LEARNING GEOMETRY AND MUSIC THROUGH COMPUTER-AIDED MUSIC ANALYSIS AND COMPOSITION: A PEDAGOGICAL APPROACH

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RÉSUMÉ

In this paper we will focus on pedagogical approaches via HexaChord, a computer-aided music analysis environment based on spatial representation of musical objects inspired by the *Tonnetz* and other geometric models. We will report some past experiences with children and adolescents during two Festival of Science. We will also suggest future laboratories for children.

1. INTRODUCTION

The results of learning geometry through interactive geometry software (IGS) and dynamic geometry software (DGS) such as Cabri and GeoGebra are well known in mathematics education [7]. At the same time, the employment of the information technology (IT) in music education shows an increasing interest [1]. Could pedagogical approaches on learning music and geometry be developed together through the IT tools? The same abstract notions in different concrete settings are useful for the development of abstraction capabilities. Although there are studies and experiments on higher education [2], interdisciplinary pedagogical approaches among mathematics, music and computer science are often scarce in primary and secondary school. We will analyze some pedagogical aspects via HexaChord [3, 4], reporting experiences with children and adolescents, and we will suggest future pedagogical laboratories.

2. GEOMETRIC MODELS IN MUSIC

2.1. Musical graphs and simplicial complexes

Geometric structures such as graphs and simplicial complexes are music-analytical tools commonly used to visualize and describe parsimonious voice leading and relations among musical objects. In music theory two different kinds of graphs are used: note-based and chord-based graphs. In the first ones each vertex is labeled by a pitch-class, in the second ones, by contrast, each vertex is labeled by a chord. Chord-based graphs are associated with note-based graphs by duality.

The most famous example of note-based graph is the *Tonnetz*. It is a graph in which pitch classes of twelve-tone equal temperament are organized along intervals of fifth in the horizontal axis, major and minor thirds in the diagonal axis. In this construction each triplet of distinct vertices, which are adjacent two by two, is a triangle representing a major or a minor triad. We can also define it as an infinite 2-dimensional simplicial complex, where 0-simplices represent pitch classes, and 2-simplices identify major and minor triads. The dual graph of the Tonnetz, known as *Chicken-wire torus* [6], is a chord-based graph in which vertices represent major and minor triads.
The edge-flips in the Tonnetz and the edges in the Chicken-wire torus represent the neo-Riemannian musical operations P (parallel), R (relative) and L (leading tone), commonly used to describe parsimonious voice leading [5].

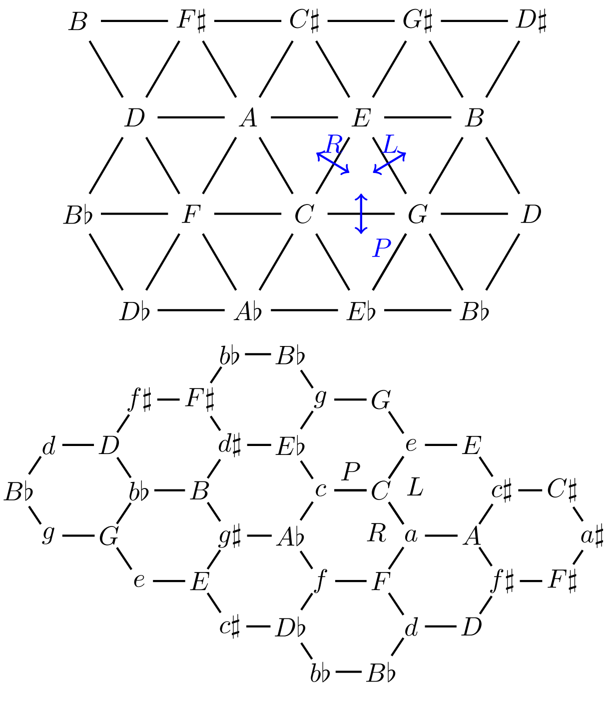

Figure 1. The Tonnetz (above) and the Chicken-wire Torus (below)

2.2. Circular geometric models in music

There are also two other well known geometric models in music: the *circle of fifths* and the *tone-clock*.
The first one is a circle in which the 12 major and minor scales, and their corresponding key signatures, are represented along a circle at a distance of fifth.

