Uncertainty components estimates in transient climate projections. Bias of moment-based estimators in the single time and time series approaches.

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ABSTRACT

In most climate impact studies, model uncertainty in projections is estimated as the empirical variance of the climate responses for the different modeling chains. These estimates are however biased. We explore the importance of the bias for a simple but classical configuration where uncertainties in projections are composed of two sources: model uncertainty and internal climate variability. We derive exact formulation of the bias. It is positive, i.e. the empirical variance tends to overestimate the true model uncertainty variance. It can be especially high when a single time ANOVA is used for the analysis. In the most critical configurations, when the number of members available for each modeling chain is small (≤ 3) and when internal variability explains most of total uncertainty variance (75% or more), the overestimation is higher than 100% of the true model uncertainty variance. The bias is considerably smaller with a time series ANOVA approach, owing to the multiple time steps accounted for. The longer the transient time period used for the analysis, the smaller the bias. When a quasi-ergodic ANOVA approach is applied to decadal data for the whole 1980-2100 period, the bias is up to 2.5 to 20 times smaller than that obtained with a single time approach, depending on the projection lead time. Whatever the approach, the bias is likely to be not negligible for a large number of climate impact studies resulting in a likely large overestimation of the contribution of model uncertainty to total variance. In many cases, classical empirical estimators of model uncertainty should be thus bias-corrected.

1. Introduction

A critical issue in climate change studies is the estimation of uncertainties in projections along with the contribution of the different uncertainty sources, namely scenario uncertainty, the different components of model uncertainty and internal variability (e.g. Hawkins and Sutton 2009). Such estimation is intended to help evaluating the significance of estimated changes or at least their value for eventual planning purposes. This is besides intended to highlight the most important uncertainty sources. This thus allows estimating the fraction of total uncertainty that could be narrowed via scenario refinement and model improvement. This also allows estimating the irreductible fraction of total uncertainty pertaining to natural variability (e.g. Hawkins and Sutton 2011; Lafaysse et al. 2014).

In the recent years, various methods have been proposed for partitioning uncertainty sources associated to Multimodel Multimember Ensembles (MM2E) of transient climate projections (Johns et al. 2011; Jacob et al. 2014). Most are based on an Analysis of Variance (ANOVA) of projections available for the considered projection lead time (Hingray et al. 2007; Yip et al. 2011; Giuntoli et al. 2015; Bosshard et al. 2013; van Pelt et al. 2015). In this single time approach, and provided multiple members are available for each modeling chain, the model uncertainty components are estimated from the dispersion between the climate responses of the different modeling chains, obtained for each chain from the multimember mean of the projections. Similarly, the internal variability component is estimated from the inter-member variance of the projections.

In recent years, long time series have become available for the large majority of climate model experiments and in turn for a large number of modeling chains. Another approach is to estimate the different uncertainty components from those times series, based for instance on a quasi-ergodic assumption for climate simulations in transient climate (QEANOVA) (Hingray and Saïd 2014). This assumption considers that if the climate response of a particular simulation chain varies over the period, this variation should be gradual and smooth, the higher frequency variations of the time series being due to internal variability alone. It assumes also that the internal variability remains constant over the considered period or that it varies as a gradual and smooth function of the climate response of the chain. These assumptions were used by Räisänen (2001); Hawkins and Sutton (2009, 2011); Charlton-Perez et al. (2010); Hingray and Saïd (2014); Bracegirdle et al. (2014); Reintges et al. (2017) for changes in different climate variables such as surface temperature, precipitation, winds or stratospheric ozone or Atlantic Meridional Overturning Circulation. In this time series approach, the noise-free signal, extracted from the time series of each simulation chain, defines the climate change response of the chain and its possible evolution with time. The climate responses of all chains can then be used to estimate the components of model uncertainty for any projection lead time. In parallel, the variance over time of the deviations from the climate response allows estimating the internal variability of each chain.

Both single time and time series approaches have been used in a number of recent climate impact studies for a number of different climate variables. In most cases, the model uncertainty components are estimated with an empirical ANOVA. In a MM2E resulting from the experiments of different Global Climate Models (GCM) for instance, the GCM uncertainty is estimated as the empirical variance of their respective climate responses. In a single prediction lead time approach, this empirical variance is however known to be a biased estimator of model uncertainty variance as shown in a more general context by Montgomery (2012) and recalled for climate projections by Northrop and Chandler (2014) and Lyu et al. (2015). The bias is positive, i.e. the empirical variance tends to overestimate the true model uncertainty variance. It is also known to be larger for small numbers of members. In a time series based approach, the empirical variance of the climate responses is also likely to be a biased estimator of model uncertainty.

In the following, we explore the importance of the bias in both the single time and the time series approaches for a simple but classical configuration where MM2E are composed of two single uncertainty sources: model uncertainty and internal variability. For this analysis, we first derive theoretical expressions for unbiased estimators of model uncertainty and internal variability variance in the general case where the climate response functions of the different modeling chains are linear combinations of functions of time (section 2). We next give the simplified expressions obtained for specific analysis configurations, including the time series approach considered in Hingray and Saïd (2014) and the single time approach considered in Yip et al. (2011) (section 3). From these expressions, we discuss the bias resulting in estimating model uncertainty variance as the empirical variance of estimated climate responses (section 4). We especially discuss the importance of the bias for different levels of internal variability contribution to total uncertainty variance and we present how the bias depends on the number of members available for the estimation. Note that most expressions and results derived in the following are general and could also apply to datasets of non climate variables.

2. Unbiased QEANOVA estimators

In this section, we derive the expressions of unbiased estimators for model uncertainty and internal variability variance when estimated from a time series analysis of some climate variable Z, for which the climate response can be expressed as a linear combination of functions of time.

a. Climate responses, model uncertainty and internal variability

Let us consider a given MM2E ensemble of transient climate experiments composed of multiple members for G different climate modeling chains. The number of available members for chain g is M_g. A chain refers for instance to a given GCM and the members to the different runs available for each GCM. A chain could also refer to a given GCM/RCM combination where a given Regional Climate Model (RCM) is used to produce regional high resolution climate projections from the outputs of a given GCM. Members would respectively refer to the potentially multiple generations obtained with the different runs for each GCM/RCM chain (e.g. Lafaysse et al. 2014).

Note $Z(g,m,t)$ the experiment outputs obtained for the m^{th} member of chain g for any given time t of the experiment period $[t_S, t_F]$. These outputs for instance correspond to the n-yr interannual mean values of annual projections for the n-yr period centered on year t. We consider that Z follows a model of the form

$$Z(g,m,t) = \varphi(g,t) + \eta(g,m,t) \tag{1}$$

for $t_S \leq t \leq t_F$ where $\eta(g,m,t)$ are i.i.d. with $\mathbb{E}\{\eta(g,m,t)\} = 0$ and $\text{Var}\{\eta(g,m,t)\} = \sigma_\eta^2$. $\varphi(g,t)$ is the climate response of chain g at time t and $\eta(g,m,t)$ is the deviation from the climate response obtained with the member m at this time as a result of internal variability. The climate response function $\varphi(g,t)$ can be expressed, for each g, as:

$$\varphi(g,t) = \mu(t) + \alpha(g,t) \tag{2}$$

where $\mu(t) = \frac{1}{G}\sum_{g=1}^{G}\varphi(g,t)$ and $\alpha(g,t) = \{\varphi(g,t) - \frac{1}{G}\sum_{g=1}^{G}\varphi(g,t)\}$ are deterministic, with $\sum_{g=1}^{G}\alpha(g,t) = 0$. $\mu(t)$ is the mean response function of the G modeling chains in the ensemble and $\alpha(g,t)$ is, for each modeling chain g, the deviation of its response function from $\mu(t)$.

