Safety Analysis for Near Rectilinear Orbit Close Approach Rendezvous in the Circular Restricted Three-Body Problem

Stéphanie Lizy-Destrez, Bastien Le Bihan, Antonino Campolo, Sara Manglativi

HAL Id: hal-01755252

https://hal.science/hal-01755252

Submitted on 30 Mar 2018

Safety Analysis for Near Rectilinear Orbit Close Approach Rendezvous in the Circular Restricted Three-Body Problem

Stephanie Lizy-Destrez[a], Bastien Le Bihan[b], Antonino Campolo[c], Sara Manglaviti[d]

[a] *ISAE/SUPAERO, France, stephanie.lizy-destrez@isae.fr*
[b] *ISAE/SUPAERO, France, bastien.le-bihan@isae.fr*
[c] *Politecnico of Milan, Italy, nino.campolo1@gmail.com (Corresponding Author)*
[d] *ISAE/SUPAEO & Politecnico of Milan, France & Italy, sara-04@hotmail.it*

Abstract

A concise and factual abstract (written in third person and in one paragraph) of no more than 400 words is required. The abstract should state briefly the purpose of the research, the principal results and major conclusions. An abstract must be stand alone and complete in itself with no references to the main body of the manuscript. References should be avoided, but if essential, then cite the author(s) and year(s). Also, non-standard or uncommon abbreviations should be avoided, but if essential they must be defined at their first mention in the abstract itself. Readers should not have to read the full text to understand the abstract. The abstract can be an updated version of the one submitted at the call-for-abstracts, but its contents must not differ substantially.

Keywords: (maximum 6 keywords)

1. Introduction

On the road to a manned exploration of Mars, several mission scenarios beyond Low Earth Orbit have been identified as significant landmarks. Two of them, "Asteroid Next" and "Moon Next", have been integrated into a single reference framework [1].

In this context, one option being considered includes an outpost at one of the libration points of the Earth-Moon system which would be used as a logistic hub for human missions in cis-lunar space − including lunar surface − and beyond. Moreover, innovative technologies could be tested onboard before being employed in deep space missions [2]. At this time, such an option is likely to involve the NASA/ESA Orion Multi-Purpose Crew Vehicle combined with the Space Launch System (SLS), designed to facilitate human exploration beyond Low Earth Orbit (LEO).

The capacity to rendezvous in cis-lunar space is by nature necessary both for the deployment and the overall operability of the Space Station. Hence, a rendezvous analysis in such an environment becomes fundamental.

Concerning the fully characterization of a deep-space rendezvous, it is at first necessary to choose within the cis-lunar orbits selected by NASA as possible locations for a future international space station (ISS). In order to privilege a particular orbit for the future ISS, it is necessary to define the characteristics and the impact on system design and mission operations of the different options at hand. After an accurate analysis, one can affirm that the Near Rectilinear Orbits (NROs) represent the most favorable option and are therefore selected for the rest of the study.

NASA and ESA have flown manned rendezvous and docking missions since Gemini 6 (1966) and many of the techniques pioneered in the mid-60's and used in Apollo and Shuttle programs are still applicable today to ATV and HTV's rendezvous with ISS. They are all based upon the assumption of two vehicles operating in a near circular orbit in a strong central gravity field. However, neither of these conditions are present in a cis-lunar environment.

Thus, the constraints and the safety procedures derived by operating in the neighbourhood of a strong central body are no longer in effect. Moreover, since the gravity field is shallow in cis-lunar orbits, the relative dynamics of proximity motion are almost straight [3]; therefore, the "carving" characteristics of LEO trajectories, which govern the ISS safety standards, cannot be used. In addition, the GPS navigation system cannot be directly applied, which is often used as a primary navigation method for far rendezvous operations in LEOs. Another navigation system, such as a ground-based tracking system like NASA's deep space network (DSN), must be utilized.

A noticeable work by Mand et al. [4] achieved to define the rendezvous strategy for a station in a EML2 Halo orbit. Mand's work has been taken by the author as a reference to compare and validate the results obtained in this article. Finally, a significant contribution to the study of relative motion in cis-lunar orbits derived from the spacecraft formation flying (SFF) analyses.

However, the SFF dynamical model resulting from the CRTBP is very different from the one implemented for the near-Earth missions. Hence, only a few missions of a single spacecraft (including ISEE-3, WIND, SOHO, ACE, Genesis) have been actually operative [5].

The author of this article did not find any literature specifically focused on NRO's rendezvous.

