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Root cause analysis of actuator fault based on invertibility of interconnected system

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Abstract: This paper addresses the problem of root cause analysis (RCA) of actuator fault. By considering an actuator as an individual dynamic subsystem connected with process dynamic subsystem in cascade, an interconnected system is then constituted. The fault detection and diagnosis (FDD) algorithm is carried out in actuator subsystem and aims at identifying the root causes of actuator faults. According to real plant, outputs of the actuator subsystem are assumed inaccessible and are reconstructed by measurements of the global system, thus providing a means of monitoring and diagnosis of the plant at both local and global level. A condition of invertibility of the interconnected system is first developed to guarantee that faults occurring in the actuator subsystem will affect the measured output of the global system distinguishably. For that, a necessary and sufficient condition is proposed to ensure invertibility of the interconnected system. Effectiveness of the proposed approach is demonstrated on an intensified HEX reactor.

Keywords: root cause analysis; RCA; interconnected system; invertibility; fault detection and diagnosis; FDD; actuator FDD; input reconstruction; left invertibility; control valve; actuator subsystem; process subsystem; adaptive observer.
Increasingly, industrial systems are becoming more integrated and complex. Each unit may consist of multiple components, including interconnected sensors, actuators, and system components. Modern control systems often consist of a large number of interacting components, which makes system analysis challenging. In these situations, a dynamical system or a subsystem can be analyzed at different levels down to the component level in estimating the reliability of the whole plant.

In this work, we focus on actuator faults. Many fault detection and diagnosis (FDD) methods are presented in the literature for nonlinear dynamic systems subject to actuator fault. One main category is system level-based diagnosis approach which aims at detecting and identifying fault existence and location from the view point of global system. Another common kind focuses on the field component level which aims at analyzing internal dynamics of a specific actuator.

Current FDD methodologies typically focus on system level where the major objective relates to performance supervision of the final product. In these methods, dynamics of field devices (i.e., actuator) is normally ignored, instead, they are treated as a component which is viewed as constant in the input/output coefficient matrix/function of the process system model. A key approach is based on residual generation. In Persis and Isidori (2001), a nonlinear FDI filter is designed for generating residuals using a geometric approach, theses residuals are affected by a particular fault and not affected by disturbances and the rest of faults. Besides, actuator fault isolation is studied by exploiting the system structure to generate dedicated residuals (see Methnani, 2013; Chen et al., 2005; Li and Dahhou, 2006).

In this approach, each residual, defined as the differences in the input/output coefficient matrix/function of the process system model. A key approach is based on residual generation. In Persis and Isidori (2001), a nonlinear FDI filter is designed for generating residuals using a geometric approach, theses residuals are affected by a particular fault and not affected by disturbances and the rest of faults. Besides, actuator fault isolation is studied by exploiting the system structure to generate dedicated residuals (see Methnani, 2013; Chen et al., 2005; Li and Dahhou, 2006).

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residuals breach their thresholds. Another approach different to residual generation is fault estimation or fault reconstruction which can determine the size, location and dynamics behaviour of the fault. The relevant literature on this topic has its roots on system inversion theory (Pinheiro and Araújo, 2013). There are several techniques available for fault reconstruction: sliding mode observers (Xu et al., 2012; Edwards et al., 2012; Martinez-Guerra et al., 2013), unknown input observers (Manaia et al., 2015; Blesa et al., 2014; Nagy-Kiss and Schutz, 2013; Zarei and Shokri, 2014), input reconstruction (Maksimov and Pandolfi, 2001; Yang et al., 2015; Szigeti et al., 2002; Schubert et al., 2012; Edelmayer et al., 2004). The key requirement is to completely decouple the faults from the effect of disturbances and also the input signals.

With respect to the above results, fault symptoms can be detected without having the capability to pinpoint the root causes of these faults. For example, Benaïssa et al. (2008) show that decrease of measured temperature in HEX/reactor may be due to decrease of fluid flowrate, this implies an actuator fault. With the help of above FDD algorithms, we can detect and isolate the actuator fault, but fail to realise the root cause of the fault in that particular actuator. The involving candidate root causes of this fault could be valve clogging, stop of utility fluid pump or leakage. A main reason that leads to this is that actuator is treated as a component, rather than a dynamic system in the process. Varying failure signatures are denoted by the changes of elements of the input matrix function. Therefore, the applications of the above FDD methodologies mainly limit to the existence and the isolation of a fault at a global level, while seldom further efforts are made for root cause analysis (RCA) of the detected fault. However, a single disturbance may indicate more than one candidate of root causes. Hence, the determination of these malfunctions of subcomponents, especially those small and incipient faults, before they become serious has important influences on safety and productivity.

In order to examine potential relationship from causes to effects of an actuator fault, efforts have been made to locate subcomponent faults for RCA from the view point of component level. There are two main kinds: self-validating and FDD method, however, both kinds are restricted to local level. The former one only self-diagnosis from local level as so called smart actuator (Yang and Clarke, 1999). Smart actuator is independent of FDD approach; it is an instrument that is designed to compensate for its own undesirable inherent characteristics, to correct from fault conditions, e.g., smart positioner in Bartyś et al. (2006). Also, there are FDD involved methods by which actuator are treated as a dynamic system. Sarosi et al. (2015) propose an algorithm to detect valve stiction for diagnosis of oscillation. Other artificial neural network-based methods for fault diagnosis can be found in Subbaraj and Kannapiran (2014), Zhang (2011) and McGhee et al. (1997). Model-based FDD approaches are proposed in Wickramatunge and Leepakprpeeda (2013), Roy et al. (1998) and Ding and Zhao (2014). For example, Puig et al. (2006) develops an interval observer-based passive fault detection method for control valve. But these available results are limited to actuator level and fail to monitor the whole process simultaneously. It is because the component level-based diagnosis method focuses only on management of the subsystems that use the local information, i.e., states/outputs of this subsystem. Another challenge when researching FDD methods locally is getting data from the subsystem being observed to develop and validate these methods, because physical measurements of actuators are often not possible or difficult owing to distances or rough environment.

In summary, there is a lack of results on actuator FDD that allows to implement advanced FDD methods capable of RCA at local level, as well as of system supervision at global level. Motivated by the above considerations, the main contribution of this paper is to combine RCA of fault from a local level with generally global system supervision. The objective is to explain how the behaviour of global output can be interpreted to identify root cause of actuator faults in actuator subsystem. We propose a left invertible interconnected nonlinear system structure with input reconstruction laws which is based on dynamic inversion, forming a new model-based actuator FDD and RCA algorithm. Actuator is viewed as subsystem connected with the process subsystem in cascade manner, thus identifying reasons of actuator faults with advancing FDD algorithm in actuator subsystem. Outputs of the actuator subsystem are assumed unmeasured and reconstructed by measured global outputs. The left invertibility of individual subsystem is required for ensuring that faults occurring in actuator subsystem can be transmitted to the process subsystem uniquely, and for reconstructing inputs of process subsystem, also outputs of actuator subsystem, from measured global outputs. The developed fault diagnosis algorithm is an effort to combine the strength of system level and the component level model-based fault diagnosis.

