A Dual Reconfigurable 4-rRUU Parallel Manipulator

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Abhilash Nayak[1], Stéphane Caro[2] and Philippe Wenger[2]

Abstract— The aim of this paper is to introduce the use of a double Hooke's joint linkage to reconfigure the base revolute joints of a 4-RUU parallel manipulator (PM). It leads to an architecturally reconfigurable 4-rRUU PM whose platform motion depends on the angle between the driving and driven shafts of the double-Hooke's joint linkage in each limb. Even when the angle is fixed, the manipulator is reconfigurable by virtue of the different operation modes it can exhibit. By keeping the angle as a variable, the constraint equations of the 4-rRUU PM are derived using Study's kinematic mapping. Subsequently, the ideal of constraint polynomials is decomposed as an intersection of primary ideals to determine the operation modes of the 4-rRUU PM for intersecting and parallel revolute joint axes in the base and the moving platform.

I. INTRODUCTION

Reconfigurability in a PM extends its employability for a variety of applications. A lower mobility PM with *dof*< 6 is reconfigurable if it has different configuration space regions with possibly different type or number of degrees of freedom. These regions are known as the operation modes of the PM and were first exemplified by Zlatanov et al. [1] for the 3-URU DYMO robot exhibiting five different types of platform motion. Its closely related SNU 3-UPU [2] and Tsai 3-UPU [3] PMs were analyzed by Walter et al. with a complete characterization of their operation modes along with the transition poses. Most of the operation modes of the 3-URU or the 3-UPU PMs are physically distinguishable unlike the 3-[PP]S PMs for which the first two joints in each limb generate a motion equivalent to two coplanar translations followed by a spherical joint. 3-RPS [4], 3-PRS [5] and 3-SPR PMs [6] are such manipulators that exhibit two operation modes each with coupled motion. Other reconfigurable PMs include the 3-RER PM (E denotes a planar joint) found to have 15 3-*dof* operation modes [7] and the 4-RUU PM with vertical base and platform revolute joint axes possessing three operation modes [8].

Besides, a PM can be reconfigurable also by changing the position and/or orientation of one or more of its constituent joints. This type of reconfigurability is named architectural reconfigurability in this paper. MaPaMan [9] is one such manipulator in which the strut can be oriented to have two different architectures of the same PM where it can transition between roll-pitch-heave and roll-pitch-yaw degrees of freedom. Gan et al. [10] introduced a novel reconfigurable

revolute joint and proposed a metamorphic 3-rRPS PM. In this paper, a double Hooke's joint linkage is used as a reconfigurable revolute joint and is used to demonstrate different types of reconfigurability of a 4-RUU PM.

The double Hooke's joint linkage is a well-known special over-constrained 6R-mechanism, where the first three and the last three joint axes are mutually perpendicular. It is also known as a Double Cardan Joint and is ubiquitous as a steering column in automobiles. Its architecture is fairly simple compared to a general over-constrained 6R mechanism which makes it easier to derive the input-output relations. There have been different approaches in the literature to derive its input-output relations proving that it is a constant velocity transmitter when the angle between the input shaft and the central yoke is equal to the angle between the central yoke and the output shaft [11], [12], [13]. The constant-velocity transmission property of a double-Hooke's joint is exploited in this paper to reconfigure the first revolute joint axis in each limb of a 4-RUU PM.

The organization of the paper is as follows: Section II presents the architecture of the dual reconfigurable 4-rRUU PM along with the architecture of its constituent double-Hooke's joint linkage. Section III deals with the derivation of constraint equations and the determination of operation modes of 4rRUU PMs for some specific orientations of the base and revolute joint axes. Section IV concludes the paper and puts forth some open issues associated with the construction of a functional 4-rRUU PM prototype.

II. THE 4-rRUU PARALLEL MANIPULATOR

A. Manipulator Architecture

The architecture of the dual reconfigurable 4-rRUU PM with a square base and a platform is shown in Fig. 1 and its constituent double-Hooke's joint linkage is shown in Fig. 2.

