Heat transfer and thermo-mechanical stresses in a gravity casting die - Influence of process parameters

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Abstract

The paper is concerned with thermal measurements in a gravity casting die of simple geometry. A inverse method, based on the Laplace Transform of the heat conduction equation, is used to determine the heat flux exchanged between the die and the aluminium product. This estimation is compared to the flux that can be determined from temperature difference between two thermocouples. A ratio between 1.5 and 2 is reported. In a specific experiment, both estimated heat flux are compared with the heat exchanged with an ice cylinder. This validates the measurements and the inverse method as far as order of magnitude is concerned. Thermal measurements are used to evaluate the thermal stresses in the mould, using a simple analytical formulation.

The influence of two process parameters, the initial temperature of the die and the coating on the moulding surface, is studied. Basically the more conductive the coating and the lower the initial temperature of the die, the higher the heat transfer and the higher the thermal stresses. Less obvious is the influence of ageing for the graphite based coating. A phenomenon is observed from the heat flux vs. time curves, that can be related to recalescence. It is discussed in terms of reactivity of the coating, heterogeneous nucleation on the coating and supercooling of the melt.

Keywords: Thermo-mechanical stress; Process parameters; Heterogeneous nucleation; Supercooling

1. Introduction

In casting processes, heat transfer is well known to be the fundamental process parameter that induce microstructural quality to the products. The higher the heat transfer, the finer the microstructure and therefore the better the mechanical properties (yield strength, strength, etc.) and the higher the productivity. This is the reason why the commercial part of the die casting industry increases every year. However the die materials usually react with the liquid metal (aluminium, zinc alloys). To protect the dies, it is necessary to coat them. This coatings performs some secondary effects: it limits the heat transfer and thus the quality of the part, but supplies a protection against thermal shock to the die. Since casting dies are frequently considered useless when heat checking (a bi-dimensional network of cracks) on the moulding die surface appears, the last effect of the coatings is not negligible.

This study is concerned firstly with measuring the temperature in a die and estimating from them the heat flux at the moulding surface. And secondly, to determine the thermal stresses within the die. The die is plate-shaped and the casting process is gravity casting. The focus is set on the influence of the initial temperature of the die and on the coating as process parameters.

The present paper explains the experimental set-up and the associated model for the heat transfer. The second part deals with a validation of the thermal model and of the measurements. To finish details of the experiments and their results in terms of temperature rise in the die, of heat flux at the moulding surface and of thermal stresses are given.

2. Description of applied methodology

2.1. Experimental set up

The geometry of the die was a plate $200 \times 200 \times 40\,\text{mm}^3$ made out of an XC35 and heat treated to 42 HRc on the surfaces. The mould cavity was $100 \times 100 \times 20\,\text{mm}^3$ and the cavity filling was lateral type. The aluminium alloy is the classical aluminium–silicon AS7G06 and is heated up to 750°C.

A temperature gauge was made using the same steel as the die. Fifteen coated thermocouples were brazed into the

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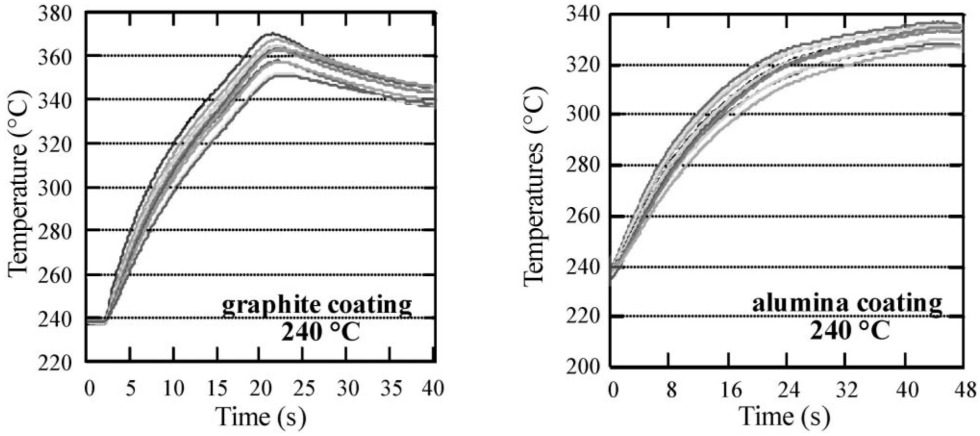