By definition, no correlation is expected between the climate responses and the deviations. The total uncertainty variance of Z at time t is $\sigma_Z^2(t) = s_\alpha^2(t) + \sigma_\eta^2(t)$ where $s_\alpha^2(t)$ and $\sigma_\eta^2(t)$ are the variances of the $\alpha's$ and the $\eta's$ at t. $s_\alpha^2(t)$ and $\sigma_\eta^2(t)$ correspond respectively to the model uncertainty and internal variability components of $\sigma_Z^2(t)$.

We further consider that, for each modeling chain g, the response function $\varphi(g,t)$ is a linear combination of P functions of t of the form:

$$\varphi(g,t) = \sum_{p=1}^{P}\Phi_{gp}f_{gp}(t)$$

with $f_{gp}(t)$ the p^{th} function of t and with Φ_{gp} the corresponding model parameter. For instance, in the case where the response function for g is a linear function of time, we would have $P = 2$ with $f_{g1}(t) = 1$ and $f_{g2}(t) = t - t_S$. In the case where it is a polynomial function of time of order $P-1$, we would have $f_{gp}(t) = (t - t_S)^{p-1}$ for $p = 1,\ldots,P$. The P functions can be different from one chain to the another but a same set of functions $\{f_p(t); p = 1,\ldots,P\}$ can also apply for all chains.

Let focus on some prediction lead time t_k, where t_k is the k^{th} time step of $t_1,\ldots,t_T$, a discretisation of $[t_S, t_F]$ into T times with $t_1 = t_S$ and $t_T = t_F$. Noting $F_{gkp} = f_{gp}(t_k)$, we can write, for any modeling chain g:

$$\varphi(g,t_k) = \sum_{p=1}^{P}F_{gkp}\Phi_{gp} \tag{3}$$

Equation (3) shows that unbiased estimators of Φ_{gp} for $p = 1,\ldots,P; g = 1,\ldots,G$ allow having unbiased estimators of the climate response of each chain g and, in turn, unbiased estimators of the mean response $\mu(t_k)$ and of the deviations $\alpha(g,t_k)$. As shown later, unbiased estimators of the Φ_{gp}'s allow additionally having an unbiased estimator of the internal variability variance $\sigma_\eta^2(t)$.

An unbiased estimator of model uncertainty is not so straightforward. The following decomposition of the sample variance $s^2_\alpha(t_k) = \frac{1}{G-1}\sum_{g=1}^{G}\{\alpha(g,t_k)\}^2$ (see Appendix A) shows that it requires unbiased estimates of the cross-products $\Phi_{gp}\Phi_{g'p'}$ for any $g, g' = 1,\ldots,G$ and $p, p' = 1,\ldots,P$. :

$$
\begin{aligned}
s^2_\alpha(t_k) \quad = \quad & \frac{1}{G}\sum_{g=1}^{G}\left[\sum_{p=1}^{P} F^2_{gkp}\Phi^2_{gp} + 2\sum_{p=1}^{P}\sum_{p'>p} F_{gkp}F_{gkp'}\Phi_{gp}\Phi_{gp'}\right] \\
& - \frac{2}{G(G-1)}\sum_{g=1}^{G}\left[\sum_{g'>g}\sum_{p=1}^{P} F^2_{gkp}\Phi_{gp}\Phi_{g'p} + \sum_{g'\neq g}\sum_{p=1}^{P}\sum_{p'>p} F_{gkp}F_{gkp'}\Phi_{gp}\Phi_{g'p'}\right]
\end{aligned}
\tag{4}
$$

In the following, unbiased estimators of the Φ_{gp}'s and of the cross-products are obtained indirectly based on the raw climate projections $Y(g,m,t)$ of the considered climate variable which are linked to Z through the simple relationships:

$$
Z(g,m,t) = Y(g,m,t) + aY(g,m,t_C), \tag{5}
$$

where t_C is some reference period included into $[t_S, t_F]$ and a is a constant. Case $a = 0$ corresponds to the case where the uncertainty analysis is carried out on the raw climate projections, i.e. $Z = Y$. Case $a = -1$ corresponds to the case where the uncertainty analysis is carried out on the climate change variable, i.e. $Z = X$ where $X(g,m,t) = Y(g,m,t) - Y(g,m,t_C)$ with t_C some reference time period. These two configurations will be considered respectively in sections 2.c and 2.d, while section 2.b gives the unbiased estimators of Y in the general case.

b. Unbiased estimation of the auxiliary linear model

Let consider the variable Y linked to Z through (5). Then Y follows necessarily a linear model of the form $Y(g,m,t) = \lambda(g,t) + \nu(g,m,t)$ for $m = 1,\ldots,M_g$ and $t_S \le t \le t_F$ where M_g is the number of members available for each g and where $\nu(g,m,t)$ are i.i.d. with $\mathbb{E}\{\nu(g,m,t)\} = 0$ and $\mathrm{Var}\{\nu(g,m,t)\} = \sigma^2_\nu$. Let also consider that for each g, $\lambda(g,t)$ is a linear combination of L functions of t of the form:

$$
\lambda(g,t) = \sum_{\ell=1}^{L} \Lambda_{g\ell} r_{g\ell}(t) \tag{6}
$$

with $r_{g\ell}(t)$ the ℓ^{th} function and $\Lambda_{g\ell}$ the corresponding model parameter.

For any chain g and member m, we can write in vector form:

$$
\begin{pmatrix} Y(g,m,t_1) \\ \vdots \\ Y(g,m,t_T) \end{pmatrix} = \mathbb{R}_g \begin{pmatrix} \Lambda_{g1} \\ \vdots \\ \Lambda_{gL} \end{pmatrix} + \begin{pmatrix} \nu(g,m,t_1) \\ \vdots \\ \nu(g,m,t_T) \end{pmatrix} \tag{7}
$$

where $\mathbb{R}_g$ is the $T \times L$ matrix of covariates which $(k,\ell)^{th}$ element is $R_{gk\ell} = r_{g\ell}(t_k)$. For every g, unbiased estimators of $\Lambda_{g\ell}$ based on all members $m = 1, \ldots, M_g$ available for g are given by the least squares estimates

$$\begin{pmatrix} \hat{\Lambda}_{g1} \\ \vdots \\ \hat{\Lambda}_{gL} \end{pmatrix} = \frac{1}{M_g} \sum_{m=1}^{M_g} (\mathbb{R}_g' \mathbb{R}_g)^{-1} \mathbb{R}_g' \begin{pmatrix} Y(g,m,t_1) \\ \vdots \\ Y(g,m,t_T) \end{pmatrix}. \tag{8}$$

Covariance matrix of the estimators $\hat{\Lambda}_{g\ell}$, $\ell = 1, \ldots, L$, is estimated by $\widehat{\sigma^2_{\nu_g}} M_g^{-1} \mathbb{V}_g$ where $\mathbb{V}_g$ is the $(L \times L)$ matrix equals to $(\mathbb{R}_g' \mathbb{R}_g)^{-1}$ and where $\widehat{\sigma^2_{\nu_g}}$ is an unbiased estimator of $\sigma^2_{\nu_g}$ given by:

$$\widehat{\sigma^2_{\nu_g}} = \frac{1}{(TM_g - L)} \sum_{k=1}^{T} \sum_{m=1}^{M_g} \left\{ Y(g,m,t_k) - \sum_{\ell=1}^{L} R_{gk\ell} \hat{\Lambda}_{g\ell} \right\}^2. \tag{9}$$

