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Abstract

This paper concerns the study of vibroacoustic problems. By considering a displacement-pressure formulation, a non-symmetric eigenvalue problem is obtained. In order to solve it, three numerical schemes are compared: the classical ARPACK solver, an indicator method (initially proposed in B. Claude *et al.* Comptes Rendus Mécaniques, 2017, 345(2)) which has the property to be null at the eigenvalues, and an original method based on the analysis of Taylor series expansions near a singularity. Numerical results show all the evaluated numerical methods give accurate results but the indicator method requires the lowest computational times. Nevertheless, the original method based on the behavior of the perturbation method close to eigenvalues seems to be a very promising technique.

Key words: Linear vibroacoustic problem, eigenvalues, pertubation method

1. Introduction

Vibroacoustic problems are encountered in many industrial applications (automotive, aerospace, ...). One of the difficulty to deal with this kind of problems is to well represent the coupling between fluid and solid. Several formulations can then be considered [1]. Among them, one can cite a pure displacement-formulation, a three fields formulation (displacement, pressure and potential for example) or a displacement-pressure formulation. In this study, this latter is chosen. One of the difficulties for this displacement (solid) - pressure (fluid) formulation is that it leads to an unsymmetric eigenvalue problem. In order to solve this problem, some numerical methods can be used : they are listed and described in Ref. [2] for different cases of structures and fluid.

This paper concerns the free vibrations of an elastic structure which is completely filled with a homogeneous, inviscid and non-weighting compressible fluid. The displacement-pressure equations are spatially discretized with the finite element method. The discrete linear non-symmetric eigenvalue problem is then solved with three numerical methods. One objective of this paper is to compare these numerical methods. The first algorithm is based on the common Arnoldi method with ARPACK solution [3]. For the two others methods, a perturbation method is used. The second method has been recently proposed in [4]. This method consists in introducing a right-hand side (r.h.s.) in the eigenvalue problem. This r.h.s. is a scalar multiplied by a random vector. This modified eigenvalue problem is then solved with the help of a perturbation method where the angular frequency is the perturbation parameter. Finding the couple of unknowns (eigenvalues and modes) consists then in finding the value of the angular frequency for which the introduced scalar is null. In the last numerical method, the forced vibration problem is considered. A perturbation method is also used to compute the response curve of the considered problem. This third numerical method is based on the behavior of the series in the vicinity of a singularity. Then according to the work presented in Ref. [5], close to a singularity (in this study the eigenvalue), a geometric progression appears in the asymptotic expansion. So determining the eigenvalues consists in finding the emergence of a geometric progression during the computation of the response curve of the forced vibration problem.

2. Governing equations

In this study, the displacement (u in the solid) and pressure (p in the fluid) formulation is chosen. The governing equations of an elastic solid (Ω_s) entirely filled with a homogeneous, inviscid and non-weighting compressible fluid (Ω_f) are the followings:

$$\left\{\begin{array}{l} \sigma_{ij,j}(u) + \omega^2 \rho_S u_i = 0 \text{ in } \Omega_\mathrm{s} \\ \triangle p + \frac{\omega^2}{c^2} p = 0 \text{ in } \Omega_\mathrm{f} \\ \sigma_{ij}(u) n_j^f = p n_j^f \text{ on } \partial\Omega_\mathrm{sf} \\ \frac{\partial p}{\partial n^f} = \nabla p.\vec{n}^f = \omega^2 \rho_f \vec{u}.\vec{n}^f \text{ on } \partial\Omega_\mathrm{sf} \end{array}\right. \quad (1)$$

where subscripts s and f stand for solid and fluid domain respectively. The symbol $\partial\Omega_\mathrm{sf}$ is the solid-fluid interface where the couplings are. The scalars ω, ρ_f, ρ_s and c are the angular frequency, the density of the fluid and of the solid and the velocity of sound respectively. By using the finite element method, the discrete form of the previous equations is obtained:

$$\left(\begin{bmatrix} \mathrm{K_s} & -\mathrm{C} \\ 0 & \mathrm{K_f} \end{bmatrix} - \lambda \begin{bmatrix} \mathrm{M_s} & 0 \\ \rho_\mathrm{f} \mathrm{C^t} & \mathrm{M_f} \end{bmatrix} \right) \begin{Bmatrix} \mathrm{u} \\ \mathrm{p} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (2)$$