Fig. 1. Typical curves temperature vs. time on each thermocouples for (left) a graphite based coating and (right) an alumina based coating.

gauge. The hot junctions were also brazed onto the coat of the thermocouple, to minimise the rise time. Their position was such that the hot junctions were set between 0.5 and 9 mm from the moulding surface. The thermocouples were positioned along a 5 mm diameter helicoidal coil. The separation between two thermocouples was either 0.5 or 1 mm (±0.05) and 26°.

The cold junctions of the 15 thermocouples are all squeezed between two copper plates, inside a polystyrene box. The temperature inside the box was lowered with ice cubes and controlled with a thermometer. It was found to be 11°C (±0.5°C).

A data acquisition card on a PC was used without any analogue filtering. The acquisition was performed at 400 Hz, and then averaged for every 40 time steps. The duration of an acquisition was 30–50 s. Fig. 1 presents the typical curves that we obtained in two different casting conditions.

2.2. Model and inverse method

We imagined thermal conduction in the mould to be a one dimension problem (this assumption will be discussed later). From now on the following notations will be adopted according to the draft in Fig. 2. The transient heat conduction problem can be written as:

$$\frac{\partial^2 T}{\partial x^2} - \frac{1}{a}\frac{\partial T}{\partial t} = 0 \quad \text{for } 0 < x < L \text{ and } t \geq 0 \tag{1a}$$

$$-\lambda \frac{\partial T}{\partial x} = q_0(t) \quad \text{at } x = 0 \text{ and for } t \geq 0 \tag{1b}$$

$$T = T_2(t) \quad \text{at } x = L \text{ and for } t \geq 0 \tag{1c}$$

$$T = T_0 \quad \text{for } t = 0 \text{ at } 0 \leq x \leq L \tag{1d}$$

where $a = \lambda/(\rho C_p)$ is the diffusivity of the die, its value is 12.7×10^{-6}–$13.4 \times 10^{-6}\ \mathrm{m^2/s}$ between 20 and 400°C [1] and λ its thermal conductivity, its value is 43.5–51 W/m K between 200 and 300°C [1]; ρ its density, its value is around 7000 kg/m^3; C_p its specific heat, its value is 520–586 J/kg K between 150 and 400°C [1]; L the die thickness.

For the sake of simplicity, we hold that the physical properties are not dependent on the temperature. When the boundary conditions $q_0(t)$ and $T_2(t)$ are known, the problem described by Eqs. (1a)–(1d) can be solved: this is the direct problem. On the contrary, the inverse problem is concerned with the determination of the heat flux $q_0(t)$ using the measured temperature in x_1 or more locations in the mould and the second boundary condition $T_2(t)$. The position $x = L$ has been chosen at the position of the last thermocouple (i.e. 9 mm depth). The position of the required temperature for the inverse problem is set at $x_1 = 0.5$.

With the assumption that the heat flux is unidirectional, it is possible to exploit the temperature vs. time curves (Fig. 1) into heat flux vs. time curves (Fig. 3). This is obtained by differentiating the temperature to the distance between two thermocouples Δx.

$$Q(x,t) = -\lambda \frac{\Delta T}{\Delta x} \tag{2a}$$

The direct problem, as well as the inverse one, requires numerical or analytical methods to be solved. We chose the

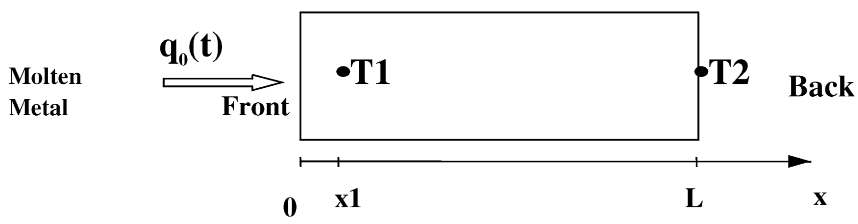

Fig. 2. Draft of the heat transfer problem.

