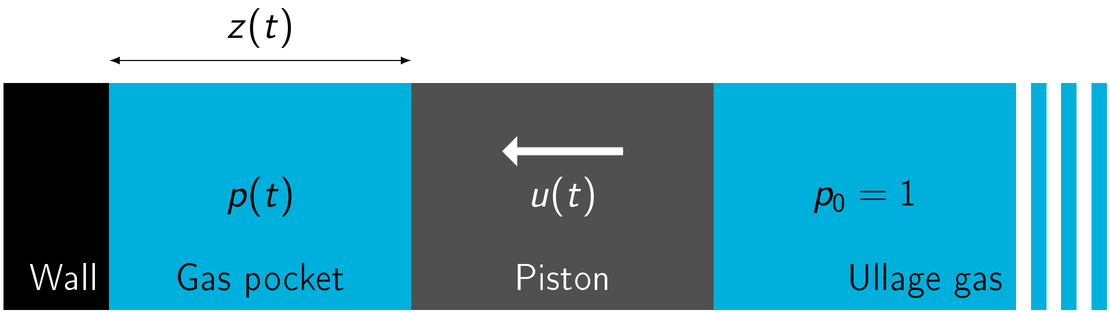
On the dimensionless parameters governing some extended Bagnold models

Matthieu Ancellin
with **Laurent Brosset** and **Jean-Michel Ghidaglia**

International Workshop on [...] Multiphase Flows, Cachan
16th October 2017

Bagnold model



- **Dimensionless Bagnold ODE model**

$$\frac{\mathrm{d}^2 z}{\mathrm{d}t^2} = p - 1, \qquad p = z^{-\gamma},$$

$$z(0) = 1, \qquad \left.\frac{\mathrm{d}z}{\mathrm{d}t}\right|_{t=0} = \pm\sqrt{S}.$$

Dimensionless parameters

Impact number S

Violence of the impact.

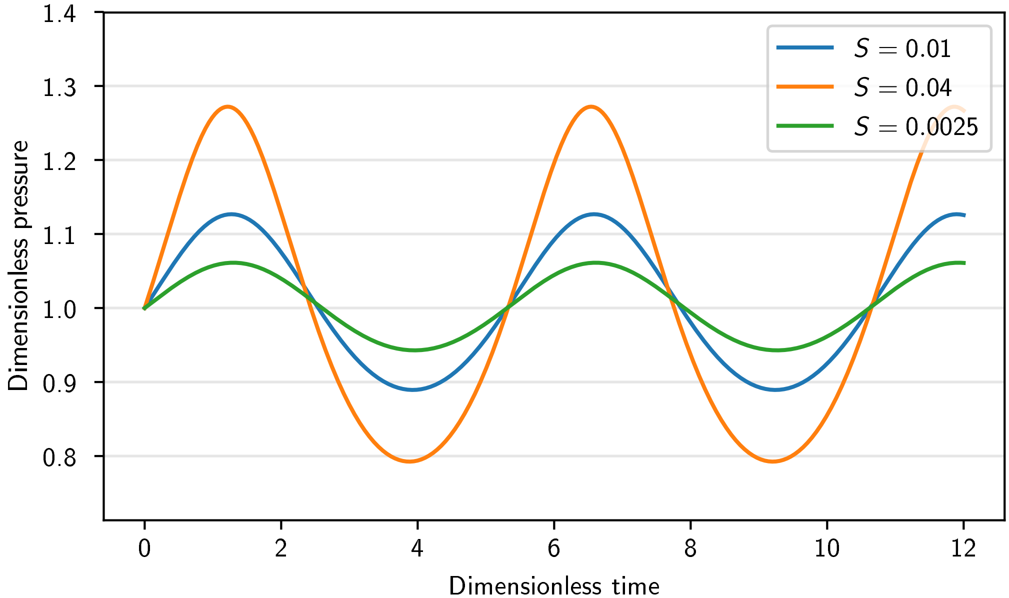
$$S = \frac{\text{Kinetic energy of piston}}{\text{Internal energy of the gas}}$$

Gas adiabatic index γ

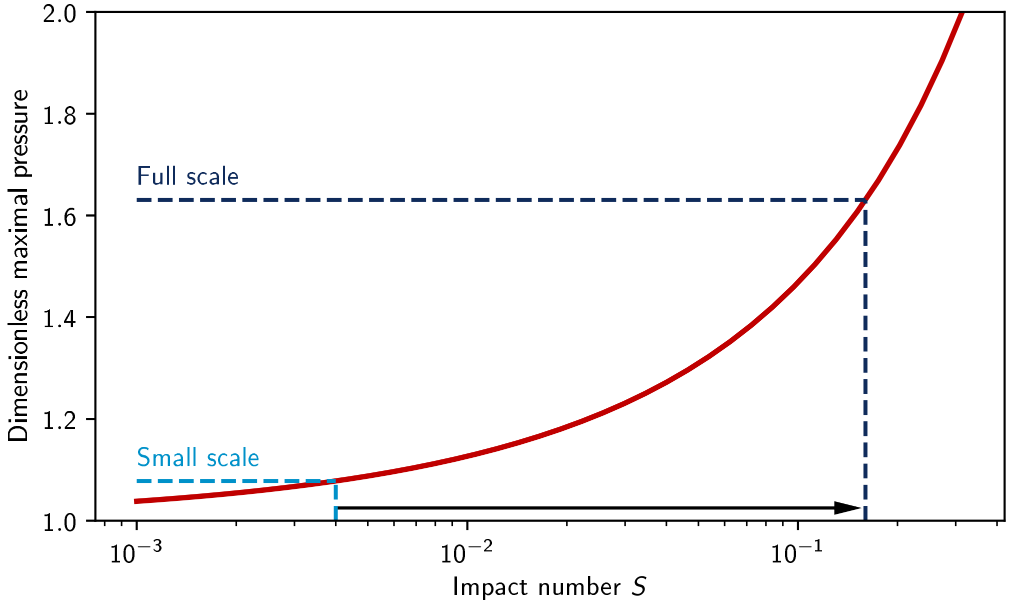
Compressibility of the gas.

(Not discussed here.)

