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# Quality-of-Service Satisfaction Games for Noncooperative Underwater Acoustic Communications

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**Abstract**—Decentralized resource sharing for multiple noncooperative underwater acoustic (UWA) communication links is investigated. Point-to-point links, considered as rational players, are involved in a noncooperative game in which they compete to find a transmission strategy satisfying individual Quality-of-Service (QoS) constraints. Examples of UWA communication scenarios with direct-sequence-spread-spectrum (DSSS) and orthogonal-frequency-division-multiplexing (OFDM) systems are formulated as satisfaction games. The transmitters adapt their parameters (power, frequency and spreading gain) autonomously and in a decentralized way, without exchanging any information. This adaptation is made possible thanks to the feedback of local information on the channel statistics provided by their receiver. A blind satisfaction response algorithm (BSRA) is also proposed, which requires only a 1-bit feedback.

Realistic assumptions are made regarding the environment. The UWA channel is modeled as random and doubly selective, its coherence time is much shorter than the propagation delays and no assumption is made regarding its distribution. The UWA communication links do not have any prior information about the interfering waveforms of competing transmitters. Extensive numerical simulations are provided and comparisons are made with the Nash equilibrium solution. Results show gains both in terms of probability of QoS satisfaction and power consumed, even when a minimal knowledge of the environment is considered.

**Index Terms**—Underwater acoustic communications, quality-of-service, interference management, game theory, resource sharing, DSSS, OFDM.

## I. INTRODUCTION

**T**HIS paper deals with decentralized resource sharing methods for underwater acoustic (UWA) communications. Unlike in radiocommunications, there are no commonly accepted regulation policies to restrict the use of the underwater acoustic spectrum. Therefore, the availability of a resource for a communication link varies with time, frequency as well as with the location of other acoustic sources in the channel [1]. Moreover, the physics of the propagation medium makes the transmission losses increase with both the transmission range and frequency, thus diminishing the available bandwidth [2]. The UWA channel offers scarce resources, and heterogeneous acoustic sources operating in the same area may anachronically interfere with each others. Interference sources can

be other communications signals as well as sonar waveforms or marine mammals calls, for instance. Several works in the past few years have reported cases of UWA communications jammed by external interferences [3], [4]. It can be expected that this interference issue between UWA communication links becomes more problematic as human activities undersea are being developed. It is thus critical to ensure that sources from different origins, or devoted to different applications, can cohabit in the UWA channel thanks to decentralized, automatic and adaptive resource sharing methods. Although the JANUS protocol [5] - standardized by NATO in March 2017 [6] - is a very useful initiative allowing interoperability, proposals of flexible, adaptive and decentralized resource sharing methods are still needed.

Some works dealing with adaptive modulations and interference mitigation [3], [4], [7]–[12] have been proposed in the past few years. Adaptive modulations are presented in [7]–[11] for the single-user context. Interference mitigation is studied in [3], [4] with prior knowledge on the interfering waveforms and [12] considers a receiver with the ability to decode messages from interfering transmitters so as to perform successive interference cancellation. These two assumptions may be valid in specific scenarios only. Medium access control schemes (MAC) have also been studied extensively for homogeneous UWA networks [13]–[17].

Recent works on noncooperative UWA communications have proposed to formulate a spectrum sharing problem within the framework of game theory [18]–[20]. By noncooperative, we mean that the communication links do not have any information about the interferences from other sources. This framework has been well investigated in the terrestrial communication community and a good survey can be found in [21]. However, its use is still very limited in the UWA community. In [18]–[20], players are UWA transmitter-receiver (TX-RX) pairs competing to access the same portion of spectrum. Orthogonal-frequency-division-multiplexing (OFDM) modulation is considered. The players strategies are power vectors allocated across frequencies and their goal is to maximize an objective (or utility) function related to their information rate. The problem is solved according to the Nash Equilibrium (NE) solution, which consists in a power allocation strategy from which no player has an incentive to deviate alone. Nash equilibria are typically reached through the use of iterative, distributed algorithms [22], [23]. In [18], it is shown how noncooperative UWA-OFDM links can share the spectrum

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very efficiently with only a local, statistical and possibly erroneous knowledge of the environment. On the basis of the theoretical work of [24], the complementary problem has also been studied in [25] where UWA-OFDM players minimize their power under a Quality-of-Service (QoS) constraint. These problems fall into the class of Generalized Nash Equilibrium problems (GNE) [26], for which solutions may not exist.

A limit of these works are the selfish behaviors inherent to the NE solutions. Solutions that are Pareto-superior to the NE can be found if the players adopt less aggressive strategies. Along this line, the concept of satisfaction games was proposed in [27]. In these games, players are no more interested in selfishly maximizing their utility (*e.g.* SINR, information rate), but only in achieving a utility threshold above which they consider themselves to be satisfied. This framework has been developed in the past few years to solve problems of QoS provisioning in decentralized wireless networks [28]–[30]. However, the satisfaction equilibrium solution proposed in [29], [30] requires that all players are simultaneously satisfied, which may not be achievable in most situations. To cover the case in which only a subset of players can satisfy their QoS constraints, the concept of Generalized Satisfaction Equilibrium (GSE) has been proposed in [28]. This solution is also reached iteratively, thanks to an algorithm distributed among the players. Nevertheless, each player must know what are the strategies that satisfy its QoS constraints given the strategies played by its opponents. In practice this would require to feed back several tenth or hundreds of bits from the receivers, depending on the information considered. Such a detailed knowledge would not be available to the transmitters if the feedback link is restricted to one or a few bits.

Our contributions are the following. We demonstrate how the reliability of point-to-point UWA communications facing interferences in a competitive, noncooperative environment can be improved with the GSE solution. Information exchange is prohibited, the links evolve outside any network structure and are selfish in the sense they consider only their individual performance. Several UWA communication scenarios are formulated under the satisfaction game framework. Practical situations targeted are any shallow water communication setup with a small number of TX-RX pairs. Realistic assumptions regarding the specificity of the UWA environment are made. The channel is randomly time-varying and frequency selective, and the attenuation and noise are frequency dependent. The feedback link required to perform adaptation at the transmitter side is subject to long delays due to the low speed of UWA waves ( $1500 \text{ m.s}^{-1}$ ). The coherence time of the channel is assumed shorter than the propagation delays, so that it becomes irrelevant to feedback instantaneous information on a channel realization. Instead, a statistical information is considered, with an integration period of several tenths of seconds. Games with direct-sequence-spread-spectrum (DSSS) and OFDM modulations are studied. The players are UWA TX-RX pairs whose transmission strategies are power levels as well as spreading gains for the DSSS modulations, and power allocation vectors on the OFDM subcarriers. They are aimed at finding a strategy satisfying an individual QoS constraint in a distributed way, and without cooperation or information

exchanges with each others. This constraint is expressed in terms of a utility function which, for a given link, depends on the channel statistics and the average level of interferences from other transmitters. Two types of feedback are considered. First, receivers feed back statistics on the channel gain and on the interference plus noise power. The asynchronous satisfaction response algorithm (ASRA) presented in [28] is used when transmitters have this detailed knowledge of their environment. A blind satisfaction response algorithm (BSRA) is proposed to fit better with the UWA context, where a low bit-rate feedback link could be preferred due to limited bandwidths. This algorithm only necessitates that the receiver feeds back a 1-bit information signal to its transmitter so as to adapt its transmission strategy. Convergence of both algorithms in all considered scenarios is discussed. Finally, a scenario considering heterogeneous systems is evaluated through simulations by merging UWA DSSS and UWA OFDM in the same game. The performance at the equilibrium are evaluated through the probability of satisfaction, the average power used when satisfied, and the average convergence time of the algorithms. To the best of our knowledge, this is the first work of this nature in the UWA context. In addition, BSRA can constitute a more general contribution.

The paper is organized as follows. Sec. II formally states the problem under consideration and develops the game-theoretic models and solutions. Specific UWA scenarios are presented in Sec. III. Numerical results illustrate the relevance and benefits of our proposal in Sec. IV. Conclusions are given in Sec. V.

**Notation:** Throughout this paper, uppercase and lowercase boldface letters, *e.g.*  $\mathbf{A}$ ,  $\mathbf{x}$ , denote matrices and vectors, respectively. The superscripts  $T$  and  $H$  denote transposition and Hermitian transposition, respectively. The entries of a matrix  $\mathbf{A}$  are denoted by  $\mathbf{A}_{k,n}$  and  $\text{Tr}[\cdot]$  denotes the trace.  $\|\cdot\|_p$  designates the  $\ell_p$ -norm. The Frobenius norm of a matrix  $\mathbf{A}$  is defined as  $\|\mathbf{A}\|_F \triangleq \sqrt{\text{tr}[\mathbf{A}\mathbf{A}^H]}$ . The power set  $\mathcal{P}(\mathcal{S})$  of a set  $\mathcal{S}$  is the set of all its subsets, including  $\mathcal{S}$  itself and  $\emptyset$ . The cardinal of a set  $\mathcal{S}$  is denoted by  $|\mathcal{S}|$ .  $\mathcal{M}_{N \times M}(\mathbb{K})$  denotes the set of matrices with  $N$  lines and  $M$  columns whose coefficients belong to  $\mathbb{K}$ . The Kronecker symbol is denoted by  $\delta_n$ . Probability spaces are denoted by  $(\Omega, \mathcal{B}, \mathbb{P})$  with  $\Omega$  the sample space,  $\mathcal{B}$  the event space, and  $\mathbb{P}$  the probability measure. Finally,  $\mathbb{E}\{\cdot\}$  denotes expectation.

## II. GAME-THEORETIC MODEL AND SOLUTIONS

### A. Problem statement

Let  $\mathcal{I} = \{1, \dots, I\}$  be a set of noncooperative UWA TX-RX links competing simultaneously for the use of the same bandwidth  $B$ . Any receiver  $i \in \mathcal{I}$  treats the signals received from transmitters  $j \neq i$  as additive coloured noise. This corresponds to the interference channel model in information theory [31]. Each TX-RX link must comply with an individual QoS constraint expressed relatively to a performance measure. In the present work, this measure is either the signal-to-interference-plus-noise ratio (SINR) or the information rate. Whether the constraint is satisfied or not depends on the choice of a transmission strategy. This choice must be made individually through repeated interactions with the environment. In this

paper, the strategies are transmission powers and/or spreading gains. The strategic choice of a transmitter impacts inevitably the performance of the others, and thus their own strategic choices. The set of TX-RX links is aimed at autonomously finding an equilibrium strategy in a decentralized manner, each link having only a local knowledge of its environment. No communication is allowed between different links, and each of them considers only its own performance and constraints when choosing its strategy. Such situations are well formalized under the framework of game theory [32], and satisfaction games [27] are particularly well suited to the problem stated.