The paper is organised as follows. Section 2 describes the proposed strategy and formulates the problem. The main results of invertibility are presented in Section 3, involving definition of invertibility for both subsystems and the cascade system, and conditions for checking invertibility. In Section 4, a dynamic model for actuator subsystem is proposed and detection observer together with RCA observer is designed for actuator FDD and RCA. In Section 5, performance of the proposed scheme is validated and discussed on an intensified HEX/reactor. Finally, conclusion is made in Section 6.

2 Architecture of the fault diagnosis system

In this section, we describe the main idea of the proposed strategy and develop the involved conditions for achieving the objectives of actuator fault detection and RCA of the detected fault.
2.1 Structure of an interconnected system and system model

As shown in Figure 1, an interconnected system Σ is considered which is composed of two subsystems: actuator Σa and process Σp. The basic idea is to identify the fault V at local level, while monitoring the plant at global level. The fault vector V indicates candidates of root causes of actuator faults.

Assuming that the MIMO process subsystem is an input affine nonlinear system, and is described by:

\[
\begin{align*}
\dot{x}(t) &= f(x) + g(x)u_a, \quad x(t_0 = x_0) \\
y(t) &= h(x)
\end{align*}
\]

where \(x(t) \in \mathbb{R}^n\) is the state of the process subsystem, \(y(t) \in \mathbb{R}^m\) is the output of the global system, which is also the output of the process subsystem, \(u_a \in \mathbb{R}^m\) is the input of process subsystem, which is also the output of the actuator subsystem, \(u_a\) is inaccessible and is reconstructed from measurement of \(y(t)\). \(f(x)\) and \(g(x)\) are smooth vector field on \(\mathbb{R}^n\) and \(g(x)\) is smooth vector field on \(\mathbb{R}^m\).

Assuming that the actuator subsystem is a nonlinear system described by:

\[
\begin{align*}
\dot{x}_a &= f_a(x_a, u, \theta_a) \\
u_a &= h_a(x_a, u, \theta_a)
\end{align*}
\]

where \(x_a \in \mathbb{R}^n\) is the state, \(u \in \mathbb{R}^l\) is the input, \(u_a \in \mathbb{R}^m\) is the output of the actuator subsystem, which is also the input of the process subsystem, \(\theta_a \in \mathbb{R}^k\) represents the parameters of the actuator subsystem, when no faults are present \(\theta_a = \theta_{a0}\) (\(\theta_{a0}\) is the nominal parameter vector). \(\theta_s \in \mathbb{R}^k\) represents the parameters in the output equation (if a sensor fault occurs \(\theta_s \neq \theta_{s0}\), where \(\theta_{s0}\) represent the nominal parameters in the output equation).

Thus, an interconnected cascade system \(\Sigma\) is then constructed by these two subsystems \(\Sigma_a\) and \(\Sigma_p\) whereby the input is vector \(u\) while output vector is \(y\).

Considering fault vector \(V = (v_1, \ldots, v_k)\) as integration of either parameters faults in \(\theta_a, \theta_s\) or other disturbance signals, then a fault model of the actuator subsystem becomes:

\[
\begin{align*}
\dot{x}_a &= f_a(x_a, u) + g_a(x_a, u)V \\
u_a &= h_a(x_a, u) + l_a(x_a, u)V
\end{align*}
\]

where \(g_a(x_a, u)\) and \(l_a(x_a, u)\) are analytic functions of the system subject to multiple, possible simultaneously faults.

The detectability of one fault in nonlinear system (3) is defined as:

Definition 1: The fault \(v_i\) \((i = 1, \ldots, k)\) is said to be non-detectable if for \(v_i \neq 0\) the relation

\[
u_a(x_{a0}, x_a, u, 0) = u_a(x_{a0}, x_a, u, 0, \ldots, v_i, \ldots, 0)
\]

is satisfied; if not, the fault \(v_i\) is detectable.

2.2 Description of proposed RCA scheme

As shown in Figure 2, the main objective is to identify the occurrence of the fault \(v_i\) in (3) independently from each other whilst monitoring the whole plant at both local and global level, as required for reliable operation of complex and high interconnected process system. Fault \(v_i\) refers to resources of an actuator fault which is related with special physical meaning, e.g., \(v_i\) represents fault caused by leakage or valve clogging of an actuator. We propose an observer-based FDD strategy at local level of actuator subsystem, thus realising these resources \((v_i, i = 1, \ldots, k)\) of actuator fault. This procedure is denoted as RCA.

Since advanced FDD strategy is performed in actuator subsystem, observers are then fed with input \(u\) and output \(u_a\) in actuator subsystem as shown in Figure 2. However, one major challenge is that the only available measurement relies on the global system output \(y\). That is because online diagnosis of actuator component is often achieved by a remote supervisory diagnostic system, therefore, it is impractical to measure \(u_a\) in realistic industrial condition. Besides, in order to monitor the plant at global level, information of global output should be included when FDD algorithm is performed at local subsystem. As shown in Figure 2, if \(u_a\) can be estimated from the global measurement \(y\) uniquely, then the above two problems are solved. In that way, advanced FDD strategy performs validation of the nominal relationships of the system, using the input \(u\) and the output \(\hat{u}_a\) which is reconstructed from measured output \(y\). Hence, a means of monitoring and diagnosis of the whole plant at both local and global level is provided.
As mentioned above, a key feature, opportunity and technical challenge of the scheme is to obtain the conditions by which the information (useful input \( u \) or faults \( V \)) provided by actuator subsystem has distinguishable effect on system output \( y \). This can be seen as problem of input observability if \( V \) is viewed as unknown input of the system. While input or fault observability are equivalent with left-invertibility of system, as shown in reference (Edelmayer et al., 2004), the input can be uniquely recovered from output and the initial state if dynamical system is left-invertible.

**Definition 2:** The fault \( v_i \) \((i = 1, \ldots, k)\) is said to be detectable and has distinguishable effect on the system output \( y \) if the interconnected dynamic system is invertible.

