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The nonlinear properties of a turbulence driven magnetic island (TDMI) are investigated. Starting from a minimal magnetohydrodynamic fluid model that provides for the generation of a TDMI and using scale separation arguments along with numerical simulation findings, we elucidate the links between the nonlinear transport properties of such magnetic islands and the characteristic features of the small scale turbulence. We also explain the phenomenon of partial pressure flattening inside the TDMI. *Published by AIP Publishing.* [http://dx.doi.org/10.1063/1.4981229]

I. INTRODUCTION

At present, large experimental tokamak devices often hit a limit in plasma β (where β is the ratio of plasma pressure to magnetic field pressure), due to the onset of neoclassical tearing modes (NTMs) that create large magnetic islands on magnetohydrodynamic (MHD) length scales. Islands grow nonlinearly due to a loss of the bootstrap current inside them. For future devices, such as ITER and other burning plasma experiments, their presence can lead to disruptions and significantly degrade both the standard ELMy H-mode operation and other advanced scenarios.[1] The degradation in the plasma stored energy due to NTMs is proportional to the radial island size,[2] and their onset is dependent on the initial β,[3] the ratio between the fluid and magnetic pressures. A prerequisite for an NTM to be excited is the existence of a seed island. Regimes of interest are characterized by a poloidal beta value much larger than the one required for such islands to be meta-stable. In other words, the triggering of NTMs in a device is not directly linked to the β levels but to the mechanisms which generate seed islands.[4] Previous research has identified several different causes for the formation of seed islands such as an unstable tearing mode,[5–7] MHD events such as sawtooth[8] and fishbones,[9] error fields,[10,11] edge localized modes (ELMS),[12] nonlinear mode coupling,[13] and turbulence.[14,15] This diversity of origins poses serious challenges for experimentally anticipating their appearance and choosing appropriate control parameters for their avoidance and suppression.[4] Present experimental efforts towards the control of NTMs rely primarily on tracking the appearance of seed islands at various mode rational surfaces and preventing their growth by replacing the missing bootstrap current inside them through externally driven helical currents. This can be achieved, for instance, by the generation of non-inductive current through the injection of radio frequency or electron cyclotron waves into the plasma (electron cyclotron current drive or ECCD).[16,17] Small size islands can also be controlled by electron cyclotron resonant heating (ECRH), which induces Ohmic currents because of the temperature dependence of the resistivity.[18,19] This strategy has led to successful results including complete stabilization of both the (3,2) and (2,1) NTMs (see, for instance, Ref. 20). However, controlling the current deposition with respect to the position of the island requires active control and is technically challenging for devices like ITER.[1] An alternative is to control the seed island triggering mechanism. For instance, ECCD could be used to control the onset of a sawtooth crash which is known to be a precursor for seed islands.

One puzzling and so far incompletely understood phenomenon is that of a large number of experimental observations where an NTM onset does not appear to be associated with any of the known mechanisms of a seed island generation. Both the 3/2 and 2/1 NTM[21,22] have been seen to grow without the existence of an MHD triggering event. The possibility of a turbulence triggered NTM onset has been suggested in the past[14,15,23,24] but remains to some extent an open question to date. It has been shown, in numerical simulations, that small scale interchange turbulence can generate a turbulence driven magnetic island (TDMI), either by a simple mode beating mechanism in the vicinity of a low order rational surface[15] or remotely by spatial transfer of energy of turbulent modes to the lowest available low order rational surface.[23] An interesting characteristic of such islands, which was pointed out in Ref. 25, was the existence of a partial pressure flattening inside a TDMI and how this could serve as a signature for their experimental identification. The flattening mechanism has been investigated in the past by using a basic quasi-linear two-scale length model, which takes into account interchange modes, to predict the condition under which a pressure flattening in a TDMI can occur. However, this model explanation suffers from some shortcomings. First, the earlier simulations on which the model was based did not have a clear separation of scales between the interchange modes and the large or MHD scales. Therefore, it was not adequate to provide a proper basis for the model. The role played by the intermediate scales, the ones in between the interchange and MHD scales, could not be tackled. Second, the reasons leading to the formation of a TDMI beyond the beating mechanism and the occurrence of a partial flattening were not theoretically explained. The link between the properties of the turbulence and such TDMI properties was not adequately addressed. In this paper, we will investigate these points theoretically and we will show that all these questions are closely linked.

This paper is organized as follows. In Section II, we present the model equations and the key mechanisms which are required to investigate this problem. In Sec. III, we focus on the first nonlinear phase where the magnetic island is generated, present a model of the interchange scales of this dynamics, evaluate transport parameters, and deduce an equation for the large scale pressure structure inside the island. In Sec. IV, we investigate these questions in the fully nonlinear but statistically stationary regime. We evaluate the critical island size above which pressure flattening occurs inside the island. In Sec. V, we show analytically why the flattening is partial and spatially uniform inside the island. In Sec. VI, we propose an explicit analytical link between the turbulence generated in a radially localized interchange band (IB) and the mean pressure flattening throughout the magnetic island. Finally, in Section VII, we present a discussion of our results and make some concluding remarks.

II. A MINIMAL DYNAMICAL MODEL FOR GENERATING A TDMI

Microinstabilities in tokamaks such as the trapped electron mode and the electron or ion temperature gradient mode include interchange type branches and are set in a bath of drift waves. They are characterized by the existence of a critical gradient length $L_{\nabla c}$ above which the modes are destabilized. Moreover, well beyond the threshold of the different instabilities ($L_{\nabla} \ll L_{\nabla c}$), a common dispersion relation governs their linear dynamics, namely,[26]

$$\gamma_{\rm int}^2 = f_t \omega_{de}\omega_{pe}^\star + \omega_{di}\omega_{pi}^\star \propto c_s^2/(RL_\nabla),$$

where $\omega_{ps}^\star = k_\theta v_{ps}^\star$ and $\omega_{ds} = 2\lambda_s k_{s\theta} v_{ds}$ are the diamagnetic and vertical drift frequencies, respectively. s refers to the particle species electrons ($s=e$) and ions ($s=i$), $k_{s\theta}$ is the poloidal wave vector, $v_{ps}^\star$ is the diamagnetic velocity, and λ_s is a number depending on the local properties of the equilibrium. f_t is the fraction of trapped electrons, and c_s is the sound speed. Thus, the interchange turbulence can be considered as a paradigm for turbulence in tokamaks.