### B. Games in satisfaction form and equilibrium

Based on the work [27]–[30], the main concepts related to games in satisfaction form are recalled. Refinements are also presented when needed.

A game in satisfaction form is a tuple

$$\mathcal{G} = (\mathcal{I}, \mathcal{A}, (f_i)_{i \in \mathcal{I}}, (\varphi_i)_{i \in \mathcal{I}}) \quad (1)$$

where

- $\mathcal{I} = \{1, \dots, I\}$  is the set of players involved in the game,
- $\mathcal{A} = \mathcal{A}_1 \times \dots \times \mathcal{A}_I$  where  $\mathcal{A}_i$  is the strategies set of a player  $i \in \mathcal{I}$ . A choice of strategy of player  $i$  is denoted  $a_i \in \mathcal{A}_i$ . The choice of all players other than  $i$  is denoted by  $a_{-i} = [a_1, \dots, a_{i-1}, a_{i+1}, \dots, a_I] \in \mathcal{A}_{-i} = \mathcal{A} \setminus \mathcal{A}_i$ . An element  $a \in \mathcal{A}$  is called a strategy profile.
- $f_i : \mathcal{A} \rightarrow \mathbb{R}$  is a performance or satisfaction measure of the player  $i$  called utility function.
- The satisfaction mapping or correspondence  $\varphi_i : \mathcal{A}_{-i} \rightarrow \mathcal{P}(\mathcal{A}_i)$  maps the strategies of all opponents of player  $i$  to a set of strategies that satisfies some performance constraints.

In the following, the QoS requirements will be expressed with a mapping  $\varphi_i$  of the form

$$\varphi_i(a_{-i}) = \{a_i \in \mathcal{A}_i : f_i(a_i, a_{-i}) \geq \Gamma_i\}. \quad (2)$$

Player  $i$  is satisfied if its utility  $f_i$  is greater than some threshold  $\Gamma_i$ .  $\varphi_i(a_{-i})$  represents the set of strategies that leads to satisfaction of player  $i$ .

The sets  $\mathcal{A}_i$  are assumed bounded and totally ordered by an order relation  $\preceq$ . We require the sets  $\mathcal{A}_i$  to have (at least) a smallest and a largest elements, denoted respectively as  $\underline{a}_i$  and  $\overline{a}_i$ . The ordering of strategies can be thought as a cost or effort that a player must pay to play a given strategy, *e.g.*, the higher the transmit power, the higher the cost. It is assumed that for all  $i \in \mathcal{I}$ ,  $f_i(a_i, a_{-i})$  is monotonically increasing in  $a_i$ .<sup>1</sup>

For any strategy profile  $a \in \mathcal{A}$ , the set of players  $\mathcal{I}$  is divided into three disjoint subsets whose union covers  $\mathcal{I}$ :

$$\mathcal{I}_S(a) = \{i \in \mathcal{I} : a_i \in \varphi_i(a_{-i})\}, \quad (3)$$

the satisfied players for whom the chosen strategy satisfies the constraints,

$$\mathcal{I}_N(a) = \{i \in \mathcal{I} : a_i \notin \varphi_i(a_{-i}), \varphi_i(a_{-i}) \neq \emptyset\}, \quad (4)$$

<sup>1</sup>This is a realistic assumption in many practical scenarios, *e.g.* when strategies spaces are powers and utility functions are SINRs or information rates.

the unsatisfied players with a non-empty set of satisfying strategies (they can profitably deviate from  $a$ ),

$$\mathcal{I}_U(a) = \{i \in \mathcal{I} : \varphi_i(a_{-i}) = \emptyset\}, \quad (5)$$

the unsatisfied players who do not have any satisfying strategies for the strategy profile  $a$ .

It is clear that if the players are rational and have common and complete knowledge of the game [32], none of them would play an strategy that drives him into the subset  $\mathcal{I}_N(a)$ . Thus, at a satisfaction equilibrium profile  $a^*$ , the set  $\mathcal{I}_N(a^*)$  should be empty, implying that no player can deviate alone profitably. We have the following definition for the *generalized satisfaction equilibrium* [28]:

**Definition 1** (Generalized Satisfaction Equilibrium). *An strategy profile  $a^* = (a_i^*)_{i \in \mathcal{I}}$  is a generalized satisfaction equilibrium (GSE) if and only if  $\mathcal{I}_N(a^*) = \emptyset$ .*

It is shown in [28, Prop. 1] that if the satisfaction mapping  $\varphi_i$  can be expressed as in (2), and if the strategy spaces of the players are finite, then there exists at least one GSE in mixed-strategies for the game  $\mathcal{G}$ .<sup>2</sup> This is because there exists at least one Nash Equilibrium (NE) in mixed-strategies in any finite normal-form game [33], and thus, in the normal-form game  $\mathcal{G}_{NE} = (\mathcal{I}, \mathcal{A}, (f_i)_{i \in \mathcal{I}})$  defined with  $f_i$  as utility functions to maximize. This equilibrium is necessarily a GSE: at any NE, the set of players can be divided as in Def. 1 and no player can increase its utility function - or be more satisfied - by an unilateral change of its strategy (by definition of the NE).

In this paper, we are interested in pure-strategies GSE. In games defined as in (1), where players have utility functions increasing in their own strategies and satisfaction correspondences defined as in (2), there exists at least a pure GSE. This equilibrium is the pure NE  $a^* = (\overline{a}_i)_{i \in \mathcal{I}}$  of  $\mathcal{G}_{NE} = (\mathcal{I}, \mathcal{A}, (f_i)_{i \in \mathcal{I}})$ , where players play their most expensive strategy.

### C. Decentralized learning of pure-strategy satisfaction equilibria

In practice, satisfaction equilibria have to be learned through iterative algorithms. The satisfaction response (SR) of a player  $i \in \mathcal{I}$  to the strategy profile  $a_{-i}$  of its opponents is a selection of strategy in the set returned by the satisfaction correspondence  $\varphi_i(a_{-i})$ . This set, in general, is not a singleton, and the question of what element to select can be posed. We define the least effort satisfaction response (LESR) as the less expensive strategic choice among the satisfying ones:

$$SR_i(a_{-i}) \triangleq \min_{a_i \in \varphi_i(a_{-i})} a_i. \quad (6)$$

Let  $\mathcal{T} = \{0, 1, 2, \dots\}$  be possible indexes in a set of times represented by an increasing sequence of real numbers, not necessarily equally spaced. At any time or iteration  $t \in \mathcal{T}$ , and while an equilibrium is not reached, some players should play a satisfaction response  $a_i \in \varphi_i(a_{-i}(t))$ .<sup>3</sup> We denote by  $\mathcal{T}_i \subseteq \mathcal{T}$

<sup>2</sup>Mixed-strategies are probability distributions on the sets of (pure-)strategies

<sup>3</sup>Note that the information player  $i$  has on the strategies of its opponents at time  $t$  can be outdated.

the set of times for which player  $i$  has to play. Algorithm 1 is proposed as a procedure to learn satisfaction equilibria. It improves the asynchronous satisfaction response algorithm proposed in [28] by selecting the least expensive satisfaction response of each player at each iteration.

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**Algorithm 1** Asynchronous satisfaction response
 

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1:  $t = 0$ 
2:  $a(t) = (\underline{a}_i)_{i \in \mathcal{I}}$ 
3: while  $\mathcal{I}_N(a(t)) \neq \emptyset$  do  $\forall i \in \mathcal{I}$ 
4:   if  $t \in \mathcal{T}_i$  then
5:     
$$a_i(t+1) = \begin{cases} SR_i(a_{-i}(t)) & \text{if } i \in \mathcal{I}_N(a(t)) \\ a_i(t) & \text{otherwise} \end{cases}$$

