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Pupil motion analysis and tracking in ophthalmic systems equipped with wavefront sensing technology

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Our eyes are constantly in motion, even during “steady” fixation. In ophthalmic systems equipped with wavefront technology, both eye and head motion potentially degrade its performance and/or increase the cost and complexity, as they induce a movement of the entrance optical pupil of the system. Here, we characterize the pupil motion in an aberrometry setting, using a custom, high-speed pupil tracker (478Hz), and draw conclusions on design considerations of future ophthalmic systems. We also demonstrate the feasibility of tracking such motion directly with a custom-built Hartmann-Shack sensor (236Hz) using a method that offers certain benefits over previously suggested approaches, thereby paving the way to an efficient and cost-effective approach. © 2016 Optical Society of America

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1. INTRODUCTION

The iris of the eye is constantly in motion, changing both in position and shape. This is because, even during “steady” fixation on a target, our eyes produce involuntary fixational eye movements (i.e., tremors, drifts, and microsaccades) [1]. Similarly spontaneous fluctuations in pupil size and accommodation occur under conditions of constant illumination and fixation [2, 3].

Understanding and accounting for eye pupil motion, generated during eye or head motion, may thus be beneficial in the design of ophthalmic systems equipped with wavefront technology, such as aberrometers, adaptive-optics (AO) retinal cameras, or wavefront-guided vision correction/stimulation systems, especially for routine clinical applications to patients where such eye motion may be exaggerated [4, 5].

Indeed, if the iris does define the entrance pupil of the system, wavefront sensing will suffer from its motion. Even more so when the system uses AO technology that needs to work in real time, and whose efficiency can be deteriorated when the sub apertures of the wavefront sensor are randomly and partially illuminated, switching on and off as the imprint of the eye’s pupil on the lenslet array wanders. This is why some teams

try to overcome the issues generated by this motion by using an optical diaphragm, smaller than most iris diameters, acting as a fixed optical pupil of the system, which also limits the aberrations to be corrected by the AO system (reducing the influence of eye pupil border and of intraocular implants). This solution trades performance (diffraction limited resolution is reduced with a smaller optical pupil) for robustness, which is a natural choice for commercial systems, such as Imagine Eyes RTX1TM [6]. Still, even in this case, it is not easy in clinical use to ensure a perfect alignment of the system on the patient eye. As a consequence, the pupil is often defined, at least partially, by the iris. Other groups, however, decide to keep the iris as the pupil of the system, either because they want to reach the highest optical resolution possible [7, 8], or because they intend to measure and exploit the iris movement itself (for instance to compute the wavefront [9]).

In this paper, we therefore wish to characterize the entrance pupil motion (both in size and position), defined by the iris, at a high temporal resolution in an aberrometry setting using a custom pupil tracker (up to 478 Hz). We also demonstrate the feasibility of tracking pupil position and size without requiring additional hardware but a custom-built Hartmann-Shack sensor (HSS), using a method that overcomes certain drawbacks

of previously suggested approaches [10, 11] (see Sect. 4). Finally, we draw conclusions on design considerations of future ophthalmic systems equipped with wavefront technology that would include pupil stabilization.

2. METHODS

A. Experimental set-up

To characterize the entrance pupil motion at a high temporal resolution in an aberrometry setting, and verify at the same time the feasibility of tracking such motion using the Hartmann-Shack sensor itself, we developed a setup incorporating a custom-built Hartmann-Shack sensor (236 Hz) and pupil motion tracker (up to 478 Hz). Figure 1 shows a schematic drawing of the experimental set-up.

