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Modeling and Control of a Rotating Flexible Spacecraft: A Port-Hamiltonian Approach

Saïd Aoues, Flávio Luiz Cardoso-Ribeiro, Denis Matignon, and Daniel Alazard

***Abstract*— In this brief, we develop a mathematical model of a flexible spacecraft system composed of a hub and two symmetrical beams using the port-Hamiltonian framework. This class of system has favorable properties, such as passivity for controller synthesis and stability analysis, where the global Hamiltonian plays the role of a Lyapunov function candidate. The spacecraft model is viewed as a power-conserving interconnection between an infinite (beam) and finite (hub) dimensional system. We show that the interconnection result has a port-Hamiltonian structure and is passive. The introduction of a nonlinear feedback term, which takes into account the beam's flexibility, is developed using the control by an interconnection approach. The closed-loop stability is proven; then, through explicitly solving the partial differential equations of the system, asymptotic stability is obtained. Finally, the experimental results are carried out to assess the validity of the proposed design methodology.**

***Index Terms*— Flexible spacecraft, Lyapunov stability, mixed port-Hamiltonian systems (PHSs), passivity-based control.**

I. Introduction

ATTITUDE maneuver of spacecraft with flexible appendages has received a significant focus in several research fields, particularly in robotics and aerospace domains. Appendages such as antennas and solar arrays attached to the central rigid body of the spacecraft are flexible. The dynamics of a flexible spacecraft is usually governed by mixed finite–infinite-dimensional equations of motion (EMs): a finite-dimensional equation [an ordinary differential equation (ODE)] for attitude maneuver, coupled with infinite-dimensional equation for the vibratory motion of flexible structures. The modeling step is a crucial issue in the design of control laws, since the performances are very sensitive to the error introduced by mathematical models.

In recent years, several studies related to the control of flexible space systems have been done; the principal challenges are: 1) a large order model; 2) weakly damped oscillating behavior of the flexible appendages; 3) inherent modeling errors due to finite-order approximation of the partial differential equation (PDE) describing the motion of the flexible parts; and 4) parameter uncertainties. Optimal controllers for linear and nonlinear models have been presented in [3] and [9]; control laws based on linearization and nonlinear inversion have been designed in [17]. The Lyapunov stability and the dissipativity theory have been used to design controllers for the maneuver of a flexible spacecraft [8]. The design of a composite adaptive control system using the backstepping technique has also been considered [11]. Recently, the discrete-time controller has been proposed using the passivity theory [2].

In the last decades [13], a new class of systems called *Port-Hamiltonian Systems* (PHSs) has been proposed. This class is very useful to model multiphysics phenomena and/or complex systems. The idea was originally developed for modeling finite-dimensional systems [14]. However, the framework has been extended to the case of infinite-dimensional systems [19]. The mixed finite–infinite dimensional PHSs are also treated in [12] and [15]. One of the most important properties of the PHS is that the interconnection of several PHSs, which results in a new PHS. This concept of interconnection is important from a control point of view, since implementing a control law or controlling a system is usually done with an external device, via external port variables [6]. Unlike other modeling techniques, the stability analysis can be easily carried out, using the total energy of the system as a Lyapunov function [18].

Our objective here is twofold: the first is to use the Hamiltonian framework to obtain the dynamic equations of flexible spacecraft, and the second is to investigate the control law based on the fundamental properties (i.e., passivity and interconnection) of this class of the system. The global model is given by a mixed finite–infinite dimensional PHS (m-PHS) viewed as the power-conserving interconnection between the central rigid body with flexible beams. Computing the energy balance equation, it is shown that the system is passive. The asymptotic stability at desired a goal position is proposed using the Proportional-Derivative (PD) control. Then, the control by interconnection is used to introduce the information about the vibrations of the beams through a nonlinear term in addition to the PD controller. Stability of the closed-loop system is proved when taking the total energy as a Lyapunov candidate function. The experimental results are carried out to assess the validity of the proposed approach. A main contribution of this brief is the design of the controller architecture with stability proofs directly on the infinite-dimensional port-Hamiltonian model, allowing a direct implementation on the experimental setup

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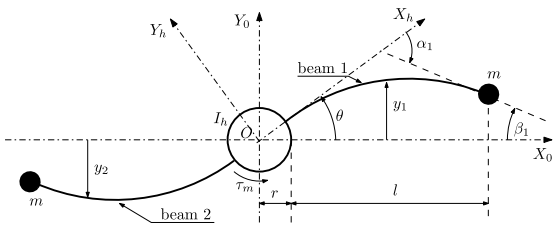

Fig. 1. Description of the spacecraft model.

and avoiding intermediate validations on a finite-dimensional simulation model.

This brief is organized as follows. In Section II, the spacecraft model with flexible appendages is briefly introduced. Section III deals with modeling of the system using the port-Hamiltonian framework. The control law and the stability result are given in Section IV. Experimental results show the validity of the proposed approach in Section V. Finally, some conclusions are outlined in Section VI.

II. Dynamics of a Flexible Spacecraft Systems

The configuration of a flexible spacecraft system considered in this brief is presented in Fig. 1. The model consists of a center hub body, two flexible beams, and two tip masses.