Sonia Cannas, *Learning Geometry and Music through Computer-aided Music Analysis and Composition : A Pedagogical Approach (poster)*, Journées d'Informatique Musicale (JIM 2018), Amiens, France, éd. L. Bigo, M. Giraud, R. Groult, F. Levé, pp. 143–146, 2018.

The *tone-clock*, also known as *musical-clock*, is a circular representation of the 12 pitch-classes at a distance of a semitone. Each pitch class is labeled with a number from 0 (in the position of the 12 in the clock) to 11, starting from $C = 0$ (see fig.3). It allows to represent chords: a 2-chord is a segment, a 3-chord a triangle, a n-chord a polygon of n vertices inscribed in a circle.

3. GEOMETRIC MODELS IN COMPUTATIONAL MUSIC ANALYSIS

There are several softwares in which musical graphs, simplicial complexes and other geometric models are integrated. One of them is HexaChord, a computer-aided music analysis environment based on spatial representation of musical objects, developed by Louis Bigo [3] [4].

The main interface is simple and easy to use, and it is represented in fig. 2. The first block is used to play music files in MIDI format. Below it, there is a second block used to visualize geometric models in music: the *Tonnetz* as a simplicial complex or other simplicial complex isomorphic to it (button "display/hide complex"), the musical-clock (button "circle 1"), the circle of fifths (button "circle 5"), and the voice-leading space (button "display graph"). During the execution of the MIDI file, it is possible to visualize in real time each chord in the different geometric models. Clicking on "trace on" it is also possible to visualize in the Tonnetz the chord sequence of the musical piece as a trajectory in the space.

In the part below the latter there is the block "Vertical compactness", to compute the compactness of the trajectory related to the musical piece in the *Tonnetz*. In the lower part there is the block "Trajectory Transformation", used to apply a geometric transformation to the original simplicial complex. One of them is the discrete translation, that musically corresponds to a transposition.

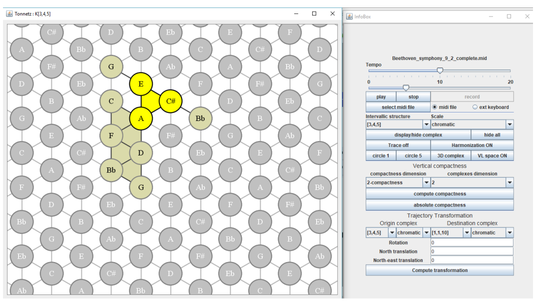

Figure 2. HexaChord

Moreover with Hexachord it is also possible to connect a piano keyboard and play/record musical pieces. All functionality are available also for the performance.

4. PEDAGOGICAL EXPERIENCES AND RESULTS

4.1. Festival of Science in Strasbourg

During the *Festival of Science* (2017) at the Vaisseau of Strasbourg, an IT-mathemusical laboratory for children has been organized. Different materials were proposed which included: a computer with HexaChord, a piano keyboard connected to it, loudspeakers, a musical-clock made with a wooden model (see fig.3), another wooden model representing both the *Tonnetz* and the *Chicken-wire Torus* (see fig.4). Many children played with the wooden musical-

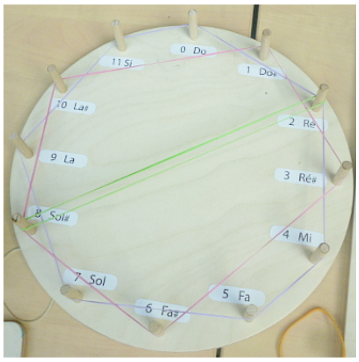

Figure 3. Wooden musical-clock. The chords are represented using a rubber band, and the twelve points are labeled with a number from 0 to 11 and the correspondent note

geometric models, more than the children who tried to play the piano and to understand HexaChord. Their attention focused more on wooden models for several reasons. First of all, since all children have already had gaming experiences with geometric shapes and rubber bands, they had no difficulties to recognize it as an entertaining game. On the contrary, the piano keyboard and HexaChord were new for most of them, and they require a first phase of comprehension in order to be used correctly. During the absence of a guide to the piano and the computer, some children tried to play with the piano looking the geometric models represented on HexaChord at the screen. Especially the younger ones did not understand how the musical sequences were displayed, then they moved on the geometric wooden models, more familiar to already known games. Moreover this underlines the different levels of learning musical and mathematical skills. The mathematical skills required in the construction of chords on geometric models are easier than the musical ones. Listening, playing, and understanding how the musical sequence is represented on the geometric model require a musical awareness that children begin to have towards around the age of 9. Furthermore, the children have to perform various tests for learning the latter, therefore it is necessary that they try one by one. On the other hand, activities on geometrical models can be carried out by several children at the same time. In the moments when a guide explained the representation of the musical sequences through Hex-