Two-dimensional “bright-pupil” tracking is employed to characterize pupil movements and geometry (IPUP module in Fig.1). Illumination is provided by a collimated superluminescent diode (SLD) with a central wavelength of 830 nm (± 50 nm), creating a point source on the retina, with the retro-reflected light back-illuminating the pupil (Injection Module Fig.1). The input beam diameter at the level of the iris is 5 mm with a 2.1 mm central obscuration. A CCD camera (iXon DV 860; Andor Technology Ltd., Belfast, UK), optically conjugated to the iris (to ensure that pupil edges are in focus), images a 1-cm field in the pupil plane (sufficient to observe eye and head motion with the subject being stabilized by a chin and forehead rest). The custom-built Hartmann-Shack sensor consists of a 30×30 micro-lenslet array and a 443×443 pixel CMOS camera, with a 14×14 array of 196 pixels allocated per lenslet (H-S Module Fig.1). Similar to the pupil tracker, the total number of lenslets enables a 9.6-mm field of view in the pupil plane. Typically, for a 7-mm diameter pupil, 22 lenslets are illuminated. A reference module (see Fig.1) is used to calibrate the system (in particular to acquire HSS reference spots).

Fig. 1. Schematic drawing of the experimental set-up, comprising a Reference Eye Module (used to acquire the reference wavefront), an Injection Module creating a point source on the retina, a pupil camera (IPUP) in parallel with a custom-made Hartmann-Shack sensor (H-S) module.

B. Model eye

An artificial “pupil,” realized by an optical fiber source and adjustable iris diaphragm (set at 7 mm), is used as a reference to

verify the ability of the IPUP module and associated fitting algorithms to follow the pupil movements, as well as to calibrate for static pupil imaging defects of the optical bench (this is different from the Reference Eye Module in Fig.1, which is used to reference the wavefront measurement and not the pupil geometry measurement). A motorized platform is used to move the artificial pupil in a controlled manner, while the IPUP camera records the introduced shifts ranging from $10 \mu\text{m}$ to $400 \mu\text{m}$ with a $10 \mu\text{m}$ step. The hysteresis that could arise from a back and forth displacement of the stage was negligible and thus did not affect the calibration.

Unlike the human eye, this model eye had no intrinsic factors that would result in rapid changes, and is thus nicely suited to determine the precision of our pupil camera based pupil tracking method. We computed the peak standard deviation of model eye motion, as a measure of precision, which is below 1 micron for pupil center coordinates and diameter.

C. Subjects

Twenty-eight normal subjects (age 22-59 years) participated in the study, which followed institutional guidelines and adhered to the tenets of the Declaration of Helsinki. Prior to the experiment, the nature and possible consequences of the study were explained, and informed consent was obtained from all subjects. Subjects were seated in front of the instrument and stabilized with a chin and forehead rest, without any pupil dilation or cycloplegia. They were asked to fixate on the 830-nm light, maintaining fixation for about 20 seconds until acquisition was complete. For three of them, only one eye was tested, leading to a total 53 eyes tested. For all 53 eyes, an acquisition with the pupil camera alone was performed at 478 Hz. A subgroup of 26 eyes was also tested with both the pupil camera and the Hartmann-Shack sensor, synchronized at 236 Hz.

The optical power of the 830-nm SLD at the subject’s pupil is always kept at $29 \mu\text{W}$ or less, more than 10 times smaller than the maximum permissible exposure established by the American National Standards Institute (ANSI) Z-136 safety standard [12] for continuous viewing at this wavelength and a beam of 0.16 cm^2 at the subject’s pupil, and in accordance with NF-EN-60825-1 (2008) safety regulations (Group 2).

D. Data analysis

Captured images with the pupil camera and Hartmann-Shack sensor are analyzed offline using custom software developed in IDL® (Exelis Visual Information Solutions, Inc., Boulder, CO) and MATLAB® (Mathworks, Inc., Natick, MA). Pupil camera images were pre-processed as follows: the background was set to 0 and the back-illuminated pupil to 1 (see Fig.2).

Fig. 2. Pre-processing of IPUP camera images : (Left) Image of the reference eye module with the IPUP camera; (Right) Corresponding binarized IPUP image.

HSS images were pre-processed as follows: the flux over each lenslet was integrated (after background subtraction, dead pixel detection and thresholding), yielding a subsampled pupil intensity image (see Fig.3), one pixel per lenslet. Next, the image is divided by a normalization factor set to 75% of the maximum. Then, values higher than 1 are set to 1. The map obtained is then an image of the illumination ratio of the lenslets: while fully illuminated lenslets have a value 1, partly illuminated lenslets (which have the pupil border running through them) have a value between 0 and 1, corresponding to the fraction of their surface located inside the pupil.