This gives additionally the following unbiased estimators:

$$\begin{aligned} \hat{\Lambda}^2_{g\ell} - \widehat{\sigma^2_{\nu_g}} M_g^{-1} V_{g\ell\ell} \quad & \text{for} \quad \Lambda^2_{g\ell}, \\ \hat{\Lambda}_{g\ell} \hat{\Lambda}_{g\ell'} - \widehat{\sigma^2_{\nu_g}} M_g^{-1} V_{g\ell\ell'} \quad & \text{for} \quad \Lambda_{g\ell} \Lambda_{g\ell'}, \\ \hat{\Lambda}_{g\ell} \hat{\Lambda}_{g'\ell'} \quad & \text{for} \quad \Lambda_{g\ell} \Lambda_{g'\ell'} \quad \text{when} \quad g \neq g', \end{aligned} \tag{10}$$

and where $V_{g\ell\ell'}$ is the element (ℓ, ℓ') of the $(L \times L)$ matrix $\mathbb{V}_g$. Using (5), the decomposition (4) and the unbiased estimators in (10) allows us to derive the QEANOVA unbiased estimators of Φ_{gp}, s^2_α, σ^2_η. We derive these expressions for the two configurations of interest in the following sections.

c. Uncertainty analysis applied on the raw variable Y

We here consider the simple case where the regression and the uncertainty analysis both apply on the raw climate variable Y. The regression model is estimated over the whole $[t_S, t_F]$ period. In this simple case, we have $a = 0$, $Z = Y$ and thus $L = P$, $\varphi(g,t) = \lambda(g,t)$, $F_{gk\ell} = R_{gk\ell}$ and $\hat{\Phi}_{g\ell} = \hat{\Lambda}_{g\ell}$ for $g = 1, \ldots, G$, $\ell = 1, \ldots, L$. Following (4) and (10), an unbiased estimator of $s^2_\alpha(t_k)$ is

$$\widehat{s^2_\alpha}(t_k) = s^2_{\hat{\alpha}}(t_k) - \frac{1}{G} \sum_{g=1}^{G} \left[\frac{\widehat{\sigma^2_{\nu_g}}}{M_g} \left(\sum_{\ell=1}^{L} R^2_{gk\ell} V_{g\ell\ell} + 2 \sum_{\ell=1}^{L} \sum_{\ell' > \ell} R_{gk\ell} R_{gk\ell'} V_{g\ell\ell'} \right) \right] \tag{11}$$

where

$$s^2_{\hat{\alpha}}(t_k) = \frac{1}{G-1} \sum_{g=1}^{G} \{\hat{\alpha}(g,t_k)\}^2 = \frac{1}{G-1} \sum_{g=1}^{G} \left(\sum_{\ell=1}^{L} R_{gk\ell} \hat{\Phi}_{g\ell} - \frac{1}{G} \sum_{\ell=1}^{L} \sum_{g'=1}^{G} R_{g'k\ell} \hat{\Phi}_{g'\ell} \right)^2$$

and where $\widehat{\sigma^2_{\nu_g}}$ is given by (9).

Note that if, for all chains $g = 1,\ldots,G$, the functions $\lambda(g,t)$ are linear combinations of the same functions $r_{g\ell}(t) = r_\ell(t)$, $\ell = 1,\ldots,L$ and if the time discretization of the interval $[t_S, t_F]$ is the same, then all chains g have the same covariate matrix $\mathbb{R}_g = \mathbb{R}$ and so $\mathbb{V}_g = \mathbb{V}$. If moreover all modeling chains g have the same number of members $M_g = M$, the expression (11) reduces to:

$$\widehat{s^2_\alpha}(t_k) = s^2_{\hat{\alpha}}(t_k) - \frac{\widehat{\sigma^2_\nu}}{M}\left(\sum_{\ell=1}^{L} R^2_{k\ell}V_{\ell\ell} + 2\sum_{\ell=1}^{L}\sum_{\ell'>\ell} R_{k\ell}R_{k\ell'}V_{\ell\ell'}\right) \tag{12}$$

where $\widehat{\sigma^2_\nu}$ is the mean of the estimates $\widehat{\sigma^2_{\nu_g}}$:

$$\widehat{\sigma^2_\nu} = \frac{1}{G}\sum_{g=1}^{G}\widehat{\sigma^2_{\nu_g}}. \tag{13}$$

In all cases, note finally that, as $\eta(g,m,t) = \nu(g,m,t)$, an unbiased estimator of $\sigma^2_{\eta_g}$ is $\widehat{\sigma^2_{\eta_g}} = \widehat{\sigma^2_{\nu_g}}$ and an unbiased estimator of the mean of $\widehat{\sigma^2_{\eta_g}}$ is simply:

$$\widehat{\sigma^2_\eta} = \widehat{\sigma^2_\nu}. \tag{14}$$

d. Uncertainty analysis applied on the change variable X.

We now consider the case when the uncertainty analysis is applied on the change variable $X(g,m,t) = Y(g,m,t) - Y(g,m,t_C)$ where $t_C \geq t_S$. The regression model is estimated on Y over the whole $[t_S, t_F]$ period. We have $a = -1$ and $Z = X$. Considering regression models with intercepts for Y and writing $r_{g1}(t) = r_{g1}$ the intercepts of each chain g, we have $P = L - 1$ and $\hat{\Phi}_{gp} = \hat{\Lambda}_{g(p+1)}$ for $g = 1,\ldots,G$, $p = 1,\ldots,P$. In this case, $\varphi(g,t) = \lambda(g,t) - \lambda(g,t_C)$. Writing K the integer such that t_C is the K^{th} time, we have thus $\hat{\varphi}(g,t_k) = \sum_{p=1}^{P} F_{gkp}\hat{\Phi}_{gp} = \sum_{\ell=2}^{L}(R_{gk\ell} - R_{gK\ell})\hat{\Lambda}_{g\ell}$ and, following again (4) and (10), an unbiased estimator of $s^2_\alpha(t_k)$ is:

$$\begin{aligned}\widehat{s^2_\alpha}(t_k) \quad = \quad & s^2_{\hat{\alpha}}(t_k) - \frac{1}{G}\sum_{g=1}^{G}\left[\frac{\widehat{\sigma^2_{\nu_g}}}{M_g}\left(\sum_{\ell=2}^{L}\left(R_{gk\ell} - R_{gK\ell}\right)^2 V_{g\ell\ell}\right.\right. \\ & \left.\left. +2\sum_{\ell=2}^{L}\sum_{\ell'>\ell}\left(R_{gk\ell} - R_{gK\ell}\right)\left(R_{gk\ell'} - R_{gK\ell'}\right)V_{g\ell\ell'}\right)\right]\end{aligned} \tag{15}$$