2. Problem Overview

2.1. Circular Restricted Three-Body Problem (CRTBP)

Consider the motion of a particle m of negligible mass moving under the gravitational influence of two masses m_1 and m_2, referred as the primary masses, or simply the primaries. In the general Three-Body Problem, all bodies are free to move in the space, while the Restricted Problem constraints the motion of the two primaries which are considered to be revolving around their center of mass under their mutual gravitational attraction. The third body is attracted by the previous two, but it does not influence their motion. This last assumption implies that the mass of the third body is much smaller than either m_1 or m_2. A further assumption is then introduced in the Circular Restricted Problem, constraining the two primaries to move on a circular orbit around their center of mass. This model fits the case in which the spacecraft is considered to be under the attractions of two big celestial bodies, such as Earth and Sun or Earth and Moon and it is in general valid for all gravitational systems generated by the Sun and the planets in the Solar System which have nearly circular orbits around the Sun [6].

The following well-known form of the equation will be particularly useful to derive equilibrium solution in the CRTBP [4]:

$$\begin{cases} \ddot{x} - 2\dot{y} = \dfrac{\partial U}{\partial x} \\ \ddot{y} + 2\dot{x} = \dfrac{\partial U}{\partial y} \\ \ddot{z} = \dfrac{\partial U}{\partial z} \end{cases} \qquad (1)$$

Although the equations of the CR3BP have no closed form of analytical solution, it is possible to determine the location of the equilibrium points, the so-called Lagrangian Points. There are five Lagrangian points: three on the x-axis of the Earth-Moon synodical reference frame, called collinear points, and two forming an equilateral triangle with the Earth and the Moon, called triangular or equilateral points.

Starting from these points, it is possible to define several families of orbits around them, and in particular the Halo orbits which are periodic orbits around the point L1 and L2. The Near Rectilinear Orbits (NROs) are defined within the Earth-Moon Three-Body framework and they can be seen as a certain class of Halo orbits if the point of the trajectory closest to the mean linear surface intersects the lunar sphere when projected on the xy plane of the Earth-Moon synodical frame.

Nowadays studies tend to target NROs as possible destinations for the next ISS because the Lagrangian points (in particular the point L2) can be seen as logistic hubs for low fuel consumption in interplanetary human spaceflight.

2.2. Reference Frames

In order to describe the absolute motion of an object in the Earth-Moon system, the Earth-Moon synodical reference frame, that is a relative reference frame, is taken into consideration. In the case of a rendezvous problem, the relative dynamics of the approaching spacecraft is defined in a reference frame relative to the target, called the Local Horizontal Local Vertical reference frame (LVLH). To construct the LVLH, it is necessary to define an inertial reference frame, that could be the Moon-Centered or the Earth-Centered Inertial frame (MCI or ECI).

The Earth-Moon synodical frame is a rotating frame centered at the Earth-Moon center of mass so that the Earth and the Moon are fixed points on the x-axis, with z-axis orthogonal to the plane of motion of these planets and y-axis completing the right-handed rule. The x and y axes are time-dependent.

The MCI (or ECI) is defined such that the origin is the center of the Moon (or the Earth respectively), the z-axis is oriented as the z-axis of the Earth-Moon synodical frame and the x and y axes are supposed to be overlapped when t=0.

The LVLH is centered on the target and it is a time-dependent frame utilized for the description of the rendezvous phase described below. The z-axis, also called the Altitude axis, is oriented in the opposite direction of the target position vector, defined with respect to one of the two previously defined inertial frames; the x-axis, called the Downrange axis, points in the direction of the target velocity and the y-axis is normal to the others two axes.

After a further analysis, the MCI has been selected in order to define the z-axis of the LVLH frame; this choice is dictated by the fact that a more suitable representation of the orbit is obtained with this inertial frame.
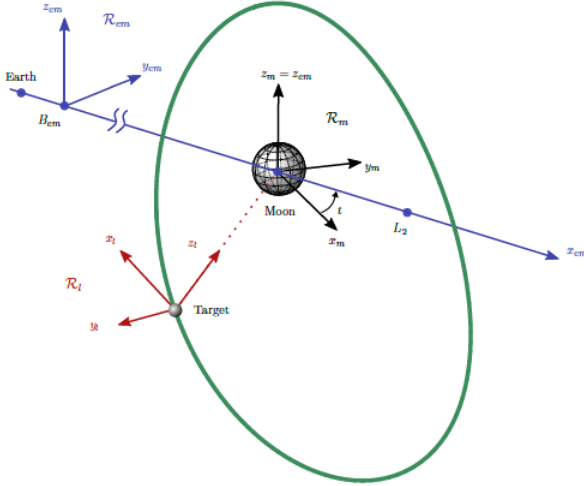
Figure 1. The main reference frames for the description of the motion in cislunar space. In blue: the Earth-Moon synodical reference frame Rem. In black: the Moon-Centered Inertial frame Rm. In red: the nro Local Vertical, Local Horizontal (LVLH) reference frame. In green: the nro of the target, as seen in Rem.