Pressure evolution



Maximal pressure



Objectives

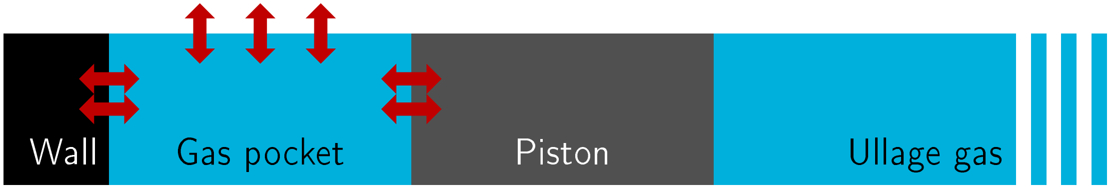
- **Influence of phase change on maximal pressure?**
  - Experimental results predict damping of the oscillations [Maillard and Brosset 2009].
  - Extended Bagnold model proposed in [Ancellin, Brosset, and Ghidaglia 2012]

- **What role do the parameters of the model play?**

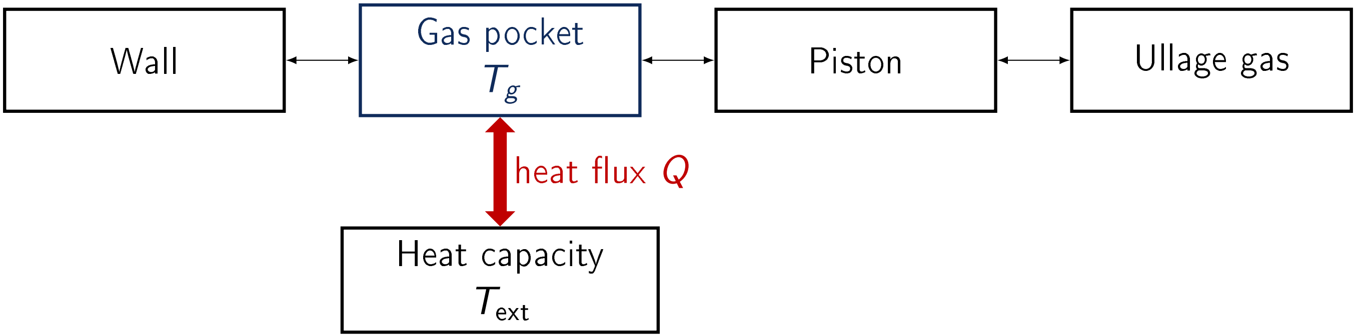
- **Content of this presentation:**
  1. Extended Bagnold model with heat exchange
  2. Extended Bagnold model with mass exchange through a porous wall
  3. Extended Bagnold model with phase change (mass and energy exchange)

1 Heat exchange

Extending the Bagnold model



Extending the Bagnold model



Classical heat flux:

$$Q \propto T_{\mathrm{ext}} - T_g ,$$

Chosen to respect the second law of thermodynamics:

$$\frac{\mathrm{d}S^{\mathrm{created}}}{\mathrm{d}t} = Q \frac{T_{\mathrm{ext}} - T_g}{T_{\mathrm{ext}} T_g} \geq 0 \, .$$

Heat exchange model

ODE system:

$$\frac{d^2 z}{dt^2} = p - 1,$$
$$\frac{d\mathcal{E}_g}{dt} = +\Omega_q(T_{ext} - T_g) - p\frac{dz}{dt},$$
$$\frac{d\mathcal{E}_{ext}}{dt} = -\Omega_q(T_{ext} - T_g),$$

Equations of state:

$$p = \frac{T_g}{z},$$
$$T_g = 1 + \mathcal{E}_g(\gamma - 1),$$
$$T_{ext} = 1 + \frac{\mathcal{E}_{ext}}{\mathfrak{C}_{ext}},$$

Initial conditions:

$$z(0) = 1, \quad \frac{dz}{dt}(0) = \pm\sqrt{S}, \quad T_g(0) = 1, \quad T_{ext}(0) = 1.$$

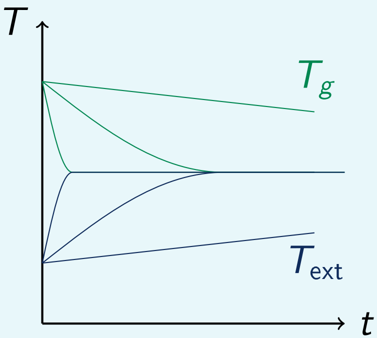
Dimensionless parameters

Impact number S

Gas adiabatic index γ

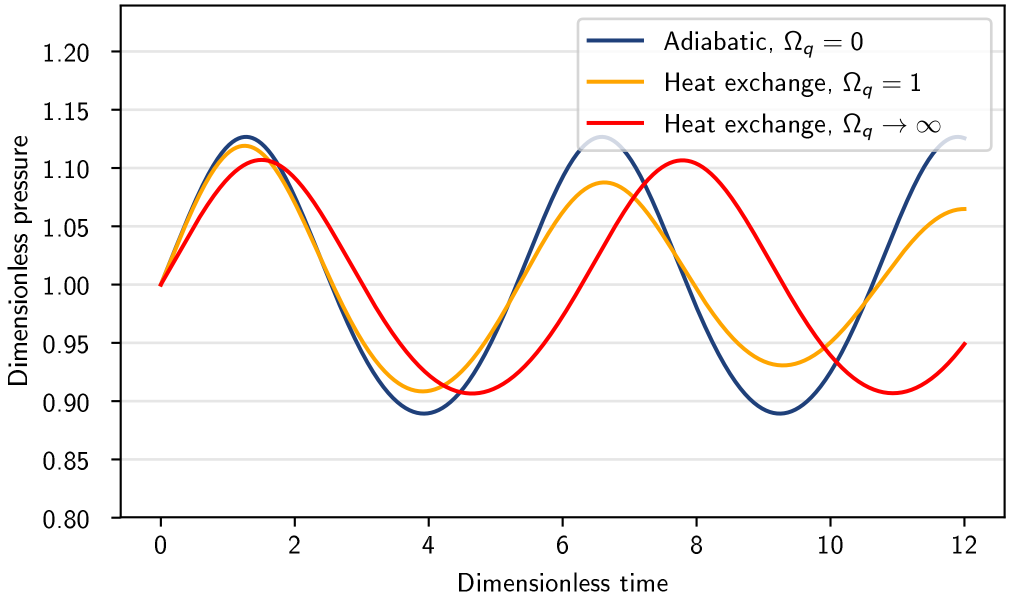
Relaxation rate Ω_q

How fast do the temperatures reach equilibrium?
(with respect to the piston oscillations)

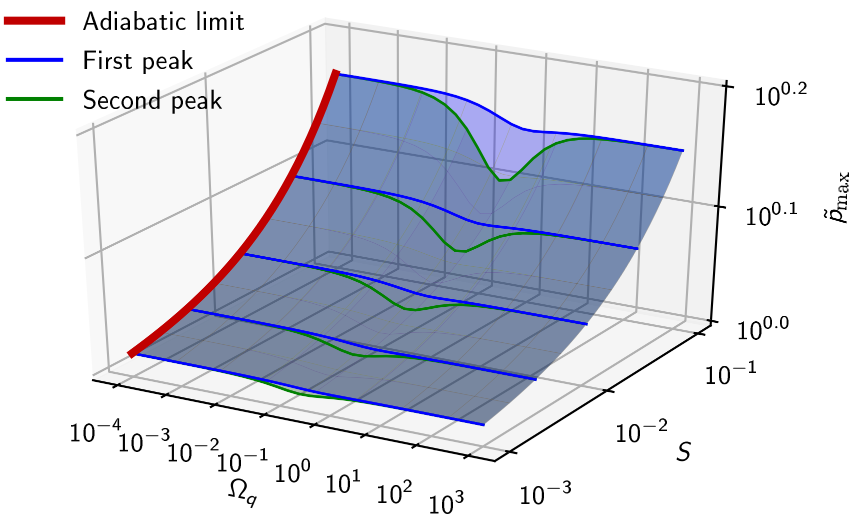


Thermal inertia $\mathfrak{C}_{ext}$

Pressure evolution



Maximal pressure



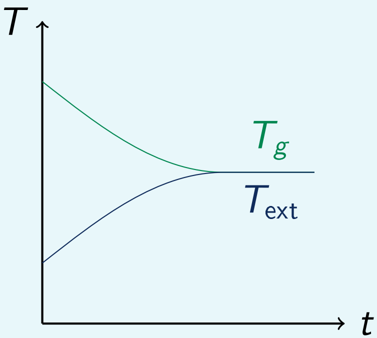
Dimensionless parameters

Impact number S

Gas adiabatic index γ

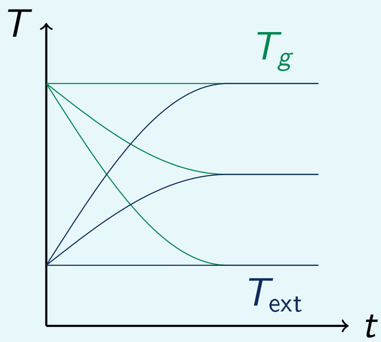
Relaxation rate Ω_q

How fast do the temperatures reach equilibrium?
(with respect to the piston oscillations)