Moreover, as shown in Figure 2, an essential requirement of the combination of individual actuator with an advanced diagnostic capability is the availability and reliability of \( u_a \), which is the output of the actuator subsystem, and also the input of the process system. This is considered as problem of input reconstruction, which is viewed as problem of system inversion (Edelmayer et al., 2004). Some issues of inversion concepts for input reconstruction were discussed, like Maksimov and Pandolfi (2001), and Szigeti et al. (2002).

In summary, based on the condition that both cascade system and process subsystem are invertible, reconstructed \( u_a \) and fault vector \( V \) has one to one relationship. In that case, advanced FDD strategy can be carried out in actuator subsystem to identify each component \( v_i \), thus, achieving RCA of the actuator fault. Therefore, the key problem is to provide condition for validating invertibility of the cascade system and the process subsystem.

### 3 System inversion and invertibility of an interconnected system

In this section, we develop the required notations and provide some background on the invertibility of nonlinear system. Based on that, we develop the definition for invertibility of an interconnected dynamic systems followed by the formal problem statement. After that, we give conditions to validate involved definitions. Finally, input reconstruction procedure is discussed.

#### 3.1 Nonlinear inverse interconnected system

In fact, for every control system with an input-output, we have an input-output map, and the left invertibility of the dynamical system basically refers to the injective and surjective of this input-output map.

**3.1.1 Nonlinear inverse system**

For simplicity, both subsystems are considered with the form as (1). Define the input-output map of process subsystem \( H_p: U_a \rightarrow Y \) for input function space \( U_a \) and the corresponding output function space generated by \( Y \). \( H_p \) maps an input \( u_a(.) \) to the output \( y(.) \) generated by the system driven by \( u_a(.) \) with an initial condition \( x_0 \). The left-invertibility is the problem of injective and bijective of the input-output map \( H_p \). We now proceed to the formal definition of invertibility for a nonlinear dynamical system as given in Isidori (1995).

**Definition 3:** Fix an output set \( Y \), the system (1) is invertible at a point \( x_0 := x(t_0) \), if for every \( y \in Y \), the equality \( H_p(u_a) = y \) implies that \( u_a = u_a \).

#### 3.1.2 Nonlinear inverse interconnected system

The inversion of an interconnected system can be thought of doing the composition invertible mapping and individual input recovery. Consequently, the basic idea for solving the invertibility problem is first to do the composition mapping by utilising the relationship among the outputs and the states of the subsystems, and then use the nonlinear structure algorithm for the corresponding subsystem to recover the input.

Considering the input-output map of actuator subsystem is \( H_a: U \rightarrow U_a \) for some input function space \( U \), and the corresponding output \( u_a \) with an initial condition \( x_{a0} \). Define composition maps \( H_aH_p: U \times U_a \rightarrow Y \) as input–output map of the interconnected system, which maps an input \( u(.) \) to the output \( y(.) \) generated by the system driven by \( u(.) \) with initial conditions \((x_{a0}, x_0)\). We now first extend the definition of invertibility to an interconnected system.

**Definition 4:** Fix an output set \( (U_a, Y) \). An interconnected system is invertible if for every \( y \in Y \) and \( u_a \in U_a \), the equality \( H_p(u_a) = u_a \) and \( H_p(u_a) = u_a \) implies that \( u_a = u_a \) and \( u_a = u_a \).

The invertibility formulated in Definition 4 may fail to hold in two ways:

a. there exits two different inputs \( u_1, u_2 \) that yield the same output \( y \).

b. there exits two different inputs \( u_1, u_2 \) that yield the same output \( u_a \).

The former refers to be non-invertibility of process subsystem while the latter possibility is due to non-invertibility of actuator subsystem.

#### 3.2 On the condition of left invertibility of an interconnected system

For checking the invertibility of a dynamic system, geometric approach (Persis and Isidori, 2001) and differential algebraic approach (Martinez-Guerra et al., 2013) are proposed.

**3.2.1 On the condition of invertibility of subsystem**

We employ the differential output rank to check system invertibility. Details of differential output rank definitions can be, for example, found in Martinez-Guerra et al. (2013).
Differential output rank is defined as the maximum number of outputs that are related by a differential polynomial equation which is independent of \( x \) and \( u \) (state and input respectively). A simple way to compute differential output rank of system (1) is defined by Corollary 1.

**Corollary 1:** Suppose \( p \) outputs, and if there exists \( r \) possible differential polynomial equations of the form

\[
P_i(y_1, y_2, ..., y_p, y_1, y_2, ..., y_p, y_1, y_2, ..., y_p, ...) = 0 \tag{4}
\]

then the differential output rank \( \rho \) is defined as \( \rho = \rho - r \), which implies \( (p - r) \) independent outputs.

**Theorem 1:** A system is left-invertible if, and only if the differential output rank \( \rho \) is equal to the total number of inputs, e.g., \( \rho = m \) in (1).

**Remark 1:** If a subsystem has more inputs than outputs, it cannot be left invertible. On the other hand, if it has more outputs than inputs, then some outputs are redundant (as far as the task of recovering the input is concerned). Thus, the case of input and output dimensions being equal is the most interesting case, e.g., \( m = p \) in (1).

### 3.2.2 On the condition of invertibility of interconnected system

From Definition 4, non-invertibility of each subsystem results in non-invertibility of the interconnected system. We now give a sufficient and necessary condition on the subsystem dynamics so that the interconnected system is invertible for set \( U, U_a \) and \( Y \).

**Theorem 2:** Consider the interconnected system \( \Sigma \) which consists of two subsystems: actuator \( \Sigma_a \) and process \( \Sigma_p \) subsystems, and an output set \( (U_a, Y) \). The interconnected system is invertible at \( (x_0, x_a_0) \) over \( (U_a, Y) \). If and only if each subsystem actuator \( \Sigma_a \) and process \( \Sigma_p \) is invertible at \( x_a_0 \) over \( U_a \), and \( x_0 \) over \( Y \), respectively.

