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Cheng-Yu Han, Alain Lambert, Michel Kieffer

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Cheng-Yu Han[1], Alain Lambert[1] and Michel Kieffer[2,3]

[1] IEF - Institut d'Electronique Fondamentale,
Bât. 220 - Centre Scientifique d'Orsay, 91405 Orsay, France
[2] L2S, CNRS-Supélec-Univ, Paris-Sud,
3 rue Joliot-Curie, 91192 Gif-sur-Yvette, France
[3] Institut Mines-Télécom, Télécom ParisTech, CNRS LTCI,
46 rue Barrault, 75634 Paris Cedex 13, France

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Introduction

In wireless sensor networks (WSN), localizing each sensor of the network is a fundamental issue, since locations are often required to process the collected information. It is also a challenging non-linear parameter estimation problem from noisy measurements. Localization is usually performed considering the different types of signals transmitted by some anchor nodes, see [2] for an overview.

This paper considers localization from Receiver Signal Strength (RSS) measurements, which has received significant attention for more than a decade. In most of the papers, the noise is assumed to be normal or log-normal, see [5], which allows to apply maximum likelihood (ML) or maximum *a posteriori* estimation techniques and asymptotic characterization of the estimator confidence region, evaluating, *e.g.*, its Cramér-Rao bound (CRB). Alternative bounded-error localization techniques have been proposed in [4] to get set estimates. Nevertheless, in practice, noise samples do not necessarily follow a Gaussian distribution, and the bounds considered in bounded-error estimation are either often violated because they are too small, or lead to huge unexploitable solution sets when the bounds are too large.

Recently, the *Leave-Out Sign-dominant Correlation Regions* (LSCR) [1] has been introduced to define non-asymptotic confidence regions (NACR) for estimators considering very mild assumptions on the noise samples corrupting the measurements. In [3], the characterization of confidence regions as defined by LSCR has been formulated as a set-inversion problem which can be solved using SIVIA.

The aim of this paper is to show that LSCR can be put at work to characterize NACR in the context of source localization from RSS measurements. Several tools borrowed from global optimization using interval analysis (contractors, monotonicity, mixed centred forms, etc.) have been analyzed to improve the efficiency of set inversion in this context. Promising results have been obtained compared to state-of-the-art techniques, however, much remains to be done.

Comparison of LSCR and ML

Fig. 1 compares the results of source localization from RSS using a classical ML approach with CRB evaluation and NACR evaluation defined by LSCR. A square region of 30m×30m is considered. $N_{\mathrm{a}} = 5$ anchor nodes, which location is represented by stars, have perfect knowledge of their location; $N = 32$ nodes of the WSN represented by '+' have to determine their location from RSS measurements of the signals broadcast by the anchors. The Okumura-Hata model is used to describe the RSS as a function of the distance between an anchor and the receiving node

$$y_{a_k} = P_0 - 10n_P \log_{10} \left(\|\boldsymbol{\theta}_0 - \boldsymbol{\theta}_{a_k}\| / d_0 \right) + \varepsilon_{a_k}, \tag{1}$$

where y_{a_k} is an RSS measurement of the signal emitted by the anchor a_k; P_0 is a known reference power measured at a distance d_0 of the anchor; n_P is the path loss exponent; ε represents the measurement noise (usually considered as log-normal). The location of the node of interest is $\boldsymbol{\theta}_0$, the location of the anchor a_k is $\boldsymbol{\theta}_{a_k}$. We assume that $\boldsymbol{\theta}_0$ and n_P are unknown and that P_0 as well as n_{p} are the same for all anchor-node pairs. The noise corrupting data is taken as Gaussian-Bernoulli-Gaussian (GBG), *i.e.*, it is $\mathcal{N}(0, \sigma_0^2)$ with a probability $1-p$ and $\mathcal{N}(0, \sigma_1^2)$ with a probability p. Here, $\sigma_0^2 = 2$, $\sigma_1^2 = 5$, and $p = 10\%$. The estimation of its location and of n_{p} is performed by each node considering $N_m = 10$ independent RSS measurements obtained from each anchor.

Fig. 1(a) shows the results obtained using the ML approach where Levenberg-Marquardt's algorithm has been used for minimization with random initialization. The small green ellipses represent the CRB corresponding to a 90% confidence region centered on the estimated location. The lines connecting the true to the estimated locations represent the localization error. Fig. 1(b) shows the projections of the approximations of the 90% confidence regions as defined by LSCR and characterized using SIVIA. The projection of the uncertain boxes is in yellow. The projection of the boxes which have been proved to be within the 90% confidence region are in green. The proposed approach is more robust to uncertainties on the noise corrupting the data than classical approaches.

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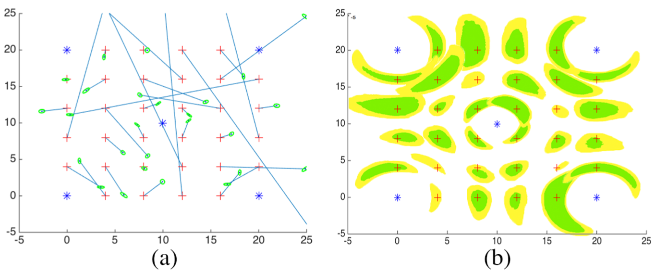

Figure 1: Node location (red crosses) estimates from RSS of waves emitted by anchors (blue stars). (a) Estimates and confidence regions from ML estimation (b) Confidence regions defined by LSCR