Fig. 3. Pre-processing of HSS camera images : (Left) Image of the reference eye module with the HSS Camera; (Center) Corresponding HSS intensity map after background subtraction, dead pixel detection and thresholding. Each pixel value corresponds to the integration of the corresponding lenslet zone in the camera image; (Right) Normalized thresholded and binarized HSS intensity map.

Then, we used the IDL procedure [13] for ellipse-based pupil tracking on both HSS and pupil camera processed images (see Figs. 2-Right and 3-Right). Use of ellipse fitting has been shown to better approximate the pupils of most eyes and compensates for geometric distortions from image projection [14]. In this procedure, first-order statistics (i.e. center of gravity) and second order statistics (the covariance matrix) are computed from which we directly derive the parameters of an ellipse fitting of the pupil. The eigenvectors of the covariance matrix determines the direction of the ellipse axes and their length can be inferred from its eigenvalues [15, 16]. The position of the pupil is taken as the center of the ellipse, and the diameter is computed as $d = \sqrt{a \cdot b}$, with a and b corresponding to the semi-major and -minor axes of the fitted ellipse. We infer from the ellipse parameters information about changes in pupil position and geometry respectively.

For further analysis, we extracted 1000 consecutive image sequences without blinks from the 478 Hz pupil camera data (~ 2 -s epochs). This resulted in 1000-point series of horizontal position (x), vertical position (y) and pupil diameter (d) data for high-speed characterization with the pupil camera (for all 53 eyes). Power spectral densities (PSDs) for x , y and d were calculated by a FFT routine using a Hamming spectral window to reduce spurious estimates of spectral power and high frequency artifacts induced by the finite duration of the temporal sequence.

Pupil camera data at the lower speed (236 Hz) was solely used as a reference, to compare and validate tracking with the HSS itself. In that case, 500-point series of x , y and d were thus obtained for both HSS and pupil camera synchronously, on a subgroup of 26 eyes.

3. RESULTS

A. Pupil tracking using the Hartmann-Shack sensor

To estimate the accuracy of our HSS-based tracking method, we calculated its error with respect to pupil-camera-based tracking, as the square root of the mean of the squared differences between the pupil camera and HSS (RMS error), which peaked 9 microns for the position and 17 microns for the diameter. Figure 4 depicts the HSS tracking error over the subgroup of 26 eyes tested with both the pupil camera and the Hartmann-Shack sensor synchronously (at 236 Hz).

Fig. 4. HSS tracking error with pupil camera as reference, calculated as the square root of the mean of the squared differences between the pupil camera and HSS. x : horizontal RMS, y : vertical RMS, d : diameter RMS

B. Characterization of entrance pupil motion using a custom, high-speed pupil tracker

B.1. Pupil size changes

In our 53 eyes, the variation of pupil diameter, during a 2-s blink-free sequence at 478 Hz, ranged from 27 to 821 microns peak to peak. A typical example of pupil diameter variation is given in Figure 5(a). Figure 5(b) shows the corresponding pupil size motion power spectrum for the same eye. Pupil motion PSDs for pupil size movements revealed a regular $\sim f^{-p}$ behavior over all subjects (mean $\pm$ SD p value of 1.8 ± 0.3 ; range: 2.2 to 0.9).

Fig. 5. (a) Variation in pupil diameter with corresponding (b) pupil size motion power spectrum displaying $\sim f^{-2}$ behavior

B.2. Pupil position changes

A typical example of a fixation trajectory for the same eye (and same subject from above; Fig.5) during a 2-s blink-free sequence at 478 Hz is depicted in Figure 6. Horizontal and vertical eye movements are shown with respect to their mean values (that is, mean values were computed for horizontal and vertical movements, respectively, and set as the center of the graph). Note that the time delay between neighboring points is about 2ms.

The series of distant points in the top right of the figure corresponds to a microsaccade (the largest and fastest component of the fixational eye movements).