**Proof:** Consider \( H_i \) as the input-output mapping of actuator \( \Sigma_a \) subsystem, while \( H_p \) is the input-output mapping of process \( \Sigma_p \) subsystem. Then, the input-output mapping of the interconnected system is the composition \( H_aH_p \).

a  **(Sufficiency):** invertibility of a dynamic system refers to bijective of the input-output mapping. Since both subsystems are invertible, the corresponding mapping \( H_a \) and mapping \( H_p \) are bijective mapping. Because composition of two bijective mappings is a bijective mapping, so input-output mapping \( H_aH_p \) of the series system is bijective. Then, the series system is invertible.

b  **(Necessity):** supposed the process subsystem is not invertible, while the actuator subsystem is invertible. Then, for the actuator subsystem (2), there exists two distinct inputs \( u_1 \neq u_2 \) that yield two distinct outputs \( H_a(u_1) = u_{a1}, H_a(u_2) = u_{a2}, u_{a1} \neq u_{a2} \). However, for the process subsystem (1), these two distinct inputs \( u_{a1} \neq u_{a2} \) may proceed two equal output

\[
H_p(u_{a1}) = H_p(u_{a2}) = y. \text{ Therefore, for the series system, these two distinct inputs } u_1 \neq u_2 \text{ result in two equal outputs } H_aH_p(u_1) = H_aH_p(u_2) = y. \text{ Thus, the series system is not invertible.}
\]

### 3.3 Input reconstruction procedure

After verifying the invertibility of individual subsystems, we will be able to construct an interconnected inverse system that can recover the original input uniquely from the global measurement, by which implies that each original input affect the global output distinguishably. In fact, if a system is invertible, the structure algorithm allows us to express the input as a function of the output, its derivatives and possibly some states. More details are given in Isidori (1995).

For invertible dynamic system described by (1), the relative degree \( r_i \) of the output \( y_i \) with respect to the input vector \( u_a \) is the smallest integer which is defined by:

\[
a_i \iff_j \exists r \iff \exists (L_f(x)h(x)) \iff \exists (L_g(x)h(x)) \iff \exists (L_h(x)h(x)) = f(x)
\]

To derive an expression for \( u_a(t) \) as a function of states and output in (1), following the inversion algorithm given by Isidori (1995), we first need to compute the derivatives of \( y_i, i = 1, ..., m \). We have:

\[
\begin{align*}
y^{(1)}(t) &= \frac{\partial h_i(x)}{\partial x} x(t) + \frac{\partial h_i(x)}{\partial x} (f(x) + g(x)u_a) \\
&= L_{x_0}^1 h_i(x) + \sum_{j=1}^{m} L_{x_0}^1 L_{x_0}^0 h_i(x) u_{aj}
\end{align*}
\]

If \( r_i = 1 \), then:

\[
y^{(1)} = L_{x_0}^1 h_i(x)
\]

We should go on this differential procedure, we have:

\[
y^{(k)} = L_{x_0}^k h_i(x); 0 \leq k < r_i - 1
\]

Until when we reach the relative degree \( r_i \), we then obtain:

\[
y^{(k)} = L_{x_0}^k h_i(x) + \sum_{j=1}^{m} L_{x_0}^j (L_{x_0}^{k-1} h_i(x)) u_{aj}, i = 1, ..., m
\]
Given the output $y_i$ for the invertible dynamic system (1),
calculating expressions for their derivatives, we get:

$$
\begin{bmatrix}
   y_1^{(1)} \\
   \vdots \\
   y_m^{(n)} \\
\end{bmatrix}
= 
\begin{bmatrix}
   L_1^1 h_1(x) \\
   \vdots \\
   L_m^1 h_m(x) \\
\end{bmatrix}
\begin{bmatrix}
   v_1 \\
   \vdots \\
   v_k \\
\end{bmatrix}
$$

$$
\begin{bmatrix}
   y_1^{(n)} \\
   \vdots \\
   y_m^{(n)} \\
\end{bmatrix}
= 
\begin{bmatrix}
   L_1^m h_1(x) \\
   \vdots \\
   L_m^m h_m(x) \\
\end{bmatrix}
\begin{bmatrix}
   v_1 \\
   \vdots \\
   v_k \\
\end{bmatrix}
$$

(5a)

the equation (5a) can be solved for $u_a$ to obtain:

$$
u_a = 
\begin{bmatrix}
   L_1^1 h_1(x) \\
   \vdots \\
   L_m^1 h_m(x) \\
\end{bmatrix}
\begin{bmatrix}
   y_1^{(1)} \\
   \vdots \\
   y_m^{(1)} \\
\end{bmatrix}
- 
\begin{bmatrix}
   \vdots \\
   \vdots \\
   \vdots \\
\end{bmatrix}
\begin{bmatrix}
   L_1^m h_1(x) \\
   \vdots \\
   L_m^m h_m(x) \\
\end{bmatrix}
\begin{bmatrix}
   y_1^{(n)} \\
   \vdots \\
   y_m^{(n)} \\
\end{bmatrix}
$$

(5b)

Although inputs of both subsystems can be reconstructed,
only process subsystem needs to reconstruct the input $u_a$ in
the proposed methodology.

4 Modelling of actuator subsystem and FDI strategy

4.1 Actuator modelling

The modelling of an actuator and possible faults is based on
understanding the physical process. The actuator
encompasses pneumatic valve in this work. Up to now,
modelling of pneumatic actuators has benefited from
researches in automation, like in Bartyś et al. (2006). The
important aspect of these approaches is the development of
a model that describes the cause and effect relationships
between the system variables using state estimation or
parameter estimation techniques. Roy et al. (1998) provide
an overview of various models of the fault mode of control
valve.

From the interesting references, despite different
actuators and various considerations, the proposed dynamic
models benefit from a parameter-affine characteristic
described in (2). If viewed unexpected variations in
parameters as unknown inputs and denoted by vector
$V = (v_1, v_2, \ldots, v_k)$, then we get an input-affine dynamic
fault model as (3). The objective of this work is to identify
$v_i$ in (3) based on system inversion. From Remark 1, we
need inputs and outputs in (1) and (3) are equal, that implies
$m = k$.

4.2 Actuator FDD and RCA

The actuator model (2) and the fault model (3) are
then incorporated with the advanced FDD, and
aimed at observing different types of postulated faults
($v_i$, $i = 1, \ldots, k$) which characterise how abnormal
measurements are related to root causes of actuator faults.

4.2.1 Detection observer design

Considering the actuator subsystem model (2), a bank of
observers in Li and Dahhou (2008) are constituted by:

$$
\begin{bmatrix}
   \hat{\xi}_j \in \mathbb{R}^n \\
   \hat{\theta}_j \in \mathbb{R}^m \\
\end{bmatrix}
= 
\begin{bmatrix}
   \mathbf{f}_j(\hat{\xi}_j, u_j, \theta_j) + \mathbf{K}_j(\tilde{u}_j - \hat{u}_j) \\
   \hat{u}_j = h_j(\hat{\xi}_j, u_j, \theta_j) \\
\end{bmatrix}
$$

(6)

where $j$ denotes the $j$th actuator, $\xi = \mathbb{R}^n$ is the state vector
of the $j$th observer, $\tilde{u}_j \in \mathbb{R}^m$ is the reconstructed input from
measured vector $y$, and $\hat{u}_j \in \mathbb{R}^m$ is the estimated output
vector. $K_j$ is the gain matrix, $\theta_j$ is the nominal value of
parameters of $j$th actuator, while $\theta_{j0}$ is the nominal
parameters in the output equation.