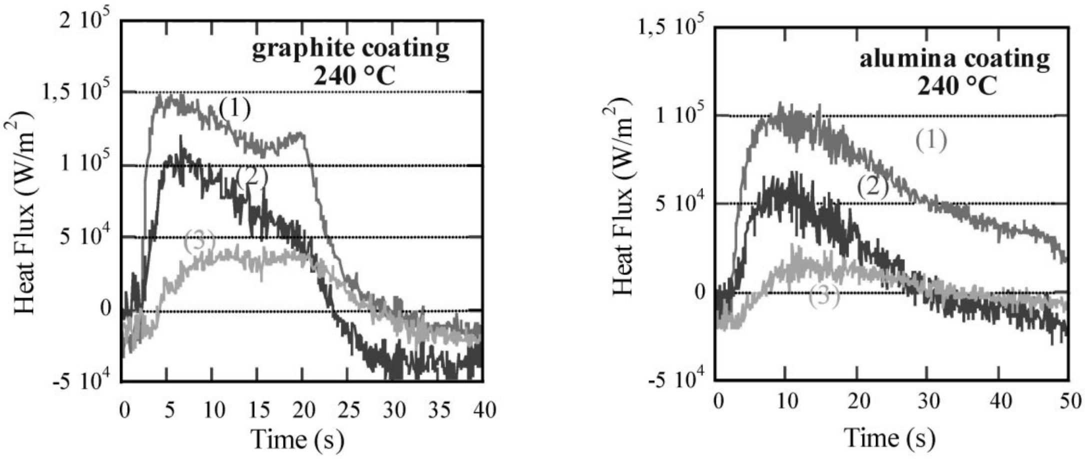

Fig. 3. Curves heat flux vs. time at different positions 1, 2 and 3 with the two coatings and the same initial temperature. 1. Being the flux derived from thermocouples 1 and 3; 2. being the flux derived from thermocouples 9 and 10; 3. being the flux derived from thermocouples 13 and 14.

semi-analytical method based on the Laplace Transform. The temperature $T(x, t)$ and the heat flux $q(x, t)$ are transformed into $\theta(x, s)$ and $\phi(x, s)$. Eqs. (1a)–(1d) are then transformed into:

$$\frac{d^2\theta}{dx^2}(x,s) = \frac{s}{a}\theta(x,s) \tag{2b}$$

$$\phi(x,s) = -\lambda\frac{d\theta(x,s)}{dx} \tag{2c}$$

These equations can be solved easily and it is common to write the solution in terms of a transfer matrix between an input ($\theta(x_i, s)$, $\phi(x_i, s)$) and an output ($\theta(x_j, s)$, $\phi(x_j, s)$) [2]

$$\begin{bmatrix}\theta(x_i,s)\\ \phi(x_i,s)\end{bmatrix} = \begin{bmatrix}A & B\\ C & D\end{bmatrix}\begin{bmatrix}\theta(x_j,s)\\ \phi(x_j,s)\end{bmatrix} \tag{3}$$

where

$$\Delta x = x_j - x_i, \qquad k = \sqrt{\frac{s}{a}}, \qquad A = D = \mathrm{ch}(k\Delta x),$$

$$B = \frac{\mathrm{sh}(k\Delta x)}{\lambda k}, \qquad C = \lambda k\,\mathrm{sh}(k\Delta x)$$