Such a model is commonly used to describe the rigid/flexible interactions governing the dynamic behavior of a spacecraft around a single degree of freedom: for instance in [16, Ch. 10] where the flexible beams are modeled by simple or multiple spring-mass systems and in [21], using a finite-dimensional model based on the assumed modes method or more recently in [5]. Nethertheless, the direct design of a controller on the infinite-dimensional model is not addressed in the space engineering context.

A. Notations and Assumptions

This test bed works in the horizontal plane $\mathcal{R}_1 = (0, X_0, Y_0)$ where $\mathcal{R}_1$ is an inertial frame, and it is composed of the following.

1) A rigid hub articulated with respect to the inertial frame by a pivot-joint around a vertical axis $(0, Z_0)$. $\mathcal{R}_h = (0, X_h, Y_h)$ is the hub body frame. The half-side and the inertia [around $(0, Z_0)$] of this hub are denoted r and I_h, respectively.
2) A torque motor, driving the hub in rotation around the vertical axis $(0, Z_0)$.
3) Two identical flexible beams (in the horizontal plane) cantilevered on the hub. The length of each beam is l, the quadratic moment of a section is I, and Young's modulus of the beam material is denoted E. The frame $\mathcal{R}_2 = (0, -X_0, -Y_0)$ is defined to describe the deflection of beam 2.
4) One local mass m fitted at the tip of each beam.

Let us denote the notations as follows.

1) τ_m $[N.m]$ is the driving torque.
2) θ $[rd]$ is the angular position of the hub.
3) $y_i(x,t)$ $[m]$ is the deflection of the beam i in the frame $\mathcal{R}_i$ at the point of abscissa $x \in [0, L]$ with length $L = r + l$.
4) $\beta_i(t) = (\partial y_i(x,t)/\partial x)|_{x=L} := \partial_x y_i(L,t)$ [rd] is the angular deviation (slope, with respect to equilibrium position) at the free-end of the beam i.
5) $\alpha_i = \beta_i - \theta$.

B. Dynamic Equations

From the Euler–Bernoulli equation [10] and the Euler principle applied to the hub, the EM and the boundary conditions (BCs) are then derived

$$\text{EM:} \begin{cases} \mathrm{EI}\partial^4_{x^4} y_i(x,t) + \rho\partial^2_{t^2} y_i(x,t) = 0, \quad i = 1,2 \\ I_h\ddot{\theta}(t) + D\dot{\theta}(t) = \tau_m(t) - \Sigma^2_{i=1}\tau_i(t) \end{cases} \tag{1a}$$

$$\underbrace{\text{BC:}}_{i=1,2} \begin{cases} \text{at } x = r : y_i(r,t) = r\theta(t); \quad \partial_x y_i(r,t) = \theta(t) \\ \mathrm{EI}\partial^2_{x^2} y_i(r,t) = -\tau_i(t) \\ \text{at } x = L : \mathrm{EI}\partial^2_{x^2} y_i(L,t) = 0 \\ \mathrm{EI}\partial^3_{x^3} y_i(L,t) = m\partial^2_{t^2} y_i(L,t). \end{cases} \tag{1b}$$

where $\tau_i(t)$ represents the bending moment at the root of the beams, and D is the damping constant of the driving mechanism.

Let us consider the change of a variable

$$y(x,t) = \frac{1}{2}(y_1(x,t) + y_2(x,t))$$
$$\bar{y}(x,t) = \frac{1}{2}(y_1(x,t) - y_2(x,t)).$$

Then, Model (1) can be reformulated as two independent subsystems Σ_1 and Σ_2 as follows:

$$\Sigma_1 \begin{cases} \mathrm{EI}\partial^4_{x^4} y(x,t) + \rho\partial^2_{t^2} y(x,t) = 0 & (2.a) \\ \text{at } x = r : y(r,t) = r\theta(t); \quad \partial_x y(r,t) = \theta(t) & (2.b) \\ \mathrm{EI}\partial^2_{x^2} y(r,t) = \frac{1}{2}(I_h\ddot{\theta}(t) + D\dot{\theta}(t) - \tau_m(t)) & (2.c) \\ \text{at } x = L : \mathrm{EI}\partial^3_{x^3} y(L,t) = 0 & (2.d) \\ \mathrm{EI}\partial^3_{x^3} y(L,t) = m\partial^2_{t^2} y(L,t) & (2.e) \end{cases} \tag{2}$$

$$\Sigma_2 \begin{cases} \mathrm{EI}\partial^4_{x^4} \bar{y}(x,t) + \rho\partial^2_{t^2} \bar{y}(x,t) = 0 \\ \text{at } x = r : \bar{y}(r,t) = 0; \quad \partial_x \bar{y}(r,t) = 0 \\ \text{at } x = L : \mathrm{EI}\partial^2_{x^2} \bar{y}(L,t) = 0 \\ \mathrm{EI}\partial^3_{x^3} \bar{y}(L,t) = m\partial^2_{t^2} \bar{y}(L,t). \end{cases} \tag{3}$$