In the previous equations, the symbols $\mathrm{K_{f\,or\,s}}$, $\mathrm{M_{f\,or\,s}}$ and C represent the stiffness matrix (for the fluid or the solid), the mass

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matrix (for the fluid or the solid) and the matrix due to the fluid-solid coupling. The scalar λ represents the eigenvalue (square of the angular frequency). For the sake of simplicity, the previous system of equations (2) is written as the following generalized eigenvalue problem:

$$[\mathrm{K} - \lambda \mathrm{M}]\,\mathrm{U} = 0 \tag{3}$$

where K and M are unsymmetric matrices and U is a mixed unknown vector (the displacement and the pressure). This problem is now solved with the help of three different numerical methods.

3. Numerical methods

3.1. Classical numerical method

In this study, all the numerical results obtained with the proposed methods are compared to the results obtained with the Arnoldi method by using ARPACK solution [3]. As it has been pointed out in Ref. [4], the system (2) is poorly conditioned and then ARPACK is unable to compute the corresponding eigenvalues. So, according to the analysis reported in [6], the system (2) needs to be preconditioned and becomes:

$$\left(\begin{bmatrix} \mathrm{K_s} & -\mathrm{C}_k \\ 0 & \hat{\mathrm{K}}_\mathrm{f} \end{bmatrix} - \lambda \begin{bmatrix} \mathrm{M_s} & 0 \\ \rho_\mathrm{f}\mathrm{C}_m^\mathrm{t} & \hat{\mathrm{M}}_\mathrm{f} \end{bmatrix}\right) \begin{Bmatrix} \mathrm{u_s} \\ \hat{\mathrm{p}} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{4}$$

with the following definitions:

$$\begin{cases} \hat{\mathrm{K}}_\mathrm{f} = ab\mathrm{K}_\mathrm{f} \quad \text{and} \quad \hat{\mathrm{M}}_\mathrm{f} = ab\mathrm{M}_\mathrm{f} \\ \mathrm{C}_k = a\mathrm{C} \quad \text{and} \quad \mathrm{C}_m = b\mathrm{C} \\ \hat{\mathrm{p}} = \frac{1}{a}\mathrm{p} \\ a = \sqrt{\frac{\|\mathrm{K_s}\|_\mathrm{F}}{\|\mathrm{M_f}\|_\mathrm{F}}} \quad \text{and} \quad b = \sqrt{\frac{\|\mathrm{M_s}\|_\mathrm{F}}{\|\mathrm{K_f}\|_\mathrm{F}}} \end{cases} \tag{5}$$

where symbol $\|\mathrm{K}\|_\mathrm{F}$ stands for the Frobenius norm of the matrix K defined by $\|\mathrm{K}\|_\mathrm{F} = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}|K_{ij}|}$. With these modifications of the initial problem, ARPACK is able to compute the eigenvalues and eigenmodes of a vibroacoustic problem [6]. These results are considered in the following as reference results.

3.2. The indicator method

The method proposed in Ref. [4] to compute the eigenvalues and eigenmodes of the system (3) is reminded. The key point of this method is to modify the Eq. (3) by introducing a right hand side μF:

$$[\mathrm{K} - \lambda \mathrm{M}]\,\mathrm{U} = \mu F \tag{6}$$

where F is a random load vector and μ is a scalar. Hence, finding the solutions (λ, U) verifying the eigenvalue problem (3) consists in determining when the scalar μ is null. In order to have a well-posed problem, the following additional equation is chosen:

$$\left\langle \mathrm{U} - \mathrm{U}^0, \mathrm{U}^0 \right\rangle = 0 \tag{7}$$