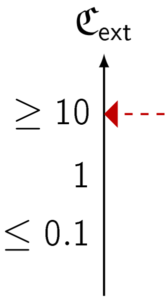


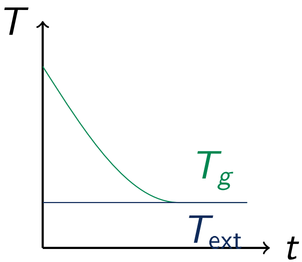
Thermal inertia $\mathfrak{C}_{\text{ext}}$

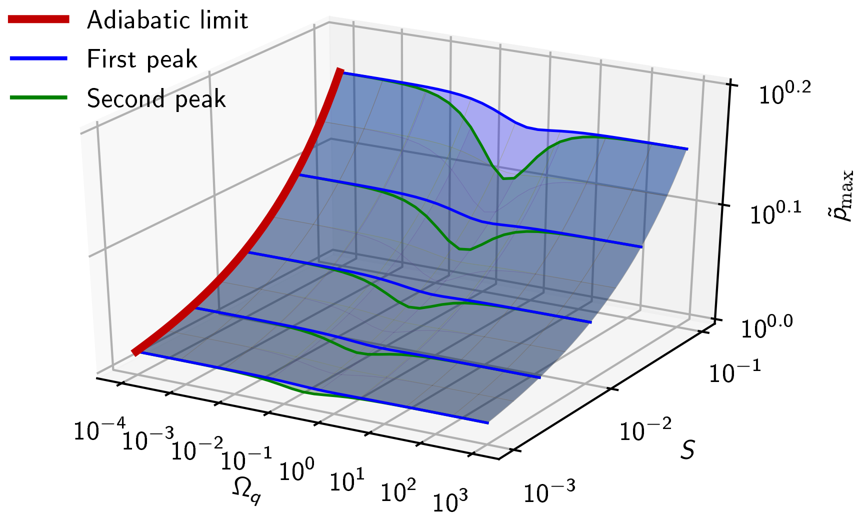
How hard is it to change T_{ext}?
(with respect to changing T_g)



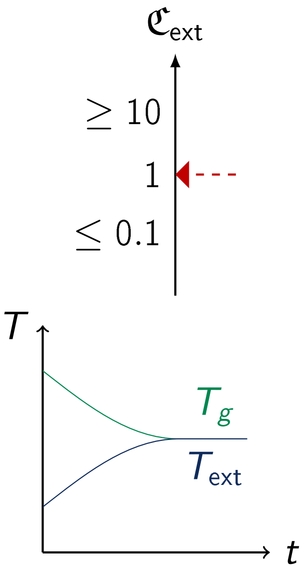
Maximal pressure

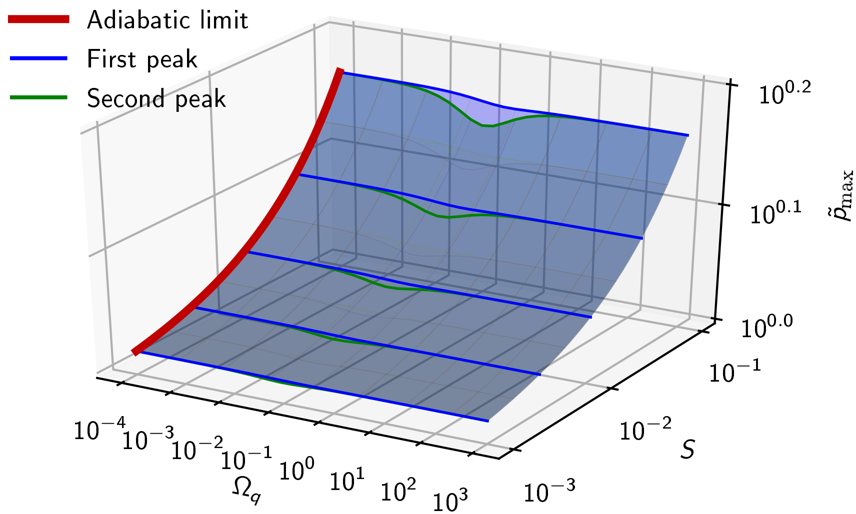




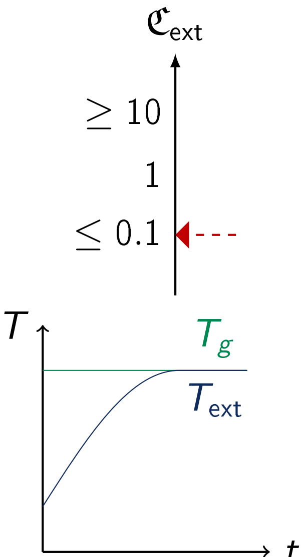


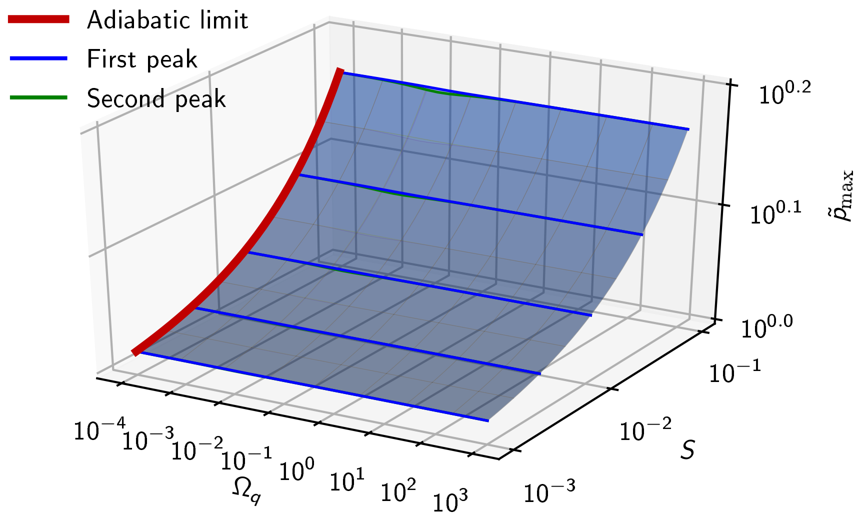
Maximal pressure





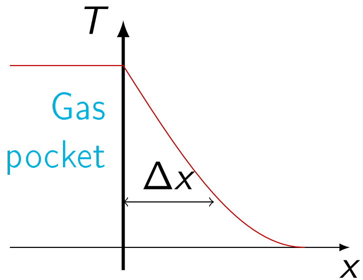
Maximal pressure





Estimating $\mathfrak{C}_{\text{ext}}$

- **Penetration depth of heat equation during Δt.**



$$\Delta x \simeq \sqrt{\frac{k_{\text{ext}}}{\rho_{\text{ext}}\, C_{v,\text{ext}}}\Delta t}$$

$$\mathfrak{C}_{\text{ext}} \simeq \frac{\sqrt{\Delta t}}{z_0} \times 10^{-2}\,\mathrm{m\,s^{-1}}$$