It is clear that Σ_1 and Σ_2 are completely independent and that Σ_2 is uncontrollable by the control signal τ_m. Σ_1 and Σ_2 are the EMs of the antisymmetric and symmetric [with respect to $(0, Y_1)$-axis] deflections of the two beams. The symmetric deflection (Σ_2) is uncontrollable (see also [1]) and will not be considered in the sequel. Thus, assuming that $y_1(x,t) = y_2(x,t) = y(x,t)$ and due to the antisymmetry of the shear forces at the two beam roots, the EM of Σ_1 (2) can be completed by the BC

$$\partial^3_{x^3} y(r,t) = 0 \tag{4}$$

and the sum of the two beam bending moments is denoted τ_r

$$\tau_r(t) = \tau_1(t) + \tau_2(t) = -2\mathrm{EI}\partial^2_{x^2} y(r,t). \tag{5}$$

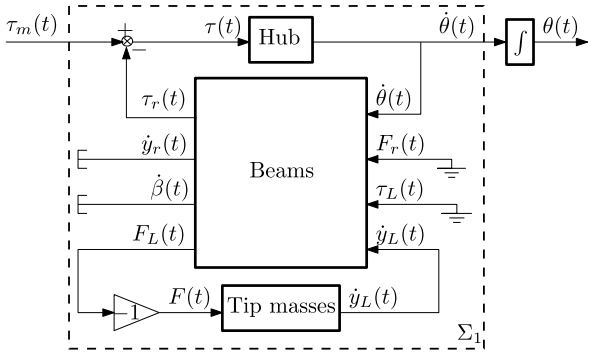

Fig. 2. Feedback interconnection between subsystems.

III. Port-Hamiltonian Model

The model of the spacecraft is composed of three subsystems: the hub (subscript h), the beams (subscript b, reduced to their antisymmetric deflections), and the tip masses (subscript m) in feedback interconnection according to the block-diagram shown in Fig. 2. Each subsystem i ($i = h, b, m$) is now characterized by its port-Hamiltonian model with its Hamiltonian H_i, its input u_i, and its conjugated output y_i. The time-derivative $\dot H_i$ is also computed to highlight the passivity property of the subsystems. It is important to note that most of the works existing in the literature about the modeling and control of flexible spacecraft use the mathematical model given by PDE equations (1) and (2) and/or the approximate models (ODE). One objective of this brief is to use the *port-Hamiltonian modeling*, which allows applying energy-based control techniques in a quite straightforward way. Moreover, the stability analysis of multiphysics-coupled systems is made easier.

Assumption 1: Since this brief is oriented to a concrete experimental application, we assume that the equations are well-posed.

A. Hub

1) *Input/Output:* $u_h = \tau$ is the total torque applied on the hub, and $y_h = \dot\theta$ is the angular velocity.
2) *ODE Model:* It shows that
$$I_h\ddot\theta(t) + D\dot\theta = \tau(t). \tag{6}$$
3) *PHS Model:* With the angular momentum $z_h(t) := I_h\dot\theta(t)$ as a state variable, we get
$$H_h(z_h) := \frac{1}{2}I_h\dot\theta^2 = \frac{1}{2I_h}z_h^2 \tag{7}$$
$$\dot z_h = -D\frac{d}{dz_h}H_h + 1\tau \quad \text{and } \dot\theta = 1\frac{d}{dz_h}H_h. \tag{8}$$
4) *Passivity:*It shows that
$$\dot H_h(z_h(t)) = \frac{1}{I_h}z_h\dot z_h = -D\dot\theta^2 + \dot\theta\tau \le y_h u_h. \tag{9}$$