A reconfigurable revolute joint (rR) and two universal joints (UU) mounted in series constitute each limb of the 4-rRUU PM. Point L_i, $i = 1, 2, 3, 4$ lies on the pivotal axis of the double-Hooke's joint linkage as shown in Fig. 2. Point A_i lies on the first revolute joint axis of the 4-rRUU PM and it can be obtained from point L_i by traversing a horizontal distance of l_i along the first revolute joint axis. Points B_i and C_i are the geometric centers of the first and the second universal joints, respectively. Points L_i and C_i form the corners of the square base and the platform, respectively. $\mathscr{F}_O$ and $\mathscr{F}_P$ are the coordinate frames attached to the fixed base and the moving platform such that their origins O and P lie at the centers of the respective squares. The revolute-joint axes vectors in i-th limb are marked $\mathbf{s}_{ij}$, $i = 1, 2, 3, 4; j = 1, ..., 5$. Vectors $\mathbf{s}_{i1}$ and $\mathbf{s}_{i2}$ are

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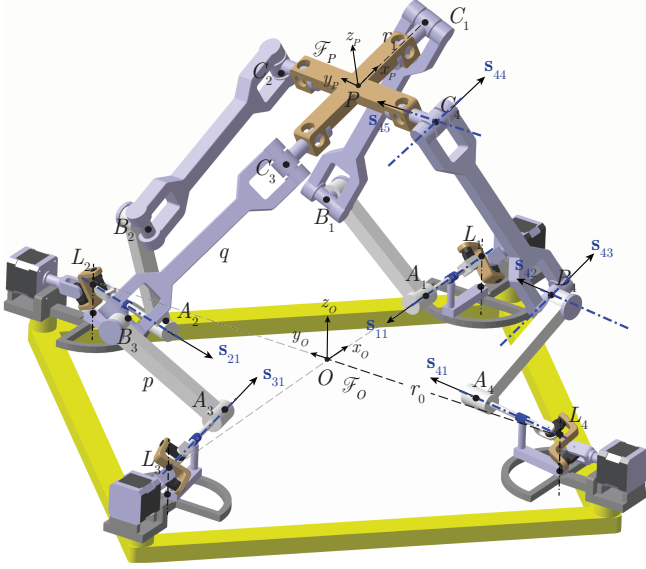
Fig. 1: A 4-rRUU parallel manipulator

Fig. 2: Double Hooke's joint

Fig. 3: Possible orientations of the base revolute joint
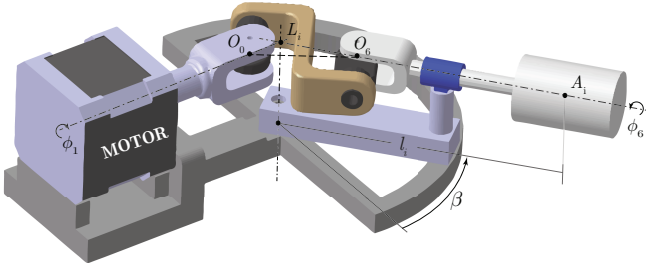
always parallel, so are vectors $\mathbf{s}_{i3}$ and $\mathbf{s}_{i4}$. For simplicity, it is assumed that the orientation of vector $\mathbf{s}_{i1}$ expressed in coordinate frame $\mathscr{F}_O$ is the same as that of $\mathbf{s}_{i5}$ expressed in coordinate frame $\mathscr{F}_P$. The position vectors of points L_i, A_i, B_i and C_i expressed in frame $\mathcal{F}_k, k \in O, P$ are denoted as ${}^k\mathbf{l}_i, {}^k\mathbf{a}_i, {}^k\mathbf{b}_i$ and ${}^k\mathbf{c}_i$, respectively. r_0 and r_1 are half the diagonals of the base and the moving platform squares, respectively. p and q are the link lengths.

