Dynamic Model for Assignment in " Sky-car " Transit System – Spatial Interactions with other Common Transport Modes

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Abstract This chapter provides Lagrangian dynamic fluid model of the traffic of personal rapid maglev transporters or personal rapid transit (PRT). The transport system using these maglev transporters - named sky-cars or sky-podcars - operate in the style of demand-responsive system. The dynamical evolution of sky-podcar travelers' demand is modelled and the problem of relocation of podcars is addressed. In a multimodal transport mobility, we describe assignment in such transit system, and its spatial interactions with other common transportation systems.

1 Introduction

The transport demand increasingly grows nowadays in business and big cities in the world, leading to more traffic incidents and traffic congestion. Researchers, engineers and practitioners always construct and deploy many tools for the traffic management, and in different scales of details and with different representations such as link-based and node-based or network-based flows, estimation or control. We observe that operators services are not quite performent to provide non-congested and fluid traffic. And that leads to Many new transport systems are being built to addressed traffic issues. We notice autonomous and automated vehicles, V2V (vehicle-to-vehicle) and V2I (vehicle-to-infrastructure) communication for traffic management. This paper focuses on a new demand-responsive transportation system, namely sky-car transportation system. It is a new generation of Personal Rapid Transit (PRT) system [7]. skyTran is a new project for the construction of rapid

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vehicles (podcars or sky-cars) using maglev system relying on magnetic levitation rather than wheels or conventional motors – to propel podcars along sky railways lanes, resulting in nearly silent transportation. The maglev system makes safe, reliable high speed travel possible at up to 150 mph (240 kph); thus allowing large local, regional, and national networks to be built. Further skyTran extends the transport network area by creating a new traffic game area at twenty feet from the ground. In order to prevent congestion on the main sky railways, podcars will be pulled off onto the side-tracks for dropping off and picking up the passengers.

Since the chosen context is futurist, on the one hand, a study has been carried out on the reliability and the efficiency of the sky-car system, and on the other hand, its impacts and spatial interactions with the common already existing transport modes such as classic cars, taxi, bus, cycling, ride and share and carpoolings, tram and train. An implementation shall obtain an evaluation of the solution proposed in this paper in sections 2 and 3.

The contents of the paper are the following: we recall graph approach for modelling network in 2 with few extend components. The section 3 addresses the fluid model of the dynamic of podcars by describing motions of the personal rapid maglev transporters. Some assumptions on how the skyTran transportation system may work efficiently have made. We establish control laws in 3.1 and design algorithms for traffic control and good interactions with other transport modes.

2 Background

Short history on Personal Rapid Transit (PRT) systems

Various Personal Rapid Transit (PRT) systems were developed internationally in the 1970s, and years afer. In the United States, a first generation PRT system serving the community of Morgantown, West Virginia and the West Virginia University campus was installed by the United States Department of Transportation. Deployment of a second generation of such this technology has emerged such as Vectus, Ultra, Sky Web Express, Coaster, and Mister. All of these systems have a spotless safety record, proving that the automation software is extremely reliable. And they prove that PRT works better than rail or buses, as they show passengers never have to wait to board a vehicle most of the time. Using automated, low speed, wheeled podcars propelled by electric motors due to contact power brushes, the podcars are accessed through automatic doors from a platform serving both arriving and departing passengers.

Sky-car network - Notion of maglev-graph

In connection with transport graph as well, the maglev-graph includes a few different components. We describe this graph, denoted by G_M, by a quadruple of sets (N_M, A_M, L_M, P_M), where N_M is the set of all nodes (that are intersections, poles

of stations, departure and arrival portals at stations) of the maglev system, A_M is the set of arcs connecting two nodes of the same maglev-lanes. There are different maglev-lanes such as deceleration lanes, non-stop guideway, acceleration non-stop guideway (or acceleration lanes) which are vertically set up above the former. We denote by L_M the set of all the lanes. P_M denotes the set of all pair of portals (departure and arrival) that physically represent the stations or the sky-car stops locations. At any station there is an "off line guideway" which keeps sky-cars that are at rest waiting for passengers to board. There are diverging and merging zones, that is to say intersection or nodes, that allow podcars for switching from "acceleration lane" onto "low nonstop guideway", and inversely.

Traffic models for demand-responsive transports

Taxi services work as demand-responsive transport services, and they have been improved to a more flexible way to respond to transport demands. A succinte review on taxi and urban demand-responsive system are given in [1, 2, 4, 3, 6, 5, 8] where Multi-agent-based model of simulation has been build for urban dynamic traffic services to addressing the problem of traffic congestion, CO_2 emissions, air pollution, accidents financial costs, and other environmental damages.