B. Beams

1) *Input/Output:* The boundary control $u_b = [\dot\theta, F_r, \tau_L, \dot y_L]^T$, and the boundary measurement $y_b = [\tau_r, \dot y_r, \dot\beta, F_L]^T$ where y_r and y_l are the deflections at the two tips ($x = r$ and $x = L$) of a beam, and F_r, τ_r, F_L, and τ_L are the shear forces and bending torques applied on the two tips of the beams.
2) *PDE Model:* Given by (2) and (4).
3) *PHS Model:* Assuming that $y_1(x,t) = y_2(x,t) = y(x,t)$, the Hamiltonian H_b, defined as the sum of the kinetic and potential energies of the two beams, reads
$$H_b = \rho\int_r^L \partial_t y(x,t)^2 dx + \mathrm{EI}\int_r^L \partial^2_{x^2} y(x,t)^2 dx. \tag{10}$$
Choosing the state variables as follows:
$$z_1(x,t) := \rho\partial_t y(x,t) \quad \text{and} \quad z_2(x,t) : \partial^2_{x^2} y(x,t) \tag{11}$$
$$H_b(z_1, z_2) = \int_r^L \frac{1}{\rho}z_1(x,t)^2 dx + \int_r^L \mathrm{EI}z_2(x,t)^2 dx \tag{12}$$
and the variational derivative of the energy $\delta_z H_b$ with respect to the state variables (11) is given by
$$\delta_z H_b := \begin{bmatrix}\delta_{z_1}H_b\\ \delta_{z_2}H_b\end{bmatrix} = \begin{bmatrix}\frac{2}{\rho}z_1\\ 2\mathrm{EI}z_2\end{bmatrix}. \tag{13}$$
Hence, the PHS model is described by $\dot z = J\delta_z H_b$, that is
$$\begin{bmatrix}\dot z_1\\ \dot z_2\end{bmatrix} = \begin{bmatrix}0 & -\frac{1}{2}\partial^2_x\\ \frac{1}{2}\partial^2_x & 0\end{bmatrix}\begin{bmatrix}\delta_{z_1}H_b\\ \delta_{z_2}H_b\end{bmatrix} \tag{14}$$
$$u_b = \begin{bmatrix}\dot\theta(t) = \partial^2_{xt}y(r,t)\\ F_r(t) = 2\mathrm{EI}\partial^3_{x^3}y(r,t)\\ \tau_L(t) = 2\mathrm{EI}\partial^2_{x^2}y(L,t)\\ \dot y_L(t) = \partial_t y(L,t)\end{bmatrix} \tag{15}$$
$$y_b = \begin{bmatrix}\tau_r(t) = -2\mathrm{EI}\partial^2_{x^2}y(r,t)\\ \dot y_r(t) = \partial_t y(r,t)\\ \dot\beta(t) = \partial^2_{xt}y(L,t)\\ F_L(t) = -2\mathrm{EI}\partial^3_{x^3}y(L,t)\end{bmatrix} \tag{16}$$
where u_b and y_b are the boundary ports (see [7, Ch. 11] for the general setting).
4) *Passivity:* Indeed, both these ports prove conjugated with respect to the Hamiltonian functional H_b, in so far as
$$\dot H_b(z_1(t), z_2(t)) = \tau_r\dot\theta + \dot y_r F_r + \dot\beta\tau_L + F_L\dot y_L = y_b^T u_b \tag{17}$$
see [4] for a careful computation of this energy balance. Note that, in our case, (4) reads $F_r = 0$, and (2.d) reads $\tau_L = 0$; hence, the conjugated ports $\dot y_r$ and $\dot\beta$ are left free.

C. Tip Masses

1) *Input/Output:* $u_m = F$ is the force (doubled) applied on the two masses, and $y_m = \dot y_L$ is the velocity.
2) *ODE Model:* It shows that
$$2m\ddot y_L(t) = F(t). \tag{18}$$

3) *PHS Model:* With the linear momentum $z_m(t) := 2m\dot{y}_L(t)$ as a state variable, we get

$$H_m(z_m) := m\dot{y}_L(t)^2 = \frac{1}{4m}z_m(t)^2 \tag{19}$$

$$\dot{z}_m = 0\frac{d}{dz_m}H_m + 1F \quad \text{and } \dot{y}_L = 1\frac{d}{dz_m}H_m. \tag{20}$$

4) *Passivity:* It shows that

$$\dot{H}_m(z_m(t)) = \frac{1}{2m}z_m\dot{z}_m = \dot{y}_L F = y_m u_m. \tag{21}$$

D. PHS Model of the Whole System

Since all three the subsystems are passive and they are interconnected with negative feedbacks, the whole system Σ_1 is passive [12].

Indeed, from Fig. 2, the interconnection equations are

$$\tau = -\tau_r + \tau_m \tag{22}$$

$$F = -F_L. \tag{23}$$

Let us define the total Hamiltonian

$$H(X) := H_h(z_h) + H_b(z_1, z_2) + H_m(z_m).$$

The passivity of the whole mixed PHS (m-PHS) Σ_1 with Hamiltonian H, variables $X := [z_1, z_2, z_h, z_m]^T$, input $\tau_m(t)$, and conjugated output $\dot{\theta}$ states that

$$\dot{H}(X(t)) = -D\dot{\theta}^2 + \dot{\theta}\tau_m \leq \dot{\theta}\tau_m. \tag{24}$$

Now our objective is to derive the controller which will drive the system to the desired equilibrium. With this in mind, we adopt an approach based on the passivity theory. We choose a desired energy H_d and we compute the controller in order to have a minimum of H_d at the desired equilibrium of the closed-loop system. This subject will be addressed in the sequel.

IV. Controller Design Using Passivity

Based upon the m-PHS dynamics, for a flexible spacecraft of Section III-D, the control law design is developed in this section. The control objective is to drive the central body to a desired angular position θ^* based on the passivity theory. In Section IV-A, we show that a pure PD controller can stabilize the m-PHS to a given equilibrium point. Generally, the system performance is not satisfactory, because the elastic vibrations cannot be effectively suppressed. From the Hamiltonian framework, we show in Section IV-B that a nonlinear term of the elastic vibrations of each appendage can be easily introduced to the PD controller via *a control by interconnection* (i.e., interconnection of several PHSs is also a PHS) in order to suppress as far as possible the residual vibration of each appendage. In all the cases, the stability and the asymptotic stability are proved.