6:   end if
7:    $t = t + 1$ 
8: end while

```

---

Algorithm 1 assumes that the transmitters can determine whether they belong to  $\mathcal{I}_S(a(t))$ ,  $\mathcal{I}_U(a(t))$ , or  $\mathcal{I}_N(a(t))$  and also know the set returned by  $\varphi_i(a_{-i}(t))$ . Such assumptions require that the transmitters have some knowledge on their environment, *e.g.*, SINR or CSI. Such a knowledge requires to transmit several tenth or hundreds of bits through a feedback link. In a UWA context, low signaling overhead is often required because of the limited bandwidth. To reduce feedback overheads, a practical policy can consist in providing only a 1-bit feedback signal informing the transmitter whether it should play another strategy or not.

#### D. Blind satisfaction response algorithm (BSRA)

We propose a blind satisfaction response algorithm where transmitters are not able to evaluate by themselves what their satisfying strategies are, *i.e.*, they do not have the knowledge of the set  $\varphi_i(a_{-i}(t))$ . Consequently, they also do not know which set of players they belong to. Nevertheless, for a given strategy profile  $a(t)$  at time  $t$ , an unsatisfied player  $i$  has a profitable deviation if and only if  $\varphi_i(a_{-i}(t))$  is not empty. In other words, only the players belonging to the set  $\mathcal{I}_N(a(t))$  will adapt their strategy during the next steps. By virtue of the assumptions made in Sec. II-B (ordered and finite strategy spaces,  $f_i(a_i, a_{-i})$  increasing in  $a_i$ , and  $\varphi_i(a_{-i})$  expressed as in (2)) we see that, for a given  $a_{-i}(t)$ , it suffices to test the condition  $f_i(\bar{a}_i, a_{-i}(t)) \geq \Gamma_i$  to check the nonemptiness of  $\varphi_i(a_{-i}(t))$ . Thus, when player  $i$  is not satisfied, the receiver can evaluate whether or not there exists a satisfying strategy to play next by testing if the most expensive strategy  $\bar{a}_i$  is satisfying and provide a 1-bit feedback to the transmitter accordingly. The feedback signal has two states, which will be denoted by ACK or NACK in the following. If a TX-RX link  $i$  belongs to the set  $\mathcal{I}_N(a(t))$ , the receiver can provide a NACK feedback to inform the transmitter that there exists a strategy  $y_i$ , more expensive than  $a_i(t)$ , which is possibly satisfying after time  $t$ . This strategy is  $\bar{a}_i$  in the worst case. Let  $\mathcal{Y}_i(t)$  be the set of strategies strictly more expensive than the strategy  $a_i(t)$  played at time  $t \in \mathcal{T}$ ,

$$\mathcal{Y}_i(t) = \{y_i \in \mathcal{A}_i : a_i(t) \prec y_i \preceq \bar{a}_i\}. \quad (7)$$

As the transmitter does not know  $\varphi_i(a_{-i}(t))$ , if it receives a NACK it randomly chooses a strategy in  $\mathcal{Y}_i(t)$  for the next iteration of the algorithm. Let

$$\Phi_t^{(i)} : \Omega \rightarrow \mathcal{Y}_i(t) \quad (8)$$

be the random variable modeling this choice. If the transmitter  $i$  receives an ACK from its receiver, the link belongs either to  $\mathcal{I}_S(a(t))$  or  $\mathcal{I}_U(a(t))$  and does not change its strategy since it would not be a profitable deviation. Algorithm 2 formalizes this procedure.

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**Algorithm 2** Blind asynchronous satisfaction response
 

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1:  $t = 0$ 
2:  $a(t) = (\underline{a}_i)_{i \in \mathcal{I}}$ 
3: while  $\mathcal{I}_N \neq \emptyset$  do  $\forall i \in \mathcal{I}$ 
4:   if  $t \in \mathcal{T}_i$  then
5:     
$$a_i(t+1) = \begin{cases} \Phi_t^{(i)}(\omega) & \text{if TX } i \text{ has received NACK} \\ a_i(t) & \text{otherwise} \end{cases}$$

6:   end if
7:    $t = t + 1$ 
8: end while

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Proposition 1 states a sufficient condition for convergence of the two algorithms. This proposition and its proof are similar to [28, Proposition 3], but is here completed for Algorithm 2. The following definitions need to be introduced before Proposition 1. Consider the order relation  $<_{\mathcal{A}}$  on  $\mathcal{A}$  defined by

$$\forall a, a' \in \mathcal{A}^2, \quad a <_{\mathcal{A}} a' \Leftrightarrow \forall i \quad a_i \preceq a'_i, \exists j \quad a_j \prec a'_j. \quad (9)$$

The mapping  $\phi_i : \mathcal{A}_{-i} \rightarrow \mathcal{A}_i$  is said to be order-preserving if, given  $a_{-i}, a'_{-i} \in \mathcal{A}_{-i}^2$  such that  $a_{-i} <_{\mathcal{A}_{-i}} a'_{-i}$ , we have

$$\phi_i(a_{-i}) \preceq \phi_i(a'_{-i}). \quad (10)$$

**Proposition 1.** *If in game (1) and  $\forall i \in \mathcal{I}$  the following conditions are satisfied:*

- 1)  $\forall t \geq 0, \exists t' > t$  such that  $t' \in \mathcal{T}_i$ ,
- 2)  $\forall a_{-i} \in \mathcal{A}_{-i}, \quad \varphi_i(a_{-i}) = \{a_i \in \mathcal{A}_i : \phi_i(a_{-i}) \leq a_i\}$ ,  
where  $\phi_i : \mathcal{A}_{-i} \rightarrow \mathcal{A}_i$  is order-preserving,
- 3)  $\forall a_i \in \mathcal{A}_i, \exists \bar{a}_i \in \mathcal{A}_i$  such that  $a_i \preceq \bar{a}_i$ ,

*then Algorithms 1 and 2 converge to a pure-strategy GSE.*

*Proof:* First, it must be emphasized that condition 1) is equivalent to assume that at any time, any player will have an opportunity to play an strategy in the future. This is not a stringent assumption.

Let  $t = 0$ , then  $a(0) = (\underline{a}_i)_{i \in \mathcal{I}}$ . If  $\mathcal{I}_N(a(0)) = \emptyset$  then  $a(0)$  is an equilibrium and the procedure terminates, otherwise there exists a player with a profitable deviation. Let  $t \geq 0$  and  $a(t) \in \mathcal{A}$ . If  $\mathcal{I}_N(a(t)) \neq \emptyset$  then  $\exists i \in \mathcal{I}$  such that  $\varphi_i(a_{-i}(t)) \neq \emptyset$  and  $a_i(t) \notin \varphi_i(a_{-i}(t))$ , which is equivalent to state that  $a_i(t) < \phi_i(a_{-i}(t))$ . Let  $\mathcal{Y}_i(t)$  be the set of strategies defined in (7). From Assumption 2) of Prop. 1, we have  $\varphi_i(a_{-i}(t)) \in \mathcal{P}(\mathcal{Y}_i(t))$ .

When  $\mathcal{I}_N(a(t)) \neq \emptyset$ , at least one player  $i \in \mathcal{I}_N(a(t))$  has to play a satisfaction response so that  $\varphi_i(a_{-i}(t)) \in \mathcal{P}(\mathcal{Y}_i(t))$ . From Assumption 1), we then deduce that,  $\forall t \geq 0$ ,

$$\exists t' > t \text{ such that } a(t) <_{\mathcal{A}} a(t') \leq_{\mathcal{A}} \bar{a}. \quad (11)$$

Thus,  $a(t)$  increases with respect to the order relation  $<_{\mathcal{A}}$ . By Assumption 3),  $a(t)$  cannot be greater than  $\bar{a}$ , which is a NE of the normal form game  $\mathcal{G}_{NE}$  and also a GSE (see Section II-B). Consequently, both Algorithms 1 and 2 converge to a GSE which is either  $\bar{a}$  or any  $a^* \leq_{\mathcal{A}} \bar{a}$  such that  $\mathcal{I}_N(a(t)) = \emptyset$ . ■

Algorithms 1 and 2 have both a maximum number of iterations of  $\mathcal{O}(I \times \max_{i \in \mathcal{I}} |\mathcal{A}_i|)$  [28]. This case occurs when all players are initially in  $\mathcal{I}_N$ , play systematically the cheapest strategy in their satisfaction correspondence, are brought back again in  $\mathcal{I}_N$  the next time they have to play, and so on until the most expensive strategy is played. For each communication link, the extra computational complexity is expected to be negligible since both algorithms can be built only on few simple statements.

### III. CASE STUDIES

In this section, the use of satisfaction games in noncooperative UWA communication problems is illustrated. Games with DSSS and OFDM modulations are discussed. DSSS point-to-point schemes provide processing gain for robust transmissions [34], [35] and OFDM is commonly used for high-rate communications [36]. For each game, two types of feedback are considered: 1-bit or statistical CSI. In the following games, the set of players  $\mathcal{I}$  is made of  $I$  UWA TX-RX links communicating at the same time in the same bandwidth  $B$ . The players are embedded with their own private strategies sets  $(\mathcal{A}_i)_{i \in \mathcal{I}}$  and utility functions  $(f_i)_{i \in \mathcal{I}}$  which model their transmission parameters and performance metric respectively. Each of them is aimed at finding, if feasible, a strategy  $a_i \in \mathcal{A}_i$  such that an individual QoS constraint  $f_i(a_i, a_{-i}) \geq \Gamma_i$  is satisfied.

We consider block transmissions as it encompasses both OFDM and DSSS schemes. Let  $\mathbf{x}_i \in \mathbb{C}^{N_i}$  be a block of  $N_i$  i.i.d zero mean complex symbols sent by the transmitter  $i$ . For any  $i \in \mathcal{I}$ , the symbols  $\mathbf{x}_{j \neq i} \in \mathbb{C}^{N_j}$  sent by any other transmitter  $j \in \mathcal{I}$ ,  $j \neq i$ , are independent of  $\mathbf{x}_i$ . All the transmitters are subject to their own power constraints so that

$$P_i^{\min} \leq P_i \leq P_i^{\max} \quad \forall i \in \mathcal{I}, \quad (12)$$