B. Double-Hooke's joint linkage

The double Hooke's joint linkage is shown in Fig. 2. The first three and the last three revolute joint axes intersect at points O_0 and O_6, respectively. The first revolute joint is driven by a motor with an input angle of ϕ_1 and the last revolute joint rotates with an output angle of ϕ_6 and their axes intersect at point $L_i, i = 1, 2, 3, 4$. It is noteworthy that for a constant-velocity transmission, the triangle $\triangle O_0 O_6 L_i$ must be isosceles with $O_0 L_i = O_6 L_i$. The angle between the input and the output shafts is denoted as $\beta \in [0, \pi]$. Since the double-Hooke's joint is known to be a constant-velocity transmitter, the following input-output relationship holds [11], [12], [13]:

$$\phi_6 = -\phi_1 \qquad (1)$$

Figure 3 shows the top view of the 4-rRUU PM without the links. For architectural reconfigurability, the reconfigurable
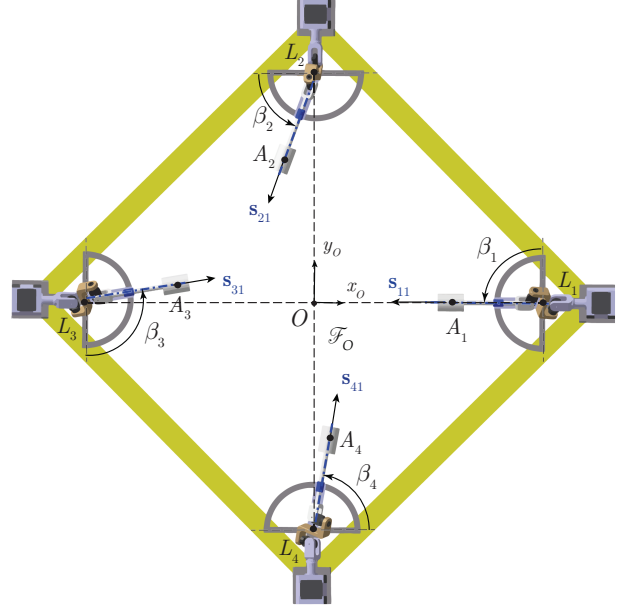
revolute joint axis in the base is allowed to have a horizontal orientation $\beta_i, i = 1, 2, 3, 4$. It is noteworthy that β_i will be changed manually in the prototype under construction.

III. OPERATION MODE ANALYSIS

A. Constraint Equations

Since the reconfigurable revolute joint is actuated, a RUU limb must satisfy the following two constraints:

1) The second revolute joint axis, the fifth revolute joint axis and link BC must lie in the same plane. In other words, the scalar triple product of the corresponding vectors must be null:

$$g_i : (\mathbf{b}_i - \mathbf{c}_i)^T (\mathbf{s}_{i2} \times \mathbf{s}_{i5}) = 0, \quad i = 1, 2, 3, 4 \quad (2)$$

2) The length of link BC must be q:

$$g_{i+4} : ||\mathbf{b}_i - \mathbf{c}_i|| - q = 0, \quad i = 1, 2, 3, 4 \quad (3)$$

Since the length of link BC does not affect the operation modes of the 4-rRUU PM, only the principal geometric constraint from Eq. (2) is considered. To express it algebraically, the homogeneous coordinates of the necessary vectors are listed below:

$${}^0\mathbf{l}_i = \mathbf{R}_z(\lambda_i) [1, r_0, 0, 0]^T \qquad (4a)$$

$${}^0\mathbf{a}_i = {}^0\mathbf{l}_i + \mathbf{R}_z(\lambda_i + \beta_i) [0, 0, l_i, 0]^T \qquad (4b)$$

$${}^0\mathbf{b}_i = {}^0\mathbf{a}_i + \mathbf{R}_z(\lambda_i + \beta_i) [0, p\cos(\theta_i), 0, p\sin(\theta_i)]^T \quad (4c)$$

$${}^0\mathbf{c}_i = \mathbf{F}\,\mathbf{R}_z(\lambda_i)[1, r_1, 0, 0]^T, \qquad (4d)$$

$${}^0\mathbf{s}_{i2} = \mathbf{R}_z(\lambda_i + \beta_i) [0, 0, 1, 0]^T, \qquad (4e)$$

$${}^0\mathbf{s}_{i5} = \mathbf{F}\,\mathbf{R}_z(\lambda_i + \beta_i)[0, 0, 1, 0]^T, \quad i = 1, 2, 3, 4. \qquad (4f)$$