A. Proportional-Derivative Controller

From an engineering point of view, the proportional-derivative controller is very easy to implement, since only measurements of θ and $\dot{\theta}$ are required. In addition, this controller is independent of the system parameters and thus possesses stability robustness to system parameters uncertainties, thanks to Theorem 2.

Theorem 2: The system Σ_1 in a closed loop with the control law

$$\tau_m = -k_d\dot{\theta} - k_p(\theta - \theta^*) \tag{25}$$

where ($k_p > 0$, $k_d > 0$) has a unique equilibrium solution defined by

$$y_e(x, t) = \theta^* x. \tag{26}$$

This equilibrium is globally asymptotically stable.

Proof: Obviously $\dot{\theta} = 0$ (the input of an integrator on the output of Σ_1 in Fig. 2 is null) at the equilibrium. Then, considering the Euler–Bernoulli equation (2.a) with the BCs (2.b), (2.d), and (4) and the equilibrium condition

$$\partial^2_{xt}y(r, t) = \dot{\theta} = 0$$

one can show (see the Appendix) that $y(x, t) = \theta_e x$ where θ_e is the equilibrium angular position of the hub.

Then, $\tau_r(t) = \partial^2_{x^2}y_e(r, t) = 0$ and $\tau_m \overset{(2).c}{=} 0 = -k_p(\theta_e - \theta^*)$. Thus, $\theta_e = \theta^*$ and $y_e(x, t) = \theta^* x$.

With $\mathcal{X} := (X, \theta)$, let us consider the Lyapunov function

$$H_d(\mathcal{X}) = H(X) + \frac{1}{2}k_p(\theta - \theta^*)^2 \tag{27}$$

$$= \frac{1}{2}I_h\partial^2_{xt}y(r, t)^2 + \rho\int_r^L \partial_t y(x, t)^2 dx + \mathrm{EI}\int_r^L \partial^2_{x^2}y(x, t)^2 dx + m\dot{y}_L^2 + \frac{1}{2}k_p(\partial_x y(r, t) - \theta^*)^2 \tag{28}$$

where H_d corresponds to the desired Hamiltonian such that H_d has the minimum at the angular position reference input θ^*. H_d enjoys the following properties.

1) $H_d(\mathcal{X}) > 0 \ \forall \mathcal{X} \neq \mathcal{X}_e$.
2) $H_d(\mathcal{X}) = 0$ if $\mathcal{X} = \mathcal{X}_e$.
3) $H_d(\mathcal{X}) \to \infty$ if $\|\mathcal{X}\| \to \infty$.
4) $\dot{H}_d(\mathcal{X}(t)) \leq 0$.

Indeed

$$\dot{H}_d(\mathcal{X}(t)) = \dot{H}(X(t)) + k_p\dot{\theta}(\theta(t) - \theta^*) \tag{29}$$

$$\leq \dot{\theta}\tau_m + k_p\dot{\theta}(\theta(t) - \theta^*) \tag{30}$$

$$\leq -k_d\dot{\theta}^2 \tag{31}$$

$$\leq 0. \tag{32}$$

Thus applying LaSalle's invariance principle [7, Ch. 8 and 9], one can conclude that the equilibrium solution is globally asymptotically stable. From a theoretical point of view, the precompactness of the trajectories of the infinite-dimensional dynamical system should be verified first, before applying LaSalles invariance principle more formally; in our

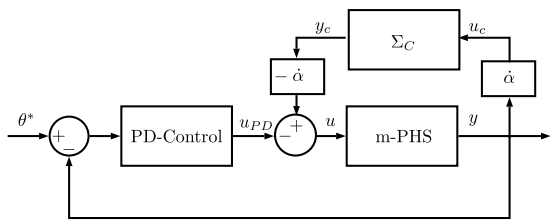

Fig. 3. Schematic of the control m-PHS.

geometric configuration though, the spatial domain is bounded, and the Rellich theorem on compactness does apply. □

Remark 3:

1) This property is independent of I_h, D, and m. Indeed, conditions (2.c) and (2.e) are not used.
2) The BC (4) is determinant in this proof and is due to the fact that Σ_1 considers only the antisymmetric motion of the beams (Σ_2 is uncontrollable). It does not hold for a spacecraft with a single beam.
3) The equilibrium solution $y_e(x,t) = \theta^* x$ corresponds to the rigid motion of the system and guarantees a response without any steady-state error.

B. Control by Interconnection

The controller (25) is able to stabilize the system Σ_1, but the vibrations of the flexible beam are not taken into account. Unlike the works that exist in the literature [1], the elastic vibrations feedback can be easily added to (25) to improve the performance of the flexible spacecraft. Wang *et al.* [20] proposed adding a nonlinear term to the classical proportional-derivative controller. This nonlinear term is given by the integral of the product of the hub angular rate $\dot{\theta}(t)$ times the angular rate of the deflection at the tip of the beam $\dot{\alpha}(t)$, given by: $k_c\dot{\alpha}(t)\int_0^t \dot{\theta}(s)\dot{\alpha}(s)ds$. This term can be seen as a controller Σ_C added to the previous controller (see Fig. 3).