where  $P_i = \text{Tr}[\mathbf{C}_i] = \text{Tr}[\mathbb{E}\{\mathbf{x}_i \mathbf{x}_i^H\}]$ . For all players  $i \in \mathcal{I}$ , the choice of a transmission strategy  $a_i \in \mathcal{A}_i$  impacts the covariance matrix  $\mathbf{C}_i$  and thus the signal  $\mathbf{x}_i$ . This will be translated in terms of powers and spreading gains for DSSS modulations and in terms of power allocations for OFDM modulations.

A realization of the block of symbols received by the receiver  $i$  can be written as

$$\mathbf{y}_i = \mathbf{H}_{ii} \mathbf{x}_i + \sum_{j \neq i} \mathbf{H}_{ji} \mathbf{x}_j + \mathbf{w}_i \quad (13)$$

$$= \mathbf{H}_{ii} \mathbf{x}_i + \mathbf{z}_i(\mathbf{x}_{-i}), \quad (14)$$

where  $\mathbf{w}_i \sim \mathcal{CN}(0, \sigma_i^2 \mathbf{I}_{N_i})$  is a Gaussian noise independent of both  $\mathbf{x}_i$  and  $\mathbf{x}_{j \neq i}$ , and  $\mathbf{H}_{ji} \in \mathcal{M}_{N_j \times N_i}(\mathbb{C})$  are channel matrices of time-varying random coefficients between transmitter  $j$  and receiver  $i$ . The receiver  $i$  only has a statistical knowledge of its direct channel  $\mathbf{H}_{ii}$ , which is assumed quasi-wide sense stationary during  $L$  blocks, and of its interference plus noise term  $\mathbf{z}_i(\mathbf{x}_{-i})$ . Nothing is known about  $\mathbf{H}_{ji}$  or  $\mathbf{x}_j$  for all  $j \neq i$ .  $L$  is assumed large enough so that sample means converge to expectations.

Regarding the only statistical knowledge of the environment available, the utility functions  $(f_i)_{i \in \mathcal{I}}$  ruling the adaptation process of each link must be either an averaged criterion or expressed as a function of these statistics. In order to enable the transmitter to adapt its parameters, the corresponding receiver must provide a feedback on its knowledge of the environment. This knowledge could be either the channel and interference plus noise statistics or a 1-bit ACK/NACK feedback indicating if the transmitter should adapt its strategy or not. In this last case, the state of this 1-bit signal is still chosen according to the channel and interference statistics since it depends on a QoS constraint expressed with the averaged performance criterion previously mentioned. Each link adapts its transmission strategy periodically. They are not supposed to be synchronized with each others. Practically, the receiver would estimate the channel and interference plus noise statistics during each period. Depending on the feedback policy chosen, two alternatives are possible at the end of a period. Let us take the perspective of some given player  $j \in \mathcal{I}$ :

- The receiver  $j$  sends directly back the statistics on the channel  $\mathbf{H}_{jj}$  and the interference  $\mathbf{z}_j(\mathbf{x}_{-j})$  to the transmitter  $j$  via a dedicated feedback link. Then, the transmitter  $j$  is able to evaluate the utility  $f_j(a_j, a_{-j})$  for any possible  $a_j \in \mathcal{A}_j$  and for the given interference, which depends on  $a_{-j}$ . It can deduce - if they exist - what are the strategies in  $\varphi_j(a_{-j})$  satisfying the QoS constraint  $f_j(a_j, a_{-j}) \geq \Gamma_j$  and chooses the least expensive for the next period. Here, the player  $j$  uses the Algorithm 1.
- In the second case, the player uses Algorithm 2. The receiver  $j$  knows the statistics on the channel and the interference and checks by itself if the constraint  $f_j(a_j, a_{-j}) \geq \Gamma_j$  is satisfied and if  $\varphi_j(a_{-j})$  is non-empty, as in the procedure described in Section II-D. If the receiver determines it belongs to the set of players  $\mathcal{I}_N(a)$ , it sends a NACK back to inform the transmitter  $j$  that a profitable deviation still exists. Transmitter  $j$  does not know exactly what strategies are profitable deviations from the actual  $a_j$ , so it chooses randomly among those that have a higher rank than  $a_j$  in its strategies set. Otherwise, the player belongs either to  $\mathcal{I}_S(a)$  or  $\mathcal{I}_U(a)$ , meaning that no other choice can be profitable. Then, the receiver sends an ACK back to transmitter.

The impact of the type of information available to the transmitters is studied through simulations in Section IV.

As stressed in [8], [18], [19], the long propagation delays typical of UWA channels combined with their random time variability could very likely lead to outdated knowledge of the instantaneous CSI/SINR at the transmitter side. As an exam-

ple, the propagation delay for a 1.5 km range is 1 s, whereas a lot of UWA channels exhibit coherence times of tenth or hundreds of milliseconds [37]–[39]. To ensure robustness and reduce the feedback activity, channels statistics should be computed on a relatively long period (several seconds or tenth of seconds, depending on the channel coherence time and transmission range).

In the next two subsections, the strategies sets and utility functions of the DSSS and OFDM players are detailed.

#### A. Model of the DSSS players

In the DSSS communication links, the vectors  $\mathbf{x}_i$  of equation (13) can be written as  $\mathbf{x}_i = S_i \times \mathbf{u}_i$  where  $\mathbf{u}_i \in \mathbb{C}^{N_i}$  is a pseudo-noise (PN) spreading sequence of  $N_i$  chips such that  $\sum_{k=1}^N u_{i,k} u_{i,n-k}^* \approx N_i \delta_n$ ,  $\mathbf{u}_i^H \mathbf{u}_i = N_i$ , and  $S_i \in \mathbb{C}$  is chosen randomly in an alphabet of equally likely symbols. Thus,  $\text{Tr}[\mathbf{C}_i] = \text{Tr}[\mathbb{E}\{\mathbf{x}_i \mathbf{x}_i^H\}] = N_i P_i$ , where  $P_i = \mathbb{E}\{|S_i|^2\}$ . The symbol time is  $T_i = N_i \times T_c$  where  $T_c \approx 1/B$  is the duration of one chip  $u_{i,n}$ . It is assumed that the length of the sequence is greater than the delay spread of the channel, which is assumed to be  $K$  chips. Any realization of the channel matrix  $\mathbf{H}_{ii}$  has the following form:

$$\mathbf{H}_{ii} = \begin{pmatrix} h_{ii}^{(0,0)} & & & & \\ h_{ii}^{(1,1)} & h_{ii}^{(0,1)} & & & \\ \vdots & & \ddots & & \\ h_{ii}^{(K-1,K-1)} & & & \ddots & \\ 0 & & & & h_{ii}^{(K-1,N_i-1)} & \dots & h_{ii}^{(0,N_i-1)} \end{pmatrix}. \quad (15)$$

Each coefficient  $h_{ii}^{(k,n)}$  is a realization at (chip) time  $n$  of a random process modelling the gain of the  $k^{\text{th}}$  path of the channel, and whose distribution is unknown by  $i$ . It is assumed that each receiver  $i$  is able to estimate the coefficients of  $\mathbf{H}_{ii}$  each time it receives a new block of  $N$  chips so as to compute the channel statistics.

The DSSS players have a finite and countable set of parameters which are pairs of transmission power  $P_i$  and spreading gain  $G_i$ , with  $G_i = N_i$ . This set is denoted by

$$\mathcal{P}_i = \left\{ \left( P_i^{(1)}, G_i^{(1)} \right), \dots, \left( P_i^{(n_i)}, G_i^{(n_i)} \right) \right\}. \quad (16)$$

The set of possible strategies for player  $i$  is the indexes selecting the parameters in  $\mathcal{A}_i$ . We denote this set by

$$\mathcal{A}_i = \{1, \dots, n_i\}, \quad (17)$$

thus, to any strategy  $a_i$  chosen by player  $i$  corresponds a pair  $\left( P_i^{(a_i)}, G_i^{(a_i)} \right)$ . This set is ordered in such a way that it should be more expensive for transmitter  $i$  to use more power, and, when the same power is used, it should also be more

expensive to increase the spreading gain.<sup>4</sup> Note that when the spreading gain is increased, the chip rate does not change so the transmission bandwidth is still  $B$ . Consequently, the symbol rate decreases with the spreading gain.

A utility function based on the SINR is proposed. After despreading, the average SINR of player  $i$  is given by<sup>5</sup>

$$f_i(a_i, \mathbf{x}_{-i}) = \frac{\mathbb{E}\left\{\|\mathbf{H}_{ii}\|_F^2\right\} P_i^{(a_i)}}{\mathbb{E}\left\{\|\mathbf{z}_i(\mathbf{x}_{-i})\|_2^2\right\}} G_i^{(a_i)} \quad (18)$$

and the satisfaction correspondence is

$$\varphi_i(\mathbf{x}_{-i}) = \{a_i \in \{1, \dots, n_i\} : f_i(a_i, \mathbf{x}_{-i}) \geq \Gamma_i\}, \quad (19)$$

where  $\Gamma_i$  is the minimum SINR levels necessary to satisfy the QoS constraint.

Although not denoted explicitly, the strategies  $a_{-i}$  governs obviously the power of the interference terms  $\mathbf{z}_i(\mathbf{x}_{-i})$  perceived by the receiver  $i$ . It is also clear from Eq. (18) that there is no need for the player  $i$  to know explicitly the strategy profile of its opponents to adapt its own strategy. Only the aggregate interference plus noise power is needed.

1) *Game with knowledge of the statistical CSI*: The receiver provides the SINR  $f_i(a_i, \mathbf{x}_{-i})$  of Eq. (18) to its transmitter, through the feedback link. Then, the transmitter can seek for the satisfying strategy having the minimal index in its strategies space. For a given interference vector  $\mathbf{x}_{-i}$ , this corresponds to its LESR (see (6)). The players use Algorithm 1.