where $\langle \bullet, \bullet \rangle$ represents the Euclidian scalar product. The vector U^0 is the solution of the problem (6) when λ is equal to zero (*i.e.* the static problem). In this case, the scalar μ is chosen equal to one. In order to solve the problem (6) with a perturbation method, the eigenvalue λ is defined by:

$$\lambda = \lambda_0 + \hat{\lambda} \tag{8}$$

where the value λ_0 is supposed to be known, equal to zero for the beginning of the calculation. The scalar $\hat{\lambda}$ is the new parameter of the problem. To determine accurate values of $\hat{\lambda}$ for which the scalar μ is null, the unknown $X = (\mathrm{U}, \mu)$ are searched as an integro-power series with respect to the parameter $\hat{\lambda}$:

$$X = \sum_{i=0}^{N} \hat{\lambda}^i X_i \tag{9}$$

where N stands for the truncation order of the asymptotic expansions. By introducing these latter in the equations (6) and (7) and balancing terms with identical powers of $\hat{\lambda}$, a set of linear problems is obtained:

$$\begin{cases} \text{Order } 0 : \\ (\mathrm{K} - \lambda_0 \mathrm{M})\,\mathrm{U}_0 = \mu_0 F \\ \left\langle U_0, \mathrm{U}^0 \right\rangle = \left\langle \mathrm{U}^0, \mathrm{U}^0 \right\rangle \\ \\ \text{Order } 1 \leq \mathrm{i} \leq N : \\ (\mathrm{K} - \lambda_0 \mathrm{M})\,\mathrm{U}_i = \mu_i F + \mathrm{M}\mathrm{U}_{i-1} \\ \left\langle \mathrm{U}_i, \mathrm{U}^0 \right\rangle = 0 \end{cases} \tag{10}$$

Once, all these linear problems have been solved, the polynomial expansion (9) is replaced by a rational equivalent one, namely Padé approximants [7, 8]:

$$\mathrm{X}_{\text{Padé, N}}(\hat{\lambda}) - \mathrm{X}_0 = \sum_{k=1}^{N-1} \frac{R_{(N-1-k)}(\hat{\lambda})}{Q_{(N-1)}(\hat{\lambda})} \hat{\lambda}^k \mathrm{X}_k \tag{11}$$

where R_k, Q_k are polynoms of degree k. Finally, finding the values for which the scalar μ is equal to zero consists in determining the roots, $\hat{\lambda}_r$, of the polynoms $R_{(N-1-k)}(\hat{\lambda})$. The accuracy of a given root is checked by the following criteriom:

$$\frac{\left({}^t\mathrm{U}_r K \mathrm{U}_r - \hat{\lambda}_r^t \mathrm{U}_r M \mathrm{U}_r\right)}{\hat{\lambda}_r} \leq \epsilon \tag{12}$$

where ϵ is a chosen user tolerance. A continuation technique [9] is used to determine all the eigenvalues between the initial value ($\lambda_0 = 0$) and the given maximum value of λ. Let us remark that this method is not sensitive at all to the poorly conditioned matrices [4].

It is finally pointed out, that this indicator method is quite similar to the numerical algorithm proposed by Avery et al. in Ref. [10] to identify the eigenvalues missed by a symmetric eigensolver. Indeed, these authors proposed to define a scalar transfer function which is a rational function whose poles are exactly the eigenvalues of the considered problem. This transfer transfer is replaced by an equivalent Padé approximant, by computing Taylor expansions around chosen initial points. Analysing the poles of the Padé approximant permits to detect and identify the eigenvalues of the considered problem.