It can then be shown that such a nonlinear controller can be represented by a port-Hamiltonian subsystem, leading to a more straightforward way to derive stability proofs. Indeed, the port-Hamiltonian representation of Σ_C reads

$$\Sigma_C : \begin{cases} \dot{\xi} = J_c \dfrac{\partial H_c}{\partial \xi} + G_c u_c \\ y_c = G_c^T \dfrac{\partial H_c}{\partial \xi} \end{cases} \tag{33}$$

where $J_c = 0$, and $G_c = G_c^T = 1$, and the energy associated to the controller $H_c(\xi)$ is

$$H_c(\xi) = \frac{1}{2}k_c\xi^2, \quad k_c > 0. \tag{34}$$

Note that solving (33) leads to

$$\xi = \int_0^t u_c(s)ds, \quad \text{and } y_c = k_c\int_0^t u_c(s)ds. \tag{35}$$

We can interconnect the flexible spacecraft in Section III-D and the controller (33) via the following power-conserving

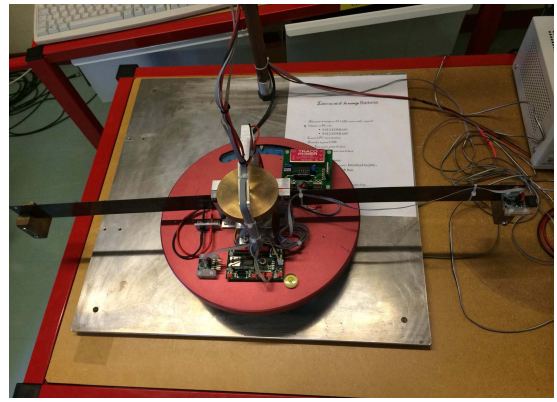

Fig. 4. Picture of a one degree-of-freedom flexible spacecraft.

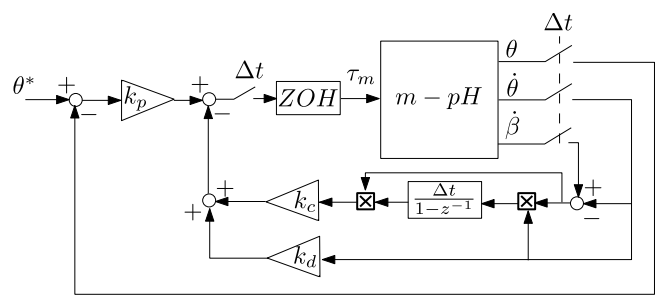

Fig. 5. Discrete-time implementation (ZOH is a zero-order hold).

feedback interconnection with some extra input u_{PD} given in (25):

$$\begin{bmatrix} u \\ u_c \end{bmatrix} = \begin{bmatrix} 0 & -\dot{\alpha} \\ \dot{\alpha} & 0 \end{bmatrix} \begin{bmatrix} y \\ y_c \end{bmatrix} + \begin{bmatrix} u_{\text{PD}} \\ 0 \end{bmatrix}. \tag{36}$$

The closed-loop system is obtained from a power-conserving interconnection of two PHSs; therefore it is still a PHS with total energy $H_{\text{tot}}(\mathbf{X})$ given by the sum of the subsystems energies defined by

$$H_{\text{tot}}(\mathbf{X}) := H(X) + \frac{1}{2}k_p(\theta - \theta^*)^2 + \frac{1}{2}k_c\xi^2 \tag{37}$$

with $\mathbf{X} := (X, \theta, \xi)$. From the interconnection of (25), (35), and (36), the nonlinear controller is obtained as

$$\begin{aligned} u &= -\dot{\alpha}y_c + u_{\text{PD}} \\ &= -k_c\dot{\alpha}\int_0^t \dot{\theta}(s)\dot{\alpha}(s)ds - k_d\dot{\theta} - k_p(\theta - \theta^*). \end{aligned} \tag{38}$$

This controller explicitly introduces the information about a flexible beam and the hub velocity, which has some effects on the reduction of elastic vibrations.

Theorem 4: The nonlinear controller in (38) can guarantee the closed-loop stability of the system [Section III-D] interconnected with Σ_C via feedback interconnection (36).