The matrix between $x = 0$ and a position x is indexed 1 and the matrix between the same position x and $x = L$ is indexed 2. A combination of the expressions (3) leads to an expression of the temperature at the position x as a function of the heat flux at the front and the temperature at the back

$$\theta(x,s) = F_1(x,s)\phi_0(s) + F_2(x,s)\theta_2(s) \tag{4}$$

where

$$F_1(x,s) = \frac{B_2}{D_1D_2 + C_1B_2} \quad \text{and} \quad F_2(x,s) = \frac{D_1}{D_1D_2 + C_1B_2} \tag{5}$$

Then, the temperature $T(x, t)$ evaluated back in the (x, t) space via the convolution form

$$T(x,t) = \int_0^t q_0(\tau)f_1(x,t-\tau)\,d\tau + \int_0^t T_2(\tau)f_2(x,t-\tau)\,d\tau \tag{6}$$

The functions f_1 and f_2 are, respectively, the inverse Laplace transform of F_1 and F_2. They are themselves estimated with a numerical method from Stehfest algorithm [3].

For the inverse problem, the determination of heat flux $q_0(t)$ requires a history of the measured temperature $T(x_1, t_i) = T_i$ at one (or eventually more) locations x_1 in the depth of the die at different times t_i. Henceforth $\Delta t = t_{i+1} - t_i$, corresponds to 0.1 s from the averaging of the experimental data. It is well known [4] that the "good solution" (determination of $q_0(t)$) depends on the location x_1 of the sensor and on the time step Δt. In what follows, the location of the sensor is 0.5 mm, which sets the Fourier ratio to 5. In this range, the inverse problem should be stable.

The inverse problem algorithm is based on the development by Fudym et al. [5] and has been adapted to our situation. It involves of transformation of the inverse problem into a minimisation problem using least-squares formulation to give a "well-posed" problem. This requires a numerical integration of Eq. (6), as

$$T(x,t_i) = \Delta t\sum_{k=1}^{i}[q_0(t_k)f_1(x,t_i-t_k) + T_2(t_k)f_2(x,t_i-t_k)]$$
$$\text{with } t_k = k\Delta t \tag{7}$$

Setting x to $x_1 = 0.5$ mm, the temperature $T(x, t_i)$ corresponds to T_i introduced earlier. This can be compared to the temperature that is measured at the same location, noted $T_i^{\exp}$. When the preceding temperatures $T_1, T_2, \ldots, T_i$ and heat flux $q_{0,1}, q_{0,2}, \ldots, q_{0,i}$ ($q_{0,i} = q(x = 0, t_i)$) are already determined, the problem is finding $q_{0,i+1}$ as a minimum of the functional (least-squares formulation) $E(q)$ [6]

$$E(q) = \sum_{k=1}^{k=m} [T_{i+k}(q) - T_{i+k}^{\exp}]^2 \tag{8}$$

where $T_{i+k}(q), k = 1, 2, \ldots, m$ is the direct problem solution at position x_1 computed with a boundary constant heat flux q ($q(t_{i+j}) = q(t_{i+1})$ $2 \le j \le m$), $T_{i+k}^{\exp}$, $k = 1, 2, \ldots, m$ the measured temperatures at position x_1, and m is an integer number, to be set to an optimum (usually around 2–5).

It follows from (7)

$$\begin{aligned} T_{i+k}(q_{i+1}) = \Delta t \sum_{j=1}^{j=i} [& q_{0,j} f_1(x_1, t_{i+k} - t_j) \\ &+ T_2(t_j) f_2(x_1, t_{i+k} - t_j)] \\ &+ \Delta t q_{i+1} \sum_{j=i+1}^{i+k} f_1(x_1, t_{i+k} - t_j) \end{aligned} \tag{9}$$

For a given, $q_{0,1}, q_{0,2}, \ldots, q_{0,j}$, T_{i+k} is a temperature dependent on the parameter q

$$\begin{aligned} T_{i+k}(q) = \Delta t \sum_{j=1}^{j=i} [q_{0,j} f_1(x_1, t_{i+k} - t_j) + T_2(t_j) f_2(x_1, t_{i+k} - t_j)] \\ + \Delta t q \sum_{j=i+1}^{i+k} f_1(x_1, t_{i+k} - t_j) \end{aligned} \tag{10}$$