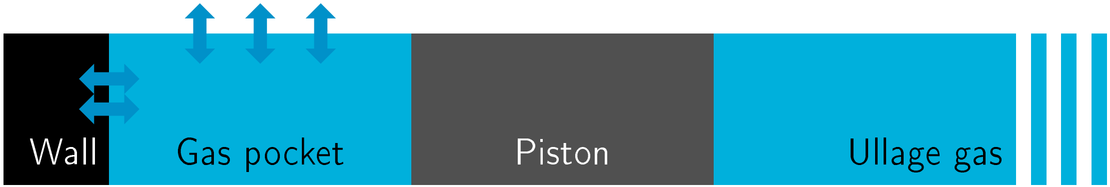
- **For macroscopic ($z_0 > 10^{-2}\,\mathrm{m}$) gas pocket**

$$\mathfrak{C}_{\text{ext}} < 10^{-4}.$$

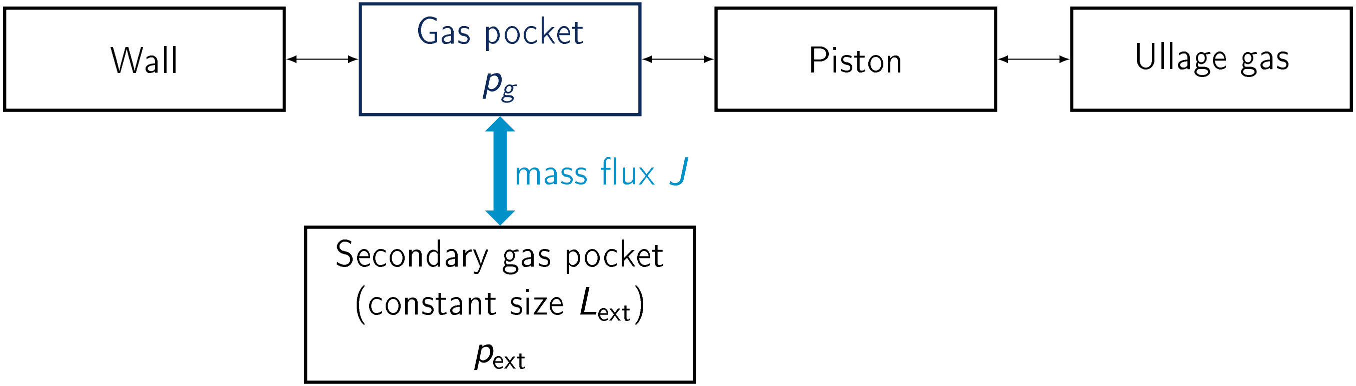
2

Mass exchange through porous wall

Porous wall model and mass flux



Porous wall model and mass flux



Mass flow rate at the porous wall:

$$J \propto p_{\text{ext}} - p.$$

Mass exchange model

ODE system

$$\frac{\mathrm{d}^2 z}{\mathrm{d}t^2} = p_g - 1,$$
$$\frac{\mathrm{d}M_g}{\mathrm{d}t} = -\Omega_p(p_g - p_{\mathrm{ext}}),$$
$$\frac{\mathrm{d}M_{\mathrm{ext}}}{\mathrm{d}t} = +\Omega_p(p_g - p_{\mathrm{ext}}),$$

Equations of state

$$p_g = \left(\frac{M_g}{z}\right)^\gamma$$
$$p_{\mathrm{ext}} = \left(\frac{M_{\mathrm{ext}}}{L_{\mathrm{ext}}}\right)^\gamma$$

Initial conditions

$$z(0) = 1, \quad \frac{\mathrm{d}z}{\mathrm{d}t}(0) = \pm\sqrt{S}, \quad p_g(0) = 1, \quad p_{\mathrm{ext}}(0) = 1.$$

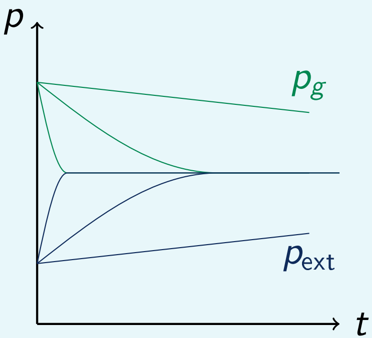
Dimensionless parameters

Impact number S

Gas adiabatic index γ

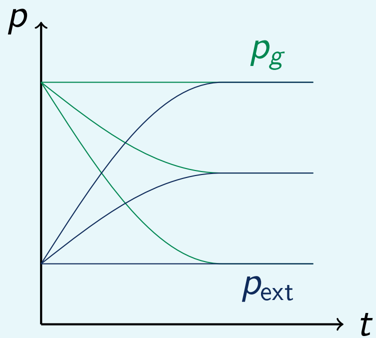
Relaxation rate Ω_p

How fast do the pressures reach equilibrium?
(with respect to the piston oscillations)

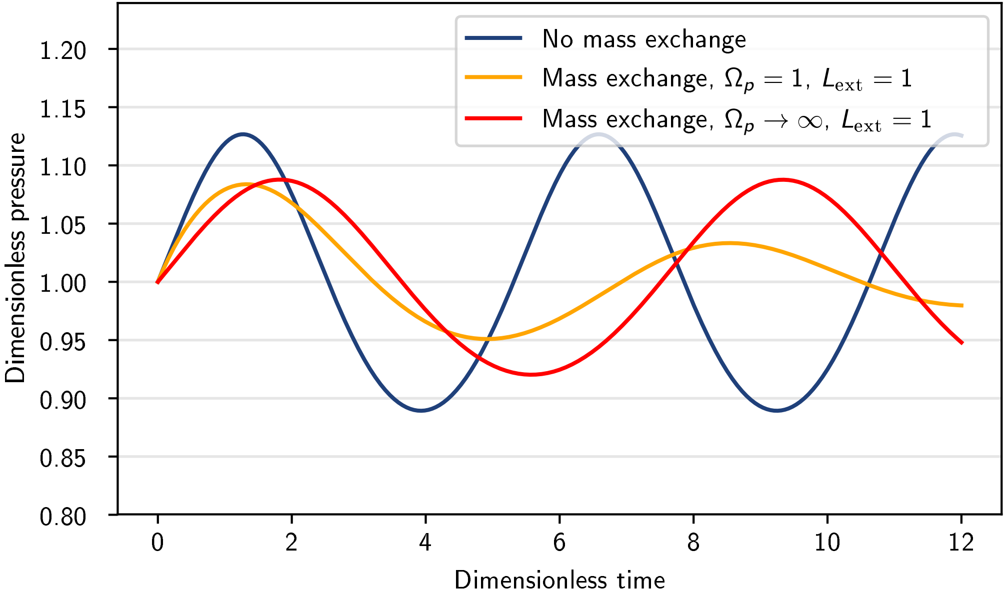


Size of the secondary pocket L_{ext}

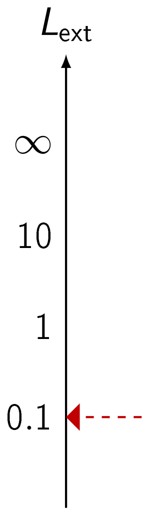
How hard is it to change p_{ext}?
(with respect to changing p_g)

