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To cite this version:
Antoine Falaize, Erwan Liberge, Aziz Hamdouni. A MULTIPHASE APPROACH TO THE CONSTRUCTION OF POD-ROM FOR FLOWS INDUCED BY ROTATING SOLIDS. Workshop CSMA Junior, Mar 2018, Gif-sur-Yvette, France. hal-01736872

HAL Id: hal-01736872
https://hal.archives-ouvertes.fr/hal-01736872
Submitted on 18 Mar 2018

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A MULTIPHASE APPROACH TO THE CONSTRUCTION OF POD-ROM FOR FLOWS INDUCED BY ROTATING SOLIDS
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INTRODUCTION

- **Objective**: Construct reduced order models (ROM) for the simulation of turbomachinery with imposed rotation velocity by proper orthogonal decomposition (POD).
- **Difficulty**: The POD yields a spatial basis from temporal correlations (here of the velocity).
- **Approach**:
  1. Extend the Navier-Stokes equations to the solid (rotor) domain by the multiphase approach. The body velocity is enforced via distributed Lagrange multipliers.
  2. Build a single POD basis for the multiphase velocity and project the governing equations.

1. MULTIPHASE APPROACH

**Level-set**

\[ \chi(x,t) = \begin{cases} +d(x,\Gamma_1(t)) & \text{if } x \in \Omega_0(t) \cup \Gamma_1(t), \\ -d(x,\Gamma_1(t)) & \text{if } x \in \Omega_1(t) \end{cases} \]

Smoothed Heaviside

\[ h_\epsilon(x) = \frac{1}{2} \left( 1 + \tanh \left( \frac{x}{\epsilon} \right) \right). \]

Membership function for \( \Omega_0(t) \)

\[ 1_{\Omega_0}(x) = h_\epsilon(\chi(x,t)). \]

**Multiphase quantities**

- Velocity field over \( \Omega = \Omega_0(t) \cup \Omega_1(t) \):
  \[ u(x,t) = \frac{1}{\Omega_0(t)} (u_0(x,t) + (I - 1_{\Omega_0}(x)) u_1(x,t)). \]

Material properties (density and viscosity):

\[ \rho(x,t) = 1_{\Omega_0}(x) \rho_0 + (I - 1_{\Omega_0}(x)) \rho_1 \]

\[ \nu(x,t) = 1_{\Omega_0}(x) \nu_0 + (I - 1_{\Omega_0}(x)) \nu_1. \]

2. FULL ORDER MODEL [1]

Denoting \( u_\omega \) the rotation velocity, \( \Lambda \) the Lagrange multiplier and \( \mu \) the test function associated with the rotation constraint, the weak form of the coupled problem is:

\[
0 = \int_{\Omega} \left( \frac{\partial u}{\partial t} + \nabla \cdot u \right) \cdot v dx + \int_{\Gamma} f \cdot v dx \\
+ \int_{\Omega} \frac{2\nu}{\rho} \nabla (D(u) : D(v)) dx \\
- \int_{\Gamma} \mu \cdot (u - u_\omega) \cdot v dx + \int_{\Omega_0(t)} \lambda \cdot (\mu \cdot v) dx
\]

with an appropriate standard functional setting.

3. STANDARD POD-ROM [2]

- **Mean field** \( \overline{u}(x,t) = \frac{1}{N_T} \sum_{n=1}^{N_T} u(x,t_n) \)
- **Fluctuating field** \( \tilde{u}(x,t) = u(x,t) - \overline{u}(x) \)
- **Data matrix** \( U = \{ \Phi(x,t_n) \}_{n=1}^{N_T} \)

**POD basis**: left singular vectors of \( U \)

\[ \Phi = \left( \phi_i(x) \right)_{1 \leq i \leq N_\Phi}, \quad N_\Phi << N_T << N_X. \]

Decomposition

\[ u(x,t) \simeq \overline{u}(x) + \sum_{i=1}^{N_\Phi} \phi_i(x) \alpha_i(t). \]

4. PROPOSED POD-ROM

\[ \Rightarrow \text{POD of the membership function} \]

**POD basis** \( \Lambda = \{ \Lambda_i(x) \}_{1 \leq i \leq N_\Lambda}, N_\Lambda << N_X. \)

**Decomposition**

\[ 1_{\Omega_0(t)}(x) \simeq \sum_{i=1}^{N_\Lambda} \Lambda_i(x) \gamma_i(t). \]

**Periodicity**

Coefficients \( \gamma_i(t) \rightarrow \tilde{\gamma}_i(t) \) determined a priori.

**Insertion in the standard POD-ROM**

\[ \tilde{A}(t) \frac{d\tilde{\alpha}_i(t)}{dt} = \tilde{B}(t) \cdot \tilde{\alpha}_i(t) + \tilde{C}(t) \cdot \alpha_i(t) + \tilde{F}(t), \]

\[ \Rightarrow \text{Matrices evaluation at each timestep (cost } \sim N_\Lambda). \]

\[ A_i^{(a)}(\cdot) = \int \tilde{a}_i(\cdot,\cdot) \tilde{a}_j(\cdot,\cdot) dz, \]

\[ B_i^{(a)}(\cdot) = \int \tilde{a}_i(\cdot,\cdot) \tilde{b}_j(\cdot,\cdot) dz, \]

\[ C_i^{(a)}(\cdot) = \int \tilde{a}_i(\cdot,\cdot) \tilde{c}_j(\cdot,\cdot) dz, \]

\[ F_i(\cdot) = \int \tilde{f}_i(\cdot,\cdot) dz. \]

5. RESULTS

**Φ**: POD modes for the velocity

**A**: POD modes for the membership function

- **Coefficients** \( \alpha_i(t) \):
  - Full order Vs. standard POD-ROM.
  - Full order Vs. proposed POD-ROM.

CONCLUSION

- **Contributions**
  - Efficient reconstruction of the velocity in both the fluid and solid domains, while substantially reducing the computational cost.
  - Very general: Any simulation code for the incompressible Navier-Stokes eq. can be used to generate the data \( (u(x,t_n))_{n=1}^{N_T} \).

- **Perspectives**
  - Cope with the reconstruction of the velocity in the solid domain at each iteration by rewriting the governing equations for a rotating subdomain.
  - Interpolate between the POD-ROMs over the Grassmann manifold (see e.g. [3]).

REFERENCES