The minimum $q_{0,i+1}$ results from the Euler equation $(\partial E/\partial q) q_{0,i+1} = 0$:

$$\sum_{k=1}^{k=m} [T_{i+k}(q_{0,i+1}) - T_{i+k}^{\exp}] \frac{\partial T_{i+k}}{\partial q}(q_{0,i+1}) = 0 \tag{11}$$

For a small variation of $\Delta q = q_{i+1} - q_i$, one can write

$$T_{i+k}(q_{0,i+1}) = T_{i+k}(q_{0,i}) + \Delta q \frac{\partial T_{i+k}}{\partial q}(q_{0,i+1}) \tag{12}$$

Differentiating (10) with respect to q

$$\frac{\partial T_{i+k}}{\partial q}(q_{0,i+1}) = \Delta t \sum_{j=i+1}^{i+k} f_1(x_1, t_{i+k} - t_j) = \Delta t \sum_{j=1}^{j=k} f_1(x_1, t_k - t_j) \tag{13}$$

Then, the incremental problem resumes to searching Δq as

$$\Delta q = \frac{\sum_{k=1}^{k=m} S_{i+k} [T_{i+k}^{\exp} - T_{i+k}(q_{0,i})]}{\sum_{k=1}^{k=m} S_{i+k}^2} \tag{14}$$

where $S_{i+k} = (\partial T_{i+k}/\partial q)(q_{0,i+1})$, $k = 1, 2, \ldots, m$ are the sensitivity coefficients and are obtained from (13).

3. Validation of the methodology

3.1. Validation of the unidirectionality of the problem

To validate the assumption of the unidirectional heat transfer, some of the casting experiments were used. The temperature is measured at the thermocouple located at $x_1 = 0.5$ mm during 30 s for every 0.1 s. Applying the inverse method, the heat flux $q_0(t)$ transferred to the mould is evaluated. This heat flux is used to solve the direct problem, and then evaluate the temperatures at different locations. They are finally compared to the actual temperature measured at the same location. Not only the error between computed and measured temperature at $x_1 = 0.5$ is negligible, but the errors at the other locations do not exceed 5°C (see Fig. 4). This strongly suggests that the heat transfer actually is unidirectional.

3.2. Accuracy of the thermal measurements

To control the validity and estimate the accuracy of the thermal measurements, a specific test has been designed. Instead of a hot thermal shock, a cold thermal shock was applied to the pre-heated die with a cylindrical ice cube. The heat flux next to the surface of the die was measured using the same procedure as described in Section 1. Additionally the length of the cube was measured before and after the test, as well as the temperature of the liquid water flowing down along the die. The ice cube was set at a temperature close to 0°C, as it tends to melt and the experiment lasted for 30 s.

From the heat flux, the heat per square meter removed from the die is estimated with the integral of the heat flux

$$E = \int_0^{30\,\mathrm{s}} q_0(t)\,\mathrm{d}t \quad \text{and} \quad E_{1-3} = -\lambda \int_0^{30\,\mathrm{s}} \frac{\Delta T_{1-3}(t)}{\Delta x}\,\mathrm{d}t \tag{15}$$

E_{1-3} was estimated with thermocouples 1 and 3, positioned, respectively, at 0.5 and 1.5 mm from the surface.

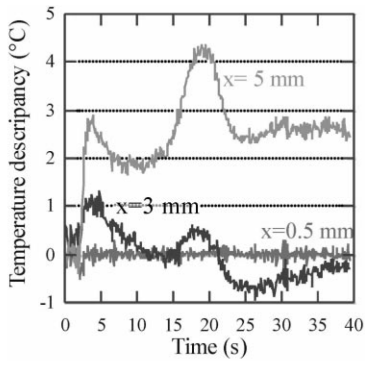

Fig. 4. Illustration of the unidirectionality of the heat flux.