3. Rendezvous on a NRO

3.1. Relative Motion

In this section, the different models to describe the CRTBP relative motion are presented and discussed.

3.1.1. Non-Linear Relative Equations

The non-dimensional non-linear relative equations of motion written in the synodic frame can be easily obtained by difference of the absolute equations of motion for the chaser and the target, respectively:

$$\ddot{x} - 2\dot{y} - x = (1-\mu)\left[\frac{x_T + \mu}{\|\mathbf{r_{1T}}\|^3} - \frac{x_T + x + \mu}{\|\mathbf{r_{1T}} + \rho\|^3}\right]$$
$$+ \mu\left[\frac{x_T + \mu - 1}{\|\mathbf{r_{2T}}\|^3} - \frac{x_T + x + \mu - 1}{\|\mathbf{r_{2T}} + \rho\|^3}\right]$$

$$\ddot{y} + 2\dot{x} - y = (1-\mu)\left[\frac{y_T}{\|\mathbf{r_{1T}}\|^3} - \frac{y_T + y}{\|\mathbf{r_{1T}} + \rho\|^3}\right] \quad (2)$$
$$+ \mu\left[\frac{y_T}{\|\mathbf{r_{2T}}\|^3} - \frac{y_T + y}{\|\mathbf{r_{2T}} + \rho\|^3}\right]$$

$$\ddot{z} = (1-\mu)\left[\frac{z_T}{\|\mathbf{r_{1T}}\|^3} - \frac{z_T + z}{\|\mathbf{r_{1T}} + \rho\|^3}\right]$$
$$+ \mu\left[\frac{z_T}{\|\mathbf{r_{2T}}\|^3} - \frac{z_T + z}{\|\mathbf{r_{2T}} + \rho\|^3}\right]$$

where

$$\mathbf{x} = [\, x \ y \ z \ \dot{x} \ \dot{y} \ \dot{z} \,] = \mathbf{x_C} - \mathbf{x_T} \quad (3)$$

is the R^6 relative state,

$$\rho = [\, x \ y \ z \,]^T \quad (4)$$

is the relative position,

$$\mathbf{x_T} = [\, x_T \ y_T \ z_T \ \dot{x_T} \ \dot{y_T} \ \dot{z_T} \,]^T \quad (5)$$

is the target absolute state,

$$\mathbf{r_{1T}} = [(x_T + \mu) \ y_T \ z_T]^T$$
$$\mathbf{r_{2T}} = [(x_T + \mu - 1) \ y_T \ z_T]^T \quad (6)$$

are the absolute non-dimensional distances of the target from the Earth and the Moon, respectively.

3.1.2. Linearized Relative Equations

Concerning the linearized relative equations (LRE), the contribution comes exclusively from the formation flying studies of Peng et al. [6] and Luquette [7].

Many previous approaches were based on the linearization with respect to a libration point, leading to a linear time-varying periodic (LTVP) system. However, applying this form of the linearized dynamics to the problem of spacecraft relative motion limits the validity and utility of the model to regions within close proximity to a libration point. Linearizing the dynamics about a reference spacecraft instead provides a generalized solution, applicable to any trajectory within the context of the CRTBP. Specifically, the approach followed by Luquette takes into account a canonical CRTBP synodic frame. The linearized equations are:

$$\dot{x} = \begin{bmatrix} 0 & I_3 \\ \{\Xi(t) - [n\times][n\times]\} & -2[n\times] \end{bmatrix} x \quad (7)$$

where

$$\Xi(t) = -\left(\frac{1-\mu}{r_{1T}^3} + \frac{\mu}{r_{2T}^3}\right)I_3 + \frac{3(1-\mu)}{r_{1T}^5}\left[\mathbf{r_{1T}}\,\mathbf{r_{1T}}^T\right] +$$
$$+ \frac{3\mu}{r_{2T}^5}\left[\mathbf{r_{2T}}\,\mathbf{r_{2T}}^T\right]$$

$$[n\times] = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (8)$$

$$[n\times][n\times] = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

3.1.3. Clohessy-Wiltshire Equations

The well-known Clohessy-Wiltshire equations (C-W) represent the classic tool to describe the relative motion in a 2-Body environment. They are based on three main hypotheses:

- Single primary mass;
- Circular target's orbit;
- Small relative distance with respect to the target-attractor distance.