where $\mathbf{R}_z(\cdot)$ is the homogeneous rotation matrix about the z-axis, λ_i for the i-th limb is given by $\lambda_1 = 0, \lambda_2 = \dfrac{\pi}{2}, \lambda_3 = \pi, \lambda_4 = \dfrac{3\pi}{2}$ and θ_i is the actuated joint angle. $\mathbf{F}$ is the

transformation matrix consisting of Study parameters x_j and y_j, $j = 0, 1, 2, 3$:

$$\mathbf{F} = \frac{1}{\Delta} \begin{bmatrix} \Delta & 0 & 0 & 0 \\ d_1 & r_{11} & r_{12} & r_{13} \\ d_2 & r_{21} & r_{22} & r_{23} \\ d_3 & r_{31} & r_{32} & r_{33} \end{bmatrix} \tag{5}$$

with $\quad \Delta = x_0{}^2 + x_1{}^2 + x_2{}^2 + x_3{}^2 \neq 0$

and $\quad r_{11} = x_0{}^2 + x_1{}^2 - x_2{}^2 - x_3{}^2$

$\quad\quad r_{12} = -2 x_0 x_3 + 2 x_1 x_2$

$\quad\quad r_{13} = 2 x_0 x_2 + 2 x_1 x_3$

$\quad\quad r_{21} = 2 x_0 x_3 + 2 x_1 x_2$

$\quad\quad r_{22} = x_0{}^2 - x_1{}^2 + x_2{}^2 - x_3{}^2$

$\quad\quad r_{23} = -2 x_0 x_1 + 2 x_2 x_3$

$\quad\quad r_{31} = -2 x_0 x_2 + 2 x_1 x_3$

$\quad\quad r_{32} = 2 x_0 x_1 + 2 x_2 x_3$

$\quad\quad r_{33} = x_0{}^2 - x_1{}^2 - x_2{}^2 + x_3{}^2$

$\quad\quad d_1 = -2 x_0 y_1 + 2 x_1 y_0 - 2 x_2 y_3 + 2 x_3 y_2,$

$\quad\quad d_2 = -2 x_0 y_2 + 2 x_1 y_3 + 2 x_2 y_0 - 2 x_3 y_1,$

$\quad\quad d_3 = -2 x_0 y_3 - 2 x_1 y_2 + 2 x_2 y_1 + 2 x_3 y_0.$

Thus, Eq. (2) is derived for each limb algebraically by substituting $t_i = \tan(\frac{\theta_i}{2})$ and $w_i = \tan(\frac{\beta_i}{2})$, $i = 1, 2, 3, 4$. The constraint polynomials $g_i, i = 1, 2, 3, 4$ form the following ideal[1]:

$$\mathcal{I} = \langle g_1, g_2, g_3, g_4 \rangle \subseteq k[x_0, x_1, x_2, x_3, y_0, y_1, y_2, y_3] \tag{6}$$

To simplify the determination of the operation modes, the 4-rRUU PM is split into two 2-rRUU PMs [8] by considering two ideals:

$$\mathcal{I}_{(I)} = \langle g_1, g_3 \rangle \tag{7a}$$
$$\mathcal{I}_{(II)} = \langle g_2, g_4 \rangle \tag{7b}$$

Even after substituting the design parameters, it was impossible to calculate the primary decomposition of ideals $\mathcal{I}_{(I)}$ and $\mathcal{I}_{(II)}$ for a general β_i and it remains an open issue. Consequently, some special configurations of the 4-rRUU PM are considered and their operation modes are determined as follows:

B. Operation Modes of some Specific 4-RUU PMs

1) $\beta_1 = \beta_2 = \beta_3 = \beta_4 = \frac{\pi}{2}$: For the PM shown in Fig. 4, the constraint equations are derived from Eqs. (2) and (4) as