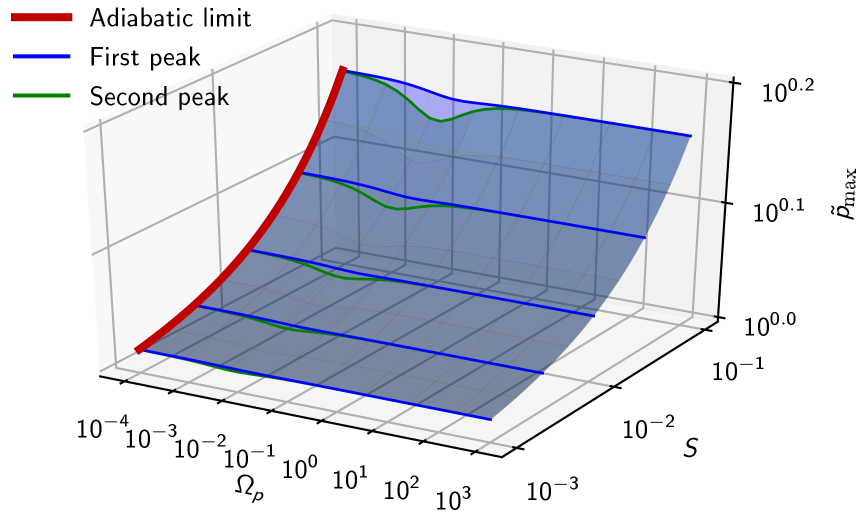


Pressure evolution

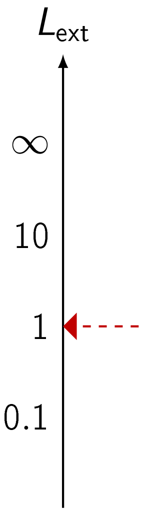


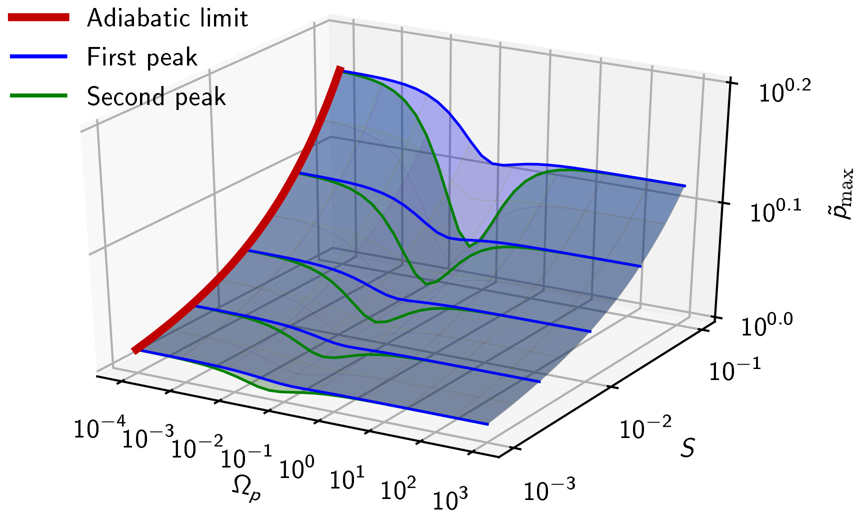
Maximal pressure



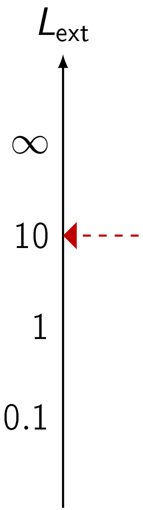


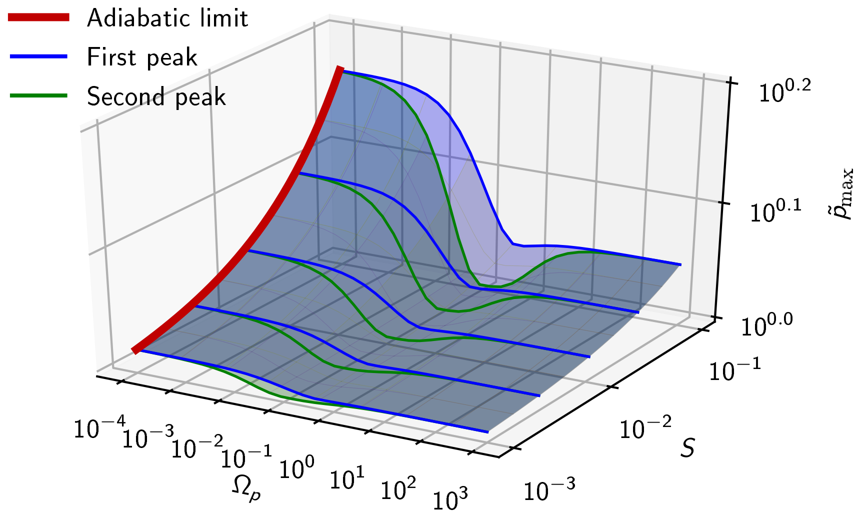
Maximal pressure



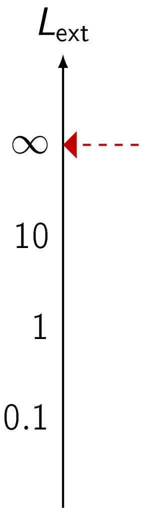


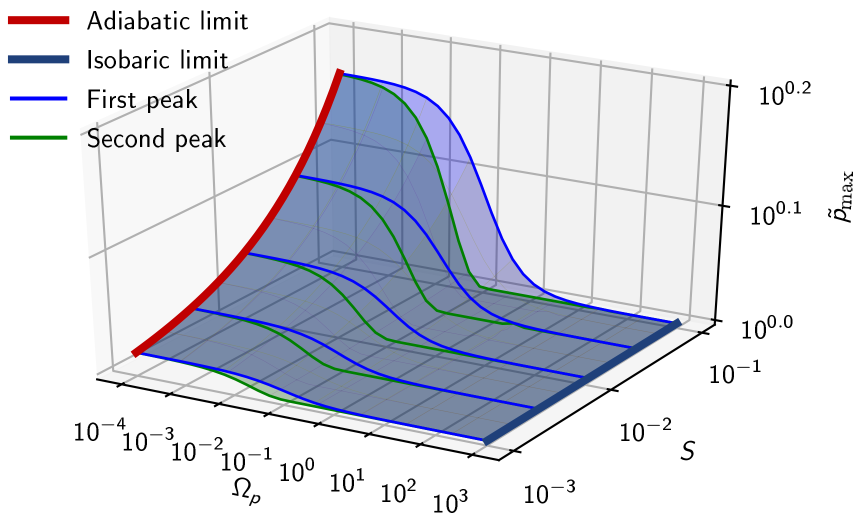
Maximal pressure





Maximal pressure





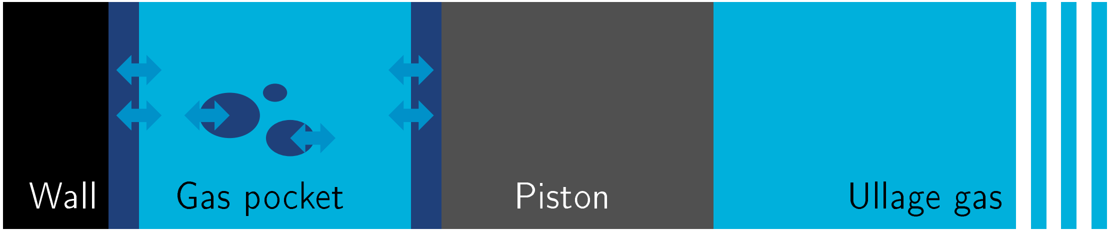
Partial conclusion

- **Not a model for gas escape before the impact...**
  - Not very well suited for the incompressible part of the gas escape.
- **... but a minimal example of phase change modelling.**

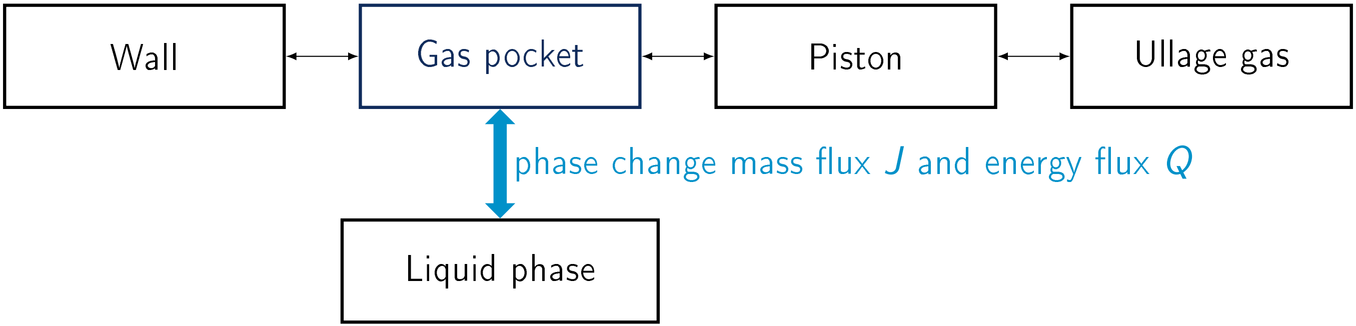
3

Phase change (mass and energy exchanges)

Extending Bagnold model



Extending Bagnold model



Second law of thermodynamics:

$$\frac{\mathrm{d}S^{\text{created}}}{\mathrm{d}t} = \left(\frac{1}{T_g} - \frac{1}{T_l}\right)(Q - h_g J) + \left(\frac{\mu_l(T_l) - \mu_g(T_l)}{T_l}\right) J\,,$$

Energy and mass flow:

$$Q = h_g J\,, \qquad J \propto \frac{\mu_\ell(T_\ell) - \mu_g(T_\ell)}{T_\ell} \propto p^{\mathrm{sat}}(T_\ell) - p$$