The residual we use to determine fault existence is:

$$
r_j(t) = \|\tilde{u}_j - \hat{u}_j\| 
$$

(7)

Through proper designing the matrix $K$, the equilibrium $r_j(t)$ is asymptotically stable. Then, through checking the bank of
$r_j(t)$, if any of them breaches the threshold, fault detection is
achieved.

4.2.2 RCA observer design

After a fault is detected, next task is to identify the causes of
the fault. Considering the fault model (3), by analysing the
fault resources $v_i$, $i = 1, \ldots, k$, we can recognise the root
cause of the detected fault. Through adaptive diagnostic
techniques, m banks of k observers proposed in Fragkoulis
et al. (2011) corresponding for all possible faulty models are
constructed and extended as below:

$$
\begin{bmatrix}
   \dot{\xi}_j \in \mathbb{R}^n \\
   \dot{\theta}_j \in \mathbb{R}^m \\
\end{bmatrix}
= 
\begin{bmatrix}
   \mathbf{f}_j(\hat{\xi}_j, u_j) + \sum_{l \neq j} g_{ij}(\hat{\xi}_j, u_l)\theta_{l0} + g_{ij}(\hat{\xi}_j, u_l)\nu_j \\
   + H_{ij}(\hat{u}_j - \hat{\xi}_j) \\
\end{bmatrix}
\begin{bmatrix}
   \nu_j \\
   \nu_j = 2\gamma_{ij}(\hat{u}_j - \hat{\xi}_j)^T P_{ij} \theta_{ij} \\
\end{bmatrix}
\begin{bmatrix}
   \tilde{u}_j \\
   \tilde{u}_j = h_j(\hat{\xi}_j, u_j) \\
\end{bmatrix}
$$

(8)

where $j$ denotes the $j$th actuator, $i$ is $i$th observer corresponding to
the $i$th fault resource candidate. $\hat{\xi}_j \in \mathbb{R}^n$ is the estimated
state vector of $i$th observer for $j$th actuator, $\nu_j$ is the fault
estimation of $v_i$ of $j$th actuator, and $\nu_j$ is the estimated
output vector of the $i$th observer for $j$th actuator. $\tilde{u}_j$ is
reconstructed output of $j$th actuator from $y$, $u_j$ is the input of
$j$th actuator. $\theta_{ij}$ is the nominal value of parameters in $j$th
actuator, subscript $l \neq i$. $f_{ij}, h_j, g_{ij}$ are analytic functions of
$j$th actuator. $H_{ij}$ is a Hurwitz matrix that can be chosen
freely with a goal to increase as much as possible the
dynamic of the observer, $\gamma_i$ is a design constant and $P_i$ is a positive definite matrix. We can calculate the matrix $P_i$ with the help of (9); where $Q_{ij}$ is a positive definite matrix that can be chosen freely.

$$H_i^T P_i + P_i H_i = -Q_{ij} \quad (9)$$

Denote $e_i^j(t)$ as the tracking error of the $i^{th}$ observer for $j^{th}$ actuator as:

$$e_i^j(t) = \hat{u}_i^j - u_i^j$$

We define the RCA residual as:

$$s_j(i) = \| e_i^j(t) \|, \quad 1 \leq i \leq k, 1 \leq j \leq m \quad (10)$$

These residuals are designed to be ‘less’ sensitive to a particular fault cause that comes from a specific actuator and sensitive to all the others actuator fault causes. For the $j^{th}$ actuator, if a fault is caused by the $i^{th}$ fault cause, then the $i^{th}$ RCA residual will leave its threshold and never comes back to zero again, but the other ($k – 1$) residuals will stay below their thresholds. So every RCA observer is designed in such a way to identify a possible fault cause in a specific actuator. The RCA of detected faults is then achieved.

5 Application to an intensified continuous
HEX/reactor

A case study is developed to test the effectiveness of the proposed scheme on an intensified HEX reactor. The pilot is made of three process plates sandwiched between five utility plates. More relative information could be found in Théron et al. (2014).

5.1 System modelling
5.1.1 Process subsystem modelling

Generally speaking, intensified HEX reactor is a continuous reactor (Théron et al., 2014), then flow modelling is therefore based on the hypothesis used for modelling of real continuous reactors, represented by a series of $N$ perfectly stirred tanks (cells). The state and evolutions of the homogeneous medium circulating inside a given cell are described by the heat balances. $T_p, T_u$ represent temperature of process fluid and utility fluid respectively. All the values of the model parameters can be found in Zhang et al. (2015). The constants and physical data used in the pilot are given in Table 1.

<table>
<thead>
<tr>
<th>Constant</th>
<th>Description</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$hA$</td>
<td>Overall heat transfer coefficient</td>
<td>214.8</td>
<td>W.K$^{-1}$</td>
</tr>
<tr>
<td>$A$</td>
<td>Reaction area</td>
<td>$4e^6$</td>
<td>m$^2$</td>
</tr>
<tr>
<td>$V_p$</td>
<td>Process fluid volume</td>
<td>2.685e$^-5$</td>
<td>m$^3$</td>
</tr>
<tr>
<td>$V_u$</td>
<td>Utility fluid volume</td>
<td>1.141e$^-4$</td>
<td>m$^3$</td>
</tr>
<tr>
<td>$\rho_p, \rho_u$</td>
<td>Fluid density</td>
<td>1,000</td>
<td>kg.m$^{-3}$</td>
</tr>
<tr>
<td>$c_{ppt}, c_{pu}$</td>
<td>Specific heat of the fluid</td>
<td>4,180</td>
<td>J.kg$^{-1}$.k$^{-1}$</td>
</tr>
<tr>
<td>$T_{ui}$</td>
<td>Utility fluid input temperature</td>
<td>15.6</td>
<td>°C</td>
</tr>
<tr>
<td>$T_{pi}$</td>
<td>Process fluid input temperature</td>
<td>76</td>
<td>°C</td>
</tr>
</tbody>
</table>