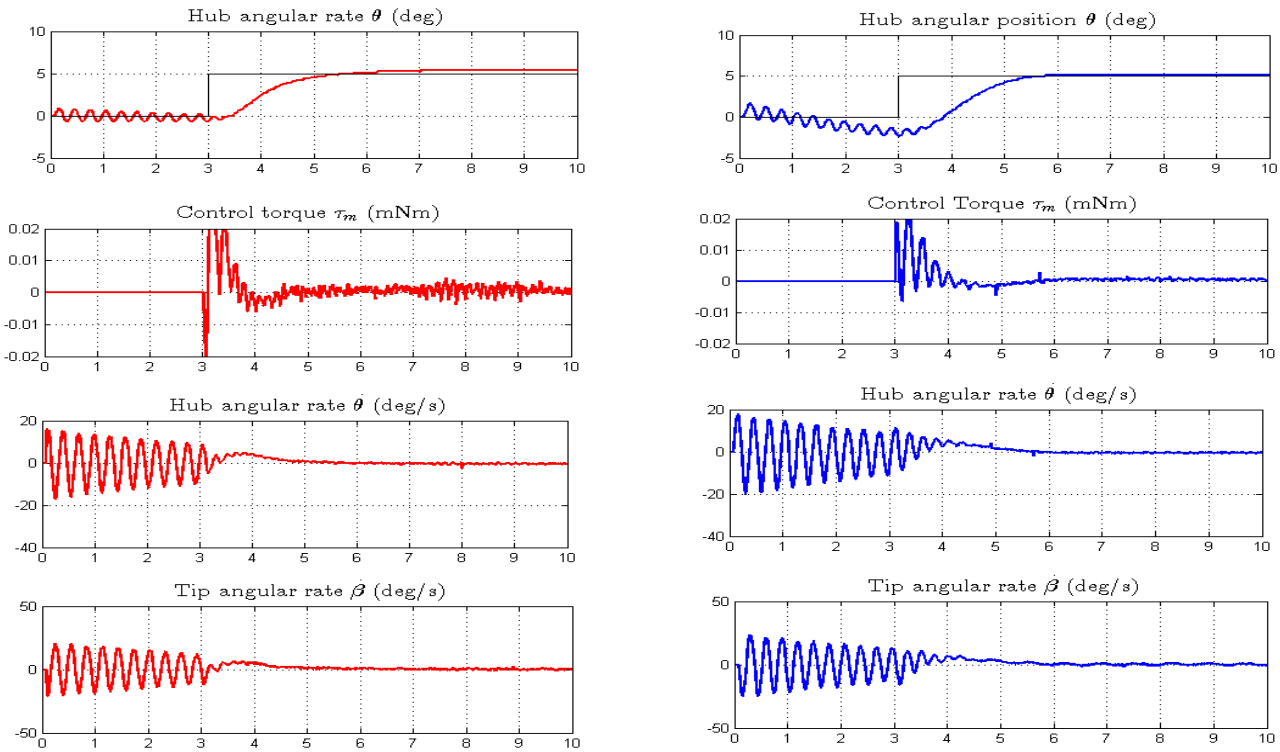

Fig. 6. Time responses with different controllers. Left column: nonlinear controller (38) (in red). Right column: PD controller (25) (in blue). In each column, we record the hub-angular position first, the applied control torque second, and the responses of the hub-angular rate and the tip-angular rate, respectively.

Proof: The time derivative of $H_{\text{tot}}(\mathbf{X})$ is

$$\begin{aligned}\dot{H}_{\text{tot}}(\mathbf{X}(t)) &= -D\dot{\theta}^2 + \dot{\theta}u + \dot{\theta}[k_p(\theta-\theta^*)] \\ &\quad + k_c\dot{\theta}\dot{\alpha}\left[\int_0^t \dot{\theta}(s)\dot{\alpha}(s)ds\right] \\ &= -D\dot{\theta}^2 \\ &\quad + \dot{\theta}\left[-k_c\dot{\alpha}\int_0^t \dot{\theta}(s)\dot{\alpha}(s)ds - k_d\dot{\theta} - k_p(\theta-\theta^*)\right] \\ &\quad + \dot{\theta}[k_p(\theta-\theta^*)] + k_c\dot{\theta}\dot{\alpha}\left[\int_0^t \dot{\theta}(s)\dot{\alpha}(s)ds\right] \\ &= -\underbrace{(D+k_d)}_{D_d>0}\dot{\theta}^2 \leq 0. \end{aligned} \tag{39}$$

Then, the system is stable. □

The asymptotic stability can be proved using the same lines as for Theorem 2. At the equilibrium point, when $\dot{H}_{\text{tot}} \equiv 0$ from (39), we have $\dot{\theta} \equiv 0$, and the nonlinear controller (38) becomes $u = -k_p(\theta_e - \theta^*)$.

V. Experimental Results

In this section, we give some experimental results to support the theoretical control method proposed in Section IV. The experimental test bed is depicted in Fig. 4 and corresponds to the scheme presented in Fig. 1 with the parametric data given in Table I. The sizing and the instrumentation of this experimental setup were chosen in order to increase the rigid/flexible interactions and to be relevant for the validation of new control strategies.

This test bed is instrumented with the following.

1) A reaction wheel to apply a torque τ_m on the hub.

TABLE I

Numerical Values of the Parameters of a Flexible Spacecraft ($\rho = 0.2$ Kg/m)

E	I_h	l	m	r	I
$200.1N/m^2$	$0.015Kgm^2$	$0.286m$	$0.3Kg$	$5cm$	$0.874mm^4$

2) An optical encoder to measure the angular position θ of the hub with respect to the inertial frame.
3) A tachymeter to measure the angular rate $\dot{\theta}(t)$ of the hub with respect to the inertial frame.
4) A gyrometer to measure the inertial angular rate $\dot{\beta}(t) = \dot{\theta}(t) + \dot{\alpha}(t)$ at one tip mass.