These hypotheses are, in general, no more valid in the CRTBP. The C-W equations written in the LVLH frame are in the form [8]:

$$
\begin{aligned}
\ddot{x} - 2n\dot{z} &= 0 \\
\ddot{y} + n^2 y &= 0 \\
\ddot{z} + 2n\dot{x} - 3n^2 z &= 0
\end{aligned}
\tag{9}
$$

and can be written in a state-space representation as:

$$
\dot{x} = \begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 2n \\
0 & -n^2 & 0 & 0 & 0 & 0 \\
0 & 0 & 3n^2 & -2n & 0 & 0
\end{bmatrix} x = A_{CW}x
\tag{10}
$$

where

$$
n = \sqrt{\frac{\mu}{r_{2T}^3}}
\tag{11}
$$

is the mean angular motion of a fictitious Lunar-centered keplerian circular orbit whose radius is equal to the target distance from the Moon center. In order to improve the precision of this linear algorithm, r_{2T} is evaluated at the beginning of each transfer, so that n is continuously updated.

3.1.4. Straight Line Equations

The Straight-Line (SL) approach is the simplest and most intuitive one. The desired Δv aims the velocity vector towards the targeted point at all times, disregarding any gravitational effect. This results in:

$$
\Delta v = \frac{r_f - r_i}{\Delta t} - v_i
\tag{12}
$$

so that the state-space representation of the system is simply:

$$
\dot{x} = \begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} x = A_{SL}x
\tag{13}
$$

In 1965, the Gemini 4 vehicle attempted the first rendezvous in history, utilizing the Straight-Line approach due to the limited knowledge of the orbital mechanics involved. It failed to rendezvous with its spent Titan II launch vehicle's upper stage, as both target and chaser were still in LEO.

3.2. Linear Targeting Evaluation

At time $t = 0^-$, the position δr_0 and the velocity δv_0^- of the chaser vehicle B relative to the target A are known. At $t = 0$ an impulsive maneuver instantaneously changes the relative velocity to δv_0^+ at $t = 0^+$. The solution of the targeting problem lies in the evaluation of $\delta v_0^+ = \{\delta u_0^+ \ \delta v_0^+ \ \delta w_0^+\}$, at the beginning of the rendezvous trajectory, so that B will arrive at the target in a specified time. The variation of velocity required to place B on the rendezvous trajectory is:

$$
\Delta v_0 = \delta v_0^+ - \delta v_0^- = \begin{pmatrix} \delta u_0^+ \\ \delta v_0^+ \\ \delta w_0^+ \end{pmatrix} - \begin{pmatrix} \delta u_0^- \\ \delta v_0^- \\ \delta w_0^- \end{pmatrix}
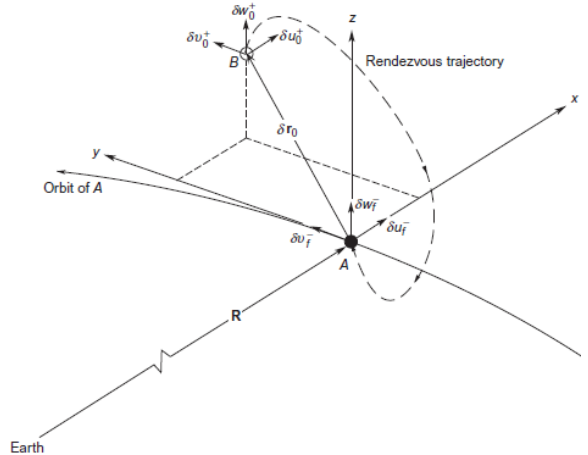\tag{14}
$$

Figure 2. Example of rendezvous trajectory [8]

It is possible to use the notion of linear targeting algorithm (defined in the next subsection) to estimate the error introduced by relative linear models with respect to non-linear models and, consequently, assess their ability to accurately model the dynamics during a NRO rendezvous. Specifically, linear and non-linear models are

compared on a single case - namely, a two-impulse transfer arc between two NROs. In this situation, both the target and the chaser spacecraft are supposed to orbit on given NROs with their own station-keeping strategy.

For the three linear models previously, namely the Linearized Relative equations, the Clohessy-Wiltshire equations and the Straight Line equations, the solution of the linear targeting problem Δv_0 has been evaluated in different scenarios. The computed relative velocity at $t = 0^+$, along with the selected initial relative position, has been then used as the initial condition for the integration of the non-linear equations 2, for a specified Time of Flight (TOF).

For $t = TOF$, the final integrated position has been compared to the desired final position and the linear error has been defined as the norm of the vectorial difference of the two. In other words:

$$E_j = ||(\mathbf{r}_{f,d} - \mathbf{r}_{f,i})||$$ (15)

where E represents the linear error, j represents the j-th model, $\mathbf{r}_{f,d}$ is the final desired position and $\mathbf{r}_{f,i}$ is the final integrated position using relative state coming from the j-th model as the initial condition.