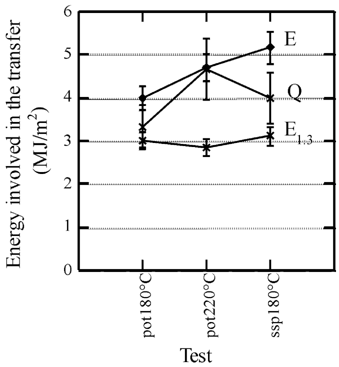

Fig. 5. Comparison between the heat measured from the temperature gauge and from melting of the ice cube in a cold thermal shock.

From the temperature of the water and the length of ice, the heat per square meter received by the ice (that makes it melt) was

$$Q = \rho \Delta H \Delta l + \rho C_p \Delta T \Delta l \quad (16)$$

with ρ is the density of water (1000 kg/m^3); C_p the specific heat of water (2 kJ/kg K); ΔH the latent heat of water (33 kJ/kg); Δl the variation in the length of the ice cube; ΔT the variation of the temperature of water.

Several experiments have been performed, changing the temperature of the die (between 180 and 220°C) and changing the coating on the die (alumina, none). The following diagram (Fig. 5) summarises the results for the three tests. The accuracy bars are set knowing the accuracy of the measurements. Concerning the temperature measurements in the die, the relative accuracy is set by the amplitude of the noise (i.e. 0.5°C) divided by the temperature difference between two consecutive thermocouples (i.e. 5°C). Additionally, the relative accuracy of the position between two consecutive thermocouples is 0.05%. That is to say, the best relative accuracy for the heat flux we face is about ±15%. As far as the ice measurement is concerned, an accuracy around ±20% would be more correct.

From the curves on Fig. 5, it is clear that our measurements are reliable as long as the order of magnitude is concerned. The heat flux estimated at 1 mm from the surface is not the same as the heat flux at the surface. In fact, a ratio around 1.5–2 exists in these experiments. In fact the heat flux were of the same magnitude as the one obtained in gravity casting conditions, thus all the preceding remarks remain valid for the experiments on casting.

4. Experiments on casting

The objective of this study was to inspect the heat transfer condition during casting and to evaluate the thermal stresses in the die. The boundary condition on the working surface and the initial condition are the main parameters (when one excepts the die physical properties, such as conductivity, Young modulus, dilatation, etc.). This is why the following experiments are performed with:

- two different coatings, to change the boundary condition, and
- three initial temperatures of the die, to change the initial condition.

The coatings are both in use in the casting industry for gravity die or low pressure die casting. Their commercial names are "Metalcote 28 and 36-1". The former has a fine ceramic base and is said to be a good thermal insulator. It has been called "alumina coating". The latter is very fine colloidal graphite in water, and is said to be a heat conductor. After deposition of the coatings, their thickness was measured with a magnetic thickness tester. The alumina coating is usually 150–200 μm thick, and the graphite 10–20 μm thick. All six experiments were performed two times with the same coating to avoid discrepancies with coating thickness.

The results of the experiments are summarised in Figs. 6 and 7 right, in terms of temperature rise at 0.5 mm depth and heat transfer at the surface. Fig. 6 shows that the temperature rises about twice as fast with the graphite coating than with the alumina. Fig. 7 confirms this fact, as the heat transfer is 1.5–3 times higher as well. Both figures also point out that the collected information is very consistent with the alumina coating but very unreliable with the graphite coating. The fact is that the experiments were performed under the following sequence for the initial temperature: 200 (1), 240 (1), 300, 280, 240 (2), 200 (2). If a sequential line was drawn through the figures it would pass through the lowest dots from left to right and then through the highest dots on the way back. This suggests that the coating ages in use. The effect of the ageing can be better interpreted from Fig. 7 (left) and comparison with the experiment performed by Schmidt [7] and by Sciama and Ribou [8]. For the first use of the coating at 240°C, the heat flux increases rapidly as the molten aluminium comes into contact with the die. A slight decrease follows, which corresponds to the cooling of the liquid and the warming of the die. A second increase can be seen, and this can be related to a recalescence phenomena. Finally a last decrease occurs when an air gap forms. For the