2) *Game with 1-bit feedback*: The players use Algorithm 2. The receiver is able to compute (18) from the channel and interference statistics. It is assumed that the receiver is also able to compute what would be (18) with maximum transmit power and spreading gain. An ACK is fed back to the transmitter if the SINR constraint  $f_i(a_i, \mathbf{x}_{-i}) \geq \Gamma_i$  is satisfied, or if the maximum power and spreading gain does not achieve the target SINR ( $f_i(n_i, \mathbf{x}_{-i}) < \Gamma_i$ ). In this case, the transmitter does not change its current strategy. Otherwise, a NACK is sent back and the transmitter chooses randomly a strategy among those having an higher rank than the current one.

For the two types of feedback, at least one GSE exists; It is the NE  $\bar{a} = (n_1, \dots, n_I)$  where all players play their most expensive strategy. From Sec. II-B, it is possible to prove that both Algorithms 1 and 2 converge for DSSS players. This proof is provided in Appendix A.

<sup>4</sup>A possible way to obtain such an order for the set  $\mathcal{A}_i$  is to impose the following condition:

$$\forall a_i, a'_i \in \mathcal{A}_i^2, \\ x_i \preceq a'_i \Leftrightarrow \begin{cases} P_i^{(a_i)} & \leq P_i^{(a'_i)} \\ P_i^{(a_i)} G_i^{(a_i)} & < P_i^{(a'_i)} G_i^{(a'_i)}. \end{cases}$$

<sup>5</sup>For the sake of clarity, the expression of this utility function is kept simple. However, in practice this utility could be modified to integrate residual inter-symbol interference.

### B. Model of the OFDM players

For OFDM players, the vectors  $\mathbf{x}_i$  of Eq. (13) are random symbols on  $N$  orthogonal subcarriers and  $\mathbf{y}_i$  is the received symbol vector after Discrete Fourier Transform and cyclic prefix removal. The subcarrier spacing of the player  $i$  is  $\Delta f_i = B/N_i$  and the symbol duration is  $T_i = \Delta f_i^{-1} + T'_i$  where  $T'_i$  is the cyclic prefix duration. Each symbol  $x_{i,n}$  on each of the  $N_i$  subcarriers has a power  $p_{i,n} = \mathbb{E}\{|x_{i,n}|^2\}$ , where  $n \in \{1, \dots, N_i\}$ . The power allocated on the whole bandwidth  $B$  is  $P_i = \sum_n p_{i,n}$ . Assuming no inter-carrier interference (ICI)<sup>6</sup>, the channel matrix  $\mathbf{H}_{ii}$  is diagonal. Each of its coefficient  $(h_{ii,n})_{n=1}^{N_i}$  is a realization of the channel gain at the  $n^{\text{th}}$  subcarrier. The distribution of  $\mathbf{H}_{ii}$  is unknown from player  $i$ . The performance criterion chosen for the OFDM systems is related to the information rate. We follow [18] and define the utility function as

$$f_i(\mathbf{p}_i, \mathbf{p}_{-i}) = \alpha_i \sum_{n=1}^{N_i} \log(1 + \gamma_{i,n}(\mathbf{p}_{-i}) p_{i,n}). \quad (20)$$

where  $\alpha_i = (N_i T_i \Delta f_i)^{-1}$  and  $\gamma_{i,n}$  is a statistical CSI on subcarrier  $n$  expressed as

$$\gamma_{i,n}(\mathbf{p}_{-i}) \triangleq \frac{e^{\mathbb{E}\{\log|h_{ii,n}|^2\}}}{\sigma_{w_i,n}^2 + \sum_{j \neq i} \mathbb{E}\{|h_{ji,n}|^2\} p_{j,n}}. \quad (21)$$

The vector  $\mathbf{p}_i = [p_i(1), \dots, p_i(N)]^T \in \mathbb{R}_+^N$  is the power allocation strategy on the  $N_i$  subcarriers of transmitter  $i$ , and  $\mathbf{p}_{-i} = [\mathbf{p}_1^T, \dots, \mathbf{p}_{i-1}^T, \mathbf{p}_{i+1}^T, \dots, \mathbf{p}_M^T]^T$  are the strategies of all the other transmitters. To compute its utility function, the receiver should estimate the coefficients  $(h_{ii,n})_{n=1}^{N_i}$  as well as the noise plus interference power spectral density  $\zeta_i(n) = \sigma_{w_i,n}^2 + \sum_{j \neq i} \mathbb{E}\{|h_{ji,n}|^2\} p_{j,n}$ . In practice, this is made possible by dedicating some subcarriers to pilots symbols.

The transmitters are constrained in power so that  $\forall i \in \mathcal{I}$ ,  $\mathbf{p}_i \in \mathcal{P}_i$  where

$$\mathcal{P}_i \triangleq \left\{ \mathbf{p}_i \in \mathbb{R}_+^{N_i} : P_i^{\min} \leq \|\mathbf{p}_i\|_1 \leq P_i^{\max} \right\} \quad (22)$$

is the set of feasible power allocations of player  $i$ . The possible power allocation choices depend on what information the players have about their environment *i.e.*, whether or not the transmitters know the statistical CSI  $\gamma_i(\mathbf{p}_{-i})$ .

1) *Game with knowledge of the statistical CSI:* Here, the players use Algorithm 1 and it is supposed that the receivers have fed the CSI  $\gamma_i(\mathbf{p}_{-i})$  back to their respective transmitters when an update is to be performed. For a transmitter  $i$  knowing its CSI  $\gamma_i(\mathbf{p}_{-i})$ , it can be shown via convex optimization tools [25], [41] that the unique solution of

$$\begin{aligned} & \underset{\mathbf{p}_i \in \mathbb{R}_+^{N_i}}{\text{minimize}} && \|\mathbf{p}_i\|_1 \\ & \text{subject to} && f_i(\mathbf{p}_i, \mathbf{p}_{-i}) \geq \Gamma_i \end{aligned} \quad (23)$$

<sup>6</sup>Considering ICI would not change the problem formalized next. As discussed in [40], ICI translates into an additional noise term in the utility function.

is a water-filling solution such that  $\forall n \in \{1, \dots, N_i\}$

$$p_{i,n} = \left[ \lambda_i - \frac{1}{\gamma_{i,n}(\mathbf{p}_{-i})} \right]^+, \quad (24)$$

where  $[x]^+$  denotes  $\max(0, x)$  and  $\lambda_i$  is a Lagrange multiplier chosen to saturate the constraint  $f_i(\mathbf{p}_i, \mathbf{p}_{-i}) \geq \Gamma_i$ . If the constraints are feasible, it can be checked from equation (6) that the solution of (23) is the unique LESR of  $i$  to the interference profile  $\mathbf{p}_{-i}$ . Moreover, for a given constraint on the  $\ell_1$ -norm of  $\mathbf{p}_i$ , the water-filling solution is also the unique maximizer of  $f_i(\mathbf{p}_i, \mathbf{p}_{-i})$ . Thus, a rational player should always play a water-filling vector computed on the basis of its statistical CSI  $\gamma_i(\mathbf{p}_{-i})$ . Any other choice of strategy is either more expensive (it has a greater norm than necessary so the transmitter uses more power) or gives less utility (and thus, less chances of being satisfied). Let

$$\begin{aligned} \mathbf{p}_i^* &= \arg \max_{\mathbf{p}_i \in \mathbb{R}_+^{N_i}} f_i(\mathbf{p}_i, \mathbf{p}_{-i}) \\ & \text{subject to} \quad \|\mathbf{p}_i\|_1 = P_i \end{aligned} \quad (25)$$

be the (water-filling) power allocation vector that maximizes the utility function when the available power is  $P_i$ . It is also true that

$$\begin{aligned} P_i &= \min_{\mathbf{p}_i \in \mathbb{R}_+^{N_i}} \|\mathbf{p}_i\|_1 \\ & \text{subject to} \quad f_i(\mathbf{p}_i, \mathbf{p}_{-i}) = f_i(\mathbf{p}_i^*, \mathbf{p}_{-i}). \end{aligned} \quad (26)$$

$\mathbf{p}_i^*$  can be written as  $\mathbf{p}_i^* = P_i \mathbf{q}_i$ , with  $\mathbf{q}_i \in \mathbb{R}_+^{N_i}$  and  $\|\mathbf{q}_i\|_1 = 1$ . The solutions of problems (23) and (25) are the same vector  $\mathbf{p}_i^*$  if the norm constraint  $P_i$  of (25) is chosen such that the maximum of utility achievable with this norm is equal to the utility constraint of problem (23) - or, reversely, if the utility constraint in (23) is such that the solution has a norm equal to the constraint of problem (25). Considering that the only power vectors that should be used are those having the water-filling property form, the strategies sets for this game can be reduced to the choice of the  $\ell_1$ -norm of these vectors. Thus,

$$\mathcal{A}_i = [P_i^{\min}, P_i^{\max}]. \quad (27)$$

The utility function then satisfies

$$f_i(P_i, \mathbf{p}_{-i}) = \alpha_i \sum_{n=1}^{N_i} \log(1 + P_i \gamma_{i,n}(\mathbf{p}_{-i}) q_{i,n}), \quad (28)$$

with  $P_i \in \mathcal{A}_i$  and the satisfaction correspondence is expressed by

$$\varphi_i(\mathbf{p}_{-i}) = \{P_i \in \mathcal{A}_i : f_i(P_i, \mathbf{p}_{-i}) \geq \Gamma_i\}. \quad (29)$$

Then, the LESR of player  $i$  becomes the smallest power in  $\mathcal{A}_i$  for which the water-filling allocation on  $N_i$  subcarriers gives a utility greater than the QoS constraint  $\Gamma_i$  (see also (6)). From a practical point of view, it can be evaluated by solving the problem (23) and checking whether the norm  $P_i$  of the solution is greater than  $P_i^{\max}$  or not. If so, it means that the set  $\varphi_i(\mathbf{p}_{-i})$  is empty. Otherwise, the water-filling vector with norm  $P_i$  that maximizes  $f_i$  complies with the QoS constraint and gives exactly  $f_i(P_i, \mathbf{p}_{-i}) = \Gamma_i$ .

Since this game is not finite, in the sense that the players do not have a finite set of strategies, the existence of a



GSE cannot be guaranteed from the arguments of Sec. II-B. However, one can verify that the strategies sets  $\mathcal{A}_i$  and utility functions  $f_i$  from Eq. (27) and (28) are compliant with the conditions of the Debreu-Glicksberg-Fan theorem [32, Theorem 1.2]. This theorem states that the existence of a pure NE is guaranteed in any game whose strategies sets are non-empty compact and convex sub-sets of an Euclidian space and whose utility functions  $f_i(a_i, a_{-i})$  are continuous in  $(a_i, a_{-i})$  and quasi-concave in  $a_i$ . As any pure NE is also a GSE, the existence of at least one GSE is guaranteed for this game. Convergence of Algorithm 1 is discussed in Appendix B-A.