3.2.1. Algorithm

If the equations of motion can be linearized to the state space representation form of:

$$\dot{\mathbf{x}} = A\mathbf{x}$$ (16)

where A is the constant system matrix and x is the relative state, the state transition matrix (STM) can be solved using the equation [9]:

$$\Phi(\Delta t) = e^{(A\Delta t)}$$ (17)

where Δt is the transfer time. Then, the state transition matrix and relative state vector can be decomposed into:

$$\Phi(\Delta t) = \begin{bmatrix} \Phi_{rr} & \Phi_{rv} \\ \Phi_{vr} & \Phi_{vv} \end{bmatrix}$$

$$\begin{bmatrix} x & y & z & \dot{x} & \dot{y} & \dot{z} \end{bmatrix}^T = \begin{bmatrix} \mathbf{r} \\ \mathbf{v} \end{bmatrix}$$ (18)

The change in velocity required at the point of the burn, Δv_0^+, can be determined to achieve any three of the six states of the final relative state, x_f, beginning with the initial relative state, x_i, as:

$$\mathbf{x}_f = \Phi(\Delta t)\mathbf{x}_i$$ (19)

To target a desired position, r_f, after a transition time, Δt, substitute the required initial velocity, $v_{i,req}$, for the current initial velocity:

$$\begin{bmatrix} \mathbf{r}_f \\ \mathbf{v}_f \end{bmatrix} = \begin{bmatrix} \Phi_{rr} & \Phi_{rv} \\ \Phi_{vr} & \Phi_{vv} \end{bmatrix} \begin{bmatrix} \mathbf{r}_i \\ \mathbf{v}_{i,req} \end{bmatrix}$$ (20)

Then, $v_{i,req}$ can be determined with respect to r_f as :

$$\mathbf{v}_{i,req} = \Phi_{rv}^{-1} \left(\mathbf{r}_f - \Phi_{rr}\mathbf{r}_i \right)$$ (21)

The desired Δv_0 can be determined by taking the difference between the vehicle's required velocity and its current velocity:

$$\Delta \mathbf{v}_0 = \mathbf{v}_{i,req} - \mathbf{v}_i$$ (22)

3.2.2. Results

Due to the very different nature of the baseline linear discussed, it has been considered necessary to evaluate their performances in three different orbital regions. Specifically, three dynamically diverse regions such as the periselene, the aposelene and a intermediate point between the two have been selected. A direct two-impulse transfer, bringing the chaser to dock with the target in a given TOF, has been selected as the tool to compare the linear and non-linear models. In order to provide a more general characterization of the problem, the linear error has been evaluated varying both the position of the chaser in a limited region around the target and the TOF of the transfer. It has been furthermore ensured that the results of the three orbital regions can be compared. According to the particular region considered, both the chaser's orbit and its angular position span have been slightly varied in order to obtain a relative distance span of [0.5 390] km, which have been considered the extrema for the close approach maneuvering. The discretization in position has been set to 0.01° while the time discretization has been set to 5 minutes, with a minimum and maximum TOF of 10 minutes and 10 hours, respectively. The position of the target has been considered fixed.

The relative position has been defined as the difference between the angular position of the chaser and the target. However, it is important to remember that these two position angles are defined in different planes, but since the orbits are very close, we can ignore this drawback.
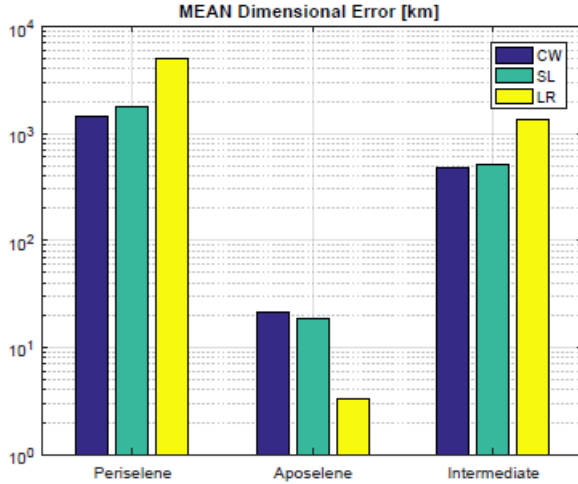
Figure 3. Mean Dimensional Error

Observing the LRE's dimensional error it becomes evident that the LRE's periselene error is the highest allowed for any linear model in any orbital location. This result can be explained by a combination of the complexity of the model and the particular orbital region considered. As a matter of fact, the LRE represent the only model, among the ones employed, designed for a Three-Body environment. Thus, its validity is limited in regions where the influence of one primary is not utterly predominant over the other, case in which the orbits degenerate in the classic Keplerian conics. This characteristic allows the use of LRE both for large Halo orbits or for the relative motion close to NRO's aposelene regions, where the high distance from the Moon or the Earth does not allow a strong influence predominance. However, for the NRO's periselenic regions, the Moon's influence is highly predominant, so that the LRE lose their fundamental hypothesis. It is therefore the simplicity and the assumptions of the C-W equations or the SL equations that allow these models to represent better baseline solutions for the shooting method in the periselene vicinity.