To describe a TDMI, we take the help of a minimal 2D fluid model that was presented in Refs. 27 and 28 and that includes the basic features of the microinstabilities. It contains interchange modes and drift waves and is characterized by a pressure gradient threshold above which an instability occurs. This model is made up of three nonlinear equations consisting of the momentum equation describing the time evolution of the electric potential ϕ, the energy equation governing the dynamics of the electronic pressure fluctuations p, and Ohm's law describing the time evolution of the total magnetic flux $\psi = \psi_{\rm eq} + \tilde{\psi}$:

$$\frac{\partial}{\partial t}\nabla_\perp^2\phi + \{\phi, \nabla_\perp^2\phi\} = \{\psi, \nabla_\perp^2\psi\} - \kappa_1\frac{\partial p}{\partial y} + \mu\nabla_\perp^4\phi, \quad (1)$$

$$\frac{\partial}{\partial t}\psi = \{\psi, \phi - p\} - v_\star\frac{\partial \psi}{\partial y} + \eta\nabla_\perp^2\tilde{\psi}, \quad (2)$$

$$\frac{\partial}{\partial t}p + \{\phi, p\} = -v_\star\left((1-\kappa_2)\frac{\partial \phi}{\partial y} + \kappa_2\frac{\partial p}{\partial y}\right) + \hat{\rho}^2\{\psi, \nabla_\perp^2\psi\} + \chi_\perp\nabla_\perp^2 p. \quad (3)$$

Eqs. (1)–(3) are normalized using the characteristic Alfvén speed v_A, the magnetic shear length $L_\perp$, and the Alfvén time $\tau_A = L_\perp/v_A$. In this model, we will consider the situation where there is only one resonant surface. For instance, when the magnetic equilibrium is given by the Harris current sheet model, namely, $\mathbf{B}_{\rm eq}(x) = \tanh\left(\frac{x}{a}\right)\hat{\mathbf{y}}$, the resonant surface, where the interchange and tearing instabilities can develop, is $x=0$. a determines the width of the profile and is linked to the value of tearing mode stability index parameter Δ'. This simplified model can capture the essentials of the interchange and tearing instabilities and as a consequence provides a very useful framework to study the large island generation mechanism involving small-scale interchange modes. Indeed, with an appropriate set of parameters and initial conditions, selecting cases where there is no tearing instability, one can numerically observe the growth and saturation of magnetic islands called TDMIs.[15] Thereafter, we will consider the evolution of the poloidal Fourier components of the fields defined as $\psi(x,y,t) = \sum_{m\in\mathbb{Z}} \psi_m(x,t) \times \exp(ik_m y)$. To focus on TDMIs, we will suppose in the following that there is no unstable tearing mode and that there is a scale separation between the island and interchange scales. In other words, we suppose that between the MHD poloidal scales and the interchange poloidal scales, there exists a range of wavelength scales that are stable to both tearing and interchange instabilities. This is, in fact, a normal situation in most tokamak plasmas.

One of the main objectives of this paper is to understand the mechanisms that sustain a finite pressure profile inside a TDMI, i.e., $dp_0/dx \neq 0$, and to propose a suitable physical model to describe it. In the small island limit, strong parallel diffusion would try to flatten the profile as one observes, for example, in islands that are not excited by turbulence. However, such a complete flattening would suppress the source of the interchange instability and therefore the growth of the TDMI. A finite pressure gradient is necessary for the existence of the TDMI. These contradictory requirements can be reconciled if one recognises that small scale fluctuations act as a source inside the island. It is therefore important to identify the key ingredients necessary to build a physical model of a TDMI that incorporates the dynamics of these fluctuations and explains the mechanism of partial pressure flattening. In the large island limit, finite perpendicular diffusivity effects would provide an additional mechanism for sustaining the finite pressure gradient.[29]

The first important ingredient is the basic physical mechanism that ensures the self-sustainment of TDMIs. We will assume the existence of a quadrupolar flow which is the fundamental mechanism that makes a magnetic structure, like an island, to become self-sustaining or growing.[30] Such a flow is observed in the case of a tearing instability and also in alternative scenarios where the Sweet-Parker and/or Petschek models apply. Basically, when magnetic reconnection occurs, a large fraction of flow penetrates into the island transversely to the magnetic field lines in the vicinity of the X point. It is expelled from the island at the O point, where the radial island extension is maximal and the magnetic tension is minimal, as shown schematically in Fig. 1. In our model, the magnetic flux is advected by $\phi - p$. Thus, we will assume the existence of a large scale quadrupolar $\phi - p$ structure as the island appears and evolves.

A second important dynamical assumption is to suppose that the condition for a partial flattening of the TDMI is satisfied. This condition is well defined for tearing driven magnetic islands, namely, that the radial width of the island w must be larger than a critical value which is proportional to the one-fourth power of the ratio of the perpendicular diffusion coefficient $\chi_\perp$ to the parallel diffusion coefficient $\chi_\parallel$.[29] In the case of TDMIs, the turbulent perpendicular diffusion coefficient $\chi_\perp^{turb}$ needs to be taken instead of $\chi_\perp$. Thus, flattening should occur when $w \gg w_c^{turb} = \sqrt{8}(\chi_\perp^{turb}/\chi_\parallel)^{1/4}\sqrt{a/k_1}$ with $\chi_\parallel = \hat{\rho}^2/\eta$.[25] In the following, we will give an estimate of $\chi_\perp^{turb}$.

The final important step is a proper recognition of the multi-scale character of the fluctuation dynamics and the nature of the turbulence. By introducing a scale hierarchy, we will quantify how interchange turbulence affects the large scales. However, this requires further analytical development. To make this point more precise, we will now introduce some modeling of the dynamics. We will analyze separately the quasi-linear and nonlinear phases of this dynamics. The quasi-linear phase follows the linear one where unstable interchange modes arise. It is characterized by an exponential growth of the interchange modes as well as the large scale length modes. The latter grow owing to a beating of the dominant interchange modes. In particular, if the most unstable interchange modes with poloidal mode numbers $m_\star \gg 1$ have a growth rate $\gamma_{m_\star} = \gamma_\star$, then the modes $m \leq 1$ have a growth rate $\gamma_m \approx 2\gamma_\star$.[15]

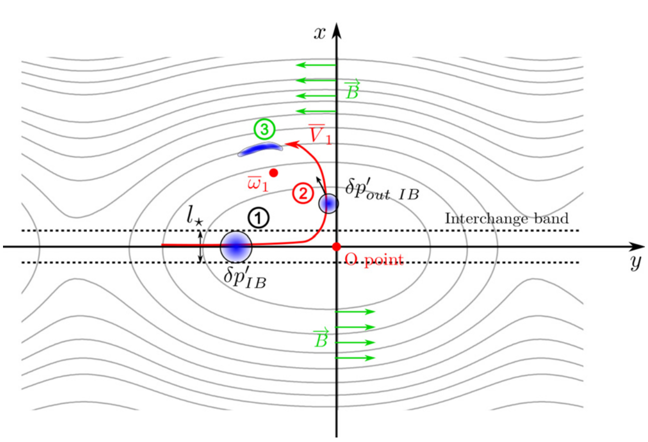

FIG. 1. Schematic representation of the transfer energy mechanism which feeds the growth of a magnetic island from interchange fluctuations: ① Interchange fluctuations are generated in the interchange band (IB). ② They are advected by the quadrupolar flow outside the IB. ③ The diffusion along the magnetic field lines of the fluctuations is enhanced outside the IB and generates large scale fluctuations on short time scales $\tau_\parallel \ll \gamma_\star$.