[1]The ideal generated by the given polynomials is the set of all combinations of these polynomials using coefficients from the polynomial ring $k[x_0, x_1, x_2, x_3, y_0, y_1, y_2, y_3]$ [14].
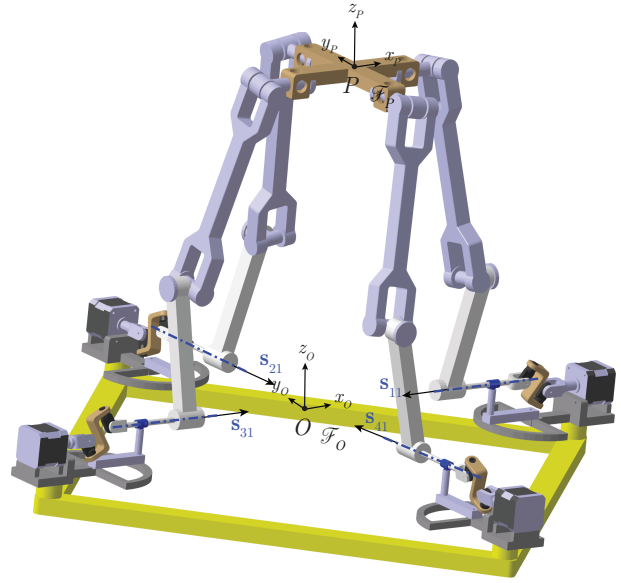

Fig. 4: A 4-rRUU PM with horizontal and intersecting base revolute joint axes

follows:

$$g_1 := \left(-p t_1{}^2 + p\right) x_0 x_2 + 2 p t_1 x_0 x_3 + 2 p t_1 x_1 x_2$$
$$+ \left(p t_1{}^2 - p\right) x_1 x_3 + \left(2 t_1{}^2 + 2\right) x_2 y_2 + (2 t_1{}^2$$
$$+ 2) x_3 y_3 = 0 \tag{8a}$$

$$g_2 := \left(-p t_2{}^2 + r_0 t_2{}^2 - r_1 t_2{}^2 + p + r_0 - r_1\right) x_0 x_2$$
$$+ 2 t_2 p x_0 x_3 + 2 t_2 p x_1 x_2 + (p t_2{}^2 - r_0 t_2{}^2 - r_1 t_2{}^2$$
$$- p - r_0 - r_1) x_1 x_3 + \left(2 t_2{}^2 + 2\right) x_2 y_2 + (2 t_2{}^2$$
$$+ 2) x_3 y_3 = 0 \tag{8b}$$

$$g_3 := \left(-p t_3{}^2 + p\right) x_0 x_2 + 2 p t_3 x_0 x_3 + 2 p t_3 x_1 x_2$$
$$+ \left(p t_3{}^2 - p\right) x_1 x_3 + \left(2 t_3{}^2 + 2\right) x_2 y_2 + (2 t_3{}^2$$
$$+ 2) x_3 y_3 = 0 \tag{8c}$$

$$g_4 := \left(-p t_4{}^2 - r_0 t_4{}^2 + r_1 t_4{}^2 + p - r_0 + r_1\right) x_0 x_2$$
$$+ 2 t_4 p x_0 x_3 + 2 t_4 p x_1 x_2 + (p t_4{}^2 + r_0 t_4{}^2 + r_1 t_4{}^2$$
$$- p + r_0 + r_1) x_1 x_3 + \left(2 t_4{}^2 + 2\right) x_2 y_2 + (2 t_4{}^2$$
$$+ 2) x_3 y_3 = 0 \tag{8d}$$

The primary decomposition of ideals $I_{(I)}$ and $I_{(II)}$ shown in Eq. (7) leads to three sub-ideals each. Among them, the third sub-ideals $\mathcal{I}_{3(I)}$ and $\mathcal{I}_{3(II)}$ correspond to a mixed mode and are of little importance in this context. The other two sub-ideals $\mathcal{I}_{k(I)}$ and $\mathcal{I}_{k(II)}$, $k = 1, 2$ are as follows:

$$\mathcal{I}_{(I)} = \mathcal{I}_{1(I)} \cap \mathcal{I}_{2(I)} \cap \mathcal{I}_{3(I)},$$
$$\text{where } \mathcal{I}_{1(I)} = \langle x_0, x_1, x_2 y_2 + x_3 y_3 \rangle$$
$$\mathcal{I}_{2(I)} = \langle x_2, x_3, x_0 y_0 + x_1 y_1 \rangle \tag{9a}$$
$$\mathcal{I}_{(II)} = \mathcal{I}_{1(II)} \cap \mathcal{I}_{2(II)} \cap \mathcal{I}_{3(II)},$$
$$\text{where } \mathcal{I}_{1(II)} = \langle x_0, x_1, x_2 y_2 + x_3 y_3 \rangle$$
$$\mathcal{I}_{2(II)} = \langle x_2, x_3, x_0 y_0 + x_1 y_1 \rangle \tag{9b}$$