Fig. 6. Pupil trajectory. The green and red circles represent the initial and final fixation points respectively

In our 53 eyes, the variation of fixational eye movements in the horizontal direction ranged from 66 to 742 microns peak to peak (mean $\pm$ standard deviation (SD) = 190 ± 121 microns), and in the vertical direction ranged from 32 to 186 microns peak to peak (mean $\pm$ SD = 93 ± 33 microns).

We then exploited these trajectories to compute pupil motion PSDs. Pupil motion PSDs for translational pupil movements were computed analogously to those for pupil size movements (by a FFT routine using a Hamming spectral window). Pupil movements exhibited a regular f^{-p} behavior for both horizontal and vertical components, with a mean $\pm$ SD p value of 2.3 ± 0.4 (see Fig. 7(a); range: 3.7 to 1.1). For 11 eyes, this f^{-2} behavior is supplemented by an f^{-5} behavior in the horizontal component (see Fig. 7(b); mean $\pm$ SD p value of 5.1 ± 1.3 ; range: 7.9 to 3.8) after around 38 Hz (mean $\pm$ SD = 37.9 ± 1.3 Hz; range: 23.0 to 49.5 Hz), while the PSD of the vertical component remains essentially unchanged (mean $\pm$ SD p value of 2.3 ± 0.8 ; range: 3.8 to 1.5) beyond 35 Hz.

Fig. 7. Pupil motion power spectra for translational pupil movements. (a) Horizontal (black trace and blue least-squares approximation) and vertical (red trace and green least-squares approximation) pupil motion PSDs following regular $\sim f^{-2}$ behavior. (b) Pupil motion PSDs showing supplementary $\sim f^{-5}$ behavior in the horizontal component after ~ 38 Hz.

C. Pupil tracking loop rate

From the pupil motion outlined above we simulated the action of a pupil stabilization loop and integrator-based control scheme with a 0.5 gain (ensuring 45° stability margin) [17], assuming a

2-frame delay [17, 18]. Figure 8 shows the performance of pupil stabilization as a function of pupil tracker speed.

The number of eyes with residual RMS errors below 25 microns ($\sim 1/10$ of a sub-pupil for an ophthalmic system with a 30×30 sub-aperture lenslet array over a 7-mm eye pupil) for different acquisition frequencies is depicted in Figure 8a. Figure 8b displays the overall distribution of residual horizontal RMS errors by means of a box plot.

(a) Number of eyes with residual RMS pupil motion below 25 microns for different acquisition frequencies, for horizontal (black trace), vertical (red trace), and pupil size (blue trace) movements.

(b) Box plot representation of horizontal RMS errors for different acquisition frequencies. The horizontal line within the box identifies the median. The length of the box represents the interquartile range (IQR), with the edges being the 25th (lower quartile) and 75th (upper quartile) percentiles. The ends of the whiskers represent minimum and maximum values. Values more than 1.5 IQRs are labeled as outliers (+). In addition, the green trace identifies mean RMS errors.

Fig. 8. Pupil stabilization as a function of tracker speed.

4. DISCUSSION AND CONCLUSION

Pupil motion characterization with the high-speed pupil tracker

In 53 eyes, we have characterized the entrance pupil motion in an aberrometry setting, using a custom, high-speed pupil tracker at 478 Hz. Based on the analysis of consecutive 2-s epochs without blinks, pupil movements did not exceed 742 microns and 821 microns in translation and size respectively. Translational pupil motion extended more in the horizontal than the vertical direction in accordance with [19–22].

Similarly, we found the power spectral density of horizontal and vertical components to be different [19]. Horizontal pupil motion PSDs fell into two categories, displaying rather regular f^{-p} decrease (similar trends are reported in [9] below 4 Hz), with p around 2, either over the entire frequency range like vertical pupil motion PSDs, or until around 38 Hz where they are supplemented by a f^{-5} behavior. Already Findlay and Eizenman observed the presence of two spectral components in the power spectrum of fixational eye motion [20, 21].