Model with phase change

ODE system

$$\frac{\mathrm{d}^2 z}{\mathrm{d}t^2} = p - 1,$$
$$\frac{\mathrm{d}\mathcal{E}_g}{\mathrm{d}t} = +h_g \frac{\mathrm{d}M_g}{\mathrm{d}t} - p\frac{\mathrm{d}z}{\mathrm{d}t},$$
$$\frac{\mathrm{d}\mathcal{E}_\ell}{\mathrm{d}t} = -h_g \frac{\mathrm{d}M_g}{\mathrm{d}t},$$
$$\frac{\mathrm{d}M_g}{\mathrm{d}t} = +\Omega_m(p^{\mathrm{sat}}(T_\ell) - p),$$
$$\frac{\mathrm{d}M_\ell}{\mathrm{d}t} = -\frac{1}{\mathfrak{M}_{\ell,0}}\Omega_m(p^{\mathrm{sat}}(T_\ell) - p),$$

Equations of state

$$p = \frac{M_g}{z} T_g, \qquad h_g = \frac{\mathcal{E}_g + pz}{M_g},$$
$$p^{\mathrm{sat}}(T) = \exp\left(\Lambda\left(1 - \frac{1}{T}\right)\right),$$
$$T_g = 1 + \left(\frac{\mathcal{E}_g}{M_g} - \Lambda + 1\right)(\gamma - 1),$$
$$T_\ell = 1 + \frac{\mathcal{E}_\ell}{M_\ell \mathfrak{C}_\ell},$$

Initial conditions

$$z(0) = 1, \quad \frac{\mathrm{d}z}{\mathrm{d}t}(0) = \pm\sqrt{S}, \quad T_g(0) = 1, \quad T_\ell(0) = 1, \quad M_g(0) = 1, \quad M_\ell(0) = 1.$$

Dimensionless parameters

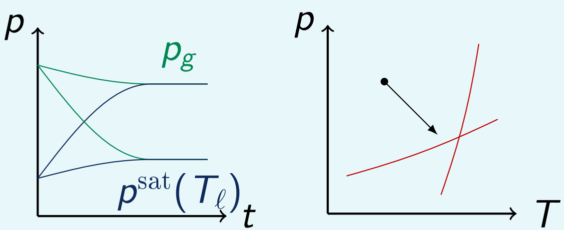
Impact number S

Relaxation rate Ω_m

Do you really want me to explain it again?