As a result, the first two operation modes of the 4-rRUU PM

shown in Fig. 4 are:

$$\mathcal{I}_1 = \mathcal{I}_{1(I)} \cup \mathcal{I}_{1(II)} = \langle x_0, x_1, x_2 y_2 + x_3 y_3 \rangle \quad (10a)$$

$$\mathcal{I}_2 = \mathcal{I}_{2(I)} \cup \mathcal{I}_{2(II)} = \langle x_2, x_3, x_0 y_0 + x_1 y_1 \rangle \quad (10b)$$

Substituting the condition $x_0 = x_1 = x_2 y_2 + x_3 y_3 = 0$ in the transformation matrix in Eq. (5) yields:

$$\mathbf{F}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\dfrac{2y_3}{x_2} & -1 & 0 & 0 \\ 2(x_2 y_0 - x_3 y_1) & 0 & x_2^2 - x_3^2 & 2x_2 x_3 \\ 2(x_2 y_1 + x_3 y_0) & 0 & 2x_2 x_3 & -x_2^2 + x_3^2 \end{bmatrix} \quad (11)$$

From the transformation matrix, it can be observed that the operation mode is a 4-*dof* Schönflies mode in which the translational motions are parametrized by y_0, y_1 and y_3 and the rotational motion is parametrized by x_2, x_3 and $x_2^2 + x_3^2 = 1$. In this operation mode, the platform is upside down with the z_P-axis pointing in a direction opposite to the z_O-axis. The rotational motion is about x_O-axis.

Similarly, substituting the condition $x_0 = x_1 = x_2 y_2 + x_3 y_3 = 0$ in the transformation matrix in Eq. (5) yields:

$$\mathbf{F}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\dfrac{2y_1}{x_0} & 1 & 0 & 0 \\ -2(x_0 y_2 - x_1 y_3) & 0 & x_0^2 - x_1^2 & -2 x_0 x_1 \\ -2(x_0 y_3 + 2 x_1 y_2) & 0 & 2 x_0 x_1 & x_0^2 - x_1^2 \end{bmatrix} \quad (12)$$

In this case, the operation mode is also a 4-*dof* Schönflies mode in which the translational motions are parametrized by y_1, y_2 and y_3 and the rotational motion is parametrized by x_0, x_1 and $x_0^2 + x_1^2 = 1$. The platform is in upright position with rotational motion about x_O-axis.
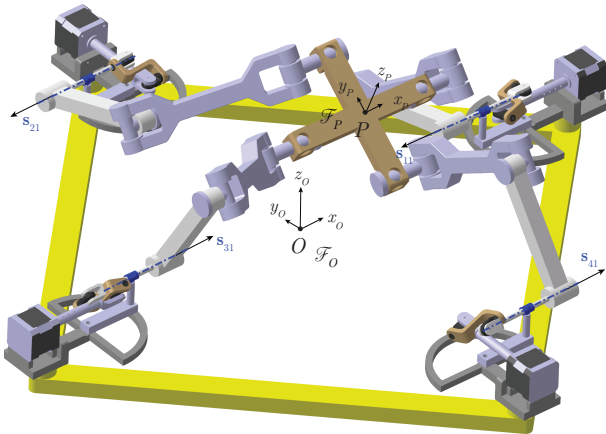