3.3. Analysis of the series

The last proposed numerical algorithm to solve the eigenproblem (3) considers the forced vibration problem:

$$[\mathrm{K} - \lambda \mathrm{M}]\,\mathrm{U} = F \tag{13}$$

where F represents the amplitude of a time periodic load vector applied to the solid part of the considered problem : $F(t) = F\cos(\omega t)$. The previous problem is also solved by using a perturbation method where the unknown X, with $X = (\mathrm{U}, \lambda)$, is defined by:

$$X = \sum_{i=0}^{N} a^i X_i \tag{14}$$

where the path parameter a can be written as:

$$a = < \mathrm{U} - \mathrm{U}_0, \mathrm{U}_1 > + (\lambda - \lambda_0)\lambda_1 \tag{15}$$

By introducing the asymptotic expansions (14) into equations (13) and (15) and by equating like powers of a, a set of linear equations is obtained:

$$\begin{cases} \text{Order } 0 : \\ (K - \lambda_0 M)\mathrm{U}_0 = F \text{ with } \lambda_0 \text{ known} \\ \\ \text{Order } 1 : \\ (K - \lambda_0 M)\mathrm{U}^\star = M\mathrm{U}_0 \\ \lambda_1 = \sqrt{\frac{1}{1 + <\mathrm{U}^\star, \mathrm{U}^\star>}} \\ \mathrm{U}_1 = \lambda_1 \mathrm{U}^\star \\ \\ \text{Order } 2 \leq i \leq N : \\ (K - \lambda_0 M)\mathrm{Uq}(i) = \sum_{ir=1}^{i-1} \lambda_{ir} M U_{(i-ir)} \\ \lambda_i = -\frac{<\mathrm{Uq}(i), \mathrm{U}_1>}{\lambda_1 + <\mathrm{U}^\star, \mathrm{U}_1>} \\ \mathrm{U}_i = \lambda_i \mathrm{U}^\star + \mathrm{Uq}(i) \end{cases} \tag{16}$$

So by solving the $(N + 1)$ equations previously defined, one can compute a part of the solution. Next, the polynomial approximation (14) is replace by the Padé approximants (11). By applying the continuation technique as previously [9], the response curve of a vibroacoustic problem submitted to a harmonic load vector is then computed in a range $\lambda \in [0, \lambda_{\max}]$ where $\lambda_{\max}$ is chosen by the user.

The question is now how can be determined the eigenvalue and eigenmode solutions of Eq. 3 ? The proposed algorithm is based on the behavior of the asymptotic expansions close to a singularity. Indeed, in a recent paper, Cochelin and Medale [5] have shown the emergence of a geometric power series in the asymptotic expansion near a singularity. In this latter reference, the singularity is a steady bifurcation. This technique has been used to detect steady bifurcation in Newtonian [5, 13] or non-Newtonian [14] fluid mechanics and Hopf bifurcation in rotating beam framework [12]. From the work presented in Ref. [5], the unknown X can be splitted into two parts:

$$X = \hat{X} + X_{\text{error}} \tag{17}$$

where $\hat{X}$ is a flawless solution and X_{error} represents the solution due to the defaults or approximation errors in the computation. In the reference [5], the authors have demonstrated that the term X_{error} in the initial series X is a geometric progression series. So determining a singularity consists in finding a geometric progression into the polynomial representation (14). Hence, this is carried out numerically by verifying a collinearity condition and a relative error test applied to the last terms of the series as follows:
For $N - 3 \leq n \leq N - 1$ compute :

$$\beta_n = \frac{< X_n, X_N >}{< X_N, X_N >} \text{ and } X_n^\perp = X_n - \beta_n X_N \tag{18}$$

$$\text{If } \sum_{n=N-3}^{N-2} \left(\frac{|\beta_n|^{1/(N-n)}}{|\beta_{N-1}|} - 1 \right)^2 \leq \delta_1 \tag{19}$$

$$\text{and } \sum_{n=N-3}^{N-1} \left(\frac{\| X_n^\perp \|}{\| X_h \|} \right) \leq \delta_2 \tag{20}$$

where the two parameters δ_1 and δ_2 are chosen equal to respectively 10^{-3} and 10^{-6} according to the numerical results presented in Ref. [5].