Findlay attributed this to the addition of an impulsive and a random component, corresponding to the saccadic and the tremor/drift portions of the spectrum, respectively [20]. Indeed, the difference in spectral behavior that we found may be linked to the presence of microsaccades (more dominant in horizontal movements [19]) during the analyzed epoch, as illustrated in Figure 9 by means of one example (same eye as in Fig. 6). For example, after the microsaccade (Fig. 9a) is removed, horizontal and vertical PSDs become similar (Figs. 9b and 9c), showing $\sim f^{-2}$ decrease. However, this warrants further investigation, which is beyond the scope of this paper.

(a) Horizontal (black trace) and vertical (red trace) pupil movements of the eye shown in Fig. 6

(b) Horizontal (black trace and blue least-squares approximation) and vertical (red trace and green least-squares approximation) pupil motion PSDs before removing the microsaccade

(c) Horizontal and vertical pupil motion PSDs after removing the microsaccade

Fig. 9. Effect of microsaccades on pupil motion PSDs

Pupil tracking with the Hartmann-Shack Sensor

For ophthalmic systems equipped with wavefront technology, the use of the Hartmann-Shack sensor (HSS) itself as a pupil-tracking device offers certain benefits and has been previously reported in the literature [10, 11]. For example, Thibos and colleagues considered using the “centroid of the centroids” of the HSS focal spots as pupil position measurement [10]. However, this method leads to erroneous readings for pupil motion below one subaperture, as noted in Ref. [11], where an alternative solution is proposed, computing the centroid of the whole HSS camera image (corresponding to Fig. 3-Left), background included. This alternative solution yields an error up to $27\mu\text{m}$ RMS (compared to a maximum $9\mu\text{m}$ RMS for our method), depending on the Signal-to-Noise Ratio (SNR) in the HSS image. As the low-level background plateau, originated by multi-diffuse light in the retina, is considered as signal in this method, low SNRs can indeed occur, even in closed-loop.

Here, we have demonstrated the feasibility of tracking pupil motion at 236 Hz in 26 eyes (that is at a higher speed and on a larger number of eyes than previously demonstrated [11]), using

a custom-built Hartmann-Shack sensor. Our method makes use of intensity maps computed from thresholded HSS images, where the low-level background plateau is removed. Only the high SNR peaks are used, which makes our method less sensitive to noise. Furthermore, the pupil position is computed on a low-resolution intensity map, that is systematically computed for sub-aperture validation purposes, with a computational cost much reduced compared to processing the total HSS image. Last, we also provide a measurement of the diameter of the pupil. Both position and diameter error remained below 5% of a lenslet width in all 26 eyes tested with our custom HSS.

Pupil tracking loop performance

Now that we have a low computational cost pupil tracking method suited to real-time pupil tracking/stabilization loops, we shall address the question of what this loop rate shall be. In other words, how fast should a pupil-tracking device run to ensure specified residual motion on the wavefront sensor? To answer this question, we simulated the action of a pupil stabilization loop and integrator-based control scheme (Figure 8).

Based on our results, it can be concluded, for example, that, for an ophthalmic system with a 30×30 sub-aperture lenslet array over a 7-mm eye pupil, a pupil-tracking device should run faster than 160 Hz to ensure a residual RMS motion below 25 microns for 100% of the subjects tested here (Fig. 8a). That frequency is reduced by almost a third for a residual error below 50 microns (Fig. 8b).

Conclusion

The pupil is a central element in ophthalmic examination via wavefront sensing technology. In this paper, we have characterized the entrance pupil motion in an aberrometry setting for 53 eyes, using a custom, high-speed pupil tracker at 478 Hz, and simulated the performance of pupil stabilization as a function of pupil tracker speed based on the results. With a pupil tracker running faster than 160 Hz, for example, an overall residual RMS motion below 25 microns on the wavefront sensor can be ensured. Tracking entrance pupil motion may thus be beneficial and improve the performance of ophthalmic systems equipped with wavefront sensing technology. We have also demonstrated the feasibility of tracking pupil motion (both in position and size) with the wavefront sensor directly, in 26 eyes, using a custom HSS at 236 Hz that overcomes drawbacks of previously suggested approaches, thereby paving the way to an efficient and cost-effective approach.

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