III. TRANSPORT PROPERTIES OF TDMIs IN THE QUASILINEAR PHASE

We denote a mean scale of the island by an overbar and write $p = \bar{p} + p'$ with $\overline{p'} = 0$. We set $\varphi = p + p_{eq} - \phi$ where $p'_{eq}(x) = -v_\star$. Thus, the nonlinear terms in the pressure equation expand according to

$$\overline{\{\phi, p\}} = \{\bar{\phi}, \bar{p}\} + \overline{\{\phi', p'\}}, \tag{4}$$

and

$$\begin{aligned} \hat{\rho}^2\{\psi, j\} &= \chi_\parallel\{\psi, \{\psi, \varphi\}\} + \hat{\rho}^2\{\psi, j_{eq}\} \\ &= \chi_\parallel\left(\overline{\nabla_\parallel}^2\bar{\varphi} + \Pi + \Theta + \{\psi', \{\psi', \varphi'\}\}\right) \\ &\quad + \hat{\rho}^2\{\psi, j_{eq}\}, \end{aligned} \tag{5}$$

$$\Pi = \{\psi', \overline{\nabla_\parallel}\bar{\varphi}\} + \overline{\nabla_\parallel}\{\psi', \bar{\varphi}\} + \overline{\nabla_\parallel}^2\varphi', \tag{6}$$

$$\Theta = \{\psi', \{\psi', \bar{\varphi}\}\} + \{\psi', \{\bar{\psi}, \varphi'\}\} + \{\bar{\psi}, \{\psi', \varphi'\}\}. \tag{7}$$

$\overline{\nabla_\parallel} = \{\bar{\psi}, .\}$ is the parallel gradient along the island-scale magnetic field. The following approximate relations (exactly in the quasilinear phase) also hold:

$$\Pi \approx \Pi' \quad \text{and} \quad \Theta \approx \bar{\Theta}, \tag{8}$$

and the fourth term in the RHS of Eq. (5) can be neglected when considering a large scale average. The equation of evolution of the mean pressure results from the averaging of (3) on island scales

$$\begin{aligned} \frac{\partial}{\partial t}\bar{p} + \overline{\{\phi, p + p_{eq}\}} &= -v_\star\left(\kappa_2\frac{\partial\bar{\phi}}{\partial y} - \kappa_2\frac{\partial\bar{p}}{\partial y}\right) \\ &\quad + \hat{\rho}^2\overline{\{\psi, j\}} + \chi_\perp\nabla_\perp^2\bar{p}. \end{aligned} \tag{9}$$

To obtain an evolution equation for the pressure at large scales, we need to provide a model of the energy transfer from the interchange to island scales in order to evaluate $\overline{\{\phi', p'\}}$ and $\bar{\Theta}$. To investigate this problem of closure of Eq. (9), we first need to describe a model which specifies how the interchange scales interfere with the island dynamics.

A. A model for the interchange scales

There is a poloidal scale separation between the island scales and the interchange scales in the quasi-linear phase where mainly two scales are present $m \leq 1$ and $m = m_\star$. In this phase, only the interchange modes transfer energy to the large/island scales through mode beating. In particular, the nonlinear bracket $\{\phi - p, \psi\}$ produces large scale fluctuations $\bar{\psi}$, leading to the growth of the magnetic island size $w(t)$. A key point is that the interchange fluctuations are linearly generated in a radially narrow band of width δ.[31] Numerically, for the set of simulations considered here, one finds $\delta \approx l_\star$, where $l_\star = L_y/m_\star$ is the poloidal length of the most unstable interchange mode. We call this band the interchange band (IB). Let us introduce the parallel diffusion time of a structure of width l at radial position x in front of the O point of the island: $\tau_\parallel^l(x) \equiv 1/(\chi_\parallel k_\parallel^2) = \chi_\parallel^{-1}/(al/x)^2$.

As soon as $w > \delta$, small scale interchange fluctuations $(\tilde{\phi}, \tilde{p})$ generated by the interchange instability in this band (step ① in Fig. 1) are advected radially by the mean flow through the terms $\{\bar{\phi}, \omega'\}$ and $\{\bar{\phi}, p'\}$ (step ② in Fig. 1). Owing to the quadrupole structure of this flow, they cross the magnetic field lines up to the separatrices. At the same time, because the characteristic parallel diffusion time within the island $\tau_\parallel \equiv \tau_\parallel^{l_\star}(w/4) = L_y^2 \chi_\parallel^{-1} B_{\rm eq}^{-2}(w/4)$ is much lower than the interchange and eddy turnover times, namely, γ^{-1} and $\bar{\omega}_1$, respectively, pressure fluctuations populate the island through parallel diffusion along the magnetic field lines and are then converted into fluctuations at island scales (step ③ in Fig. 1). The parallel diffusion time of potential fluctuations $\tilde{\tau}_\parallel = L_y^2/(\eta k_\star)^2 B_{\rm eq}^{-2}(w/2) = \tau_\parallel/(k_\star \hat{\rho})^2$ is of the same order of magnitude as $\tau_\parallel$. Note that the parallel diffusion time becomes infinite at the resonant surface $x = 0$, and therefore, this mechanism is inefficient in the IB. Thus, only the fraction advected outside the IB of the small scale interchange fluctuations generated in this band populates the island and feeds large scales. To evaluate this fraction f, let us first compute the level of fluctuations advected outside the IB. For this purpose, one has to consider only the advection mechanism of the fluctuations by the quadrupole. Let us write the dominant fluctuations in the IB as

$$\delta p'①(x,y) = \hat{p}_\star(x)\exp(\gamma_\star t)\exp(i(k_\star y - \omega_\star t)), \tag{10}$$

where $\hat{p}_\star(x)$ is the eigenfunction of the mode with wavenumber $m_\star$, and $\omega_\star$ is the frequency of the mode. An estimate of the quadratic pressure fluctuations in the IB is given by

$$E① = \int_{\rm IB} dxdy\, |\delta p'①|^2 = \exp(2\gamma_\star t)\int_{-\delta/2}^{\delta/2} dx\, \hat{p}_\star(x)^2. \tag{11}$$