When all modeling chains g have the same covariate matrix $\mathbb{R}$ and the same number of members M, the expression again simplifies to:

$$\widehat{s^2_\alpha}(t_k) = s^2_{\hat{\alpha}}(t_k) - \frac{\widehat{\sigma^2_\nu}}{M}\left(\sum_{\ell=2}^{L}(R_{k\ell}-R_{K\ell})^2 V_{\ell\ell} + 2\sum_{\ell=2}^{L}\sum_{\ell'>\ell}(R_{k\ell}-R_{K\ell})(R_{k\ell'}-R_{K\ell'})V_{\ell\ell'}\right) \quad (16)$$

where $\widehat{\sigma^2_\nu}$ is given by (13). Finally, as $\eta(g,m,t) = \nu(g,m,t) - \nu(g,m,t_C)$, for each g, an unbiased estimator of $\sigma^2_{\eta_g}$ is $\widehat{\sigma^2_{\eta_g}} = 2\widehat{\sigma^2_{\nu_g}}$ and an unbiased estimator of the mean of the estimates $\widehat{\sigma^2_{\eta_g}}$ is:

$$\widehat{\sigma^2_\eta} = 2\widehat{\sigma^2_\nu}$$

3. Particular cases

In this section, we give the simplified expressions of $\widehat{s^2_\alpha}(t_k)$ obtained for specific analysis configurations, including the QEANOVA approach considered in Hingray and Saïd (2014) and the local-QEANOVA approach considered in (Hingray et al. submitted). We additionally recall the expressions for the single time approach considered in Yip et al. (2011).

a. When climate responses are linear functions of time over a transient period

We detail here the derivation of $\widehat{s^2_\alpha}(t_k)$ in a particular case of climate responses, similar to that considered in Hingray and Saïd (2014). The climate response function $\lambda(g,t)$ fitted to the raw variable Y is here assumed to be a linear function of time over $[t_S,t_F]$. The climate response function for a given chain therefore reads:

$$\lambda(g,t) = \Lambda_{g1} + \Lambda_{g2}(t-t_S). \quad (17)$$

We have thus $\lambda(g,t) = \Lambda_{g1}r_1(t) + \Lambda_{g2}r_2(t)$ with $r_1(t) = 1$ and $r_2(t) = t - t_S$. For all modeling chains, $L = 2$, $\mathbb{R}_g = \mathbb{R}$ and considering $t_1,\ldots,t_T$ regularly spaced on $[t_S,t_F]$ every dt units (i.e. $dt = (t_F - t_S)/(T-1)$, e.g. $dt = 10$ for decadal values), the matrix $\mathbb{R}$ reads:

$$\mathbb{R} = \begin{pmatrix} 1 & 0 \\ \vdots & \vdots \\ 1 & (T-1)dt \end{pmatrix} \begin{array}{ll} \text{line } 1 & (t=t_1) \\ & \\ \text{line } T & (t=t_T) \end{array} \quad (18)$$

We have then the following expression for $\mathbb{V} = (\mathbb{R}'\mathbb{R})^{-1}$:

$$\mathbb{V} = \frac{1}{(dt)^2 T(T^2-1)}\begin{pmatrix} 2(dt)^2(T-1)(2T-1) & -6dt(T-1) \\ -6dt(T-1) & 12 \end{pmatrix}. \quad (19)$$

When the QEANOVA analysis applies on the raw variable $Y(g,m,t)$ as in section 2.c, we have, for any time t_k in $[t_S, t_F]$:

$$F_{gk\ell} = R_{gk\ell} = \begin{cases} r_{g1}(t_k) = 1 & \text{if } \ell = 1 \\ r_{g2}(t_k) = t_k - t_1 & \text{if } \ell = 2 \end{cases} \tag{20}$$

and using (12) and (18), an unbiased estimator of $s_\alpha^2(t_k)$ at t_k is

$$\widehat{s_\alpha^2}(t_k) = s_{\hat{\alpha}}^2(t_k) - \left(V_{11} + (t_k - t_1)^2 V_{22} + 2(t_k - t_1)V_{12}\right)\left(\frac{1}{G}\sum_{g=1}^{G}\frac{\widehat{\sigma_{\nu_g}^2}}{M_g}\right)$$

where V_{11}, V_{12} and V_{22} are the elements $(1,1)$, $(1,2)$ and $(2,2)$ of $\mathbb{V}$ and where $\widehat{\sigma_{\nu_g}^2}$ is given by (9). With the expressions of V_{11}, V_{12} and V_{22} in (19), this expression simplifies to

$$\widehat{s_\alpha^2}(t_k) = s_{\hat{\alpha}}^2(t_k) - \frac{1}{T}\left(1 + 12\frac{T-1}{T+1}\left(\frac{t_k - t^\star}{t_T - t_1}\right)^2\right)\left(\frac{1}{G}\sum_{g=1}^{G}\frac{\widehat{\sigma_{\nu_g}^2}}{M_g}\right) \tag{21}$$

where $t^\star = (t_1 + t_T)/2$.

When the QEANOVA analysis is applied on the change variable $X(g,m,t) = Y(g,m,t) - Y(g,m,t_C)$ with $t_C \geq t_S$, using the same notations as in section 2.d, we have $P = 1$ and $F_{k1} = R_{k2} - R_{K2} = r_2(t_k) - r_2(t_K) = t_k - t_K$ if $\ell = 2$.

Using (15) and (19), an unbiased estimator of $s_\alpha^2(t_k)$ at t_k is

$$\begin{aligned} \widehat{s_\alpha^2}(t_k) &= s_{\hat{\alpha}}^2(t_k) - (t_k - t_K)^2 V_{22}\left(\frac{1}{G}\sum_{g=1}^{G}\frac{\widehat{\sigma_{\nu_g}^2}}{M_g}\right) \\ &= s_{\hat{\alpha}}^2(t_k) - \frac{12}{T}\frac{T-1}{T+1}\left(\frac{t_k - t_K}{t_T - t_1}\right)^2\left(\frac{1}{G}\sum_{g=1}^{G}\frac{\widehat{\sigma_{\nu_g}^2}}{M_g}\right). \end{aligned} \tag{22}$$

b. When climate responses are locally linear in time

This case corresponds to the local QEANOVA configuration presented in (Hingray et al. submitted). We still consider the change variable $X(g,m,t) = Y(g,m,t) - Y(g,m,t_C)$ where $t_C \geq t_S$ and the regression model is fitted on Y but $\lambda(g,t)$ is only locally linear in time, in the neighborhoods $\Omega(t)$ of t and $\Omega(t_C)$ of t_C respectively. When prediction lead time t_E is of interest, two local linear models are thus considered, one on $[t_C - \omega, t_C + \omega]$ and one on $[t_E - \omega, t_E + \omega]$. The response function for Y can thus be expressed as:

$$\lambda(g,t) = \begin{cases} \Lambda_{g1,C} + (t - t_C)\Lambda_{g2,C} & \text{for } t_C - \omega \leq t \leq t_C + \omega \\ \Lambda_{g1,E} + (t - t_E)\Lambda_{g2,E} & \text{for } t_E - \omega \leq t \leq t_E + \omega \end{cases} \tag{23}$$

If each interval $[t_C - \omega, t_C + \omega]$ and $[t_E - \omega, t_E + \omega]$ is discretized into $T^\star$ regular times, with $T^\star$ odd, and provided both intervals do not overlap, an unbiased estimator of the sample variance of α at $t_k = t_E$ is (see Appendix B for details):

$$\widehat{s^2_\alpha}(t_k) = s^2_{\hat{\alpha}}(t_k) - \frac{2}{T^\star}\left(\frac{1}{G}\sum_{g=1}^{G}\frac{\widehat{\sigma^2_{\nu_g}}}{M_g}\right) \tag{24}$$