3 Model of the system - Lagrangian coordinates

3.1 Sky-car motion formulation

To set ideas, the following assumptions have made on the functioning of the sky-car personal rapid transit system.

Assumptions

1. The number of sky-car stations in the system can be as high as desired to reach all transport demands.
2. Sky-cars do not takeover each other except at internal intersection where vertical avoidance may be possible for changing direction or for overtaking by using avoidance lane (which is an offline guideway).
3. A sky-car capacity is limited, taking finite number of passengers of the same profile.
4. Any sky-car is available for only one transport demand.
5. For one transport demand in the system, more than one passenger can enter in the sky-car at the starter station to a target destination station.
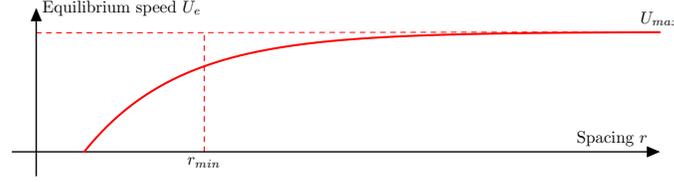
Fig. 1: Spacing-equilibrium speed fundamental diagram of sky-car motion

6. Two maglev-lanes of same level cannot intersect with each other at any station; maglev-lanes of different levels interconnect together with poles. That allow avoidance of collisions. The transfer of traffic is made between two lanes of different levels through poles by switches.
7. Generally, we assume that the considered demand-responsive transportation system is equipped with an adaptive cruise control that increases the driving comfort, reducing traffic accidents and increasing the traffic flow throughput.
8. We assume that accelerations of pods do not cost anything while they are offset by decelerations, in term of energy consumption of the system. However, we do not address this aspect which even look at the management of the system.

Sky-car motion with a control law - How skyTran works

Let S be the set of all stations $s \in S$ of the maglev-system. Let x denotes the position of podcar, $t \geqslant 0$ the time and a the podcar index. $\{x_a(t), t \in \mathbb{R}^+\}$ refers to the trajectory of the podcar $a \in \Lambda$, Λ being the total number of podcars of the transit system, and $x_a^j(t)$ referring to the position of a on the arc $(j) \in A_M$. Let $u_a(t)$ be the speed of the podcar a at the time t, $w_a(t)$ the acceleartion-deceleration of the pod a at the time t. Assuming that vehicles are labelled according to a snapshot of line from downstream to upstream, the podcar labels will increase with the position x. Therefore, $r_a(t) = x_{a-1}(t) - x_a(t)$ is the spacing between vehicle a and its leader $a-1$ at time t, and $v_a(t) = \dot{r}_a(t) = \dot{x}_{a-1}(t) - \dot{x}_a(t) = u_{a-1}(t) - u_a(t)$ is its relative velocity at the same time t.

Let us describe the dynamic of the vehicles along the sky transit network. Along the same lane, and on a line section without intersection and switches poles, the dynamic of sky-car is governed by the following system of equations, in simple case: $\forall t \geq 0, \forall a \in \Lambda, \exists!(j) \in A_M$ such that:

$$\begin{cases} x_a^j(t+1) = x_a^j(t) + \delta t u_a^j(t) \\ u_a^j(t) = \min\left(U_e\left(r_a^j(t)\right), u_{p_a}\left(x_a^j(t)\right)\right). \end{cases} \tag{1}$$

$U_e\left(r_a^j(t)\right)$ is the speed equilibrium relationship depicted by the figure Fig. 1. $u_{p_a}\left(x_a^j(t)\right)$ is the sky-car velocity profile that may depend on the vehicle a, the charge of the current link (j) or arc (j) and the mission of the podcar a. In the sake of capturing
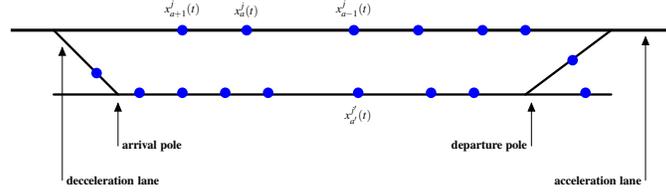
Fig. 2: Notations of sky-car following model

all situations, one may applied a following dynamic system (2) instead of the system of governing equations (1):