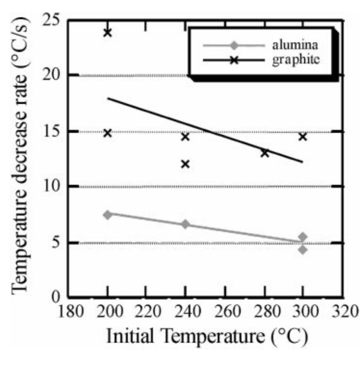

Fig. 6. Influence of the coating and of the initial temperature on the temperature increase at the very first instant.

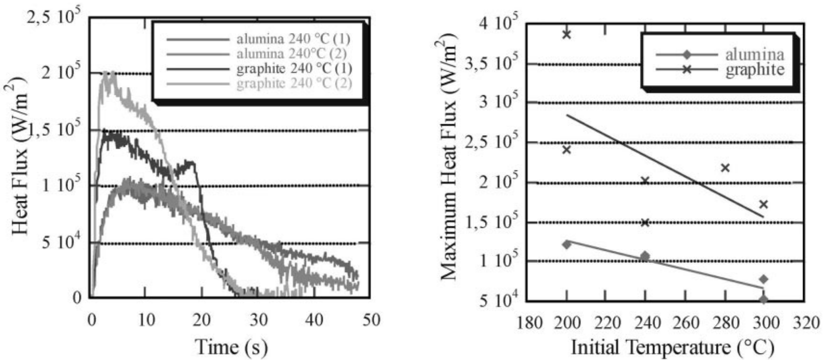

Fig. 7. Influence of the time on the heat flux at 0.5 mm inside the die (left). Influence of both the coating and the initial temperature on the maximum heat flux estimated on the moulding surface (right).

second use of the coating at 240°C, the recalescence disappears. The final drop of the heat transfer is not significantly affected by the recalescence phenomena.

Recalescence is well known to be connected to nucleation under a high supercooling. The ageing of the coating could be explained by a change of its reactivity: when new, the heterogeneous nucleation of new grains on the coating is slow to occur and a high supercooling is needed. On the contrary, the aged coating is much more reactive and a lower undercooling is necessary for new grains to nucleate. Further investigation is necessary to confirm this interpretation.

The thermal stresses within the die can be estimated with the expressions (17)–(20). Such an expression is deduced from a thermo-elastic analysis of a semi-infinite plate [9]. The mechanical conditions on the plate are such that it is free to distort. This distortion could be evaluated in terms of curvature, however, this is not the objective of the present paper. The stress tensor in such a plate is diagonal and depends on the depth inside the die and on time. It can be summarised as Eq. (17), with x being the depth axis and the (y, z) plane corresponding to the moulding surface

$$\bar{\sigma}(x,t) = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \sigma(x,t) & 0 \\ 0 & 0 & \sigma(x,t) \end{bmatrix} \quad (17)$$

with

$$\sigma(x,t) = \frac{1}{1-v}\left\{\alpha E\left[-T(x,t) + \frac{1}{2h}N_t + \frac{3(x+h)}{2h^3}\Pi_t\right]\right\} \quad (18)$$

$$N_t = \int_0^{2h} T(x,t)\,\mathrm{d}x \quad (19)$$

$$\Pi_t = \int_0^{2h} (x+h)T(x,t)\,\mathrm{d}x \quad (20)$$

Fig. 8 (left) shows the thermal stresses within the die at the instant when its value at the moulding surface ($x = 0$) is at its maximum (in absolute value). This instant is not the same for the two coatings, nor the temperature at 0.5 mm (and then on the surface). It should be noted that the stress is compressive on the working surface and tensile deeper in the die. The amplitudes are, however, very different. Between the two coatings the maximum compressive stress differs by a factor 2. The maximum compressive stress also depends on the initial temperature of the die, as Fig. 8 (right) demonstrates. Basically, one can summarise by: the less the initial temperature gap between the aluminium and the die and the more conductive the coating, the larger the compressive stresses on the moulding surface. The same interpretation