2) *Game with 1-bit feedback*: The players use Algorithm 2. In this game, the transmitters do not have access to the values of the statistical CSI  $\gamma_i(\mathbf{p}_{-i})$ . Instead, they observe an ACK/NACK signal fed back by the receiver. Since they cannot compute a water-filling solution, it will be considered that their choices are reduced to the number of subchannels on which the transmitter allocate a non-zero, constant power. The strategies sets are defined as

$$\mathcal{A}_i = \{1, \dots, N_i\}. \quad (30)$$

The corresponding power allocation vectors possibly chosen are

$$\mathcal{P}_i(a_i) = \left\{ \mathbf{p}_i \in \{0, P_i\}^{N_i} : \|\mathbf{p}_i\|_0 = a_i \right\} \quad (31)$$

where  $P_i = P_i^{\max}/N_i$  and  $a_i \in \mathcal{A}_i$ .

Each transmitter is initialized with its least expensive strategy, which consists in any vector with only one non-zero component chosen randomly. According to Algorithm 2, when the transmitter receives a NACK, it means that it must randomly choose a more expensive strategy than the one previously played. Here, it will consists in randomly choosing some subchannels to allocate with power  $P_i$  among those that are not already used. This can be formalized as follows. For a given power allocation vector  $\mathbf{p}_i(t) \in \mathcal{P}_i(a_i(t))$ , chosen by player  $i$  at time  $t \geq 0$ , let

$$\mathcal{N}_i(t) = \{n \in \{1, \dots, N_i\} : p_{i,n}(t) \neq 0\} \quad (32)$$

be the set of subcarriers allocated with power  $P_i$ . At time  $t+1$ , this set is extended such that  $|\mathcal{N}_i(t+1)| > |\mathcal{N}_i(t)|$  with  $\mathcal{N}_i(t) \subset \mathcal{N}_i(t+1)$ .

The existence of at least one pure GSE for this game is guaranteed by the arguments of Sec. II-A. Similarly to the DSSS game, this GSE is the pure NE  $\bar{a} = (N_1, \dots, N_I)$  where every player transmits at its maximum power. A proof that Algorithm 2 converges is given in Appendix B-B.

#### IV. NUMERICAL EXAMPLES

The framework of satisfaction games and GSE learning is now illustrated with numerical results obtained on synthetic UWA channels. Considerations for a practical application of this framework are discussed at the end of this section.

We consider UWA communication links transmitting at the same time and using the same bandwidth  $B = 6$  kHz, centered at  $f_c = 12$  kHz. First, the DSSS and OFDM scenarios are studied separately. The two different types of feedback are

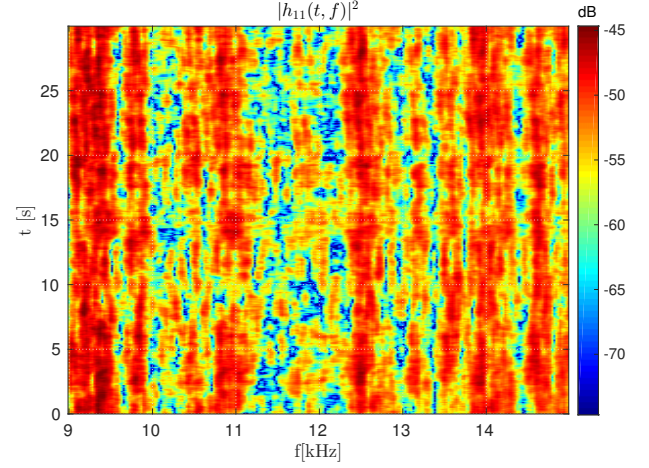


Fig. 1. A channel frequency response from the UWA channel simulator from [42], [43].

considered, namely, average SINR or statistical CSI and 1-bit feedback. A last scenario mixing DSSS and OFDM with 1-bit feedback is also presented. The performance is evaluated in terms of probability of satisfaction per player. The average power used per satisfied player at the equilibrium and the convergence speed to an equilibrium are also considered.

In the simulations, transmitters and receivers are randomly immersed inside a 1.5 km radius circle. The water depth is set to 50 meters, and the minimum and maximum immersion depths are 5 and 20 meters, respectively. For each single game played, the location and immersion depths are randomly drawn with a minimal distance of 500 meters between terminals. The UWA channel simulator described in [42], [43] is used to produce the time-varying impulse responses of the channels between each possible TX-RX link. The channels statistics needed by the players to compute their utility functions (18) and (28) are extracted from these responses. On this basis, a game is run for several iterations during which the players update their transmission strategy according either to Algorithm 1 or 2, depending on the type of feedback returned by the RXs. The probability of satisfaction, average power consumption of the satisfied transmitters and average number of iterations results from averages over 500 game realizations.

The channel simulator produces synthetic Rice fading<sup>7</sup> UWA channels and proceeds as follows: parameters describing the transmission geometry (ranges and depths) are used as inputs of a ray-tracing model to obtain a static impulse response around which time fluctuations are then generated by entropy maximization of the Doppler spectrum. The channel fluctuations are parameterized by the mean Doppler spread  $\sigma_D$  and the Rice factor of the main path  $K_{max}$ , given as constraints of the entropy maximization problem. The Rice factors of the secondary paths decrease exponentially with their arrival times. For all the simulation scenarios described here, the Doppler spread and Rice factor are set to  $\sigma_D = 1$  Hz and  $K_{max} = 20$  dB, respectively. The simulator also integrates

<sup>7</sup>Rice fading is assumed here for the needs of the simulations. This assumption is made in agreement with [39], [44], [45] but note that there exists other statistical models for UWA channels [46], [47].

physical propagation models for path losses and frequency-dependent attenuation (given by Thorp's formula) [2]. A sample realization of a simulated UWA channel frequency response corresponding to the described setup is shown in Figure 1. Other examples can be found in [42, Fig. 2], [19, Fig. 1] or [18, Fig. 1]. We implicitly assume that the channel statistics are stationary during a game, or at least until an equilibrium is reached. This is an approximation as in practice the taps statistics can be time-varying [39], [48]. This point is discussed in Sec. IV-D. A reference signal-to-noise ratio (SNR) of 25 dB is set for a 1 km transmission range at maximum power without interference. If the transmission range changes during the simulations, the SNR varies accordingly to the path loss. The noise power spectral density is modelled as a linear decay of 18 dB/decade on the frequency scale [49].

#### A. Game 1: DSSS modulation

The minimum SINR to satisfy the QoS constraint (19) is set to  $\Gamma_i^{(\text{dB})} = 15$  dB for all players. Their minimum power is fixed to  $P_i^{\text{min, dB}} = 170$  dB ref  $\mu\text{Pa}$  @ 1m and their strategies sets are depicted in Table I.

TABLE I  
ACTION SPACE OF THE DSSS PLAYERS

$a_i$	$P_i^{(x_i)}$	$G_i^{(x_i)}$
1	$1 \times P_i^{\text{min}}$	1
2	$1 \times P_i^{\text{min}}$	4
3	$4 \times P_i^{\text{min}}$	4
4	$4 \times P_i^{\text{min}}$	16
5	$16 \times P_i^{\text{min}}$	16
6	$16 \times P_i^{\text{min}}$	64
7	$64 \times P_i^{\text{min}}$	64

Note that the same strategies set is used for all players in the simulation, but the proposed framework allows heterogeneous strategies sets. The symbol time is  $T_i = N_i \times T_c$  with constant chip rate  $T_c \approx 1/B \approx 166 \mu\text{s}$  and  $N_i$  the size of the PN sequence. The useful rate depends on  $N_i$  and therefore changes with the strategy chosen.

The NE of this game is  $a_{NE} = [a_1 = 7, \dots, a_I = 7]$ , which corresponds to a situation where all transmitters use the highest power and spreading gain. In the following, the probability of satisfaction  $\mathbb{P}[f_i(a_i, a_{-i}) \geq \Gamma_i]$  is evaluated and comparison is made between the NE strategies and the GSE learned through Algorithms 1 and 2. The two algorithms are initialized with all players using their minimum power and spreading gain, i.e.  $(a_i = 1)_{i \in \mathcal{I}}$ . The power used on the average by satisfied players and the convergence time to a GSE in terms of iterations are also considered. Results are shown in Fig. 2 and 3.

1) *Games with SINR feedback*: Results of the DSSS games with average SINR feedback and Algorithm 1 are shown in Fig. 2

Fig. 2-(a) depicts the probability of satisfaction for each player at a GSE and a comparison is made with the NE strategy profile. An improvement can be seen as the number of players in the game increases. At the NE, the interference

is maximum since all players transmit at maximum power. Whereas at a GSE, unsatisfied players are more likely to use lower levels of power. Therefore, if there are many players in the game, the specific equilibrium where none of the player has satisfied its QoS constraint occurs more often when players learn a NE than when they learn a GSE. For a small number of players, Fig. 2-(b) shows that for a similar probability of satisfaction between a NE and GSE, the consumed power at a GSE is smaller than at a NE. The number of iterations before convergence translates into convergence time only when put in relation with the update scheme and the update period. For instance, consider a sequential update where players start their transmission at different times and have the same update period  $T_{\text{obs}} = 20$  seconds.<sup>8</sup> They play one after the other and there are 20 seconds between two updates of a given player. As shown in Fig. 2-(c) and considering this update scheme, the corresponding convergence time is then around or below 40 seconds on average. With the exception of the two player game, players update their strategy not more than once on average. The same kind of analysis can be conducted for all the other games.

