2806.400\hat{q}^{(2)} + 22.164\hat{q}\alpha + 0.004\alpha^2; \tag{146}$$
$$C^{(2)}_{\mathrm{Z}\hat{q}}(\alpha,\hat{q}) = -0.010 - 32.148\hat{q} + 0.019\alpha + 1120.800\hat{q}^{(2)} - 11.921\hat{q}\alpha + 0.004\alpha^2; \tag{147}$$
$$C^{(3)}_{\mathrm{Z}\hat{q}}(\alpha,\hat{q}) = -0.014 - 36.128\hat{q} + 0.029\alpha + 507.060\hat{q}^{(2)} - 3.943\hat{q}\alpha + 0.004\alpha^2; \tag{148}$$
$$C^{(4)}_{\mathrm{Z}\hat{q}}(\alpha,\hat{q}) = -0.010 - 40.109\hat{q} + 0.019\alpha + 1120.800\hat{q}^{(2)} + 4.036\hat{q}\alpha + 0.004\alpha^2; \tag{149}$$
$$C^{(5)}_{\mathrm{Z}\hat{q}}(\alpha,\hat{q}) = -0.008 - 46.443\hat{q} + 0.138\alpha + 2806.400\hat{q}^{(2)} - 30.049\hat{q}\alpha + 0.004\alpha^2; \tag{150}$$

$$C^{(1)}_{\mathrm{m}\hat{q}}(\alpha,\hat{q}) = 0.008 - 41.631\hat{q} - 0.013\alpha + 300.350\hat{q}^{(2)} - 10.716\hat{q}\alpha - 0.081\alpha^2; \tag{151}$$
$$C^{(2)}_{\mathrm{m}\hat{q}}(\alpha,\hat{q}) = 0.009 - 40.268\hat{q} + 0.009\alpha + 632.510\hat{q}^{(2)} - 4.284\hat{q}\alpha - 0.081\alpha^2; \tag{152}$$
$$C^{(3)}_{\mathrm{m}\hat{q}}(\alpha,\hat{q}) = 0.009 - 40.445\hat{q} + 0.012\alpha + 674.850\hat{q}^{(2)} - 2.169\hat{q}\alpha - 0.081\alpha^2; \tag{153}$$
$$C^{(4)}_{\mathrm{m}\hat{q}}(\alpha,\hat{q}) = 0.009 - 39.916\hat{q} + 0.009\alpha + 381.400\hat{q}^{(2)} - 0.193\hat{q}\alpha - 0.081\alpha^2; \tag{154}$$
$$C^{(5)}_{\mathrm{m}\hat{q}}(\alpha,\hat{q}) = -0.001 - 35.258\hat{q} + 0.008\alpha - 208.640\hat{q}^{(2)} + 0.192\hat{q}\alpha - 0.081\alpha^2; \tag{155}$$