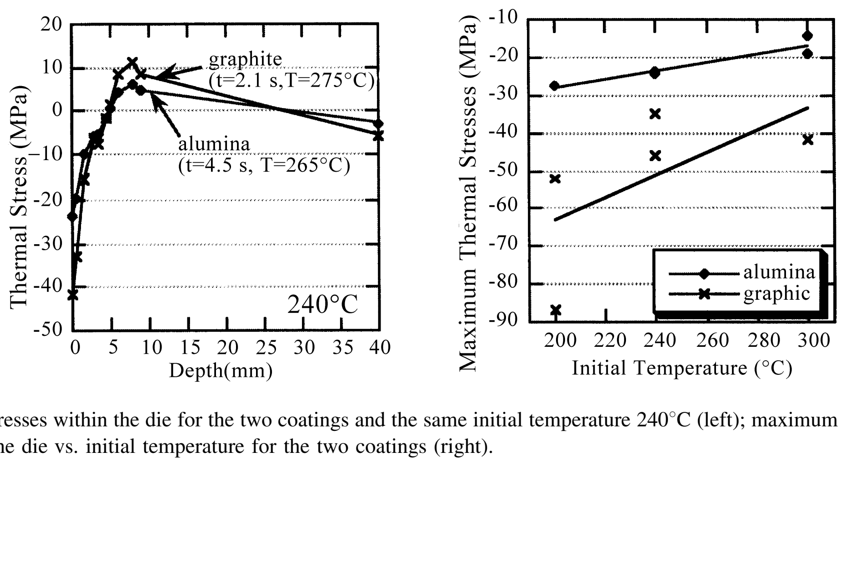

Fig. 8. Maximum thermal stresses within the die for the two coatings and the same initial temperature 240°C (left); maximum compressive thermal stresses at the moulding surface of the die vs. initial temperature for the two coatings (right).

regarding the repetition of the results for the alumina and the graphite coatings on the heat transfer applies for the thermal stresses.

5. Conclusion

A parallelepiped die was used and equipped with a specially designed gauge in order to measure the temperature field through the die at different times. The measurements, were performed with 15 thermocouples brazed within an inserted cylinder. The hot junctions were positioned along an helicoidal coil.

From these measurements it is possible to determine the heat flux transferred from the melt to the die. Such an evaluation requires an inverse method, that has been previously developed in the neighbouring Energy-Environment Research Centre and adapted to the casting conditions. The measured heat flux has been compared to the heat exchanged with the part in contact with the die in a specific experiment. The comparison is favourable, and a good correlation exists between the measurements.

From the temperature measurements it is possible to estimate the thermal stresses in the die. Such an evaluation requires a classical thermo-elastic analysis and is formulated in forms of an analytical expression with $T(z, t)$ as a variable.

The effect of two process parameters on the heat transfer and on the thermal stresses is analysed. Basically one can state: the lower the temperature of the mould and the more conductive the coating, the higher the heat transfer and the higher the stresses. However, the coating may age, with a dramatic effect on the thermal stresses (a factor 2 can be expected) and on the heat transfer. Nevertheless, the formation of the air gap, that definitely affects the heat transfer, does not appear to be much affected.

The perspective of this work is threefold:

- To perform measurements for other casting processes involving higher heat transfer, such as high pressure die casting and counter pressure die casting. It should be possible to get thermal stresses close to the yield stress of the die steels.
- To correlate our estimates of thermal stresses on a thermo-mechanical experiment and control the assumption of thermo-elasticity. On the other hand, a visco-plastic behaviour of the die materials should be adapted and modelling of thermal fatigue should follow.
- To give an interpretation of the ''curves anomaly'', as can be seen in Fig. 7. The reactivity of the coating could be used as a protection of the die against thermal fatigue without affecting dramatically the global heat transfer (as the air gap formation is not affected).

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