2) *Games with 1-bit feedback*: Results of the DSSS games with 1-bit feedback and Algorithm 2 are shown in Fig. 3.

Except for the one and two player games, no significant difference can be seen between Fig. 2-(b) and 3-(b). In games with more than two UA communication systems, the considered area of 1.5 km radius is small enough to make the players use their highest strategies to be satisfied, because of the perceived level of interference. At a GSE, they often choose  $a_i = 6$  or  $a_i = 7$ . When there are only one or two systems in the area, they could use less power to satisfy the constraint but they have less information than in the previous game. With SINR feedback, the power used is lower because players choose systematically the least effort strategy. There is no difference between Fig. 2-(a) and 3-(a) in terms of successful channel access probability. When the number of players exceeds two, they converge almost always toward similar GSEs with maximum powers and spreading gains. The difference in convergence times between the SINR and the ACK/NACK feedback is explained by the fact that, in the second case, players choose a random strategy among those that are more expensive than the previous one. Consequently, they may chose an strategy that would not be returned by their satisfaction correspondence and this slows down the process of GSE learning. However, using the same sequential update scheme as presented in Sec. IV-A1, the convergence time is below 60 seconds on average.

#### B. Game 2: OFDM modulation

In OFDM games, the bandwidth  $B = 6$  kHz is divided  $\forall i \in \mathcal{I}$  in  $N_i = N = 256$  orthogonal subcarriers so that  $B = N \times \Delta f$ . The OFDM symbol duration is  $T = \Delta f^{-1} + T' = 57.7$  ms where  $T' = 15$  ms is the cyclic prefix duration. The Qos constraint (29) is set to  $\Gamma_i = 1$  bit/s/Hz. For the simulation

<sup>8</sup>This is only a particular case of the asynchronous algorithms formalized in Sec. II-C

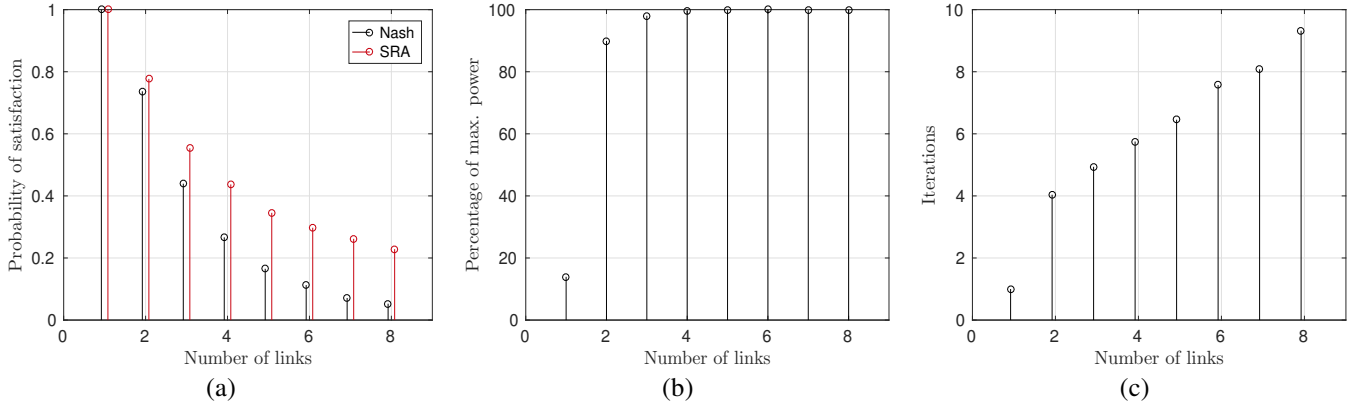


Fig. 2. DSSS game with SINR feedback and Algorithm 1,  $\Gamma_i^{(\text{dB})} = 15 \text{ dB}$ ,  $\forall i \in I$ . From left to right: probability of satisfaction, average power used by satisfied players, average number of iterations to a GSE.

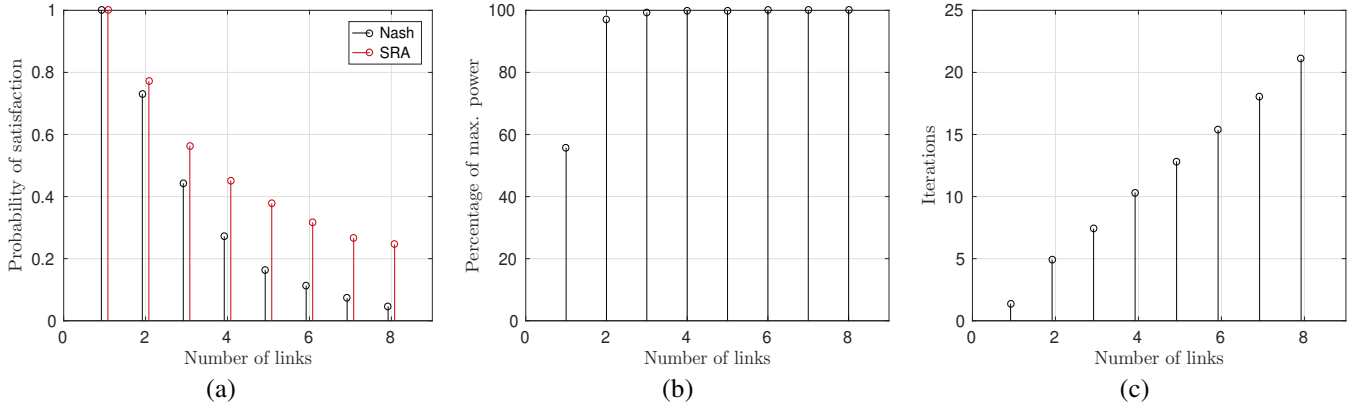


Fig. 3. DSSS game with 1-bit feedback and Algorithm 2,  $\Gamma_i^{(\text{dB})} = 15 \text{ dB}$ ,  $\forall i \in I$ . From left to right: probability of satisfaction, average power used by satisfied players, average number of iterations to a GSE.

$N_i = N$ ,  $\forall i \in \mathcal{I}$ , but note that the proposed framework can account for players with heterogeneous configurations.

1) *Games with statistical CSI feedback:* In this game, the strategies set of any player  $i \in \mathcal{I}$  is the power at which it constrains its water-filling power allocation vector (27). The minimum and maximum transmission powers  $P_i^{\min}$  and  $P_i^{\max}$ ,  $\forall i \in \mathcal{I}$  are set to 170 and 190 dB ref.  $\mu\text{Pa}$  @ 1m, respectively. Results of the OFDM games with statistical CSI and Algorithm 1 are depicted in Fig. 4.

In Fig. 4-(a), the performance in terms of successful access probability is compared to the NE solution which corresponds to the water-filling power allocation at power  $P_i^{\max}$ . Fig. 4-(a) and (b) show that the performance gain compared to the NE solutions is mostly significant in terms of power savings. When there are less than four players, the probability of satisfaction obtained at the NE remains high because of the choice of the QoS constraints along with the reference SNR and radius of the transmission area. These factors combined may allow the systems to comply with their QoS constraint of 1 bits/s/Hz when using the water-filling power allocation with power  $P_i^{\max}$ , provided that there are not too many interfering systems in the area. The good performance in terms of power used (always less than 70% of  $P_i^{\max}$  on average) is not surprising since Algorithm 1 enforces the transmitters to use their minimal transmission power. The number of iterations

necessary to converge is quite small, with, on average, two or three iterations per player.

2) *Games with 1-bit feedback:* Each player has a different number  $N_i$  of subchannels, uniformly chosen between  $\{4, 8, 16, 32\}$  at a new realization of a game. This consists in dividing the  $N = 256$  previous subcarriers in  $\{8, 16, 32, 64\}$  groups of adjacent subcarriers. The power allocated by a player  $i \in \mathcal{I}$  on each subcarrier/subchannel is either  $P_i$  or 0, with  $P_i = P_i^{\max}/N_i = P_i^{\min}$ . The strategies sets are the number of subchannels on which a non-zero power is allocated. The maximum power  $P_i^{\max}$  is set to 190 dB ref.  $\mu\text{Pa}$  @ 1m,  $\forall i$ . Results of the OFDM games with 1-bit feedback and Algorithm 2 are shown in Fig. 5.

Fig. 5-(a) shows that the probability of satisfaction per player is significantly improved compared to the NE strategy. Since less information is available at the TXs, the performance are not as good as in the game with knowledge of the statistical CSI. In Fig. 4-(a), the small gap in probability of satisfaction between NE and SRA, compared to Fig. 5-(a), is explained by a better correlation of the power allocation strategies to the channel and to the external interferences for both NE and SRA. In the 1-bit feedback case, the power allocation vector is chosen randomly on the sole basis of a one bit information and the NE of this game is a power allocation with no adaptation neither to the channel nor to the interferences. As the power

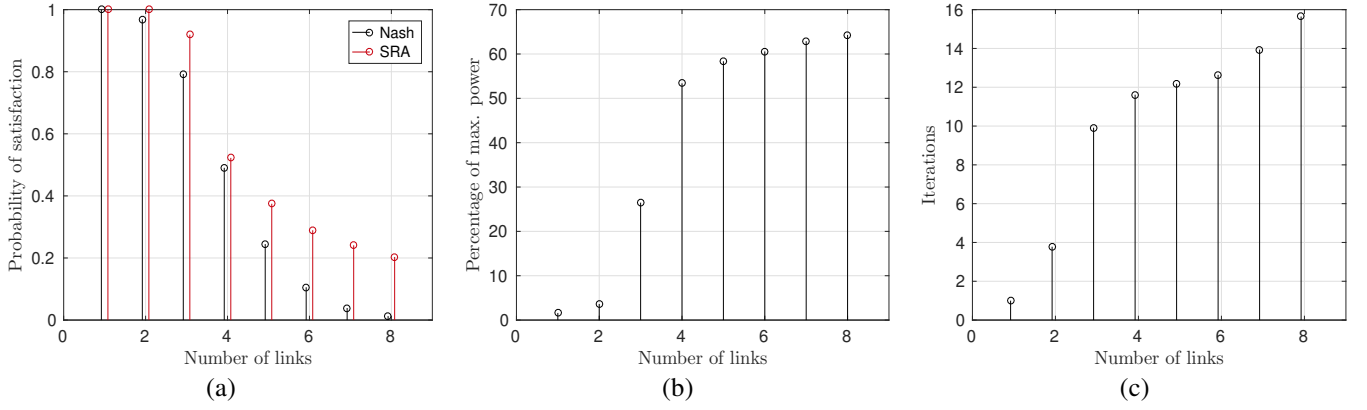


Fig. 4. OFDM game with CSI feedback and Algorithm 1,  $\Gamma_i = 1$  bit/s/Hz,  $\forall i \in I$ . From left to right : probability of satisfaction, average power used by satisfied players, average number of iterations to a GSE.

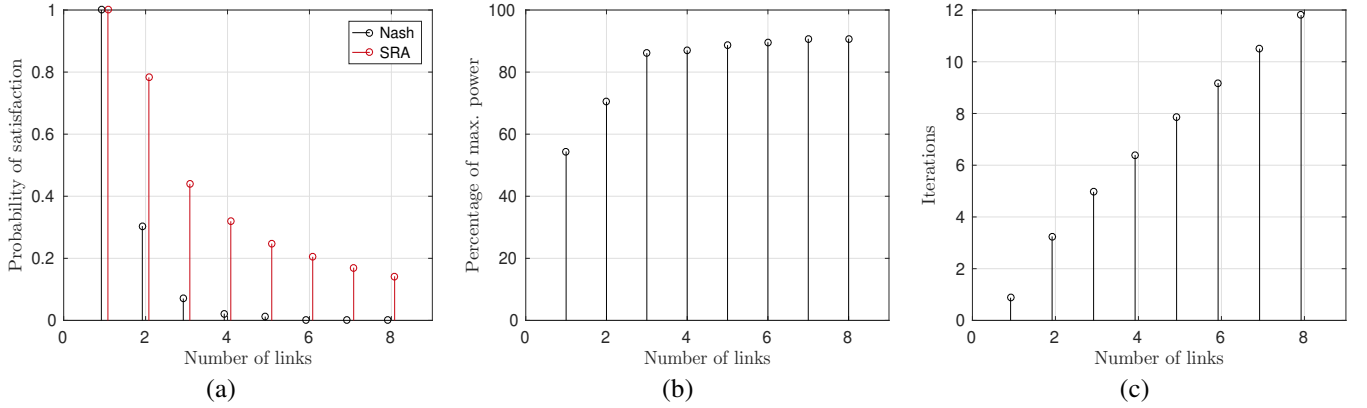


Fig. 5. OFDM game with 1-bit feedback and Algorithm 2,  $\Gamma_i = 1$  bit/s/Hz,  $\forall i \in I$ . From left to right: probability of satisfaction, average power used by satisfied players, average number of iterations to a GSE.

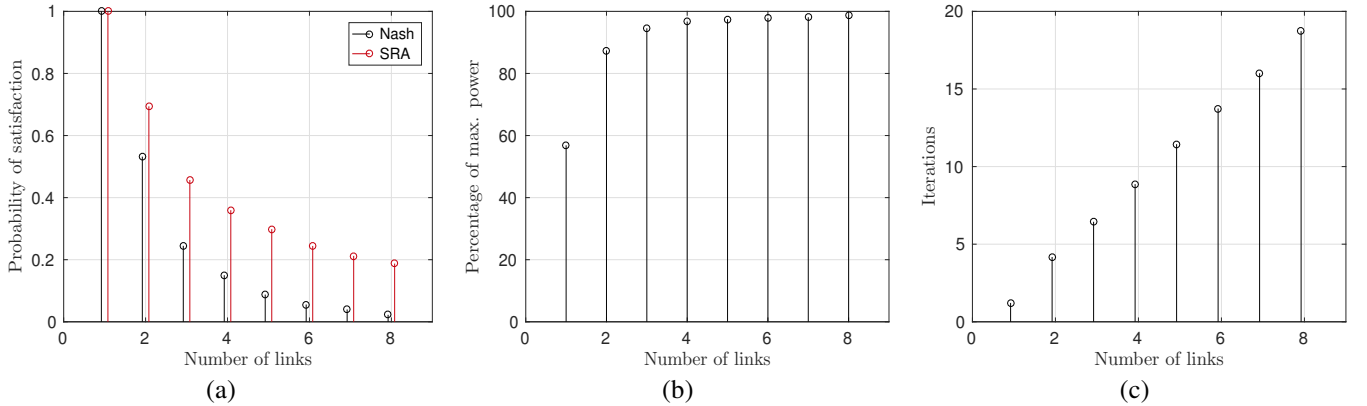


Fig. 6. Game with heterogeneous systems, 1-bit feedback and Algorithm 2,  $\Gamma_i^{(\text{dB})} = 15$  dB for DSSS players and  $\Gamma_i = 1$  bit/s/Hz for OFDM players. From left to right: probability of satisfaction, average power used by satisfied players, average number of iterations to a GSE.

level allocated to a single subcarrier cannot be tuned, it is also natural that the power used by satisfied players is higher than in the other game. The results in terms of convergence time are of the same order.

### C. Game 3: heterogeneous systems with 1-bit feedback

In a last setup, we evaluate the performance of Algorithm 2 when implemented with heterogeneous systems. The simulation process is the same as in the previous games and focus is put on the ACK/NACK feedback. Each time a game is played,

a player is randomly assigned a type between OFDM or DSSS. Fig. 6 depicts the results obtained. As in the other games, improvement in terms of probability of satisfaction can be seen compared to transmission at maximum power. Note that in this game, DSSS players can benefit from the spreading gain and will generate a high level of interference to OFDM players. The average power budget used at the GSE by satisfied players has the same shape as in the DSSS and OFDM games with ACK/NACK feedback. Naturally, it seems to be the average of the two performances since, at each game played, the type

of players is drawn uniformly. The number of iterations to converge to an equilibrium is also small, with slightly more than two iterations per player in the game.