3.3. Non-Linear Targeting Evaluation

Considering non-linear models, where an analytic solution is not available, it is necessary to implement a recursive algorithm to evaluate Δv_0. The rendezvous problem defined in chapter 3.2. can be transposed in the following manner: given an initial relative state, a final relative state and a TOF, it is necessary to evaluate the variation of initial velocity so that the chaser docks with the target in the specified time. This formulation of the problem represents the well-known Lambert's problem that, in the frame considered, must be transposed to a three-body environment.

A Lambert's three-body problem is stated as the search of a path between two given points $r1 = \{x1, y1, z1\}$ and $r2 = \{x2, y2, z2\}$ with a given Time of Flight. It represents a typical two-points boundary value problem (2PBVP) that requires seven conditions (r1, r2, TOF) to be solved because, if for instance the TOF is not given, there are infinite trajectories linking r1 and r2 [10].

3.3.1. Shooting Method

When a generalized multi-variable Newton method [11] is employed to solve the Lambert's problem as a means of computing orbits in multi-body dynamical regimes subject to a set of user-defined constraints, it is called shooting method. Fundamentally, in this approach, a trajectory is propagated for a specified length of time and, based on a set of defined constraints, an error is computed. The error is then utilized in conjunction with dynamical gradient information to iteratively adjust the trajectory, i.e., one or more states along the path, until it satisfies all constraints. The differential equations of motion are solved using explicit integration techniques to propagate trajectories in the RTBP. The state transition matrix supplies significant gradient information that is necessary for differential corrections in the context of the shooting problems. The simplest application of the shooting method is termed "single shooting" because it utilizes a single integrated trajectory segment to solve the two-point boundary value problem. Suppose a spacecraft is initially located at point in space associated with the six-dimensional state vector, x0, and, given its initial velocity, arrives at point xt at time t = t0 + T. For any number of reasons, it may be necessary to alter the velocity at x0, i.e., perform Δv maneuver, such that the spacecraft arrives at an alternate location to be denoted as the desired point, rd. The first step in any multi-body trajectory design problem in this investigation is the definition of the free variables, X, that can be modified or adjusted. In this example, the spacecraft initial position is fixed and only the velocity is allowed to vary, thus,

$$X = \begin{bmatrix} \dot{x}_0 \\ \dot{y}_0 \\ \dot{z}_0 \end{bmatrix} \tag{23}$$

The constraint vector, F(X), is constructed to require that the final integrated spacecraft position be equal to the desired final position, rd. Mathematically, these constraints are expressed as

$$F(X) = \begin{bmatrix} x^t - x_d \\ y^t - y_d \\ z^t - z_d \end{bmatrix} = 0 \tag{24}$$

The 3×3 Jacobian matrix, DF(X) represents the partial derivatives of the constraints with respect to the free variables,

$$DF(X) = \begin{bmatrix} \Phi_{14} & \Phi_{15} & \Phi_{16} \\ \Phi_{24} & \Phi_{25} & \Phi_{26} \\ \Phi_{34} & \Phi_{35} & \Phi_{36} \end{bmatrix} = \Phi_{rv} \quad (25)$$

and consists of a 3×3 submatrix of STM elements. While the stated goal is to drive the vector constraint equation to zero, in practice, the procedure is numerical in nature and the problem is simply iterated until $\|F(X^{j+1})\|$ is reduced below a specified convergence tolerance, ε,

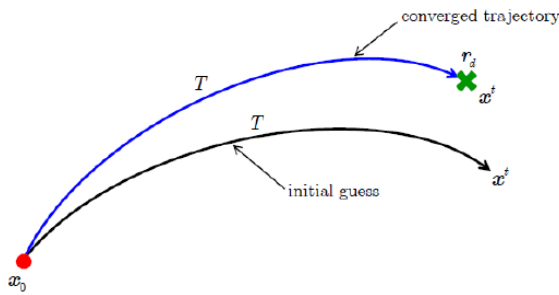$$\|F(X^{j+1})\| < \epsilon \quad (26)$$

Figure 4. Single Shooting Example

3.4. Numerical Implementation

In general, the algorithm is conceived so that the user can employ all the linear methods to provide a first guess for the shooting method.