If both previous criteria are checked, then the common ratio of the geometric progression (β_{N-1}) is exactly the distance to the singularity, denoted by a_0, and leads to the computation of all the terms $\hat{X}_n$ of the flawless solution:

$$\text{For } 1 \leq n \leq N - 1 : \hat{X}_n = X_n - a_0^{(N-n)} X_N \tag{21}$$

These terms are then used to compute a "cleaned" solution. Finally, once a geometric progression has been numerically detected, the eigenvalue and the eigenmode can be computed by using the following expression:

$$X_c = X_0 + \sum_{i=1}^{N-1} a_0^i \hat{X}_i \tag{22}$$

Moreover, it is known from [5] that the vector X_N is related to tangents at singular solutions. So, when the continuation is performed on the fundamental branch and in the case of a pitchfork bifurcation, X_N is exactly the direct mode at the singluar solution, see Ref. [13]. Using the Lyapunov-Schmidt reduction [13], one shows that this direct mode is the orthogonal tangent to the fundamental branch.

In the considered vibroacoustic problem, the flawless solution (*i.e.* $\hat{X}$) is the solution of the generalized eigenvalue problem (3) and the "error" solution (solution with default) is due to the load applied to the structure (Eq. 13). So, close to the eigenvalue, a geometric progression should appear in the asymptotic expansions (14).

4. Numerical results

The three previous numerical methods are now applied to determine the eigenvalues and eigenmodes of a steel cavity (linear elastic structure) which is filled with an inviscid compressible fluid (air or water) (see Figure 1). This problem has been solved

in references [4] and [11]. Linear quadrilateral elements are used both for solid and fluid domains. Solid elements have eight degrees of freedom (two displacements per node) and fluid elements only four (one pressure per node). The fluid-solid interface elements have twelve d.o.f. The first ten eigenvalues for

Steel:
E=1.44 10^{11} Pa
ν= 0.35
ρ_s = 7.7 10^3 kg.m^{-3}
Air's properties:
c_f = 340 m.s^{-1}
ρ_f = 1 kg.m^{-3}
Water's properties
c_f = 1430 m.s^{-1}
ρ_f = 1000 kg.m^{-3}

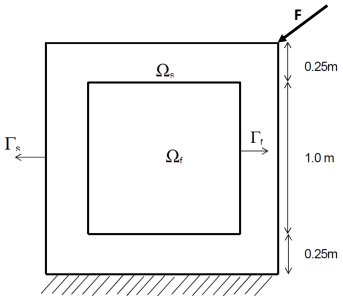


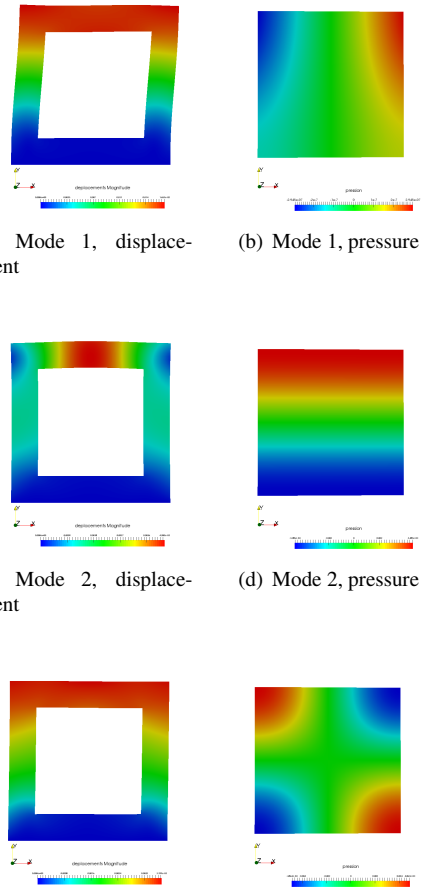
Figure 1: Geometry and mechanical properties for the steel cavity [11].

air and water obtained with ARPACK and the indicator method are given in Table 1 and compared to those from Ref. [11]. The

Air		Water	
PMs	Reference[11]	PMs	Reference[11]
676.926	664.121	654.159	641.837
1068.562	1068.129	2159.301	2116.398
1068.607	1068.152	3445.498	3201.475
1511.191	1510.589	3907.321	3804.124
2139.448	2136.102	4221.192	4211.62
2139.707	2136.240	4710.677	4687.927
2304.012	2258.686	5168.735	5155.246
2391.688	2388.418	5454.176	5385.805
2391.734	2388.539	6280.978	6239.332
3026.00	-	7597.443	-