The sampling period of the on-board computer is $\Delta t = 10$ ms. The controller (38) to be implemented involves three gains k_p, k_d, and k_c. In this application: k_p and k_d are tuned on the system assumed rigid to assign the rigid mode to a second order with a frequency of $\omega = 1$ rd/s and damping ratio of 0.7, that is: $k_p = I_t\omega^2$ and $k_d = 1.4I_t\omega$ ($I_t = I_h + 2m(l+r)^2$). The last gain $k_c = 0.324$ corresponding to the nonlinear integral term in (38) is chosen as great as possible considering limitation of the real-time system (phase lags due to discrete-time sampling and actuator bandwidth). Finally, the discrete-time controller is implemented according to the sketch of Fig. 5.

The experimental results are obtained in the following conditions.

1) At time $t = 0$, the hand-made disturbance is applied on the system to excite the flexible mode. The open-loop response is recorded during 3 s.

2) At time $t = 3$ s, the feedback loop is closed on the controller with a reference input $\theta^* = 5°$.

Two controllers are analyzed: the PD controller (25) and the nonlinear controller (38). Fig. 6 shows the hub-angular position θ, the control torque τ_m, the hub-angular rate $\dot{\theta}$, and the tip-angular rate $\dot{\beta}$. Comparing the different responses, we can conclude the following.

1) The open-loop responses (from 0 to 3 s) exhibit the very low damping behavior of the flexible mode.
2) Both controllers allow the reference input θ^* to be tracked efficiently.
3) But, from the tip-angular rate response and during the transient response (between 3 and 5 s), it can be observed that the nonlinear controller reduces all the elastic vibrations of the flexible spacecraft by comparing with a simple PD controller.

The nonlinear controller part of the controller feedback $\int \dot{\theta}(t)\dot{\alpha}(t)dt$ allows to attenuate the oscillations, which are present on the response with the PD controller in the steady state.

VI. Conclusion

In this brief, an m-PHS of a flexible spacecraft system is presented, based on the power-conserving interconnection property. Then, the passivity theory is used to develop the nonlinear controller, which takes into consideration and/or suppresses the beam's flexibility. The asymptotic stability result is proved using the total Hamiltonian as a Lyapunov function candidate. The experimental results are carried out to assess the validity of the proposed design methodology.

Appendix

Equilibrium Conditions

The change of variables

$$v(x,t) := y(x+r,t)$$

in the Euler–Bernoulli equation (2.a), the BCs (2.b), (2.d), and (4) and the equilibrium condition

$$\partial^2_{xt} y(r,t) = \dot{\theta} = 0 \Leftrightarrow \partial_x y(r,t) = cst = \theta_e$$

lead to the PDE problem

$$\rho \partial^2_{t^2} v(x,t) + \mathrm{EI}\partial^4_{x^4} v(x,t) = 0 \tag{40}$$

with the following four unusual BCs:

$$v(0,t) = \theta_e r \tag{41}$$

$$\partial_x v(0,t) = \theta_e \tag{42}$$

$$\partial^3_{x^3} v(0,t) = 0 \tag{43}$$

$$\partial^2_{x^2} v(l,t) = 0 \quad (l = L - r). \tag{44}$$

Applying the method of separation of variables, the elementary solutions are looked for

$$v(x,t) = \phi(x)\eta(t)$$

with

$$\ddot{\eta} + \omega^2 \eta = 0 \Rightarrow \eta(t) = E e^{j\omega t} \tag{45}$$

$$\phi''''(x) - \lambda^4 \phi(x) = 0 \quad \text{with } \lambda^4 = \omega^2 \frac{\rho}{\mathrm{EI}} \tag{46}$$

and with two kinds of solutions (or any linear combination of these).

1) The *rigid* solution

$$\text{if } \omega = 0, \quad \text{then } \phi_r(x) = A_0 + A_1 x + A_2 x^2 + A_3 x^3.$$

2) The *flexible* solutions

$$\text{if } \omega \neq 0, \quad \text{then } \phi_f(x) = A\cos\lambda x + B\sin\lambda x + C\cosh\lambda x + D\sinh\lambda x.$$

The equilibrium condition reads

$$\partial^2_{xt} v(0,t) = \phi'(0)\dot{\eta}(t) = \phi'(0) E j\omega e^{j\omega t} = 0.$$

with the following two cases.

1) $\omega = 0$, then the BCs (41)–(44) lead to

$$A_0 = \theta_e r, \quad A_1 = \theta_e, \quad A_3 = 0, \quad A_2 = 0$$
$$\Rightarrow \phi_r(x) = \theta_e(x + r).$$

2) $\omega \neq 0$, then $\phi'(0) = \theta_e = 0$ and the BCs (41)–(44) lead to

$$\begin{aligned} A + C &= 0 \\ \lambda(B + D) &= 0 \\ \lambda^3(-B + D) &= 0 \\ \lambda^2(-A\cos\lambda l + C\cosh\lambda l) &= 0. \end{aligned}$$

Since $\cos\lambda l + \cosh\lambda l = 0$ has no real solution

$$A = B = C = D = 0 \quad \text{and } \phi_f(x) = 0.$$

Thus, the only solution is: $v(x,t) = \theta_e(x + r)$, that is

$$y(x,t) = \theta_e x.$$

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