Note that the total number of time steps considered for the analysis is $T = 2T^\star$.

c. When single time steps are considered

We still consider the change variable $X(g,m,t) = Y(g,m,t) - Y(g,m,t_C)$ where $t_C \geq t_S$ but the analysis is a single time step analysis. That is, the analysis for a given future period $t = t_E$ only accounts for data available for t_C and for t_E respectively. The response function for Y here simply reduces to:

$$\lambda(g,t) = \begin{cases} \Lambda_{g1,C} & \text{for } t = t_C \\ \Lambda_{g1,E} & \text{for } t = t_E \end{cases}$$

The regression reduces to the estimation of the mean values of Y at t_C and t_E respectively. This is a particular case of section 3.b with $\omega = 0$, $T^\star = 1$ (i.e. $T = 2$), thus an unbiased estimator of s^2_α at $t_k = t_E$ is

$$\widehat{s^2_\alpha}(t_k) = s^2_{\hat{\alpha}}(t_k) - 2\left(\frac{1}{G}\sum_{g=1}^{G}\frac{\widehat{\sigma^2_{\nu_g}}}{M_g}\right) \tag{25}$$

with $\widehat{\sigma^2_{\nu_g}}$ given by (9). When all models have moreover the same number of runs M, it reads:

$$\widehat{s^2_\alpha}(t_k) = s^2_{\hat{\alpha}}(t_k) - \frac{2\widehat{\sigma^2_\nu}}{M}. \tag{26}$$

Note that $2\widehat{\sigma^2_\nu} = \widehat{\sigma^2_\eta}$, where σ^2_η is the internal variability of the change variable X. Substituting $\widehat{\sigma^2_\eta}$ to $2\widehat{\sigma^2_\nu}$, the above expression thus corresponds to that of a classical 1-way ANOVA applied on some variable Z, as presented for instance in Montgomery (2012), where Z corresponds here directly to the change variable X. This case corresponds to that described in Yip et al. (2011).

4. Bias in empirical estimates of model uncertainty

As highlighted by the different expressions derived previously, the mean sum $s^2_{\hat{\alpha}}(t_k)$ is a biased estimator of the sample variance $s^2_{\alpha}(t_k)$ of the α's at time t_k. The expressions show that the bias obtained when using $s^2_{\hat{\alpha}}$ in place of the unbiased estimator $\widehat{s^2_{\alpha}}$ increases with the value of the internal variability variance. It conversely decreases with the size of the dataset used for the estimation. The more members for each chain and/or the more time steps considered in the analysis, the lower the bias.

In the following, we illustrate and discuss the importance of the bias for the case where the analysis is applied on the change variable $X(g,m,t) = Y(g,m,t) - Y(g,m,t_C)$. We do not discuss the case of an analysis applied to the raw projections. Results are actually very similar due to the similar forms obtained for the unbiased estimators in both cases (see equations 21 and 22).

We consider in turn the three specific analysis configurations presented in sections 3.a, 3.b and 3.c: the single time ANOVA, the local QEANOVA and the full QEANOVA. For simplification, the number of members is assumed to be the same for all modeling chains ($M_g = M$). We also only consider the theoretical bias, that is the bias that would be obtained in the case of a perfect estimate of the internal variability variance. The quality of this estimator is further discussed in Hingray et al. (submitted).

For each variable, we consider the relative bias (RB) for s^2_{α} at time t, expressed as:

$$\mathrm{RB}(t) \quad = \quad \frac{s^2_{\hat{\alpha}}(t) - \widehat{s^2_{\alpha}}(t)}{\widehat{s^2_{\alpha}}(t)} \tag{27}$$

Following the expressions of $\widehat{s^2_{\alpha}}(t)$ derived in equations (22), (24) and (26) and using $\widehat{\sigma^2_{\eta}} = 2\widehat{\sigma^2_{\nu}}$, the RB can be expressed as:

$$\mathrm{RB}(t) = \frac{A(t,\mathscr{C})}{M}\frac{\widehat{\sigma^2_{\eta}}(t)}{\widehat{s^2_{\alpha}}(t)} = \frac{A(t,\mathscr{C})}{M}\;\frac{F_{\eta}(t)}{1 - F_{\eta}(t)} \tag{28}$$

where $A(t,\mathscr{C})$ is a constant, function of future period t and of the configuration analysis $\mathscr{C}$, and where $F_{\eta}(t) = \widehat{\sigma^2_{\eta}}(t)/\widehat{\sigma^2_{X}}(t)$ is the estimated fractional variance associated to internal variability, i.e. the estimated proportion of total variance $\widehat{\sigma^2_{X}}(t) = \widehat{s^2_{\alpha}}(t) + \widehat{\sigma^2_{\eta}}(t)$ explained by the estimated internal variability variance.

In the case of the single time step ANOVA discussed in section 3.c, and according to equation (26), we have $A(t,\mathscr{C}) = 1$. The relative bias $\mathrm{RB}(t)$ thus only depends on M and $F_{\eta}(t)$. As shown in Figure 1, $\mathrm{RB}(t)$ is logically a decreasing function of M and an increasing function of $F_{\eta}(t)$. The empirical mean sum of squares $s^2_{\hat{\alpha}}(t)$ overestimates the true model uncertainty variance $\widehat{s^2_{\alpha}}(t)$ by 100% (i.e. $\mathrm{RB}(t) \geq 1$) or even more in the most critical configurations, i.e. when the number of members is small ($M \leq 3$) and when internal variability explains the main part of total uncertainty variance ($F_\eta(t) \geq 75\%$). For small numbers of members ($M \leq 3$), the overestimation is actually greater than 25% as soon as $F_\eta(t) \geq 40\%$. It remains relatively moderate (+10%) only when internal variability explains a small to very small part of total uncertainty (i.e. when $F_\eta(t) \leq 20\%$). When only 2 members are available, the overestimation is larger than 50% as soon as $F_\eta(t) \geq 50\%$; it exceeds +200% when $F_\eta(t) \geq 80\%$.

[FIGURE 1 HERE]

Let now consider the case where the uncertainty analysis is carried out with a time series analysis as described in the general case in section 2. In this case, the relative bias expected for a given (F_η, M) configuration is simply $A(t,\mathscr{C})$ times the one obtained for the same configuration in the case of the single time ANOVA. As shown in the following, $A(t,\mathscr{C})$ is actually always smaller or equal to 1 using the local QEANOVA analysis and the specific QEANOVA analysis described in section 3. In those cases, the relative bias is thus always smaller or equal to that obtained with the single time ANOVA. The importance of the bias reduction factor $A(t,\mathscr{C})$ is discussed below.

Let consider first the local QEANOVA analysis (section 3.b). According to equation (24) and because $\widehat{\sigma}^2_\eta(t) = 2\widehat{\sigma}^2_\nu(t)$, $A(t,\mathscr{C})$ is independent on the projection lead time t and simply reads:

$$A(t,\mathscr{C}) = 1/T^\star$$

with $T^\star$ the number of time steps considered around each t_C and t_k. Thus the higher the value of $T^\star$, the smaller the bias. Using two time steps around each t_C and t_E ($T^\star = 3$), the relative bias is already one third of that with the single time ANOVA. It drops to one fifth when four time steps are considered around t_C and t_E ($T^\star = 5$). Despite this significant reduction, the relative bias is still high in the critical (F_η, M) configurations: it amounts 33% for $T^\star = 3$ (resp. 20% for $T^\star = 5$) with $M = 3$ and $F_\eta = 75\%$. This is much less than the 100% of the single time case, but still too high for the empirical variance $s^2_\alpha(t)$ to be used in practice. The unbiased variance estimator $\widehat{s^2_\alpha}(t)$ is thus here again required.