$\forall t \geq 0, \forall a \in \Lambda, \exists!(j) \in A_M$ such that,

$$
\begin{cases}
x_a^j(t+1) = x_a^j(t) + \delta t u_a^j(t) \\
u_a^j(t+1) = \min\left(u_a^j(t) + \delta t w_a^j(t), U_e\left(r_a^j(t)\right), u_{p_a}\left(x_a^j(t)\right)\right) \\
w_a^j(t) = f_a\left(r_a^j(t), u_a^j(t)\right).
\end{cases}
\tag{2}
$$

Let us briefly describe how merges and diverges work. Diverges are trivial. Let a be the first podcar on an upstream link (j). Let (d_1) be the next link pertaining to the mission of the podcar a, nd_1 the last podcar on this lane. Then the motion of the podcar a is given by (2) with $r_a^j(t)$ being the sum of the distance from a to the intersection plus the distance from the intersection to the podcar nd_1. This can be stated as: $r_a^j(t) = |x_a(t) - x_I|_{(j)} + |x_i - x_{nd_1}|_{(d_1)}$.

Let us now consider a merge with two upstream links (i_1), (u_2), (d) the downstream link, a_1 and a_2 the first podcars on links (i_1), (u_2) respectively, and a_d the last podcar on the downstream link. The two upstream podcars are liable to compete for passage through the intersection (I). The first issue to be solved is to determine which podcar will cross first the intersection (I). For each podcar a_i we calculate $\Delta t_i = |x_{a_i}(t) - x_I|_{(u_i)}/u_{a_i}^{u_i}(t)$, time required to reach the intersection. The velocity $u_{a_i}^{u_i}(t)$ is calculated following (2), with the distance $r_{a_i}^j(t)$ being the sum of the distance from a_i to the intersection plus the distance from the intersection to the podcar a_d. Once the order of passage is decided it is not changed. The trajectory of the first podcar to pass is calculated by (2) with respect to the podcar a_d. Let a_1 be this podcar. The trajectory of the second podcar is calculated with respect to the podcar a_d but with an additional term forcing passage in second. This term is applied as long as the podcar a_1 has not exited link (a_1). Thus the velocity of the podcar a_2 is $u_{a_2}^{u_2}(t+1)$:

$$
u_{a_2}^{u_2}(t+1) = \min\left(u_{a_2}^{u_2}(t) + \delta t w_{a_2}^{u_2}(t), U_e\left(r_{a_2}^{u_2}(t)\right), u_{p_a}\left(x_{a_2}^{u_2}(t)\right), \alpha \frac{|x_{a_2} - x_I|_{(a_2)}}{|x_{a_1} - x_I|_{(a_1)}} u_{a_1}^{u_1}\right).
\tag{3}
$$

3.2 Travelers' Demand optimization

Notations:

- k, ℓ, m : the attributes for stations or destinations or origines points of the trips.
- $T_{m\ell}(t)$: the demand of displacement from ℓ to m : $(\ell \to m)$ between the instants t and $t + \delta t$. So there is $T_{m\ell}(t)\delta t$ customers that want to travel from the station ℓ to the station m.
- $N_\ell(t)$: the number of sky-cars at the station ℓ.
- $K_\ell(t)$: the maximal capacity in term of number of travellers that can board at the station ℓ, and at the time t. The following relation holds : $N_\ell(t) * K_R = K_\ell(t)$, with K_R the residual capacity of any sky-car.
- $K_\sigma(t)$: the capacity of the sky-car σ at time t. This varaible changes only at stations due to the passengers boarding in σ and the passengers exiting the σ. So K_R constrains $K_\sigma(t)$ such that $K_\sigma(t) \leqslant K_R, \forall t$.
- $S_W(\ell, t)$: the ordered set of sky-cars waiting to board passengers at station ℓ.
- $D_\ell(t)$: the demand at the station ℓ and at the time t.
- $n_{\ell m}(t)$: the number of sky-cars that want to go to the stop m from the station ℓ at time t. This number represents the total demand at ℓ to m.
- $U_\ell(\sigma, t)$: the set of achievable sky-car stations from the station ℓ.
- $\tau_{\ell m}$: the travel time from ℓ to the achievable-station m.
- $v_{\ell m \sigma}$ the number of travellers from ℓ to m using the sky-car σ at the instant t. This the number of travellers that is transfering exactly from the stop ℓ to m. That is to say the performed travellers' demand by the system.
- Let denoted by $N_{\ell m}(t)$: the performed demand. Hence, the following holds : $N_{\ell m}(t) = \min\left(n_{\ell m}(t), K_{\ell m}(t)\right)$ with $N_{\ell m}(t) = \sum\limits_{\sigma \in S(\ell,t)\,;\,m \in U_\ell(\sigma,t)} K_\sigma(t)$.
- $\sigma \in S(\ell, t)$ means that σ is at the station ℓ at time t.
- $m \in U_\ell(\sigma, .)$ means that m is in the neighborhood of ℓ and is easily reachable from ℓ using the sky-car σ, when departurting at time t at ℓ.