#### D. Practical aspects

To make the most of the satisfaction game framework, the channel statistics must remain quite stable while Algorithm 1 or 2 has not reached the equilibrium. With an update period  $T_{obs} = 20$  seconds and a sequential update, equilibria can be reached within 40 to 60 seconds. Assuming (quasi) wide-sense stationarity over this period of time is often fine for fixed or slowly drifting acoustic sources. With moving sources such as AUVs, the channel statistics may fluctuate quicker. In that case,  $T_{obs}$  can be reduced and/or the utility functions changed to take into account the non-stationarity of the channel. For this scenario, the power consumed at the equilibrium may not be an adequate performance metric. It may be better to consider the total energy consumed during the whole period of the game. The period  $T_{obs}$  should also be long compared to the propagation delays to take advantage of the most recent strategies updates as well as to avoid outdated knowledge of the channel statistics at the transmitter side. The ACK/NACK feedback can be implemented with a robust, low bit-rate link from the receiver so as to ensure a low error probability. If a feedback loss occurs in practice, it could be considered as a NACK after a timer expires at the transmitter side. In this case, the timeout period should be of the same order of magnitude as the update period  $T_{obs}$ .

For the sake of clarity, the expressions of the utility functions (18) and (20) have been kept simple. However, the game-theoretic framework allows these functions to be modified in order to take into account implementation-specific issues. For instance, (18) and (20) could be modified to integrate residual inter-symbol or ICI interference terms.

#### V. CONCLUSIONS AND PERSPECTIVES

We have shown that efficient decentralized spectrum sharing is possible for UWA communications within the framework of satisfaction games. In such games, transmitters are modeled by players that seek to satisfy their individual QoS constraints. Each player adapts its transmission strategy based on some channel knowledge obtained through a feedback link. The original framework of satisfaction games has been extended to consider more practical scenarios where transmitters cannot have perfect knowledge of their satisfaction correspondence. More precisely, a blind satisfaction response algorithm has been proposed to deal with situations where only a 1-bit feedback is available. Spectrum sharing scenarios with multiple UWA DSSS and/or UWA OFDM links have been studied in details. The assumed strategy space for DSSS players has been the transmit power combined with the spreading gain. For OFDM players, this space has been the power allocation vector across frequencies. Depending on the game, the QoS has been formulated either as a SINR or information rate constraint. Two types of feedback have been implemented, namely the channel statistics or a 1-bit ACK/NACK. Both types of feedback are compatible with the specificity of the

UWA environment. Numerical experiments have shown that (generalized) satisfaction equilibria leads to Pareto-superior solutions compared to Nash equilibria where players aim to maximize their individual benefit. This performance improvement translates into power savings and/or higher probability of QoS satisfaction. The number of iterations to converge to a satisfaction equilibrium is also small, with around two iterations per player in the game. Possible extensions of this work include considering other strategy spaces (space and/or time and/or frequency), adapting this framework to highly non (wide-sense) stationary channels, modeling the interactions with non-communication UWA systems such as sonars, and conducting at-sea trials.

#### APPENDIX A

##### CONVERGENCE OF ALGORITHMS 1 AND 2 FOR DSSS PLAYERS

To prove the convergence of Algorithms 1 and 2, it suffices to check that all the conditions of Prop. 1 are met. Conditions 1) and 3) are true by hypothesis. Now, let  $\phi_i : \mathcal{A}_{-i} \rightarrow \mathcal{A}_i$  be defined as

$$\phi_i(\mathbf{x}_{-i}) = \inf_{a_i \in \mathcal{A}_i} \{a_i : f_i(a_i, \mathbf{x}_{-i}) \geq \Gamma_i\}, \quad (33)$$

with  $f_i(a_i, \mathbf{x}_{-i}) = \mathbb{E} \{\|\mathbf{H}_{ii}\|_F^2\} P_i^{(a_i)} G_i^{(a_i)} / \mathbb{E} \{\|\mathbf{z}_i(\mathbf{x}_{-i})\|_2^2\}$ . The satisfaction correspondence  $\phi_i$  then satisfies

$$\phi_i(\mathbf{x}_{-i}) = \{a_i \in \mathcal{A}_i : \phi_i(\mathbf{x}_{-i}) \preceq a_i\}. \quad (34)$$

To prove the convergence of Algorithms 1 and 2, we have to show that  $\phi_i$  is order-preserving.

Since  $\mathbf{z}_i(\mathbf{x}_{-i})$  represents the interference plus noise perceived by player  $i$ ,  $\mathbf{x}_{-i} < \mathbf{x}'_{-i}$  implies  $\mathbb{E} \{\|\mathbf{z}_i(\mathbf{x}_{-i})\|_2^2\} \leq \mathbb{E} \{\|\mathbf{z}_i(\mathbf{x}'_{-i})\|_2^2\}$ , which in turns implies  $f_i(a_i, \mathbf{x}_{-i}) \geq f_i(a_i, \mathbf{x}'_{-i})$ . From (33), we therefore conclude that  $\phi_i(\mathbf{x}_{-i}) \leq \phi_i(\mathbf{x}'_{-i})$  so that  $\phi_i$  is order-preserving, *i.e.* the more the opponents of  $i$  interfere on receiver  $i$ , the more expensive the satisfying strategy of player  $i$  needs to be. ■

#### APPENDIX B

##### CONVERGENCE OF ALGORITHMS 1 AND 2 FOR OFDM PLAYERS

###### A. Case 1: water-filling power allocation

By virtue of the arguments given in Sec. III-B1, the strategies set of any player  $i$  are the possible transmission powers

$$\mathcal{A}_i = [P_i^{\min}, P_i^{\max}], \quad (35)$$

and the only rational power allocation vectors on  $N_i$  subcarriers are water-filling vectors. These vectors are the solution  $\mathbf{p}_i^*$  given by (25) and are such that  $\mathbf{p}_i^* = P_i \mathbf{q}_i$  with  $\|\mathbf{q}_i\|_1 = 1$ . The satisfaction of any player  $i$  is then given by

$$\phi_i(\mathbf{p}_{-i}) = \left\{ P_i \in \mathcal{A}_i : \alpha_i \sum_{n=1}^{N_i} \log(1 + P_i \gamma_{i,n}(\mathbf{p}_{-i}) q_{i,n}) \geq \Gamma_i \right\}, \quad (36)$$

where  $\gamma_{i,n}(\mathbf{p}_{-i})$  is defined in (21).

From Prop. 1, if  $\varphi_i(\mathbf{p}_{-i})$  is expressed with an order-preserving function  $\phi_i$  then Algorithm 1 converges. Finding a function  $\phi_i$  that is order-preserving for any configuration of the game is not possible. However, a sufficient condition can be found. More precisely, the satisfaction correspondence can be written as  $\varphi_i(\mathbf{p}_{-i}) = \{P_i \in \mathcal{A}_i : \phi_i(\mathbf{p}_{-i}) \preceq P_i\}$  with  $\phi_i(\mathbf{p}_{-i}) = \inf_{P_i \in \mathcal{A}_i} \left\{ P_i : \alpha_i \sum_{n=1}^{N_i} \log(1 + P_i \gamma_{i,n}(\mathbf{p}_{-i}) q_{i,n}) \geq \Gamma_i \right\}$ . In configurations where  $\gamma_{i,n}(\mathbf{p}'_{-i}) \leq \gamma_{i,n}(\mathbf{p}_{-i})$  for any  $\|\mathbf{p}_{-i}\| < \|\mathbf{p}'_{-i}\|$ , then  $\phi_i$  is order-preserving. Such a situation occurs when the SINR perceived by player  $i$  decreases as its opponents use more power. Although this happens very often in practice, such a condition is not always satisfied. For instance, at a given iteration of Algorithm 1, an opponent of player  $i$  may increase its average power but, as a result of the water-filling procedure, may also allocate less power in the frequency band used by player  $i$ .

Therefore, for this game we are only able to provide sufficient conditions for convergence of Algorithm 1. However, note that during the simulations presented in Sec. IV-B, Algorithm 1 has always converged.

### B. Case 2: ON/OFF power allocation

The strategies set of an OFDM player is the number of subcarriers allocated with a power  $P_i = P_i^{\max}/N_i$ , i.e.

$$\mathcal{A}_i = \{1, \dots, N_i\}, \quad (37)$$

the corresponding possible power allocation vectors are

$$\mathcal{P}_i(a_i) = \{\mathbf{p}_i \in \{0, P_i\}^{N_i} : \|\mathbf{p}_i\|_0 = a_i\}. \quad (38)$$

We denote the strategic choice of  $i$  by  $a_i \in \mathcal{A}_i$  and the corresponding power vector by  $\mathbf{p}_i(a_i)$ . Accordingly,  $a_{-i} = (a_j)_{j \in \mathcal{I}, j \neq i} \in \mathcal{A}_{-i}$  and  $\mathbf{p}_{-i}(a_{-i})$  are the strategies and the corresponding power allocation vectors of the opponents of  $i$ . The utility function given in (20) can be expressed as

$$f_i(a_i, a_{-i}) = \sum_{n=1}^{N_i} \log(1 + \gamma_{i,n}(\mathbf{p}_{-i}(a_{-i})) p_{i,n}(a_i)) \quad (39)$$

with

$$\gamma_{i,n}(\mathbf{p}_{-i}(a_{-i})) \triangleq \frac{e^{\mathbb{E}\{\log|h_{ii,n}|^2\}}}{\sigma_{w_i,n}^2 + \sum_{j \neq i} \mathbb{E}\{|h_{ji,n}|^2\} p_{j,n}(a_j)}. \quad (40)$$

The satisfaction correspondence  $\varphi_i$  satisfies

$$\varphi_i(a_{-i}) = \{a_i \in \mathcal{A}_i : \phi_i(a_{-i}) \preceq a_i\} \quad (41)$$

with  $\phi_i(a_{-i}) = \inf_{a_i \in \mathcal{A}_i} \{a_i : f_i(a_i, a_{-i}) \geq \Gamma_i\}$ .

For  $a_{-i} < a'_{-i}$ , we have  $\gamma_{i,n}(\mathbf{p}_{-i}(a'_{-i})) \leq \gamma_{i,n}(\mathbf{p}_{-i}(a_{-i}))$ , i.e. when the opponents of player  $i$  increase their number of allocated subcarriers, the SINR perceived by player  $i$  will decrease or remain the same. In this case,  $f_i(a_i, a_{-i}) \geq f_i(a_i, a'_{-i})$  so that  $\phi_i$  is order-preserving. Therefore, the conditions of Prop. 1 are satisfied so that Algorithm 2 converges to a GSE.

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