Let now consider the full QEANOVA case when it is applied over a transient period with effects expressed as linear functions of time (section 3.a). According to equation (22) and reminding that $\widehat{\sigma}^2_\eta(t) = 2\widehat{\sigma}^2_\nu(t)$, $A(t,\mathscr{C})$ now reads:

$$A(t,\mathscr{C}) = \frac{6(T-1)}{T(T+1)}\tau(t)^2 \tag{29}$$

where $\tau(t) = (t - t_K) \div (t_T - t_1)$. $\tau(t)$ increases with the temporal distance of t to the reference period t_K considered for the change variable. As both t and t_K belong to $[t_1, t_T]$, $\tau(t)^2$ is always smaller than 1. $\tau(t)$ is actually zero when $t_k = t_K$. It is maximal when $t = t_T$. Whatever the value for T, the relative bias is thus zero when the considered lead time corresponds to the reference period and it is maximal when t corresponds to the last time step of the transient period used for the regression. The first term of equation 29 is a function of T, the total number of time steps in

$[t_S, t_F]$. It amounts 1 for both $T = 2$ and $T = 3$. It is a decreasing function of T for $T \geq 3$, with an asymptotic behavior as $6/T$ for large T values.

With the full QEANOVA analysis, $A(t, \mathscr{C})$ is thus smaller or equal to 1 whatever the length of the transient period and whatever the projection lead time t_k for which model uncertainty is estimated. When compared to the relative bias obtained from the single time ANOVA approach, the bias with this QEANOVA approach can be much smaller, as illustrated in Figure 2 for the case where the first time step of the period used for the regression is also the reference period (i.e. $t_K = t_1$). The value of $A(t_k, \mathscr{C})$ is presented as a function of $\tau(t_k)$ and T, and also, for clarity, as a function of k and T. It has to be compared to the value $A(t, \mathscr{C}) = 1$ obtained in the single time ANOVA approach. Let consider for instance an analysis applied on 1980-2100 transient climate projections, with $t_C = 1980$. If the analysis applies for some decadal climate variable, we have $dt = 10$, $T = 13$. If the target prediction lead time is the 2040's decade, we next have $\tau(t_k) = 0.5$, so $A(t, \mathscr{C}) \simeq 0.1$. As a consequence, the relative bias obtained when using $s^2_{\hat{\alpha}}(t)$ instead of $\widehat{s^2_{\alpha}}(t)$ is relatively moderate in this case, even in the most critical $(F_\eta(t), M)$ configurations. For instance, $s^2_{\hat{\alpha}}(t)$ at $t = 2040$ overestimates the true model uncertainty variance $\widehat{s^2_{\alpha}}(t)$ by only 10% when $M = 3$ and $F_\eta(t) = 75\%$, which is still significant but much lower than the 100% overestimation in the single time ANOVA case. It is also lower than the 33 or 20% obtained with the local QEANOVA approach when $T^\star = 3$ or 5 respectively.

As mentioned above, the bias increases when the target prediction lead time gets further the reference period (i.e. when $\tau(t_k)$ increases). $A(t, \mathscr{C}) \simeq 0.39$ for instance when $t = 2100$ in the previous configuration, giving an overestimation of 39% of the true model uncertainty variance when $M = 3$ and $F_\eta(t) = 75\%$. In this case, the overestimation is rather large even if still much lower than the 100% overestimation of the single time ANOVA case. It is similar to the overestimation of the local QEANOVA approach when $T^\star = 3$ but becomes larger to the local QEANOVA overestimation when $T^\star = 5$. In the QEANOVA approach also, the unbiased variance estimator $\widehat{s^2_{\alpha}}(t)$ is thus here again required, instead of the biased $s^2_{\hat{\alpha}}(t)$.

[FIGURE 2 HERE]

5. Discussion and conclusions

Numerous studies have been recently presented for partitioning model uncertainty and internal variability variance in climate projections. Most of them are based on a single time ANOVA analysis, the other being based on a time series approach such as the QEANOVA approach. In most cases, the estimate of model uncertainty is obtained from the empirical variance of the main effects in the different climate responses obtained respectively for the different modeling chains.

In the present work, we recall the expressions for unbiased estimates of model uncertainty in the single time ANOVA case and derive these expressions in the general case of a time series ANOVA approach where the climate responses of the different chains are linear combinations of functions of time. We next discuss the importance of the bias when empirical estimates are used instead of unbiased estimates. The bias is shown to be always positive. The empirical estimates thus systematically overestimate model uncertainty. The contribution of model uncertainty to total uncertainty as well as the total uncertainty are in turn also systematically overestimated.

The positive bias of empirical estimates can be especially high with the single time ANOVA analysis. Its largest values are obtained for small numbers of members and/or large contribution of internal variability variance to total uncertainty. In recent climate impact studies, very different values have been obtained for $F_\eta(t)$, depending on the climate variable under consideration. F_η tend to be larger for higher spatial and/or temporal resolution data and for closer prediction lead times. For instance, for decadal mean precipitation projections in the 2050's, the F_η value obtained by Hawkins and Sutton (2011) was lower than 5% at the global scale, but greater than 50% for the European region. Even higher values can be obtained when annual and/or local scale data are considered, as illustrated by the 80% value obtained for annual precipitation in Southern-France by Hingray and Saïd (2014). Whatever the value for F_η, the number of members available for any given modeling chain is classically lower than three, as a consequence of the small number of runs classically available for climate models. In a large number of climate impact studies, the contribution of model uncertainty to total variance estimated from a single time ANOVA analysis is thus likely to be significantly overestimated if the empirical variance $s^2_{\hat{\alpha}}$ is not corrected for bias, i.e. if it is used instead of the unbiased estimator $\widehat{s^2_\alpha}$.

The bias of empirical estimates is considerably smaller with a time series approach, owing to the multiple time steps accounted for in the analysis. The larger the number of time steps accounted for, the smaller the bias. With a local QEANOVA approach, the bias is inversely proportional to the size of the temporal neighborhood considered for the analysis. The bias is for instance reduced by a factor of 3 (resp. 5) when the 2 (resp. 4) time steps adjacent to both the reference and the future lead time are considered. The size of the neigborhood actually acts as a multiplier of the number of members M available for each modeling chain. Let for instance consider a MM2E with 2 members for each modeling chain. The bias obtained for an analysis with 3 time steps in the neigbhorhood would be the same as the bias obtained for a MM2E with 3×2 members for each chain.

A full time series analysis leads to an even smaller bias of the model uncertainty variance. When a QEANOVA approach (with responses functions being linear function of times) is used for decadal data covering the whole 1980-2100 period, the bias is 2.5 to 20 times smaller than that of the single time ANOVA analysis, depending on the lead time under consideration. Again the time series approach acts as a multiplier of the number of members available for each individual chain. A time series analysis allows thus having smaller biases in empirical estimates of model uncertainty in climate projections. In the two specific cases studied here however, the bias potentially remains not negligible, calling in that case also for the unbiased estimators instead of the empirical ones.