The initial linear error is evaluated and the baseline model which presents the lower initial error is further selected to develop the shooting method. This procedure is justified by two considerations. First, it is assumed that the solution space is such that the velocity of convergence of the shooting method is directly dependent from the initial error, i.e. if the initial error is lower, the method will converge faster. Second, selecting the model with a lower initial error, it is more likely to obtain the "good" Lambert's solution. Indeed, two solutions exist for a given transfer time: one in the clockwise direction and one in the counter clockwise direction [12]. Thus, being the natural motion of the chaser clockwise the classical solution "follows" the natural motion of the chaser while the symmetric solution requires a breaking maneuver which completely reverses the natural motion of the chaser. Hence, in order to save fuel, the solution which follows the natural motion of the chaser has to be selected. If the selected baseline model does not allow the convergence of the shooting method or the Lambert's solution is not in the same direction of the orbital

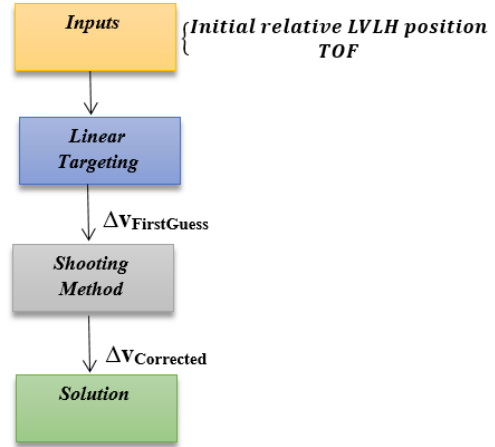natural motion, a continuation algorithm is employed to solve the Lambert's problem.

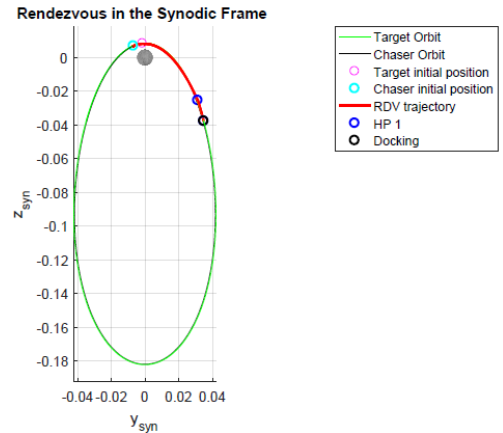Figure 5. General Structure of the Algorithm

Figure 6. Lambert's solution using C-W or SL equations as baseline models

In the following it will be shown two examples of trajectory design: Line of Sight Corridor and Line of Sight Glide [4].

With Line of Sight Corridor, the trajectory is obtained by defining an approach cone, in which the chaser has to be located; moreover it has to maneuver every time (n) that it will reach one side of the corridor. Four angles have to be selected: θ is the approach angle, α and β are the trigger angles and Φ is an offset angle that has to respect the conditions $\Phi < \alpha$ and $\Phi < \beta$. If α and β are equal, a symmetric cone is obtained.

Assuming rectilinear motion, these maneuvers create a series of similar triangles. Utilizing the formula deduced from geometric considerations, it is possible to obtain the following trajectory example:
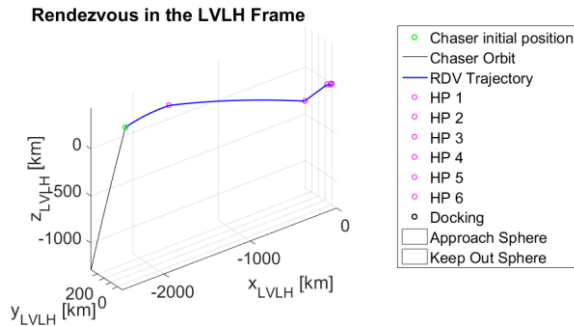
Figure 7. Rendezvous Trajectory: Line of Sight Corridor with random angles.

Where the Approach Sphere (AS) and the Keep Out Sphere (KOS) are safety regions defined around the target. The AS is centered on the target and has a radius of 2 km and the KOS is also centered on the target and has a radius of 200 m.

Line of Sight Glide allows a relatively safe approach, with the chaser always within sight but between only one trigger angle and an offset angle. Moreover, the offset angle is not set across the line of sight between the chaser and the target, but on the same side as the trigger angle. This choice will increase the number of maneuvers but it will drastically decrease the fuel consumption and consequently the mission cost.

Three angles have to be selected: θ is the approach angle, α is the trigger angle and Φ is an offset angle that has to respect the condition $\Phi < \alpha$.

