

# Kernel estimation of extreme regression risk measures

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**Abstract.** The Regression Conditional Tail Moment (RCTM) is the risk measure defined as the moment of order  $b \geq 0$  of a loss distribution above the upper  $\alpha$ -quantile where  $\alpha \in (0, 1)$  and when a covariate information is available. The purpose of this communication is first to establish the asymptotic properties of the RCTM in case of extreme losses, *i.e* when  $\alpha \rightarrow 0$  is no longer fixed, under general extreme-value conditions on their distribution tail. In particular, no assumption is made on the sign of the associated extreme-value index. Second, the asymptotic normality of a kernel estimator of the RCTM is established, which allows to derive similar results for estimators of related risk measures such as the Regression Conditional Tail Expectation/Variance/Skewness. When the distribution tail is upper bounded, an application to frontier estimation is also proposed. The results are illustrated both on simulated data and on a real dataset in the field of nuclear reactors reliability.

**Keywords:** Conditional tail moment, Kernel estimator, Asymptotic normality, Risk measures, Extreme-value index, Extreme-value analysis.