Dimensionless latent heat Λ

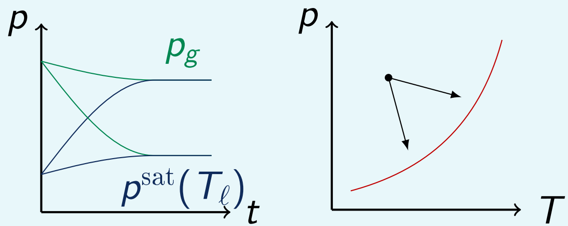
How hard is it to change p^{sat}?
(with respect to p_g and T_g)



Gas adiabatic index γ

Thermal inertia of the liquid $\mathfrak{C}_\ell$

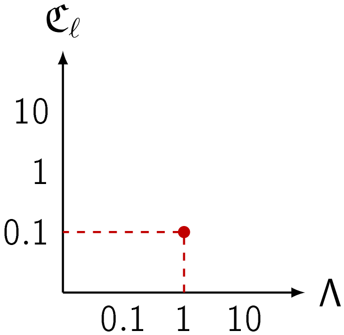
How hard is it to change T_ℓ?
(with respect to p_g and T_g)

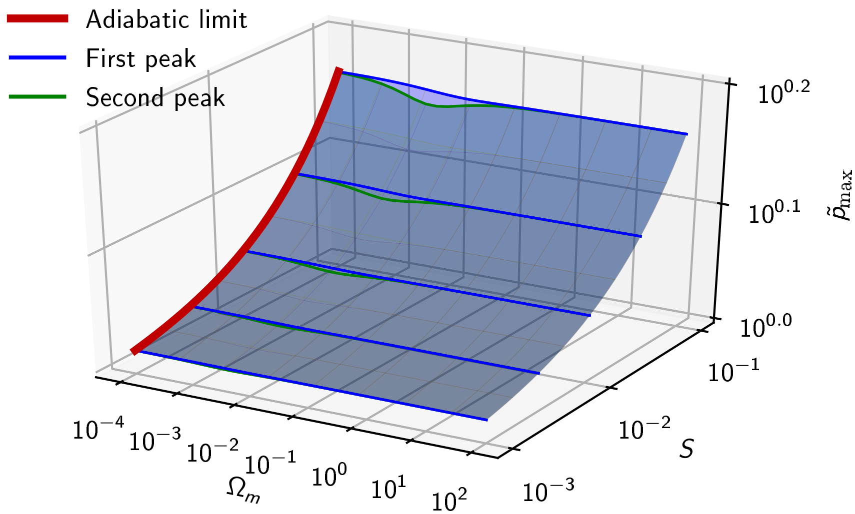


Dimensionless mass of liquid $\mathfrak{M}_{\ell,0}$

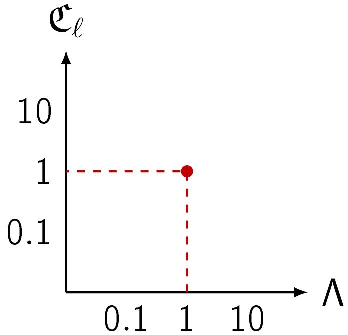
Variation of $\mathfrak{C}_\ell$ due to mass exchange.