In the regime $\delta < w \ll L_y$, the fluctuations are advected by the quadrupole, predominantly along and in the vicinity of the line $y = y_O$ crossing the O point and pile up outside the IB. As such, first, the quadratic level of pressure fluctuations outside the IB, $E② = \int_{\rm out\, IB} dxdy\, |\delta p'②|^2$, is due to the flux crossing the IB boundary. Second, this flux is approximately localized in a band of width $\delta \approx l_\star$ around the O point where $\bar{\mathbf{v}}_1$ is roughly in the x direction. Thus, one can deduce that while leaving the IB by crossing $x = \pm\delta/2$, the pressure fluctuations roughly rotate by $\pi/2$ (see Fig. 1) and, thus, satisfy $\delta p'②(x,y) = \delta p'①(y,x)$. It follows that:

$$\frac{\partial}{\partial t} E② \approx 2\int_{-\delta/2}^{\delta/2} dy\, |\hat{p}_\star(y)|^2 \exp(2\gamma_\star t)\bar{\mathbf{v}}_{\rm adv}(t), \tag{12}$$

where $\bar{\mathbf{v}}_{\rm adv}(t) = \bar{\mathbf{v}}_1(\delta/2, y_O, t) = \bar{\mathbf{v}}_{\rm adv}(0)\exp(2\gamma_\star t) \approx \frac{2\delta}{w}\frac{2\pi}{L_y}\hat{\phi}_1(w/2)$ (see Fig. 1). Thus, if one does not yet consider the parallel diffusion of the fluctuations, the ratio f of the amplitudes of interchange fluctuations advected outside the interchange band to the ones in the band satisfies

$$f^2 = \frac{E②}{E①} \approx \frac{1}{2}\frac{\bar{\mathbf{v}}_{\rm adv}}{\gamma_\star L_y}. \tag{13}$$

Let us note that, as advection occurs, the parallel diffusion of the localized fluctuations $\delta p'②$ occurs along the perturbed magnetic field lines, including the island structure perturbation. Thus, $\delta p'② \to \delta p③ \equiv \delta \bar{p}$: a small scale blob of pressure is elongated along field lines on short time scales $\tau_\parallel^\delta(w/4) \ll \gamma^{-1}$ and only a large scale structure along the field line remains.

The mechanisms discussed above indicate that small scale pressure fluctuations act differently in the IB band where there is almost no parallel diffusion and outside the IB band where they are simultaneously advected and elongated along the island field line structures. More specifically, let us consider a blob of pressure fluctuations with amplitude $\delta p'$, generated by interchange mechanisms in the IB. The energy content of this blob is proportional to $\delta p'^2 l_\star \delta \approx \delta p'^2 l_\star^2$. However, a fraction of interchange free energy that has led to the growth of the blob has been nonlinearly transferred to large scales through the mechanism discussed above. First, according to Eq. (13), in terms of the level of pressure, considering the presence of a magnetic island of width w, this fraction at a given time is given by $g = \delta p'②/\delta p'① \approx f\sqrt{\delta/w}$. Second, as the total pressure of the blob is conserved when the stretching of the blob occurs along the magnetic field lines of the island, one has $\delta p'② l_\star \delta \approx \delta p'② l_\star^2 \approx l_\star L_y \delta\bar{p}$ where one has made use of the condition $w/L_y \ll 1$. Thus, one can write

$$\delta p' l_\star^2 \to \delta\bar{p} L_y l_\star + \delta p'_{①} l_\star^2, \tag{14}$$

where $\to$ implies a mean on fast time scales, i.e., larger than $\tau_{\rm adv} \equiv \bar{v}_{\rm adv}(t)\delta \approx \gamma_\star^{-1} \gg \tau_\parallel^\delta(w/4)$. More precisely, one finds numerically that $\tau_{\rm adv}$ is much larger than $\gamma_\star^{-1}$ at the beginning of the quasilinear phase and decreases down to a few $\gamma_\star^{-1}$. This mean allows one to include cumulative effects of the advection and elongations of interchange fluctuations (see Eq. (12)).

B. Closure in the quasilinear phase

From this two-scale model approximation, it turns out that the convective term can be written as

$$\overline{\{\phi', p'\}} \to -\chi_\perp^{\rm turb}\nabla_\perp^2\bar{p} - \tilde{\chi}_\perp^{\rm turb}\nabla_\perp^2\bar{\phi} + \bar{S}_{\phi',p'}. \tag{15}$$

By construction, p' is a sum on all the cells or blobs $\delta p'$. $\bar{S}_{\phi',p'}$ is the local coupling term from interchange modes to large scales, in the vicinity of the resonance, with radial extension $l_\star$. It is the source of large scale fluctuations, including the local transport term. It should be noted that $\chi_\perp^{\rm turb}$ is not a local quantity. In fact, the operator $\chi_\perp^{\rm turb}\nabla_\perp^2$ acts, by assumption, on island scales. This model gives the following estimate of the turbulent transport coefficients and the source:

$$\chi_\perp^{\rm turb} \approx rA_{\phi'}, \tag{16}$$

$$\tilde{\chi}_\perp^{\rm turb} \approx rA_{p'}, \tag{17}$$

$$\bar{S}_{\phi',p'} \approx \overline{k'^2\phi'①p'①}, \tag{18}$$

where $r = 2\pi f\sqrt{w/l_\star}$, $A_{\phi'}$ is the amplitude of the interchange fluctuation $\phi'①$, and $A_{p'}$ represents the amplitude of the pressure fluctuations. The expressions given in Eqs. (16) and (17) are estimates of the amplitudes of $\chi_\perp^{\rm turb}$ and $\tilde\chi_\perp^{\rm turb}$, respectively. They have been taken to be constant for simplicity. We have used a small island width hypothesis $k_1 w \ll 1$ and assumed $\nabla_\perp^2 \sim 1/w^2$. In the following, the notation ϕ' will correspond to the quantity ϕ'① which is localized in the interchange band of radial extension $l_\star$. Keeping in mind that in the quasilinear phase, wave beating is at play and that the drive of the $m=0$ mode is due to the convective terms in Eq. (3),[25] one gets $2\gamma_\star A_{p_0} \approx A_{\phi'}A_{p'}k_\star l_\star^{-1}$. In the vorticity equation (1), the drive is due to both the convective and Maxwell terms, but none of these two non-linearities dominate. Contributing at the same order of magnitude, we also have $2\gamma_\star A_{\phi_0} \approx A_{\phi'}A_{\phi'}k_\star l_\star^{-1}$ which gives an estimate of A'_p/A'_ϕ and

$$\tilde\chi_\perp^{\rm turb} \approx \chi_\perp^{\rm turb}|p'_0(0)/\phi'_0(0)|. \tag{19}$$