Nomenclature

α	=	Angle of attack (rad);
α_0	=	Low-angle of attack boundary (°);
β	=	Side-slip angle (rad);
γ_{A}	=	Air-path inclination angle (rad);
ζ	=	Rudder deflection (rad), negative if leading to positive yaw moment;
η	=	Elevator deflection (rad), negative if leading to positive pitch moment;
μ_{A}	=	Air-path bank angle (rad);
ξ	=	Aileron deflection (rad), negative if leading to positive roll moment;
ϱ	=	Air density ($\varrho = 1.200\,\mathrm{kg\,m^{-3}}$);
χ_A	=	Air-path azimuth angle (rad);
Θ	=	Pitch angle (rad);
Φ	=	Bank angle (rad);
Ψ	=	Azimuth angle (rad);
b	=	Reference aerodynamic span ($b = 2.088\,\mathrm{m}$);
c_{A}	=	Aerodynamic mean chord ($c_{\mathrm{A}} = 0.280\,\mathrm{m}$);
g	=	Standard gravitational acceleration ($g \approx 9.810\,\mathrm{m\,s^{-2}}$);
l_{t}	=	Engine vertical displacement, positive along z_{f}-axis ($l_{\mathrm{t}} = 0.100\,\mathrm{m}$);
m	=	Aircraft mass ($m = 26.190\,\mathrm{kg}$);
p	=	Roll rate ($\mathrm{rad\,s^{-1}}$);
q	=	Pitch rate ($\mathrm{rad\,s^{-1}}$);
r	=	Yaw rate ($\mathrm{rad\,s^{-1}}$);
$x_{\mathrm{cg}}, z_{\mathrm{cg}}$	=	Longitudinal position center of gravity ($x_{\mathrm{cg}} = -1.450\,\mathrm{m}$, $z_{\mathrm{cg}} = -0.300\,\mathrm{m}$);
$x_{\mathrm{cg}}^{\mathrm{ref}}, z_{\mathrm{cg}}^{\mathrm{ref}}$	=	Longitudinal position reference center of gravity ($x_{\mathrm{cg}}^{\mathrm{ref}} = -1.460\,\mathrm{m}$, $z_{\mathrm{cg}}^{\mathrm{ref}} = -0.290\,\mathrm{m}$);
C_{l}	=	Aerodynamic coefficient moment body x_{f}-axis $(\cdot)$;
C_{m}	=	Aerodynamic coefficient moment body y_{f}-axis $(\cdot)$;
C_{n}	=	Aerodynamic coefficient moment body z_{f}-axis $(\cdot)$;
C_{D}	=	Aerodynamic drag coefficient, force negative air-path x_{a}-axis $(\cdot)$;
C_{L}	=	Aerodynamic lift coefficient, force negative air-path z_{a}-axis $(\cdot)$;
C_{X}	=	Aerodynamic coefficient force body x_{f}-axis $(\cdot)$;
C_{Y}	=	Aerodynamic coefficient force body y_{f}-axis $(\cdot)$;
C_{Z}	=	Aerodynamic coefficient force body z_{f}-axis $(\cdot)$;
D	=	Drag force, positive along negative air-path X_{a}-axis ($L = -X_{\mathrm{a}}^{\mathrm{A}}$, N);
F	=	Thrust force (N), positive along body x_{f}-axis;
L	=	Lift force, positive along negative air-path Z_{a}-axis ($L = -Z_{\mathrm{a}}^{\mathrm{A}}$, N);
L_{f}	=	Roll moment (N m), mathematically positive around x_{f}-axis;
M_{f}	=	Pitch moment (N m), mathematically positive around y_{f}-axis;
N_{f}	=	Yaw moment (N m), mathematically positive around z_{f}-axis;
S	=	Wing area ($S = 0.550\,\mathrm{m^2}$);
$V_{\mathrm{A}}, \mathbf{V}_{\mathrm{A}}$	=	Aircraft speed and velocity *relative to air* ($V_{\mathrm{A}} = \lVert\mathbf{V}_{\mathrm{A}}\rVert_2$, $\mathrm{m\,s^{-1}}$);
X^{A}	=	Resulting force along air-path x_{a}-axis (N);
$X_{\mathrm{f}}^{\mathrm{A}}$	=	Aerodynamic force along body x_{f}-axis (N);
$X_{\mathrm{f}}^{\mathrm{F}}$	=	Thrust force along body x_{f}-axis (N);
Y^{A}	=	Resulting force along air-path y_{a}-axis (N);
$Y_{\mathrm{f}}^{\mathrm{A}}$	=	Aerodynamic force along body y_{f}-axis (N);
Z^{A}	=	Resulting force along air-path z_{a}-axis (N);
$Z_{\mathrm{f}}^{\mathrm{A}}$	=	Aerodynamic force along body z_{f}-axis (N);
$(\cdot)^{post}$	=	Domain of high angle of attack;
$(\cdot)^{pre}$	=	Domain of low angle of attack;
$x_{\mathrm{a}}, y_{\mathrm{a}}, z_{\mathrm{a}}$	=	Air-path axis system;
$x_{\mathrm{f}}, y_{\mathrm{f}}, z_{\mathrm{f}}$	=	Body axis system;
$x_{\mathrm{g}}, y_{\mathrm{g}}, z_{\mathrm{g}}$	=	Normal earth-fixed axis system;