Table 1: Comparison of the first ten angular frequencies obtained with the proposed methods (PMs) and those given in Ref. [11] for the elastic cavity presented in Fig. 1.

solutions obtained with the indicator method (Section 3.2) and those obtained with ARPACK are exactly the same. Some of the eigenmodes (Mode 1, 2 and 4) are represented in Fig. 2 for a cavity filled with air. One considers now the forced vibration problem of an elastic cavity filled with water. The response curve is given in Fig. 3. This curve is obtained by choosing a truncation order N of the asymptotic expansion (14) equal to 20. More than one hundred steps of the continuation technique



(a) Mode 1, displacement (b) Mode 1, pressure

(c) Mode 2, displacement (d) Mode 2, pressure

(e) Mode 4, displacement (f) Mode 4, pressure

Figure 2: Eigenmodes for the elastic cavity filled with air (from reference [4]).

is required to compute the solution up to an angular frequency equal to 3000 rad/s. The interesting feature is the step accumulation near the eigenfrequencies, ω_1 =654 rad/s and ω_2 = 2159 rad/s in Fig. 3. Each step of the continuation technique is represented by a cross symbol in Fig. 3. This behavior of the perturbation method close to singular values is classical and has been used in some applications as a visual indicator to detect instabilities. The algorithm presented in Section 3.3 and based on the detection of a geometric progression is now applied to detect eigenvalues by analysing the series. Then, when the latter method is used, only 55 steps of the continuation technique are needed to computed the solution for an angular frequency in the range $[0, 8000]$. This permits to compute the first ten eigenvalues (for the water case) given in Table 1. These eigenvalues are exactly the same as those computed with ARPACK or with the indicator method. With the analysis technique of series, the eigenmodes are easily carried out by introducing the distance to the closest singularity (a_0) into the expression (22). These modes are plotted in Fig. 4 for the air-filled cavity. In this figure, one can clearly see that these modes are not the same that those obtained with ARPACK or with the indicator method (see Fig 2). For a more precise idea of the accuracy

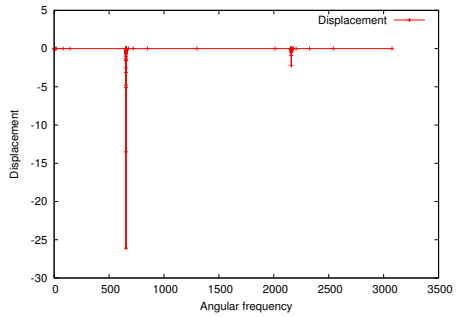
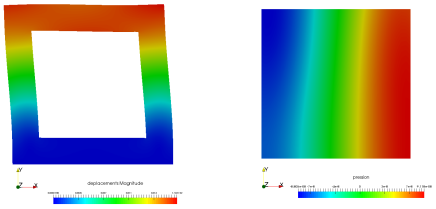
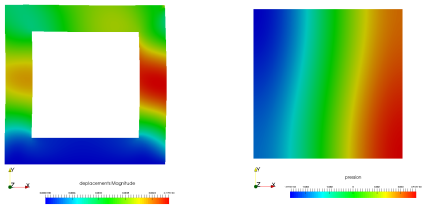


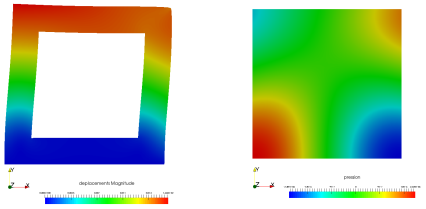
Figure 3: Response curve of the forced vibration problem for the elastic cavity filled with water. The displacement corresponds to the point where the load is applied (see Fig. 1).