Note that the evaluation of the ratio A'_p/A'_ϕ seems to remain true numerically in the nonlinear regime after the quasilinear phase, which can be surmised, for instance, from Figures 7(b) and 9(b) of Ref. 25. In the vicinity of the resonance, one may link the level of the source to the mean pressure gradient. Indeed, in the quasilinear phase, first, the interchange mechanism transfers energy linearly from the mean pressure gradient to small scale fluctuations. Second, interchange fluctuations nonlinearly transfer the energy to the mean pressure gradient through the brackets $\{\phi, p\}$[25] in the IB

$$2\gamma_\star p_0 \sim \bar S_{\phi',p'}, \tag{20}$$

where p_0 refers to the $k=0$ fluctuation of the pressure. Third, the energy on the large scales is partly transferred back to the interchange scales and tends to offset the energy generated by interchange turbulence

$$\gamma_\star p' \sim [p_0, \phi'] \sim k_\star \phi' \frac{dp_0}{dx}. \tag{21}$$

It gives

$$\chi_\perp^{\rm turb} \approx r\sqrt{2}\gamma_\star/k_\star^2, \tag{22}$$

$$\bar S_{\phi',p'} \approx k_\star^2 \overline{\phi_\star p_\star} \approx 2\gamma_\star p_0. \tag{23}$$

To evaluate the turbulent diffusivity, we have made use of the fact that p_0 is driven by the interchange fluctuations in the IB and $p'_0 \sim k_\star p_0$. However, we do not know dp_0/dx outside the IB at this stage.

To go further, we need to consider the pressure equation at island scales and specifically the regime, $w_c^{\rm turb} < \delta \ll w$. Using Eq. (15), one obtains that $\overline{\{a', \{\bar b, c'\}\}} = \overline{\{a', \{b', \bar c\}\}} = 0$. Thus,

$$\bar\Theta = \overline{\nabla_\parallel}\, \overline{\{\psi', \varphi'\}} = \bar\Theta_0 + \overline{\nabla_\parallel}\, \bar S_{\psi',\varphi'}, \tag{24}$$

where by construction $\bar\Theta_0 = \bar\Theta(\bar S_{\psi',\varphi'} = 0)$ heats the plasma throughout the entire island. Let us introduce $S_{\rm int} = \hat\rho^2 \overline{\nabla_\parallel}\, \bar S_{\psi',\varphi'} + \bar S_{\phi',p'}$. $S_{\rm int}$ is a radially localized source inside the island, i.e., it cancels out outside the IB. At island scales, the pressure equation simplifies to

$$\begin{aligned}&\frac{\partial}{\partial t}\bar p + \{\bar\phi, \bar p\} - \hat\rho^2 \bar\Theta_0 - \hat\rho^2\{\bar\psi, j_{\rm eq}\} \\ &\quad + v_\star\left((1-\kappa_2)\frac{\partial\bar\phi}{\partial y} + \kappa_2 \frac{\partial \bar p}{\partial y}\right) \\ &\quad = \chi_\parallel \overline{\nabla_\parallel}^2 \bar\varphi + \left(\chi_\perp + \chi_\perp^{\rm turb}\right)\nabla_\perp^2 \bar p - \tilde\chi_\perp^{\rm turb}\nabla_\perp^2\bar\phi + S_{\rm int}.\end{aligned} \tag{25}$$

At such scales, the system is linearly stable and the linear curvature terms have been neglected. The diamagnetic term can be omitted by transforming to a frame in which the island is rotating at the diamagnetic velocity.

Let us emphasize that, first, the pressure and the potential interchange mode structures in the IB have an even parity and the magnetic flux has an odd parity. $\bar\psi$ is even because it is generated by mode beating. Therefore, $S_{\rm int}$, which has the parity of $k_\star\partial_x\phi' p'$, is anti-symmetric, i.e., odd with regard to x and localized in the vicinity of the resonance. Second, numerically, one finds $|p'_0(0)/\phi'_0(0)| \ll 1$ and $\nabla_\parallel^2\bar\phi \ll \nabla_\parallel^2 \bar p$ in the island frame. Thus, $\tilde\chi_\perp^{\rm turb} \ll \chi_\perp^{\rm turb}$ and $\bar\varphi \approx \bar p + p_{\rm eq}$. Third, in the quasilinear phase, by definition, the ordering $\bar g \ll g' \ll 1 \approx j_{\rm eq}$ is true for any field g. One can check that the fast dynamics are the dissipative terms on the RHS of Equation (25) and the localized source $S_{\rm int} \approx \bar S_{\phi',p'}$. Setting $\bar P = \bar p + p_{\rm eq}$, it follows that:

$$\chi_\parallel \overline{\nabla_\parallel}^2 \bar P + \chi_\perp^{\rm turb}\nabla_\perp^2 \bar P + S_{\rm int} = 0. \tag{26}$$

For the sake of simplicity, and because, in general, $\chi_\perp^{\rm turb} \gg \chi_\perp$, we have set $\chi_\perp^{\rm turb} \equiv \chi_\perp + \chi_\perp^{\rm turb}$.

IV. TRANSPORT PROPERTIES OF TDMIs IN THE NONLINEAR PHASE

There is also a natural poloidal scale separation in the nonlinear regime as evident from the energy spectral properties. Indeed, the spectra present some very specific properties observed for a wide range of parameters, including cases where $w/w_c \gg 1$[25] without clear scale separation and cases where w is of the order of a few w_c[32] but where intermediate scales are linearly stable. To be more explicit, we introduce a spatial scale decomposition of a field $p = \bar p + \tilde p + p'$ and specify it now $m \le 2$

(i) corresponds to scales where energy tends to pile up and will correspond to large island scales as characterized by $\bar p$. They dominate energetically and have, basically, the tearing parity. In other words, there is a magnetic island. The energy spectra are strongly decreasing functions of the mode number, and there is typically a difference of 2 orders of magnitude in the energy between $m=1$ and $m=3$ modes.