Fig. 5: A 4-RUU PM with horizontal and parallel base revolute joint axes

2) $\beta_1 = \beta_3 = \dfrac{\pi}{2}, \beta_3 = \beta_4 = 0$: For the PM shown in Fig. 5 with $w_1 = w_3 = 1$ and $w_2 = w_4 = 0$, the

constraint equations are derived according to Eqs. (2) and (4) as follows:

$$\begin{aligned} g_1 := & \left(-p t_1^2 + p \right) x_0 x_2 + 2 p t_1 x_0 x_3 + 2 p t_1 x_1 x_2 \\ & + \left(p t_1^2 - p \right) x_1 x_3 + \left(2 t_1^2 + 2 \right) x_2 y_2 + (2 t_1^2 \\ & + 2) x_3 y_3 = 0 \end{aligned} \quad (13a)$$

$$\begin{aligned} g_2 := & \left(-p t_2^2 + p \right) x_0 x_1 + 2 p t_2 x_0 x_3 - 2 p t_2 x_1 x_2 \\ & + \left(-p t_2^2 + p \right) x_2 x_3 + \left(2 t_2^2 + 2 \right) x_1 y_1 + (2 t_2^2 \\ & + 2) x_3 y_3 = 0 \end{aligned} \quad (13b)$$

$$\begin{aligned} g_3 := & \left(-p t_3^2 + p \right) x_0 x_2 + 2 p t_3 x_0 x_3 + 2 p t_3 x_1 x_2 \\ & + \left(p t_3^2 - p \right) x_1 x_3 + \left(2 t_3^2 + 2 \right) x_2 y_2 + (2 t_3^2 \\ & + 2) x_3 y_3 = 0 \end{aligned} \quad (13c)$$

$$\begin{aligned} g_4 := & \left(-p t_4^2 + p \right) x_0 x_1 + 2 p t_4 x_0 x_3 - 2 p t_4 x_1 x_2 \\ & + \left(-p t_4^2 + p \right) x_2 x_3 + \left(2 t_4^2 + 2 \right) x_1 y_1 + (2 t_4^2 \\ & + 2) x_3 y_3 = 0 \end{aligned} \quad (13d)$$

The primary decomposition of ideals $I_{(I)}$ and $I_{(II)}$ shown in Eq. (7) leads to three sub-ideals each, of which the two sub-ideals $\mathcal{I}_{k(I)}$ and $\mathcal{I}_{k(II)}$, $k = 1, 2$ are as follows:

$$\begin{aligned} \mathcal{I}_{(I)} = \mathcal{I}_{1(I)} & \cap \mathcal{I}_{2(I)} \cap \mathcal{I}_{3(I)}, \\ \text{where } \mathcal{I}_{1(I)} &= \langle x_0, x_1, x_2 y_2 + x_3 y_3 \rangle \\ \mathcal{I}_{2(I)} &= \langle x_2, x_3, x_0 y_0 + x_1 y_1 \rangle \end{aligned} \quad (14a)$$

$$\begin{aligned} \mathcal{I}_{(II)} = \mathcal{I}_{1(II)} & \cap \mathcal{I}_{2(II)} \cap \mathcal{I}_{3(II)}, \\ \text{where } \mathcal{I}_{1(II)} &= \langle x_0, x_2, x_1 y_1 + x_3 y_3 \rangle \\ \mathcal{I}_{2(II)} &= \langle x_1, x_3, x_0 y_0 + x_2 y_2 \rangle \end{aligned} \quad (14b)$$

As a result, the first two operation modes of the 4-RUU PM are:

$$\mathcal{I}_1 = \mathcal{I}_{1(I)} \cup \mathcal{I}_{1(II)} = \langle x_0, x_1, x_2, y_3 \rangle \quad (15a)$$

$$\mathcal{I}_2 = \mathcal{I}_{2(I)} \cup \mathcal{I}_{2(II)} = \langle x_1, x_2, x_3, y_0 \rangle \quad (15b)$$

Substituting the condition $x_0 = x_1 = x_2 = y_3 = 0$ in the transformation matrix in Eq. (5) yields:

$$\mathbf{F}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \dfrac{2y_2}{x_3} & -1 & 0 & 0 \\ -\dfrac{2y_1}{x_3} & 0 & -1 & 0 \\ \dfrac{2y_0}{x_3} & 0 & 0 & 1 \end{bmatrix} \quad (16)$$

From the transformation matrix, it can be deduced that the operation mode is a 3-*dof* pure translational mode parametrized by y_0, y_1 and y_2 when $x_3 = 1$. In this operation mode, the platform is upside down with the z_P-axis pointing downwards.