5.1.2 Actuator description and Fault scenario

In this work, we need actuators to control flowrate of both process fluid and utility fluid. Pneumatic control valve is employed to act as actuator in this system. The main function of this pneumatic valve is to regulate the flow rate in a pipe line. By application of Bernoulli’s continuous flow law of incompressible fluids, we have:

$$F = C_h f(X) \sqrt{\frac{\Delta P}{sg}}$$

where $F$ is flow rate (m$^3$s$^{-1}$), $\Delta P$ is the fluid pressure drop across the valve (Pa), $sg$ is specific gravity of fluid and equals 1 for pure water(kgm$^{-3}$), $X$ is the valve opening or valve ‘lift’ ($X = 1$ for max flow), $C_h$ is valve coefficient (m$^3$s$^{-1}$) which is given by manufacturer, $f(X)$ is flow characteristic which is defined as the relationship between valve capacity and fluid travel through the valve. There are three flow characteristics to choose from: linear valve control; quick opening valve control; equal percentage valve control. For linear valve, $f(X) = X$, the valve opening is related to stem displacement. In Bartyś et al. (2006) and Roy et al. (1998), a pneumatic control valve has a dynamic model of the type:

$$p_c A_s = m \frac{d^2 X}{dt^2} + \mu \frac{dX}{dt} + kX$$

Define the state vector as $x^T = [x_1, x_2]^T = [T_p, T_u]^T$, the control input $u_i^j = [u_{ia}, u_{ib}]^T = [F_p, F_u]^T$, the output vector of measurable variables $y^T = [y_1, y_2] = [T_p, T_u]^T$, then above two the equations can be rewritten in the following state-space form:

$$\begin{align*}
\dot{x}_1 &= \frac{u_{ia}}{V_p} (T_{pi} - x_1) + a(x_2 - x_1) \\
\dot{x}_2 &= \frac{u_{ib}}{V_u} (T_{ui} - x_2) + b(x_1 - x_2)
\end{align*} \quad (11)$$

where $a = \frac{hA \rho_p c_{ppt} V_p}{\rho_p c_{pp}}, b = \frac{hA \rho_u c_{pu} V_u}{\rho_u c_{pu}}$, $y_1 = x_1, y_2 = x_2$. 

Table 1

<table>
<thead>
<tr>
<th>Physical data used in the pilot</th>
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<tbody>
<tr>
<td>Constant</td>
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</tr>
<tr>
<td>$hA$</td>
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<tr>
<td>$\rho_p, \rho_u$</td>
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<tr>
<td>$c_{ppt}, c_{pu}$</td>
</tr>
<tr>
<td>$T_{ui}$</td>
</tr>
<tr>
<td>$T_{pi}$</td>
</tr>
</tbody>
</table>
where $A_d$ is the diaphragm area on which the pneumatic pressure acts, $p_c$ is the pneumatic pressure, $m$ is the mass of the control valve stem, $\mu$ is the friction of the valve stem, $k$ is the spring compliance, and $X$ is the stem displacement or percentage opening of the valve.

Totally, there are 19 kinds of faults that may occur as shown in Bartyś et al. (2006); the causes of each fault are given in Manninen (2012).

Four kinds of fault influencing dynamics of the valve are considered in this paper:

1 fault f1: valve clogging, it occurs when the servomotor stem is blocked by an external event of a mechanical nature. It results in limitation of the piston movement in both direction, and therefore the flow cannot drop below a certain value.

2 fault f2: change of pressure drop across valve, results in $\Delta p + \Delta p'$.

3 fault f3: bellow-seal leakage due to leak, resulting in $p_cA + \Delta p$ changed; valve internal leakage is a common malfunction with industrial control valves. The causes of such leakage are numerous, including damaged plug or seat, insufficient seat load or reduced spring rate.

4 fault f4: control valve diaphragm perforation due to pinhole cracks in the periphery, resulting in $k$ changed. According to Bartyś et al. (2006), for most parts, single actuator faults are observed in industrial practice whilst multiple faults rarely occur. This characteristic is suited to the situation considered by the scheme proposed in this paper.

5.1.3 Actuator subsystem modelling

Define subscript 1 to denote the actuator of process fluid, then parameters $X_1, p_{c1}, \Delta P_1, k_1, \mu_1, F_1$ represent the opening percentage of the valve, the pneumatic pressure, and the fluid pressure drop across the valve, the spring compliance process fluid, the friction of the valve stem and flowrate of the process fluid. And denote subscript 2 for the utility fluid, then one gets parameters $X_2, p_{c2}, \Delta P_2, k_2, \mu_2, F_2$ which represent the same physical meaning for actuator of utility fluid. Since the control valves utilised in this work are linear, then

\[
\begin{align*}
(f_1) & f_1 = f(X_1) = f(X_2) = X_2. \\
\text{In order to evaluate the proposed strategy, the dynamic model should have the form given by (2), so the vector state } x_a, \text{ input } u, \text{ and output } u_a \text{ are defined as:} \\
x_a^T = [x_{a1} \ x_{a2} \ x_{a3} \ x_{a4} \ x_{a5} \ x_{a6}] \\
u^T = [u_1 \ u_2] = [p_{c1} \ p_{c2}], \\
u_a^T = [F_1 \ F_2] = [C_v \sqrt{\Delta P_1}X_1 \ C_v \sqrt{\Delta P_2}X_2].
\end{align*}
\]

The actuator subsystem is then described by four states, two inputs and two outputs, as:

\[
\begin{align*}
x_a = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -k_1/m & -\mu_1/m & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -k_2/m & \mu_2/h \\ \end{bmatrix} x_a + \begin{bmatrix} \Delta P_1 \\ 0 \\ 0 \\ \Delta P_2 \\ \end{bmatrix} u
\end{align*}
\]

As above description shown, actuator fault may be caused by parameters $\mu, k, \Delta p$, then there are eight related parameters in two actuators: $k_1, \mu_1, k_2, \mu_2, p_{c1}, p_{c2}, \Delta P_1, \Delta P_2$. The process of RCA is to identify abnormal variations of these eight parameters.

For the sake of RCA purpose, we should rewrite the above dynamic model into fault model as format (3). Therefore, we extend the state, input and output vector as follows:

\[
\begin{align*}
x_a^T = [x_{a1} \ x_{a2} \ x_{a3} \ x_{a4} \ x_{a5} \ x_{a6}] \\
\begin{bmatrix} X_1 \ dX_1/dt \ X_2 \ dX_2/dt \ C_v \sqrt{\Delta P_1}X_1 \ C_v \sqrt{\Delta P_2}X_2 \end{bmatrix} \\
V = [v_1 \ v_2 \ v_3 \ v_4 \ v_5 \ v_6 \ v_7 \ v_8]^T \\
= [k_1 \ \mu_1 \ k_2 \ \mu_2 \ p_{c1} \ p_{c2} \ \Delta P_1 \ \Delta P_2]^T \\
u_a^T = [u_{a1} \ u_{a2}] = [F_1 \ F_2] = [x_{a5} \ x_{a6}] \\
= [C_v \sqrt{\Delta P_1}X_1 \ C_v \sqrt{\Delta P_2}X_2].
\end{align*}
\]

Then, the actuator subsystem is with six states, eight unknown inputs and two outputs. These two outputs are unmeasured which need to be constructed by the global measured outputs. The augmented actuator subsystem is:

\[
\begin{align*}
x_a = f_a(x_a) + \sum_{i=1}^{8} g_{ai}(x_a)v_i \\
u_a = C_xu_a
\end{align*}
\]

From (13), there are two outputs. According to Remark 2, in order to guarantee invertibility of (13), there should be two inputs maximum. However, more than two parameters are in (13), therefore, we can only recognise two possible parameters faults simultaneously.