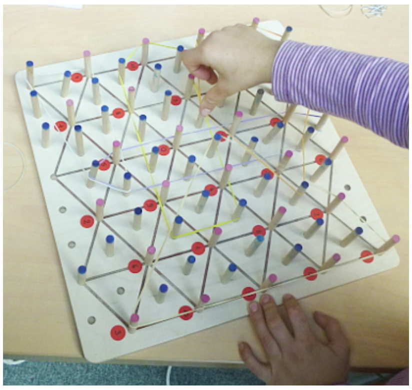

Figure 4. Wooden Tonnetz

aChord, the children showed interest and they understood very quickly its behavior. By testing them directly on the piano and observing the screen, their learning has become active. It is in fact well known that experiential learning is more efficient than passive learning such as reading or listening.

4.2. Festival of Science in Cagliari (Italy)

In the last edition of the *Festival of Science* in Cagliari (2017) I presented geometric models in music and their usefulness in music analysis through HexaChord. Most of the audience consisted of two classes of students belonging to the last year of a high school (19 years old) with a musical program. Thanks to their musical background, they easily understood the features of HexaChord and its utility for music analysis. During their musical studies they studied traditional music analysis, and their reactions to computational approaches to it based on geometric models were very positive. In addition to the surprise of knowing computer-aided music analysis, their greatest surprise was to discover that mathematics can also be a useful tool for music. Often, at school level, mathematics is taught and perceived as a set of many formulas and rules, whose meaning is unclear. Interdisciplinary learning can be an additional tool to overcome this misunderstanding of mathematics as arid subject without meaning. In conclusion, it was a very positive experience for the students: on the one hand they discovered a new type of musical analysis that brings together music, computer science and mathematics, on the other hand they discovered the beauty and utility of geometry.

5. FUTURE LABORATORIES FOR CHILDREN

The laboratories we are planning include some different activities, designed with different levels of difficultly. We want to propose game activities for children, in order to develop musical and geometric-spatial skills.
For all activities we need: a computer with which to use HexaChord, a piano keyboard connected to it, and a wooden musical-clock or a wooden Tonnetz. The aims of these activities are different. The children will become familiar with the musical-geometric models and the representation of the notes on them through HexaChord. They will learn the correspondence between number of notes of a chord and number of vertices of the correspondent figure on the musical-clock. They will start to become aware of the musical meaning of chord. Moreover they will observe that in the musical-clock and in the piano keyboard the notes are repeated in the same way, therefore they will become aware of the fact that one can work using only one octave. They will also begin to become familiar with the piano keyboard.

5.1. Reconstruction of chords

These laboratories are useful for children at the age of 9 and up.

5.1.1. First activity

The first activity consists in listening to a single chord on HexaChord, observing its geometric shape in the musical-clock integrated in the software and reproducing the latter in the wooden musical-clock with the rubber band. One can choose whether to prepare various MIDI files of individual chords to be played, or play them directly on the piano keyboard during the activity.
After that, children can do the contrary. In the wooden musical-clock a plane geometric figure is represented with the rubber band, the children have to reproduce it using the piano keyboard. The visualization on the musical-clock of HexaChord will help to this scope.

5.1.2. Second activity

The aim of the second activity is the same as the first. The children have to reproduce a chord in the wooden musical-clock but, this time, they will visualize the representation of the chord on the *Tonnetz*. To reach the aim, children can use the piano keyboard and observe their chord reproduced on the Tonnetz. This activity can be proposed if the children have already some musical background, otherwise it is recommended to carry out the first activity.

5.2. Musical sequences

This laboratory is useful for children at the age of 7 and up.
The aim is to recognize that a musical sequence in the *Tonnetz* and its transpositions have trajectories with the same shape. The children will listen a musical phrase (played in real time or previously recorded) and they will see the trajectory on the Tonnetz integrated in HexaChord. They will have to sing it and to represent the trajectory in the wooden Tonnetz. Afterwards, the musical phrase will be transposed, and the children will have to sing it and to represent the new trajectory. After some examples, the children will have to represent the new trajectory without displaying it on HexaChord and having only the initial note.

6. REFERENCES

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