Similarly, substituting the condition $x_1 = x_2 = x_3 = $

$y_0 = 0$ in the transformation matrix in Eq. (5) yields:

$$\mathbf{F}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\dfrac{2y_1}{x_0} & 1 & 0 & 0 \\ -\dfrac{2y_2}{x_0} & 0 & 1 & 0 \\ -\dfrac{2y_3}{x_0} & 0 & 0 & 1 \end{bmatrix} \qquad (17)$$

In this case, the operation mode is also a 3-*dof* translational mode parametrized by y_1, y_2 and y_3 when $x_0 = 1$. Since the rotation matrix is identity, the platform is in upright position with z_P-axis pointing upwards.
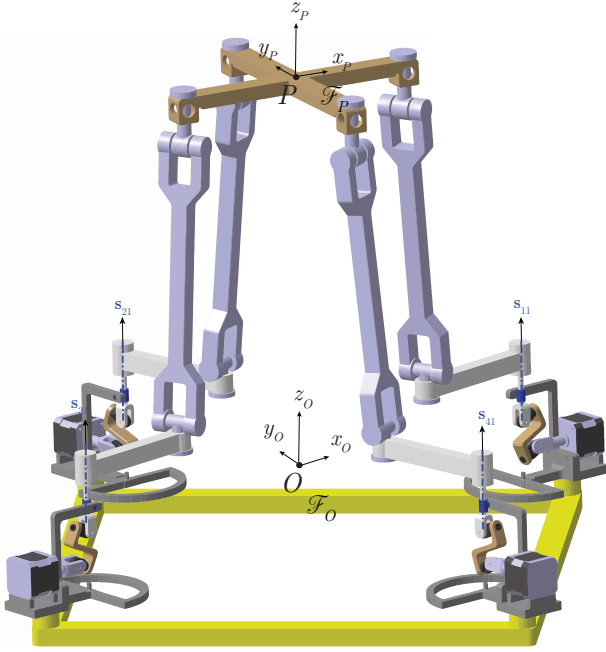

Fig. 6: A dual reconfigurable 4-rRUU PM with vertical base revolute joint axes

3) Vertical base and platform revolute joint axes: The double-Hooke's joint allows a planar transmission and hence the 4-RUU PM can have any orientation of the base revolute joints such that $\beta_i \in [0, \pi]$. Additionally, with the help of a L-fixture, it is possible to have a vertical orientation of the base revolute joint axes as shown in Fig. 6.

Reconfiguration analysis of this mechanism already exists in the literature [8], where it was shown to have three operation modes. The first operation mode is a 4-*dof* Schönflies mode in which the platform is upside down and the rotational axis is parallel to z_O-axis. The second operation mode is a 4-*dof* Schönflies mode with the rotational axis parallel to z_O-axis, but in this case, the posture of the platform is upright. The third operation mode is a 2-*dof* coupled motion mode and is less relevant from a practical view point.

IV. CONCLUSIONS AND FUTURE WORK

A dual reconfigurable 4-rRUU PM consisting of a reconfigurable revolute joint based on the double-Hooke's joint linkage was presented in this paper. It was shown how a double-Hooke's joint linkage can be exploited to impart an architectural reconfigurability to a 4-RUU PM. The resulting dual reconfigurable 4-rRUU PM was shown to exhibit at least the following operation modes: a pure translational operation mode and Schönflies motion modes with different axes of rotation depending on the orientation of base revolute joint axes.

As a part of the future work, the operation modes will be determined as a function of the angle β which will assist in recognizing all possible platform motion types of the 4-rRUU PM for $\beta \in [0, \pi]$. Furthermore, a detailed design of the 4-rRUU PM will be performed in order to construct a functional prototype exhibiting different types of reconfigurability. It should be noted that in the prototype, the orientation of the revolute joint axes in the moving platform will have to be changed manually and the choice of a joint to have all planar orientations is still an open issue.

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