The Reduced Basis Method Applied to Aerothermal Simulations
Jean-Baptiste Wahl, Christophe Prud’Homme, Y. Hoarau

To cite this version:
Jean-Baptiste Wahl, Christophe Prud’Homme, Y. Hoarau. The Reduced Basis Method Applied to Aerothermal Simulations. CANUM, May 2018, Cap d’Agde, France. <hal-01799506>

HAL Id: hal-01799506
https://hal.archives-ouvertes.fr/hal-01799506
Submitted on 24 May 2018

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
**The Reduced Basis Method Applied to Aerothermal Simulations**

JB. Wahl, C. Prud’homme, Y. Hoarau

**The Reduced Basis Method**

**Motivations**
- Modeling: multi-physics non-linear models, complex geometries, genericity
- Uncertainty management / Risk analysis
- Optimization in early design, certification or operating phases

**Objective 1: Fast**
- Complex geometries
- Large number of dof
- Uncertainty quantification
- Large number of runs

**Objective 2: Reliable**
- Field quality
- Design optimization
- Certified bounds
- Reach material limits

**Main Idea**
Weak formulation of the model: \( a(u(x), v, \mu) - f(v, \mu) \)

**FEM Approximation:**
\[
X^N = \text{span}(\varphi_1, \ldots, \varphi_N) \rightarrow u^N(\mu) = \sum_{i=1}^N N_i^N(\mu) \varphi_i \rightarrow a_i^N(\mu) \varphi_i = f_i^N(\mu)
\]

**RB Approximation:** \( u^R(\mu) = \sum_{i=1}^N N_i^R(\mu) \varphi_i \): linear combination of FEM solution

\[
\begin{align*}
&\mathcal{X}^N = \text{span}(\varphi_1, \ldots, \varphi_N) \\
&\mathcal{X}^R = \text{span}(\varphi_1, \ldots, \varphi_N) \\
&\mathcal{X}^R = \text{span}(\varphi_1, \ldots, \varphi_N)
\end{align*}
\]

**Ingredients**
- Set of parameters: \( \mathcal{D}^P \)
- FEM ‘truth’ approximation
- \( \mathcal{X}^N \): finite element approximation space of dimension \( N > 1 \)
- \( u^N(\mu) \in \mathcal{X}^N \) is solution of \( a(u^N(\mu), v, \mu) - f(v, \mu) \) \( \forall v \in \mathcal{X}^N \)

**Efficient offline-online strategy**
\[
a^R(\mu) = \sum_{i=1}^N N_i^R(\mu) \varphi_i
\]

\( N \times N \) system to solve:
\[
\sum_{i=1}^N a_i^R(\mu) N_i^R(\mu) = f_i^N(\mu), \quad 1 \leq i \leq N
\]

if the parameter dependence can be expressed as an affine combination:
\[
a(u, v; \mu) = \sum_{i=1}^N a_i^R(\mu) N_i^R(v)
\]

**Solving Strategy**
- Finite Element Discretization
- Newton Algorithm with transient continuation
- Parallel implementation using Feel++ Library: http://www.feelpp.org/

**Mathematical Model**

\[
\begin{align*}
&\mu \cdot \nabla u + \nabla p - \beta \Delta u = \rho(T - T_0)q & \text{in } \Omega \times [0, T], \\
&\nabla \cdot u = 0 & \text{in } \Omega \times [0, T], \\
&u \cdot \nabla u - \kappa \Delta T = 0 & \text{in } \Omega \times [0, T], \\
&\text{Boundary Conditions}
\end{align*}
\]

**Challenge and Dificulties**
- Multi-physic coupled model: simultaneous construction of the different reduced spaces
- High Reynolds flow: use of stabilization methods (SUPG/GLS) in the FEM and the RB model
- Non-Linearity: Newton algorithm with an affine decomposition of the Jacobian/Residual
- Non-affine terms: use of Empirical Interpolation Method (EIM) for discrete operators
- Non-Linearity: Use of Simultaneous EIM and RB (SER) algorithm to generate an affine approximation of the non-linear terms (stabilization terms)
- Complex Formulation: Due to geometric parameters, use of EIM to automatically recover the affine decomposition.

**Perspectives**
- Development of Efficient Error Estimators for the Reduced Model
- Reduction of the Coupled Turbulence Model

**Numerical Results: Cooling of a Printed Circuit Board, Reduced Model**

**Physical Model**
- Air thermal diffusivity: \( \kappa = 2.7 \times 10^{-5} \)
- Air kinematic viscosity: \( \nu = 1.9 \times 10^{-5} \)

**Parameters**
- \( h_1 \text{ and } h_2 \): Heat sources from the two integrated circuits: [0, 106]
- \( \lambda \): Thermal conductivity of the two integrated circuits: [0.2, 150]
- \( D \): The inlet rate: [5 \times 10^{-6}, 10^{-2}] 

**Sponsor**

MSO4SC: Mathematical Modeling, Simulation and Optimization for Societal Challenges with Scientific Computing

The main objective of this project is to construct an infrastructure that provides, in a user-driven, integrative way, tailored access to the necessary services, resources and even tools for the fast prototyping, providing the service producers with the mathematical frameworks as well.

**References**

- [Cemosis](http://www.cemosis.fr/)
- Feel++ Library: [http://www.feelpp.org/]
- Cécile Daversin and Christophe Prud’Homme.
- Cécile Daversin Catty.
  - Reduced basis method applied to large non-linear multi-physics problems: application to high field magnets design.
  - Theses, Université de Strasbourg, September 2010. URL: [https://tel.archives-ouvertes.fr/tel-01361722](https://tel.archives-ouvertes.fr/tel-01361722).
- Federico Negri, Andrea Merlino, and David Amsallem.
  - Efficient model reduction of parametrized systems by matrix discrete empirical interpolation.
- T Tison.
  - Reduced basis method (RBM) for non-affine elliptic parametrized pdes (phd).
  - Örebro University, 2012.