The work presented here is based on different simplifications and hypotheses that may not always fit to the MM2E under consideration. The expressions presented previously correspond for instance to the case where the differences in the climate responses obtained for the different modeling chains are due to only one factor, this factor being for instance the climate model. In practice, the differences in climate responses obtained for the different climate experiments of the considered MM2E are often due to multiple factors including emission scenarios and the different models chained in cascade to derive the required projections (e.g. G different global climate models $\times$ R different regional climate models $\times$ H different hydrological models). In this more general case, the different components of model uncertainty, associated respectively to the different factors of uncertainty, can be also partitioned with a single time or a time series ANOVA analysis. The empirical variance of the main effects of the different models for each factor are again classically used for estimating the corresponding model uncertainty components (see e.g. Yip et al. 2011; Hingray and Saïd 2014). These expressions lead logically also to biased estimates. It is easy to show that these empirical expressions can also be corrected for bias with the same terms as those derived in the present work where only one factor is accounted for.

Our work finally highlighted the large systematic errors that may obtained in uncertainty partitioning when empirical variance are used to estimate the model uncertainty component. A relevant uncertainty analysis obviously requires unbiased estimators of the different uncertainty components, such as those proposed here. To discuss the importance of the bias, we considered the idealistic configuration where a perfect estimate of the internal variability variance is known. A poor estimate of this uncertainty component is also expected to lead to a poor estimate of the model uncertainty component. Evaluating the quality of these estimates, and especially their robustness, would be also required, at least worthwhile. A more relevant interpretation of estimated uncertainty components likely requires knowing the confidence interval associated to each estimate. The robustness of model uncertainty and internal variability estimates is explored in Hingray et al. (submitted).

The empirical approaches presented above are very commonly used to assess the different components of uncertainty in ensembles of climate projections. The main advantage of those approaches is that they give simple and non-iterative estimators of variance components. An alternative for estimating uncertainty components is to rely on more modern likelihood-based methods such as maximum likelihood and restricted maximum likelihood or Bayesian methods (see e.g. Northrop and Chandler 2014). They demand much more computational efforts but they are expected to give unbiased estimates of uncertainty components. They allow moreover having a estimate of the precision of the uncertainty components estimates (see e.g. Geinitz et al. 2015; Evin and Hingray submitted) which may be especially relevant in configurations where the contribution of internal variability to total variance is large and/or when the number of members for each simulation chain is small (see e.g. Hingray et al. submitted).

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APPENDIX A

Decomposition of $s_{\alpha}^2(t_k)$

We here present the main steps followed for the decomposition of $s^2_\alpha(t_k)$ presented in equation 4. We have

$$
\begin{aligned}
s^2_\alpha(t_k) &= \sum_{g=1}^{G} \left(\alpha(g,t_k)\right)^2 \\
&= \sum_{g=1}^{G} \left(\sum_{p=1}^{P} F_{gkp}\Phi_{gp} - \frac{1}{G}\sum_{p=1}^{P}\sum_{g=1}^{G} F_{gkp}\Phi_{gp} \right)^2 \\
&= \sum_{g=1}^{G} \left(\left\{ \sum_{p=1}^{P} F_{gkp}\Phi_{gp} \right\}^2 + \frac{1}{G^2}\left\{ \sum_{p=1}^{P}\sum_{g=1}^{G} F_{gkp}\Phi_{gp} \right\}^2 - \frac{2}{G}\left\{ \sum_{p=1}^{P} F_{gkp}\Phi_{gp} \right\}\left\{ \sum_{p=1}^{P}\sum_{g=1}^{G} F_{gkp}\Phi_{gp} \right\} \right) \\
&= \sum_{g=1}^{G} \left\{ \sum_{p=1}^{P} F_{gkp}\Phi_{gp} \right\}^2 + \frac{1}{G}\left\{ \sum_{p=1}^{P}\sum_{g=1}^{G} F_{gkp}\Phi_{gp} \right\}^2 - \frac{2}{G}\left\{ \sum_{p=1}^{P}\sum_{g=1}^{G} F_{gkp}\Phi_{gp} \right\}^2 \\
&= \sum_{g=1}^{G} \left\{ \sum_{p=1}^{P} F_{gkp}\Phi_{gp} \right\}^2 - \frac{1}{G}\left\{ \sum_{p=1}^{P}\sum_{g=1}^{G} F_{gkp}\Phi_{gp} \right\}^2 \\
&= \sum_{g=1}^{G} \left\{ \sum_{p=1}^{P} F^2_{gkp}\Phi^2_{gp} + 2\sum_{p=1}^{P}\sum_{p'>p} F_{gkp}F_{gkp'}\Phi_{gp}\Phi_{gp'} \right\} \\
&\quad -\frac{1}{G}\left\{ \sum_{p=1}^{P}\sum_{g=1}^{G} F^2_{gkp}\Phi^2_{gp} + 2\sum_{p=1}^{P}\sum_{p'>p}\sum_{g=1}^{G} F_{gkp}F_{gkp'}\Phi_{gp}\Phi_{gp'} \right. \\
&\quad \left. +2\sum_{p=1}^{P}\sum_{g=1}^{G}\sum_{g'>g} F^2_{gkp}\Phi_{gp}\Phi_{g'p} + 2\sum_{p=1}^{P}\sum_{p'>p}\sum_{g=1}^{G}\sum_{g'\neq g} F_{gkp}F_{gkp'}\Phi_{gp}\Phi_{g'p'} \right\} \\
&= (1-\frac{1}{G})\sum_{g=1}^{G} \left\{ \sum_{p=1}^{P} F^2_{gkp}\Phi^2_{gp} + 2\sum_{p=1}^{P}\sum_{p'>p} F_{gkp}F_{gkp'}\Phi_{gp}\Phi_{gp'} \right\} \\
&\quad -\frac{2}{G}\sum_{g=1}^{G} \left\{ \sum_{g'>g}\sum_{p=1}^{P} F^2_{gkp}\Phi_{gp}\Phi_{g'p} + \sum_{g'\neq g}\sum_{p=1}^{P}\sum_{p'>p} F_{gkp}F_{gkp'}\Phi_{gp}\Phi_{g'p'} \right\}
\end{aligned}
$$

APPENDIX B

Local QEANOVA

The local-QEANOVA analysis presented in section (3.b) is applied on the change variable $X(g,m,t) = Y(g,m,t) - Y(g,m,t_C)$ where $t_C \geq t_S$ and the regression model is fitted on Y (we have thus $Z = X$). The response function for the raw variable Y is assumed to be only locally a linear function of time. Following the notations of (23), the response function for Y can be written as: $\lambda(g,t) = \Lambda_{g1,C}r_1(t) + \Lambda_{g2,C}r_2(t) + \Lambda_{g1,E}r_3(t) + \Lambda_{g2,E}r_4(t)$ with $r_1(t) = 1$ and $r_2(t) = t - t_C$

for $t \in [t_C - \omega, t_C + \omega]$, $r_1(t) = r_2(t) = 0$ otherwise and with $r_3(t) = 1$ and $r_4(t) = t - t_E$ for $t \in [t_E - \omega, t_E + \omega]$, $r_3(t) = r_4(t) = 0$ otherwise.

Thus $L = 4$ and the functions $r_{g\ell}(t)$ in (6) are the same for all modeling chains g.