(ii) Intermediate scales are characterized by a tendency to attain an equipartition between the pressure and kinetic energies, both of which are lower than the magnetic energy. The latter corresponds to the nonlinear magnetic island structure. They are denoted as $\tilde p$ and correspond to poloidal mode numbers in the range of $3 \le m < m_{\rm th.}$ with $m_{\rm th.} \gg 3$. It turns out that the mode threshold $m_{\rm th.}$ always satisfies $m_{\rm th.} < m_\star$.

This equipartition can be associated with the adiabatic character of the response of the system at those scales $\tilde{p} \approx -\tilde{\phi}$. Indeed, equipartition implies that $0 = \frac{d}{dt}\left(\tilde{E}_k - \tilde{E}_p\right) \approx \int dS(p+\phi)\widetilde{\{\psi, j\}}$, which cancels in the case of a strict adiabatic response. $\tilde{E}_p$ and $\tilde{E}_k$ denote the pressure and kinetic energy at the intermediate scales.

The adiabatic trend of the system at intermediate scales, except in the vicinity of the external part of the island ($x \approx w/2$), is well observed numerically.[32] Thus, we will adopt the hypothesis that intermediate scales are almost adiabatic.

(iii) From the mode number larger than $m_{\text{th.}}$, it is found that there is a fast transition towards an equipartition between magnetic and pressure energies. The transition to this equipartition spectrum property includes the dominant interchange scales $|m - m_\star| \leq \Delta m$. The kinetic, pressure, and magnetic spectral energies at those scales are larger than those at intermediate scales.

(iv) Such an equipartition persists at the smallest scales $m > m_\star + \Delta m$. It corresponds to Alfvén modes driven by the term proportional to the Maxwell stress in the pressure equation, with weak or marginal interactions. Indeed for $m \gg m_\star$, the kinetic energy is negligible compared to the magnetic energy. In fact, for $m \gg m_\star$, the spectral energies are weak and we will neglect the impact of those scales on the nonlinear island dynamics.

A. Closure in the statistically stationary nonlinear phase

As a consequence of the adiabatic response at the intermediate scales, and because dominant interchange scales concentrate a large fraction of spectral energy compared to adiabatic scales, when considering the impact of small scales on large scales by quadratic interactions and long time dynamics, we can write

$$\overline{\{\phi, p\}} \approx \overline{\{\bar{\phi} + \tilde{\phi} + \phi', \bar{p} + \tilde{p} + p'\}} \approx \{\bar{\phi}, \bar{p}\} + \overline{\{\phi', p'\}}. \tag{27}$$

This makes explicit a natural separation of scales in terms of nonlinear interactions, with adiabatic interchange scales being inactive on average. This also indicates that the IB band still plays a specific role in the dynamics. Indeed, the interchange instability is, of course, not suppressed in the nonlinear phase: if one computes the growth rate from the mean profile after saturation of the island size, one finds a growth rate $\gamma_\star^s$ of the order of $\gamma_\star$ ($\gamma_\star^s \approx 0.4\gamma_\star$ in Ref. 25) and corresponding to a mode number $k_\star^s \approx k_\star$. However, the radial extension of the dominant interchange mode can differ from the linear or quasi-linear width δ, but it remains of the same order of magnitude, $\delta \approx L_y/(m_\star - \Delta m)$.

This discussion leads to our first hypothesis: we suppose that interchange free energy is mostly provided by spatial poloidal scales in an extended interchange band including modes p', ϕ', and ψ' with the radial width of the order of δ. From an energetic point of view, the physics in the extended IB is to a large extent unchanged: the linear part provides the interchange free energy, and the nonlinearities transfer this energy to large scales and, thus, control the mean pressure gradient in the IB. Finally, the large scales also advect interchange fluctuations outside the IB. These large scales transfer back a fraction of their energy to the adiabatic, interchange, and dissipative scales. The dissipative scales correspond to wave numbers larger than interchange ones.

A second hypothesis is that, when the size of the island is modified, the mean pressure gradient is also adjusted in the vicinity of the separatrices. Indeed, this is a well observed fact in Ref. 25 where the dynamics of large TDMI has been studied. More precisely, the drive of the mean pressure gradient fluctuations is made by the nonlinear convective term in the IB and advected to the separatrices. In other words, on time scales $\tau_{NL} \gg \left(\gamma_\star^{-1}, \bar{\omega}_1^{-1}\right)$

$$p_0/\tau_{NL} \approx \langle \bar{S}_{\phi',p'} \rangle, \tag{28}$$

where $\langle . \rangle$ indicates a mean on time scales τ_{NL}. The choice of the convective term is based on the observation that this is the dominant nonlinearity which transfers energy to large scales.[33] τ_{NL} is the characteristic nonlinear time, i.e., the time for the generated interchange fluctuations to operate a cycle through nonlinearities from interchange scales to dissipative scales by going to the largest scales. One may expect such a characteristic time to be of the order of the duration of the quasi-linear phase or equivalently to be of the order of the characteristic time for the island to reach a maximum at the end of this phase $\tau_{NL}^{-1} \approx d \ln W/dt \ll 2\gamma_\star^s$. As discussed previously, the intermediate scales are assumed to be insignificant with regard to the drive of the mean pressure gradient. Thus,

$$\bar{S}_{\phi',p'} \approx \overline{k'^2 \phi' p'}. \tag{29}$$

A third hypothesis is based on stationarity: on time scales τ_{NL}, the interchange free energy supplied to the system through the linear terms is balanced by the energy nonlinearly transferred to the dissipative scales: $\langle \partial_t p' \rangle = \langle \text{Linear} + \text{Dissipative} \rangle$. Stationarity means $\langle \bar{g} \rangle = \bar{g}$, $\partial \bar{g}/\partial t = 0$. In other words, ultimately, the interchange energy source feeds the dissipative scales. Thus,

$$\begin{aligned}\langle \{\bar{\phi}, \bar{p}\} \rangle &- \hat{\rho}^2 \langle \bar{\Theta}_0 \rangle - \hat{\rho}^2 \{\bar{\psi}, j_{\text{eq}}\} \\ &= \langle \chi_\parallel \overline{\nabla_\parallel}^2 \bar{\varphi} \rangle + \chi_\perp^{\text{turb}} \nabla_\perp^2 \bar{p} - \tilde{\chi}_\perp^{\text{turb}} \nabla_\perp^2 \bar{\phi} + \langle S_{\text{int}} \rangle. \end{aligned} \tag{30}$$

As the ordering in this phase is $g' \ll \bar{g} \ll 1$ for any field g and, moreover, as there is also an equipartition up to interchange scales, which satisfies $\psi' \gg \phi' \approx p'/\hat{\rho}$, the fast dynamics pressure equation can be again simplified to

$$\chi_\parallel \Delta_\parallel \bar{P} + \chi_\perp^{\text{turb}} \Delta_\perp \bar{P} + \langle S_{\text{int}} \rangle = 0. \tag{31}$$

We have made the assumption that $\langle \chi_\parallel \overline{\nabla_\parallel}^2 \bar{P} \rangle \approx \chi_\parallel \overline{\nabla_\parallel}^2 P$. Note that in the limit where there is no source term, we recover the equation used by Fitzpatrick[29] to study the flattening of the pressure in an island without turbulence.