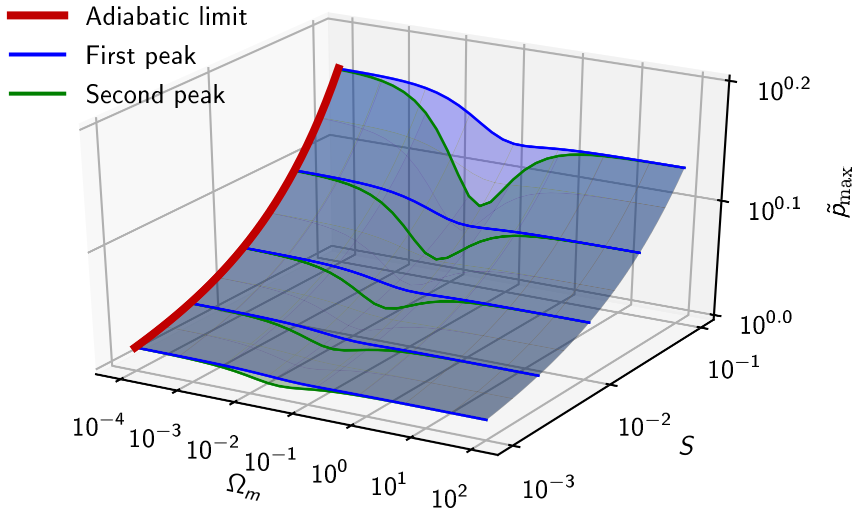
Maximal pressure



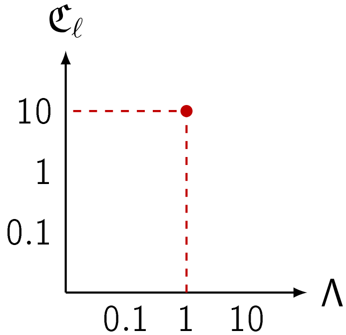


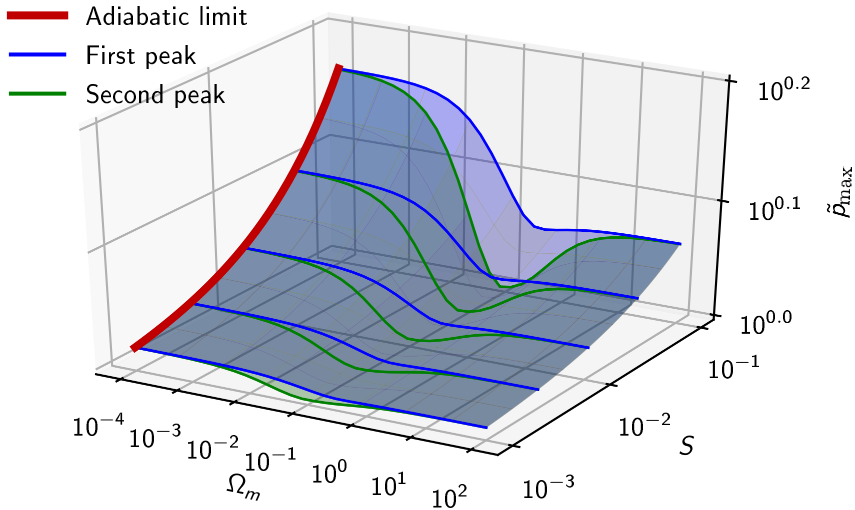
Maximal pressure



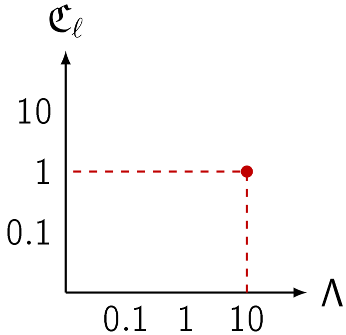


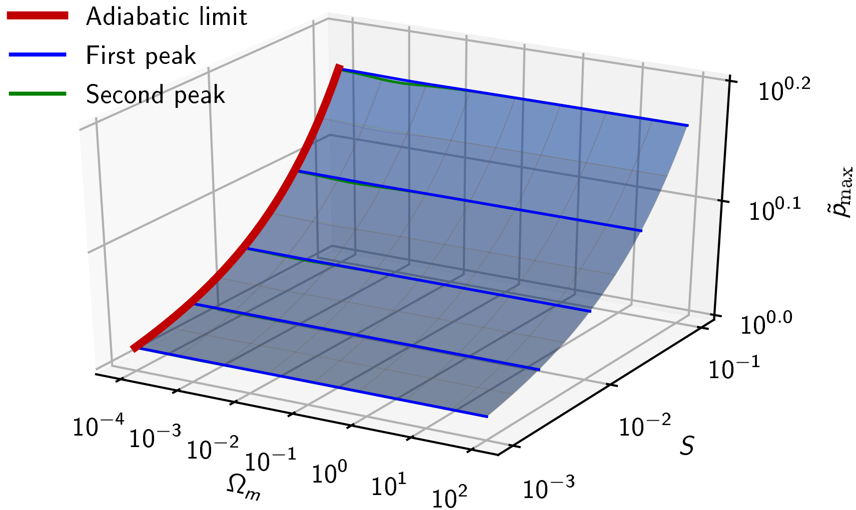
Maximal pressure



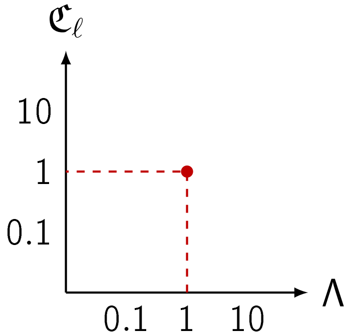


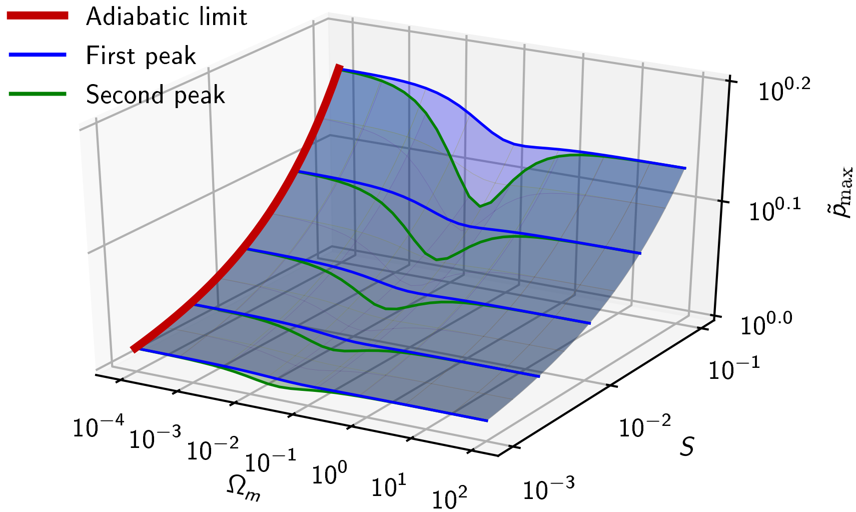
Maximal pressure



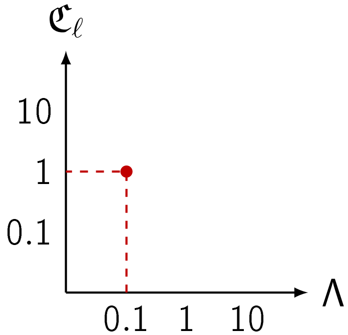


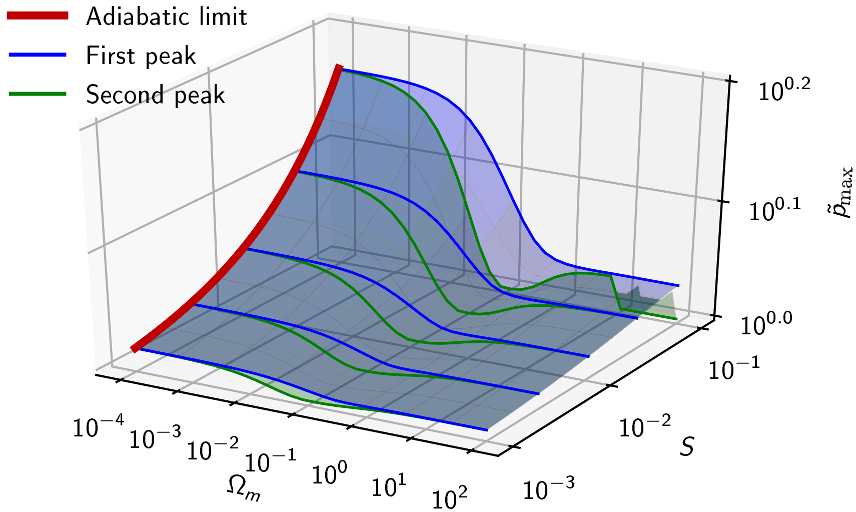
Maximal pressure





Maximal pressure





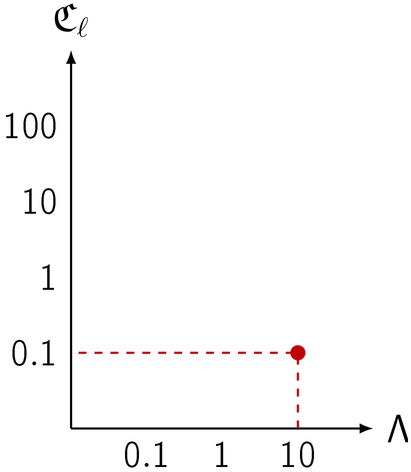
Estimating the parameters

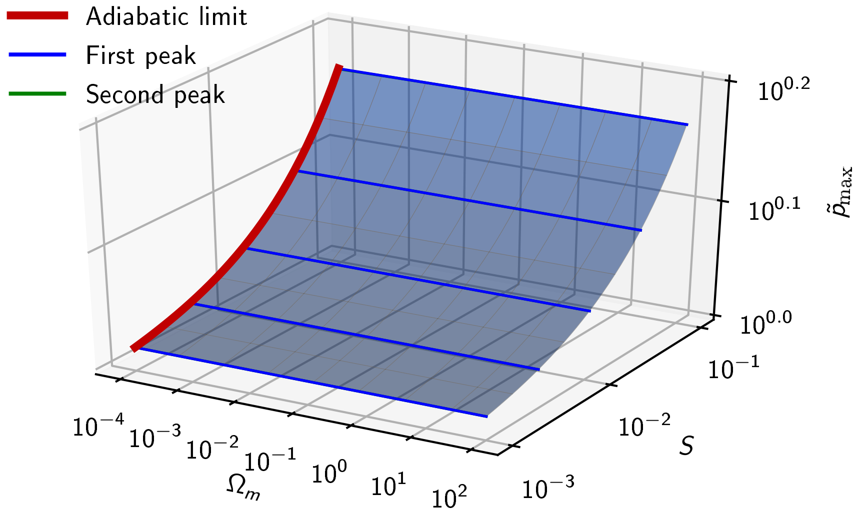
- **Ω_m is unknown**
  - Could be close to 1, thus explaining the experimentally observed damping.
- **Λ is intrinsic to a chemical species.**
  - $\Lambda \simeq 20$ for water, $\Lambda \simeq 7$ for methane.
- **$\mathfrak{C}_\ell$: How much liquid is interacting with the gas?**
  - Same arguments as for heat exchange.

$$\mathfrak{C}_\ell \simeq \frac{\sqrt{\Delta t}}{z_0} \times 10^{-2}\,\mathrm{m\,s^{-1}}$$

  - Phase change negligible for macroscopic gas pockets.

Maximal pressure





4 Conclusion

Conclusion

- **Two main parameters for the extended Bagnold models:**
  - Relaxation rate (cause of the oscillation damping);
  - Distance to equilibrium (may limit the quantitative effect).

- **Phase change = simple gas escape + fancy thermodynamics.**

- **To be generalized to all phase change during wave impacts?**

Thank you for your attention!

Matthieu Ancellin
matthieu.ancellin@ucd.ie