The maneuvers obtained with the Glide approach create a set of similar triangles, whose rotation depends on the step number n. Utilizing the formula deduced from geometric considerations, it is possible to obtain the following trajectory example:
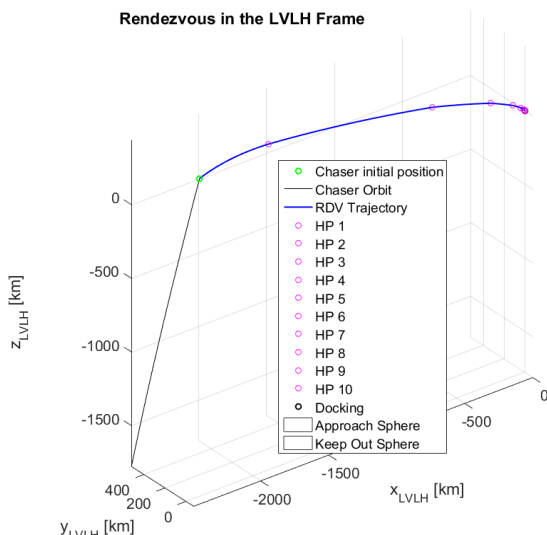

Figure 8. Rendezvous Trajectory: Line of Sight Glide with random angles.

4. Conclusions and Future Work

This work represents an attempt to define the Near Rectilinear Orbits, in the Circular Restricted Three-Body domain, under a rendezvous point of view, as no previous works on the matter were found in literature. At first, NRO relative dynamics were investigated. In comparison with other cislunar orbits, different relative frames and relative models were analysed. A Near Rectilinear Orbit Local Vertical Local Horizontal reference frame was defined and used to describe NRO relative motion. After having introduced the rendezvous targeting problem, linear models were employed to generate relative trajectories. However, linear models and linear targeting algorithms did not provide sufficiently accurate results for a NRO rendezvous. Thus, a non-linear targeting algorithm, based on the shooting method, was implemented.

Finally, an example of a safety trajectory was computed. As no previous works on the matter were found in literature, future work possibilities are vast. For example, the far rendezvous phase can be analysed in detail and reconnected to the close approach phase defined in this work. The size of the initial target and chaser orbits, before the far rendezvous phase, has to be properly fixed.

Furthermore, an optimization and a dispersion analysis can be realized in order to well define a rendezvous strategy design.

References

[1] Naomi Murakami, Satoshi Ueda et al.: Practical Rendezvous Scenario for Transportation Missions to Cis-Lunar Station in Earth-Moon L2 Halo Orbit, Proceedings of ISSFD, Munich, Germany, 2015

[2] Stéphanie Lizy-Destrez: Rendezvous Optimization with an Inhabited Space Station at EML2, Proceedings of 25th International Symposium on Space Flight Dynamics ISSFD 2015, 19 October 2015 – 23 October 2015, Munich, Germany

[3] Heather D. Hinkel, Scott P. Cryan et al.: Rendezvous and Docking Strategy for Crewed Segment of the Asteroid Redirect Mission, SpaceOps Conferences, 5-9 May 2014, Pasadena, CA, USA

[4] Kuljit Mand: Rendezvous and Proximity Operations at the Earth-Moon L2 Lagrange Point: Navigation Analysis for Preliminary Trajectory Design, MSc Thesis, Rice University, Houston, TX, USA, May 2014

[5] Haijun Peng, Jun Zhao et al.: Optimal Periodic Controller for Formation Flying on Libration Point Orbits, Acta Astronautica 69, 537-550, 2011

[6] F. Ferrari: Formation Flying Dynamics in Circular Restricted Three-Body Problem and Applications to Binary Asteroid Systems, MSc Thesis, Politecnico di Milano, 2013

[7] R. J. Luquette: Nonlinear Control Design Techniques for Precision Formation Flying at Lagrange Points, PhD thesis, University of Maryland, November 21st, 2006.

[8] Curtis, H. D., Orbital mechanics for engineering students Amsterdam, Elsevier Butterworth Heinemann (2005)

[9] F. Golnaraghi and B. C. Kuo: Automatic Control Systems, 9th Edition, John Wiley & Sons, New Jersey, 2010.

[10] Bernelli F., Topputo F. and Massari M.: Assessment of Mission Design Including Utilization of Libration Points and Weak Stability Boundaries, Final Report, ESTEC Contract No. 18147/04/NL/MV, 2004

[11] Thomas A. Pevlak, Trajectory Design and Orbit Maintenance Strategies in Multi-Body Dynamical Regimes, Ph.D. Thesis, Purdue University, May 2013

[12] B.D.A. Prado: Traveling between the Lagrangian Points and the Earth, Acta Astronautica, Vol. 39, No. 7, pp. 483-486, 1996