B. Transport properties and critical island size for a partial pressure flattening

An order of magnitude of the perpendicular turbulent diffusion coefficient can also be obtained in the limit $w_c^{\rm turb} < \delta \ll w$: it is well established that a basic mixing length estimate $\chi_\perp^{\rm turb} \approx \gamma_\star / {k_\star}^2$ underestimates transport coefficients of interchange turbulence in the nonlinear phase.[31] Quasilinear diffusion transport models give better estimates[26] but a precise estimate is out of the scope of this work. However, noting that the perpendicular diffusion is enhanced by the interchange turbulence, and more specifically, by the interchange structures advected outside the IB, and taking into account the fact that the quasi-adiabatic character of the mode persists at interchange scales, one finds

$$\chi_\perp^{\rm turb} \approx \sum_{|m-m_\star|\le \Delta m} |g_m|^2 \gamma_m^s / {k_m}^2. \tag{32}$$

The index s indicates that the growth rate γ_m^s is computed from the nonlinear profile and not the initial equilibrium. The coefficients g_m represent the nonadiabatic effects on the modes and are close to 1. Thus, one obtains

$$\begin{aligned} w_c^{\rm turb} &\approx \sqrt{8}\left(\chi_\parallel^{-1} \sum_{|m-m_\star|\le \Delta m} |g_m|^2 \gamma_m^s {k_m}^{-2}\right)^{1/4} \sqrt{a/k_y} \\ &\approx \sqrt{8}\left(\frac{\Delta_m \gamma_\star}{\chi_\parallel k_\star}\right)^{1/4} \sqrt{a/k_y}. \end{aligned} \tag{33}$$

Let us emphasize that the above considerations give some hints at the reason for which the mean pressure gradient is constant, at least in the IB and when the island is large enough. Indeed, at the end of the quasi-linear phase, the amplitudes of the interchange scale fluctuations saturate. The amplitudes of the modes which are concatenated in p' and ϕ' are therefore to a large extent constant. Neglecting, statistically, the time evolution of the phase shifts in between the interchange modes p'_m and ϕ'_m, Eq. (29) implies that the source term in the IB is constant on time scales τ_{NL}. According to the model, the source controls the mean pressure gradient, both in the IB in the quasilinear phase and at the separatrices in the subsequent nonlinear phase. Consequently, according to Eq. (28), the mean pressure gradient is space independent inside the island. In the limit where statistically $\langle \bar{S}_{\phi',p'} \rangle$ is constant and balanced by dissipative effects on dissipative time scales, we can infer that the mean pressure gradient should also be time independent in the limit $w \gg w_c^{\rm turb}$. For a large enough island or high turbulence level, one may expect this to be true as far as the system enters into the quasi-linear phase (see simulations in Ref. 25) and therefore observe a constancy from the birth of the TDMI, including into the IB.

From this model, there is evidence to support that the role played by the IB in the nonlinear phase is similar to the one played in the quasi-linear phase although it is not dominated by only one mode. Outside the IB, the reason for which the pressure gradient is constant has been discussed from a dynamics point of view and observed numerically but is still not proved at this stage. Indeed, this should result from the model we have derived and in particular, from Eq. (31).

V. PRESSURE FLATTENING BY FAST DYNAMICS

Equations (26) and (31) are similar and valid in the regime, $w_c^{\rm turb} < \delta \ll w$. In order to solve this equation, we use the condition that the width of the nonlinear island w is smaller than the characteristic equilibrium current length $w/a < 1$. In this limit,[34] the flux function and its first derivatives can be approximated by $\psi = x^2/(2a) + \psi_1(t)\cos(k_1 y)$. This is a valid hypothesis in our simulations. The island width is $w = 4\sqrt{\psi_1/a}$. Following Fitzpatrick,[29] we introduce the quantities $\xi = k_1 y$, $\Omega = \psi/\psi_1$, and

$$Z(\Omega,\xi) = \sqrt{8}x/w = \sqrt{\Omega - \cos\xi}\ (x>0),$$

where $\Omega = -1$ at the O point and $\Omega = +1$ at the X point and/or along the separatrices. In the small island, aspect ratio limit $k_1 w \ll 1$, $\Delta_\perp \approx \partial^2/\partial x^2$. Let us also set

$$\bar{P}(x,y) = p_{\rm eq}(0) + \tilde{p}(x,y).$$

In the space variables (Ω,ξ), the diffusion equation (26) can now be written as

$$\frac{1}{4}\left(\frac{w}{w_c}\right)^4 Z\frac{\partial}{\partial\xi}Z\frac{\partial}{\partial\xi}\tilde{p} + Z\frac{\partial}{\partial\Omega}Z\frac{\partial}{\partial\Omega}\tilde{p} = -\frac{1}{2}\left(\frac{w}{w_c}\right)^2 \frac{S_{\rm int}}{\chi_\perp}. \tag{34}$$

Let us first consider the solution for $x \ge \delta/2$ where $S_{\rm int} = 0$ by construction. The large island limit $w/w_c \gg 1$ gives that $\bar{p}$ is a function of the island flux surfaces on both the sides of the band $|x| \le \delta$

$$\tilde{p} = \begin{cases} \tilde{p}_+(\Omega) & \text{if } x \ge +\delta/2 \\ \tilde{p}_-(\Omega) & \text{if } x \le -\delta/2. \end{cases} \tag{35}$$

In the limit $S_{\rm int} \to 0$, $\delta \to 0$ and the pressure is a function of the island flux surface $\tilde{p} = \tilde{p}(\Omega)$ inside the separatrix $\Omega \le 1$. By construction, $\Omega(x,y)$ is symmetric about the rational surface $x=0$. Thus, this is also the case for $\tilde{p}$. As discussed in Ref. 29, $\tilde{p}$ has to be antisymmetric about the rational surface $x=0$. It implies that $\tilde{p} = 0$ inside the island. In the case $S_{\rm int} \ne 0$, $S_{\rm int}$ is an odd localized function with respect to x. Thus, by taking into account also the structure of the island, for $\Omega \lesssim 1$, the integration of the source along a given flux surface $\Omega = \Omega_\cdot$ satisfies

$$\oint_{\Omega=\Omega_\cdot} d\xi S_{\rm int} = 0 \quad \text{and} \quad \oint_{\Omega=\Omega_\cdot} d\xi Z^{-1}(\Omega,\xi) S_{\rm int} = 0\,.$$