We consider $T \geq 2$ such that $T = 2T^\star$ where $T^\star$ is odd (i.e. $T \in \{2, 6, 10, \ldots\}$). Discretizing each $[t_C - \omega, t_C + \omega]$ and $[t_E - \omega, t_E + \omega]$ into $T^\star$ regular periods of length $dt = 2\omega/(T^\star - 1)$ and writing $n = (T^\star - 1)/2$, $\mathbb{R}_g$ in (7) is given by

$$\mathbb{R}_g = \mathbb{R} = \begin{pmatrix} 1 & -ndt & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & ndt & 0 & 0 \\ 0 & 0 & 1 & -ndt \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 1 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 1 & ndt \end{pmatrix} \begin{array}{ll} \text{line } 1 & (t = t_C - \omega) \\ & \\ \text{line } n+1 & (t = t_C) \\ & \\ \text{line } 2n+1 = T/2 & (t = t_C + \omega) \\ \text{line } 2n+2 & (t = t_E - \omega) \\ & \\ \text{line } 3n+2 & (t = t_E) \\ & \\ \text{line } 4n+2 = T & (t = t_E + \omega) \end{array}$$

Then $\mathbb{V} = (\mathbb{R}'\mathbb{R})^{-1}$ is diagonal with $V_{11} = V_{33} = 2/T$ and $V_{22} = V_{44} = 96/\{T(T^2 - 4)(dt)^2\}$.

Let consider t_k, the k^{th} time of the discretization with $k \geq T/2$, i.e. $t_k \in [t_E - \omega, t_E + \omega]$ (namely $t_k = t_E - \omega + 4\omega(k - T/2 - 1)/(T - 2)$ if $k > T/2$). Focusing on the uncertainty components for X at t_k, we have:

$$\begin{aligned} \varphi(g, t_k) &= \lambda(g, t_k) - \lambda(g, t_C) \\ &= (\Lambda_{g1,E} - \Lambda_{g1,C}) + (t_k - t_E)\Lambda_{g2,E} \\ &= F_{k1}\Phi_{g1} + F_{k2}\Phi_{g2} \end{aligned}$$

with $F_{k1} = 1$, $F_{k2} = t_k - t_E$, $\Phi_{g1} = \Lambda_{g1,E} - \Lambda_{g1,C}$ and $\Phi_{g2} = \Lambda_{g2,E}$.

Following (10), an unbiased estimator of Φ_{g2}^2 is $\hat{\Phi}_{g2}^2 - \hat{\sigma}_{\nu g}^2 M_g^{-1} V_{44}$ and as $(\Lambda_{g1,E} - \Lambda_{g1,C})^2 = \Lambda_{g1,E}^2 + \Lambda_{g1,C}^2 - 2\Lambda_{g1,E}\Lambda_{g1,C}$, an unbiased estimator of Φ_{g1}^2 is $(\hat{\Lambda}_{g1,E} - \hat{\Lambda}_{g1,C})^2 - \widehat{\sigma_{\nu_g}^2} M_g^{-1}(V_{11} + V_{33})$. Using equation (4), an unbiased estimator of the sample variance of α at

t_k is finally:

$$\begin{aligned}\widehat{s_{\alpha}^2}(t_k) &= s_{\hat{\alpha}}^2(t_k) - \left\{V_{11} + V_{33} + (t_k - t_E)^2 V_{44}\right\} \left(\frac{1}{G} \sum_{g=1}^{G} \frac{\widehat{\sigma_{\nu_g}^2}}{M_g} \right) \\ &= s_{\hat{\alpha}}^2(t_k) - \frac{4}{T} \left\{ 1 + \frac{24(t_k - t_E)^2}{(dt)^2(T^2-4)} \right\} \left(\frac{1}{G} \sum_{g=1}^{G} \frac{\widehat{\sigma_{\nu_g}^2}}{M_g} \right)\end{aligned}$$

A special case of using the local QEANOVA approach is when time $t_k = t_E$ at which we have the following unbiased estimator:

$$\widehat{s_{\alpha}^2}(t_k) = s_{\hat{\alpha}}^2(t_k) - \frac{4}{T} \left(\frac{1}{G} \sum_{g=1}^{G} \frac{\widehat{\sigma_{\nu_g}^2}}{M_g} \right).$$

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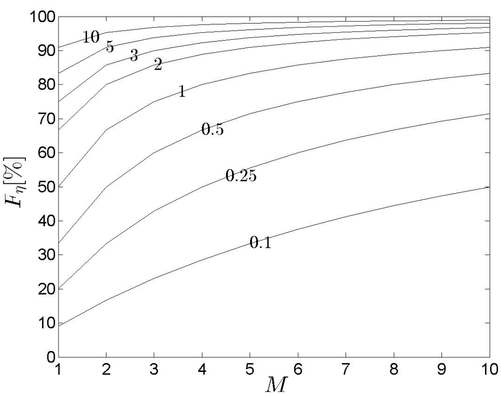
List of Figures



497 Figure 1. **Relative Bias (RB) expected for the estimate of model uncertainty variance (s^2_α) when obtained with a single time ANOVA approach**. RB values are given as a function of the number of members available for each modeling chain (M) and of the fraction of total variance explained by internal variability (F_η). For each (M, F_η) configuration, RB values are obtained from equation 26.

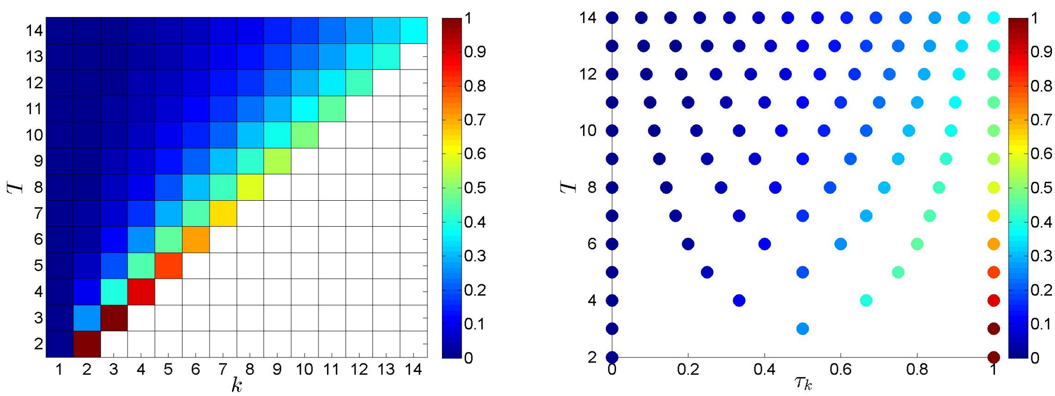


Figure 2. **Reduction factor $A(t_k,\mathscr{C})$ when the estimate is obtained with a QEANOVA approach when it applies over a single transient period with effects expressed as linear functions of time (equation 29**). Left : $A(t_k,\mathscr{C})$ values are given as a function of T, the total number of time steps of the transient climate period used for fitting the linear regression model of the climate response functions, and as a function of k, where $k \leq T$ is the time step corresponding to the time t_k for which the uncertainty component are considered. Right : $A(t_k,\mathscr{C})$ values are given as a function of T, and as a function of $\tau_k = \frac{t_k - t_K}{t_T - t_1}$. Figures correspond to the case where the time step of the reference period (t_K) corresponds to the first time step (t_1) of the transient time period used for the regression ($[t_1, t_T]$). In this case, τ_k is the proportion of time-steps separating t_k and the reference period t_C: $\tau(t_k) = 0$ if $t_k = t_1$ (and thus $k = 1$) and $\tau(t_k) = 1$ if $t_k = t_T$ (and thus $k = T$).