5.2 Invertibility checking and input reconstruction

The foundation of the proposed scheme is based on invertibility of an interconnected system. Thus, invertibility
of individual subsystem is necessary, that means invertibility of both process and actuator subsystem in this work respectively. Besides, the key difficulty and technical challenge is to reconstruct output of actuator system from output of the interconnected system.

5.2.1 Invertibility checking:
To check if the process subsystem, modelled by (11), is invertible, we have to check whether the output differential rank is equal to the number of the inputs. There are two inputs in this work: flowrate of process fluid $F_p$ and flowrate of utility fluid $F_u$ which are denoted by $u_{a1}$, $u_{a2}$ in (11) respectively. To compute the output differential rank, we first need to derive an explicit expression for the input in terms of the output $y$ by computing the derivatives of $y$. When it comes to (11), two outputs are temperature of process fluid $T_p$ and utility fluid $T_u$, which are denoted by $y_1$, $y_2$ in (11), respectively. As above mentioned, there are two inputs in this work, the computed output differential rank is equal to the total number of inputs, then the process subsystem is invertible. The same steps could easily be obtained in actuator subsystem. Therefore, the global series system is invertible.

5.2.2 Input reconstruction
Thanks to the invertibility of the process subsystem, we can reconstruct the inputs as a function of the output and its derivatives. From the above equation, an expression for the two inputs can be derived as $\tilde{u}_a = [\tilde{u}_{a1} \ \tilde{u}_{a2}]$:

$$\tilde{u}_{a1} = \frac{V_p}{T_{p0} - y_1} (\dot{y}_1 - ay_2 + ay_1)$$

$$\tilde{u}_{a2} = \frac{V_u}{T_{u0} - y_2} (\dot{y}_1 - by_2 + by_1)$$

5.3 Detection observer and RCA observer design

5.3.1 Detection observer design
There are two actuators, then a bank of two detection observers are generated with the form given by (6).

$$\begin{align*}
1 \leq i \leq 2, &
\dot{x}_{ai} = f_a (\dot{x}_u) + g_a (\dot{x}_u) u - K (\tilde{u}_a - \tilde{u}_{ai}) \\
u_{ai} &= C x_{ai}
\end{align*}$$

(14)

Objective of detection observer is to recognise existence of actuator fault. Therefore, we suppose that all the conditions in the observer are in their nominal values, then if faults occur, $\tilde{u}_a$ reconstructed from measured $T_{p0}$, $T_{u0}$ may be different from $\hat{u}_a$ estimated by observers. After that, detection residuals rule (7) is employed to produce detection residual $r = [r_1 \ r_2]$. Though checking $r_1$, faults in actuator of process fluid can be detected, and similar method can form $r_2$ for achieving fault detection in actuator of utility fluid.

5.3.2 RCA observer design
As mentioned before, faults influence $\mu$, $k$, $\Delta P$, $P_c$ related to four possible faults resources $f_1$, $f_2$, $f_3$, and $f_4$. Now, let us construct two banks of four observers as (8) for recognising those four possible faults in each control valve.

$$\begin{align*}
1 \leq j \leq 2, 1 \leq i \leq 4,
\dot{x}_i^j &= f_a (\dot{x}_u) + \sum_{l=i}^4 g_a (\dot{x}_u) \theta_l^j + g_a (\dot{x}_u) \nu_l^j + H_i (\tilde{u}_u - \tilde{u}_u^j) \\
\nu_l^j &= 2\gamma (\tilde{u}_u - \tilde{u}_u^j)^T P g_a (\dot{x}_u) \\
\tilde{u}_u^j &= \bar{C} x_u^j
\end{align*}$$

(15)

where $\dot{x}_i^j$ is the estimated state vector, $\nu_l^j$ is the estimation of root causes and $\tilde{u}_u^j$ is the estimated output. The above RCA observers aim at generating two banks of four residuals for those above mentioned fault causes. One bank of residuals are $s_{1i}$, $s_{12}$, $s_{13}$, $s_{14}$, aimed at identifying fault causes $f_1$, $f_2$, $f_3$, and $f_4$ in actuator of process fluid, the other bank are $s_{21}$, $s_{22}$, $s_{23}$, $s_{24}$, aimed at identifying fault causes $f_1$, $f_2$, $f_3$, and $f_4$ in actuator of utility fluid respectively. These residuals are constructed under rules (10), if any of these residuals breaches its threshold, the fault is caused by the corresponding fault causes.

5.4 Simulation results
The simulation results validate the proposed strategy. We first give the operating conditions of the simulation. The input of the inlet flow rate of the utility fluid $F_u$ is $4.22e^{-3} m^3 s^{-1}$, and inlet flow rate of the process fluid $F_p$ is $4.17e^{-3} m^3 s^{-1}$. Initial condition for observers supposed to be 0. Parameters in actuator subsystem are: $m = 2$ kg, $A_0 = 0.029 m^2$, $\mu = 1,500 Nm^{-1}$ and $k = 6,089 Nm^{-1}$, $P_c$ for utility fluid is 1 MPa, 1.2 MPa for process fluid, pressur drop $\Delta P$ in utility fluid is 0.6 MPa and 60 kPa in process fluid.

As above mentioned, for most part in practical situation, single fault is observed while multiple faults rarely occur on each actuator. So we consider each actuator is subject to only one fault, then, two faults may occur simultaneously in the actuator subsystem. Two cases are considered to illustrate: noise free and noise corrupted.

5.4.1 Noise free case, fault $f_3$ exists in actuator of process fluid and fault $f_4$ exists in actuator of utility fluid
In this part, two faults are considered. For actuator of process fluid, fault $f_3$ is supposed to occur at 80 s due to unexpected pressure drop across the valve, and for actuator of utility fluid, fault $f_4$ is supposed to occur at 60 s. We supposed that an expected 50 kPa pressure drop adds to the nominal pressure drop across the valve at time 80 s. While because of erosion, the gland packing of the valve may loosen, which leads to stem vibration, a failure value of
1,000 nm$^{-1}$ is added to the spring compliance $k$. Simulation results are listed in Figure 3 to Figure 6.