(a) Mode 1, displacement (b) Mode 1, pressure



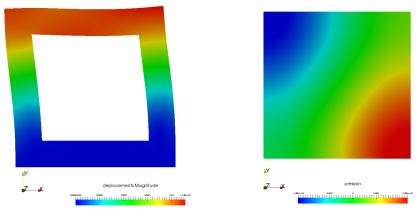
(c) Mode 2, displacement (d) Mode 2, pressure



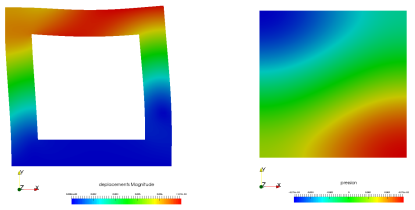
(e) Mode 4, displacement (f) Mode 4, pressure

Figure 4: Eigenmodes for the air-filled cavity by using expression (22).

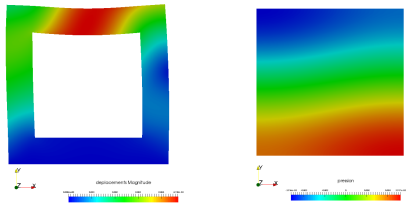
of the modes computed by using expression (22), the logarithm of the residual of the mode 2 has been calculated by using Eq. (12) and is equal to 7.83. This confirms that the modes obtained with expression (22) are not solution of the initial eigenvalues problem (3). This is quite surprising compared to the previous applications of this technique for computing steady bifurcations for example. Nevertheless, there exists another way to compute



(a) N=1, U_1 , displacement (b) N=1, U_1, pressure



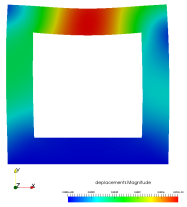
(c) N=2, U_2, displacement (d) N=2, U_2, pressure



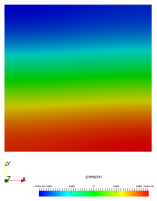
(e) N=3, U_3, displacement (f) N=3, U_3, pressure

Figure 5: Terms of the asymptotic expansions close to the second eigenvalue for an air-filled cavity according to the truncation order.

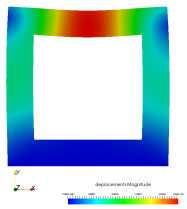
these eigenmodes. One considers for example a computation (the forced vibration case) carried out near the second eigenvalue of the air-filled cavity (close to $\omega_0 = 1000$ rad/s with $\lambda_0 = \omega_0^2$ in Eq. (16)). For this value of the angular frequency, a geometric progression has been detected and the next eigenvalue is then found. So, for this computation, some of the terms of the asymptotic expansion (the component U_i of the mixed vector X_i of Eq. 14) are plotted in Figures 5 and 6 for several truncation orders. These figures show that the component U_i (displacement and pressure) becomes similar to the mode 2 (see Figures 2(c) and 2(d)) when the truncation order increases. To underline this behavior, the evolution of the residual (12) of the vector U_i is plotted versus the evolution of the truncation order in Fig. 7. This figure shows that the vector U_i is exactly the eigenmode when the truncation order increases. The accuracy of the vector U_i seems to linearly depend on the truncation order. With this property, accurate eigenmodes can be then computed instead of using expression (22). In a way, Dirac delta functions in the response curve (Fig. 3) act as second famillies of solution crossing the fundamental branch at the singular solutions. As those functions are orthogonal to the curves, it is consistent to



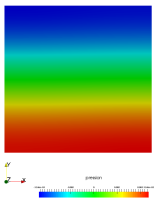
(a) N=4, U_4, displacement



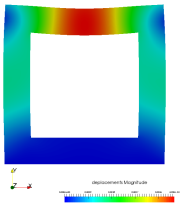
(b) N=4, U_4, pressure



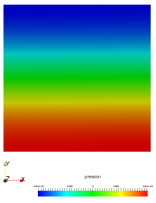
(c) N=5, U_5, displacement



(d) N=5, U_5, pressure



(e) N=6, U_6, displacement



(f) N=6, U_6, pressure

Figure 6: Terms of the asymptotic expansions close to the second eigenvalue for an air-filled cavity according to the truncation order.