Thus, the average of Eq. (34) on the phase angle ξ on a flux surface gives

$$\frac{d}{d\Omega}\left(\oint_{\Omega=\Omega_\cdot} d\xi \sqrt{\Omega - \cos\xi}\, \frac{d\tilde{p}}{d\Omega}(\Omega,t)\right) = 0. \tag{36}$$

This equation is valid, both inside and outside the island, with the restriction $|x| \ge \delta/2$. It follows that, for any x_0 ($|x_0| > \delta/2$),

$$\bar{P}_0'(x_0) = \frac{d}{dx}\oint_{x=x_0} \frac{d\xi}{2\pi}\tilde{p}(x,\xi) = \oint_{x=x_0} \frac{d\xi}{2\pi}\frac{\partial\Omega}{\partial x}\frac{d\tilde{p}}{d\Omega}$$
$$= 4\sqrt{2}\frac{A_\pm(t)}{w(t)} I(x_0/w), \quad (37)$$

where

$$A_\pm(t) = \frac{d\tilde{p}_\pm}{d\Omega}(1,t)\int_0^{2\pi}\frac{d\xi}{2\pi}\sqrt{1-\cos\xi}, \quad (38)$$

$$\frac{w}{\sqrt{8}x_0} I(x_0/w) = \oint_{x=x_0} d\xi \frac{1}{\oint_{\Omega=\Omega(x_0,\xi)} d\tilde{\xi}\sqrt{\Omega - \cos\tilde{\xi}}}$$
$$= \oint_{x=x_0} d\xi \frac{1}{\oint_{\Omega=\Omega(x_0,\xi)} d\tilde{\xi}\sqrt{8\frac{x_0^2}{w^2} + \cos\xi - \cos\tilde{\xi}}}. \quad (39)$$

By symmetry, $d\tilde{p}_+/d\Omega(1) = d\tilde{p}_-/d\Omega(1)$, and thus, $A_+ = A_-$. The graph of $I(z)$ is drawn in Fig. 2 which shows that this function is almost constant, more precisely $I(z) = 1 \pm 10\%$. Note that obviously $\lim_{z\to+\infty} I(z) = 1$. In other words, one finds that $P_0'(x) = \bar{P}_0'(x)$ is constant in space. We will denote this spatially constant quantity $\nabla P_0^{\rm isl}$.

However, in Eq. (37), $\nabla P_0^{\rm isl} \equiv P_0'(x)$ is not explicitly linked to the intensity of the source and/or the turbulence. Yet, Eq. (36) is valid as far as the flux surface $\Omega = \Omega_\star$, over which the averaging is done, including points where $x \geq \delta$. Thus, considering the surface $\Omega_\star = 8\delta^2/w^2 - 1$, we obtain the interface condition

$$\nabla P_0^{\rm isl} = P_0'(x) = P_0'(\delta/2) \quad \text{for } |x| \geq l_\star/2. \quad (40)$$

As a result, the mean pressure gradient is constant in the vicinity of the island. In other words, the localization of the interchange modes all along the resonance imposes a pressure profile gradient over the entire width of the island. The source inhibits the complete flattening along the field lines as observed in Ref. 25.

VI. AN EXPLICIT LINK BETWEEN THE TURBULENCE AND PRESSURE PROFILE INSIDE THE ISLAND

Eqs. (29) and (28) give a formal link between the mean pressure gradient inside the island and the interchange turbulence activity. Moreover, so far, it has been assumed that only the active interchange scales drive the flattening through direct nonlinear coupling, which eliminates the possibility of cascade or more complex nonlinear transfer of energy. According to the model proposed, first, the dynamical flattening of the pressure inside the island occurs in the vicinity of the separatrices. Second, this process is fed energetically by the beating of interchange modes in the IB. Third, the pressure gradient is constant in space inside the island. One can thus write $P_0(w/2) = \nabla P_0^{\rm isl} w/2$. Using Eqs. (28) and (29), it follows that

$$\nabla P_0^{\rm isl}(t) = -v_\star + \beta\left\langle \frac{2}{w}\int_0^t dt \langle \bar{S}_{\phi',p'}\rangle_x \right\rangle. \quad (41)$$

The integrated term in brackets $\langle . \rangle_x$ corresponds to $\bar{S}_{\phi',p'}$. The coefficient $\beta \leq 1$ has been introduced in the model to specify that only a fraction of $\bar{S}_{\phi',p'}$ contributes to the evolution of flattening of the pressure in the vicinity of the separatrices. Indeed, because of the dissipative mechanisms, one should expect that a fraction $1-\beta$ maintains the flattening into the core of the island. In that context, one may expect this fraction to be of the order of the ratio between the radial extension of the IB and the size of the island. The spatial mean $\langle . \rangle_x$ corresponds to a mean value inside the interchange band IB. Of course, Eq. (41) should be validated numerically to evaluate the robustness of the model developed in this paper.

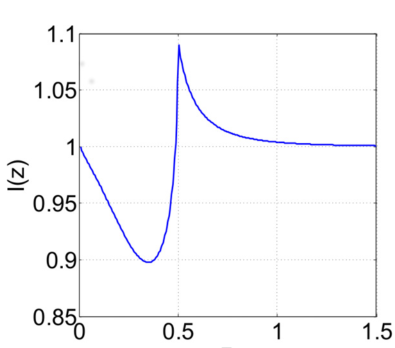

FIG. 2. Graph of the function I(z).

VII. CONCLUSIONS

In summary, we have developed a model to evaluate the impact of the interchange scales on the salient properties of TDMIs. This model calculates the turbulent perpendicular diffusivity and evaluates the critical island size above which one should observe a flattening inside the island. It also introduces a closure on the dynamics of the large scale. On the basis of a multiscale analysis, we obtain an anisotropic diffusion equation for the large scale pressure equation where interchange fluctuations act as a localized source. We show that the solution of this equation implies the presence of a constant mean pressure gradient inside the island. This is in agreement with previous numerical observations done in the limit $w/w_c \gg 1$.[25] Finally, we have proposed an explicit link between the level of the interchange source, the island size, and the pressure flattening inside the island. These analytical predictions were tested in Part II,[32] and we found a fairly good agreement with the numerical results.

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