At a station ℓ, the demand $D_\ell(t)$ at time t collapses in: $D_\ell(t) = \sum\limits_m \left\{ \left\lfloor \frac{T_{\ell m}(t)\delta t}{K_R} \right\rfloor + 1 \right\}$.

Therefore, the estimation of the travel time, from ℓ to m at time t, is suggested to the below optimization problem (4):

$$\min_{\sigma \in S(\ell,t)\,;\,m \in U_\ell(\sigma,t)} \sum v_{\ell m \sigma} * \tau_{\ell m}(\sigma, t)$$

$$\text{s.t.} \begin{cases} N_{\ell m}(t) = \sum\limits_{\sigma/m \in U_\ell(\sigma,t)} v_{\ell m \sigma}, & \forall \ell, \\[2ex] \sum\limits_m v_{\ell m \sigma} \leqslant K_\sigma, & \forall \sigma, \\[2ex] v_{\ell m \sigma} \in \mathbb{N}, & \forall \ell, m, \forall \sigma, \forall t. \end{cases} \tag{4}$$

The residues from ℓ to m, is $r_{\ell m}(t) = n_{\ell m}(t) - N_{\ell m}(t)$. This, added to the arrival of sky-cars at time t, denoted by $a_{\ell m}(t)$, we get $n_{\ell m}(t+1)$.

3.3 Mutimodality - Spatial interactions with other modes

Reactive dynamic assignment based on user mode and path choices during multimodal trips

Let considered multimodal transport system with the sky-car transit system. We assume that the whole multimodal system is semi-computerized, that is to say there is advanced information for travellers about traffic conditions, for each network mode. For the sky-car transit system, there is dynamic allocation of podcars on stations with respect to the following:

- the known cumulative demands,
- the stocks of sky-cars at stations on the off line guideway, and
- the foreseeable demands induced by passengers travel orders for some future times.

We propose a logit model for the modal choice in general, and user paths choice. In the case of this PRT system, it is only up to the system manager to make the path choice according to the origin, the destination and the traffic state of system itself. We assume only three choices for users in our considered multimodal system comprising only road network (by referring to private vehicles) and personal maglev network. Any *OD* pair could be joined with the below choices:

- mode m_1 : the use of road vehicle, then parking search availability to park and parking, and pedestrian walk for attending final destination, or
- mode m_2 : the use of sky-car and pedestrian walk, or
- mode m_3 : the use of modes m_1 and mode m_2.

For $\forall p \in \{1,2,3\}$ (*p* being the index of the mode), and for $\forall w = (o,d)$, the Logit-based rules are reduced to:

$$\pi_{od}^{p} = P[choice = m_p \mid (o,d) = w \in W] = \frac{\exp(-\theta C_{od}^{m_p})}{\sum_{p' \in \{1,2,3\}\,;\,(o,d)=w \in W}\ \exp\left(-\theta C_{od}^{m_{p'}}\right)} \quad (5)$$

The probability of choosing one mode *p* from an origin *o* to a destination *d* is set by (6) below:

$$\begin{cases} 0 \le \pi_{od}^{p} \le 1, & \forall p = 1,2,3\,, \forall w = (o,d) \in W, \\ \sum_{p=1}^{3} P[choice = m_p] = 1. \end{cases} \quad (6)$$

as well. $C_{od}^{m_p}$ is the cost of the mode m_p from origin *o* to destination *d*, cost which depends on the monetary cost, the predicted time that will be spent in the system (given by a system of information), and the search time of availability of parking-car (in case when using partially car mode), and the walking time.

4 Conclusions and perspectives

The implementation of the model and the sky-car demand responsive system shall show its real performance. The proposed system of equations which is a queue approach with adaptive cruise control on the dynamic of sky-cars. The model proposed in this chapter focussed on Lagrangian coordinates of the motion of sky-cars in the whole PRT system while discussing about multimodal trips taking in account the new responsive autonomated transport system. The relocation for sky-cars shall be addressed to respond to extra transport demands at stations were there are no available sky-cars to board passengers, and were re-routing of other sky-cars that are at rest in other locations is relevent.

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