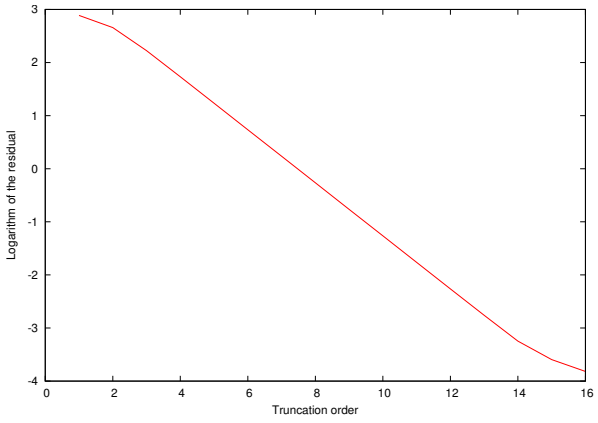


Figure 7: Evolution of the logarithm of the residual vector (Eq. 12) versus the truncation order for the vector U_i close to the second eigenvalue for an air-filled cavity.

see the eigenmode in the high order terms of the series.

The last point concerns the computational times required for each presented technique. So the computation times needed to

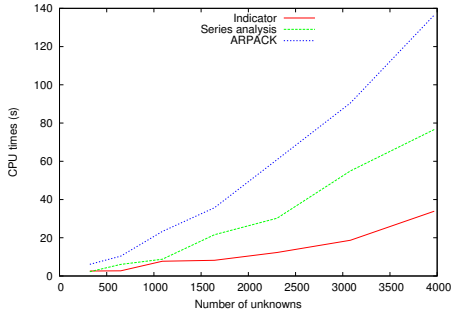


Figure 8: Comparison of CPU times obtained with ARPACK and the proposed methods to get the ten first eigenvalues (Table 1) for the elastic cavity filled with air [4, 11].

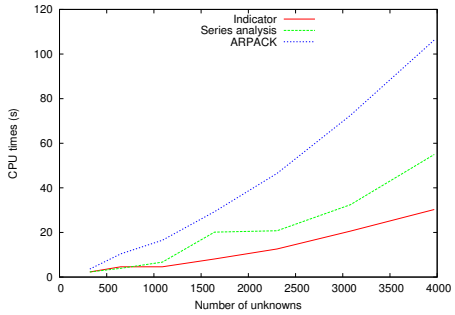


Figure 9: Comparison of CPU times obtained with ARPACK and the proposed methods to get the ten first eigenvalues (Table 1) for the elastic cavity filled with water [4, 11].

compute the first ten eigenvalues are plotted versus the number of unknowns considered in Figures 8 and 9 for air and water respectively. These figures show that the algorithm based on the analysis of the series is less time consuming than the ARPACK method. The indicator technique is the method requiring the lowest computational times. The numerical tests (not presented in this work) have shown that, on the contrary of the indicator technique, the analysis of series method is sensitive to the fact that matrices are ill-conditioned. So for this latter, as with ARPACK in section (3.1), all the matrices have been modified according to the expressions (5).

5. Conclusion

In this paper, three numerical techniques to compute the eigenvalues of a vibroacoustic interior coupled problem have been compared. A classical technique using the Arnoldi method with ARPACK solution is compared to two techniques using a perturbation method. A first method which has been initially proposed in [4] consists in determining the null values of an indicator. A second technique, proposed in a forced vibration framework, is based on the analysis of the behavior of the series close to singularities. Thanks to the detection of a geometric progression in the series, accurate eigenvalues can be determined. When a geometric progression is found, the vibration modes are the terms of the Taylor expansions at a high order (around 15 according to the presented results). The computa-

tional times are lowest with the indicator technique. Nevertheless, the method using the analysis of the series close to the eigenvalues seems to be a promising technique. Some works are in progress to improve this technique, for example by extracting singularities in the vicinity of the eigenvalues [15] or by using Euler transform [16]. All these techniques are based on Domb-Sykes plot [17, 18] which easily leads to the closest singularity of the initial point of the perturbation method.

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