**Figure 3** Reconstructed input $F_p$, $F_u$ from output $T_p$, $T_u$ in case (a)

From Figure 3, after a short transient time, reconstructed inputs $\tilde{u}_1 = \tilde{F}_p$, $\tilde{u}_2 = \tilde{F}_u$ give an accurate estimation value to the real inputs $u_1 = F_p$, $u_2 = F_u$. At 80 s, the reconstructed $\tilde{F}_p$ unexpectedly increases, and finally it stabilises at a new level. This increase implies fault occurs and no further variations illustrate no additional fault occurs. The similar result is obtained in the reconstructed $\tilde{F}_u$ of utility fluid. The simulation curve indicates that the input reconstruction proposed in this paper is proper for recovering unknown inputs.

As shown in Figure 4, at 80 s and 60 s, the detection residuals $(r_1, r_2)$ break through their thresholds, respectively. It implies that at time 80 s, a fault occurs at actuator of process fluid, it takes 0.1 s to detect the fault. While for actuator of utility fluid, at time 60 s, the detection residual $r_2$ no longer remains zero which indicates a fault occurs, fault detection time is 0.2 s.

**Figure 4** Detection residual in case (a)

Then, we recognise that, for each actuator, one fault exists. The main contribution of this paper is that we can not only detect and locate the fault, but also can analyse its root cause. Next, we focus on identifying the causes of these two faults.

**Figure 5** Residuals for identifying fault cause in process fluid in case (a)

Figure 5 aims at recognising root cause of the fault at actuator of process fluid. RCA residuals $s_{11}$, $s_{12}$, $s_{13}$, $s_{14}$ are designed to recognise four possible candidates: $f_1$, $f_2$, $f_3$, and $f_4$. Detection residual $r_1$ has already implied fault occurred at 80 s. It is obviously in Figure 5 that, only $s_{13}$ breaks through and remains above the threshold, which means that fault in actuator of process fluid is caused by fault $f_3$. Isolation time is 0.5 s which is ideal.
In Figure 6, RCA residuals $s_{21}$, $s_{22}$, $s_{23}$, $s_{24}$ are designed to recognise fault $f_1$, $f_2$, $f_3$, and $f_4$ in actuator of utility fluid. Detection residual $r_2$ in Figure 4 has already implied fault occurred at 60 s. It is obviously in Figure 6 that only $s_{24}$ breaches the threshold and never comes back, which means that fault in actuator of utility fluid is caused by fault $f_4$. In this case, isolation time is about 0.2 s. The differences in isolation time is due to magnitude of fault and the effect it has impacted the system.

5.4.2 Noise corrupted case, fault $f_2$ exists in actuator of process fluid and fault $f_3$ exists in actuator of utility fluid

To illustrate the robustness of the proposed scheme, external disturbance or measurement noise is considered in this case. Suppose the output measurement $y$ is corrupted by a coloured noise. The coloured noise is generated with a second order AR filter excited by a Gaussian white noise with zero mean and unitary variance. The standard deviation of the coloured noise is about 3.5.

The same situation as case (a), one fault is considered on each actuator separately. For actuator of process fluid, fault $f_2$ is supposed to be leakage, and reasons that can lead to the leakage are: valve tightness, leaky bushing, and terminals. Fault $f_3$ is supposed in actuator of utility fluid, fault $f_3$ is caused by valve clogging, and it is a commonly encountered fault. If not properly repaired, this kind of fault may cause severe impacts on system performance. Simulation results are demonstrated in Figure 7 to Figure 10.

It can be seen from Figure 7 that although noise exists, the developed input reconstruction techniques can provide reconstructed inputs with a good accuracy. At actuator of process fluid, sudden decrease occurs at 60 s which indicates occurrence of a fault, and it takes 4 s to steady at...
new value. For actuator of utility fluid, the reconstructed value increases from 40 s, and is stable after about 3 s. A fault is detected due to the unexpected increase.

As illustrated in Figure 8, detection residual $r_1$ indicates a fault in actuator of process fluid at 60 s, it takes 1.2 s to determine the occurrence of the fault. Detection residual $r_2$ refers to a fault in actuator of utility fluid at time 40 s, and it takes 1.5 s to detect it. We can shorten the detection time and detect smaller fault by employing larger gain for the detection observers or adopt a smaller threshold. However, larger gain or larger threshold may fail to detect the fault correctly, since observer with larger gain is too sensitive to noise and smaller threshold may lead to be undistinguished from noise. Therefore a trade between detectability and sensitivity should be made in order to detect the fault correctly. In summary, a small magnitude fault may not be detected within the existence of the noise. Again, after detection of the faults, we have to identify their root causes.

We can see from Figure 9 that only RCA residual $s_{12}$ breaks through its threshold and remains beyond it, the other three RCA residuals are below their thresholds, then, the fault resource $f_2$ of actuator of process fluid is identified. In case of RCA residuals for actuator of utility fluid in Figure 10, only $s_{23}$ is beyond its threshold which verifies the occurrence of fault cause $f_3$.

From the above simulation results, we can see that the proposed strategy is able to detect and locate a fault correctly, and RCA for each detected fault is achieved with a good accuracy. Encouraging simulation results are obtained thanks to the robustness of the proposed scheme.

6 Conclusions
The main contribution of this paper lies on the integration of both system level and component level-based FDI approaches to facilitate FDI and RCA of subcomponents actuators. A left invertible interconnected nonlinear system
structure is developed which guarantees that faults occurring in actuator subsystem will affect the measured output of the global system uniquely and distinguishably. The new system structure, together with the fault diagnosis algorithm, is the first to emphasise the importance of RCA of field devices fault, as well as the influences of local internal dynamics on the global dynamics. Since local measurement is assumed inaccessible and estimated from global output, it is more realistic due to remote distance or physical availability considerations for an industrial application. Therefore, this approach not only completes the theory but is also of great importance to narrow the gap between industry and academia. Simulated results are included to demonstrate the applicability and robustness of the proposed method and encouraging results are obtained.

For future work, one interesting research direction is to develop an input estimator which avoids using computation of successive derivatives of outputs to estimate the inputs of the process subsystem. It is because successive derivatives are considered unrealistic in practical applications since measurements suffer noise and disturbances. Another consideration is to approach the problem of parameter observability and diagnosability at local subsystem from the viewpoint of global output, and to investigate a strategy of multi parameters identification.

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References


