Lyapunov Global Stability for a Reactive Mobile Robot Navigation in Presence of Obstacles
Ahmed Benzerrouk, Lounis Adouane, Philippe Martinet

To cite this version:

HAL Id: hal-01714866
https://hal.archives-ouvertes.fr/hal-01714866
Submitted on 23 Feb 2018

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers. L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
Lyapunov Global Stability for a Reactive Mobile Robot Navigation in Presence of Obstacles

Ahmed Benzerrouk, Lounis Adouane and Philippe Martinet
LASMEA, Blaise Pascal University
24, Avenue des Landais, 63177 Aubière, France.
firstname.lastname@lasmea.univ-bpclermont.fr

Abstract—This paper deals with the navigation of a mobile robot in unknown environment. The robot has to reach a final target while avoiding obstacles. It is proposed to break the task complexity by dividing it into a set of basic tasks: Attraction to a target and obstacle avoidance. Each basic task is accomplished through the corresponding elementary controller. The activation of one controller for another is done according to the priority task. To ensure the overall stability of the control system, especially at the switch moments, properties of hybrid systems are used. Hybrid systems allow switching between continuous states in presence of discrete events. In this paper, it is proposed to act on the gain of the proposed control law. The aim is to ensure the convergence of a common Lyapunov function to all the controllers. This ensures the stability of the overall control. Simulation results confirm the theoretical study.

I. INTRODUCTION

The control of a mobile robot navigating in a cluttered environment is a fundamental problem and is receiving much attention in the robotics community. The purpose is mainly to ensure to the mobile robot a suitable and a safe navigation (avoiding a risk of collision, respecting its structural constraints, etc.).

Some of the literature considers that the robot control is entirely based on the methods of path planning while involving the total or partial knowledge of its environment: Voronoi diagrams and visibility graphs [1] or Artificial potential fields functions containing all the information on the target [2] and the robot environment are among these methods. Another community is interested by the ability of the robot to achieve the control laws according to its constraints (structural constraints, jerk-control, etc.). Even if cognitive methods of path planning and replanning [3], [4], can also be found here, more reactive methods (based on sensors information rather than a prior knowledge of the environment) are more common [5], [6] or [7]. The proposed work falls into the latter approach.

To ensure the robot’s ability to accomplish a reactive task, it is proposed to explore behavioral control architectures originally proposed by Brooks [8]. This kind of architecture of control breaks the complexity of the overall task by dividing it into several basic tasks. Each basic task is accomplished with its corresponding controllers. There are two major principles of coordinating them: the action selection [8] and merging actions [9]. In the first, only one controller selected from the basic controllers is applied to the robot at every sample time. In the second case, the control applied to the robot is a result of merging all or a part of available controllers in the control architecture.

We note that the action selection is more interesting. Indeed, one controller is applied to the mobile robot at a given time. It is then easier to examine individual stability of each controller. However, random switch from one controller to another (avoiding obstacles, follow a trajectory, reaching a target, etc.) may cause instability of the global control law, even if each individual controller is stable [10].

Stability proof of this kind of control architecture has been little explored in the literature: in [5], a merging action node is introduced to the control automaton in order to smoothly switch between the two controllers. The advantage of studying each controller alone is then lost, since we have also to study the merging action node. Controlling a mobile robot to follow a trajectory in presence of obstacles, based on the theorem of multiple Lyapunov functions [10] was established in [11]: A third secondary controller was then introduced to satisfy this theorem. However, this control architecture is not suitable for any cluttered environment.

Finding a common Lyapunov function to the basic systems forming a hybrid system is not a simple task [12]. In this paper, we propose to deal with this problem by ensuring overall stability of our control architecture with a single Lyapunov function. Here we are interested by a mobile robot reaching a target while avoiding obstacles: this task is then divided into two basic tasks: attraction to a target and obstacle avoidance.

The rest of the paper is organized as follows: in next section, the basic controllers and the proposed control law are introduced. The proposed control architecture is exposed in Section III. Simulation results are given in IV. Finally, we conclude and give some prospects in Section V.

II. ROBOT MODEL AND TASKS TO ACHIEVE

Before introducing attraction to the target controller, obstacle avoidance and the proposed control law, we recall that the kinematic model of the used unicycle mobile robot used is expressed by the well-known equations:

\[
\begin{align*}
\dot{x} &= v \cos(\theta) \\
\dot{y} &= v \sin(\theta) \\
\dot{\theta} &= \omega
\end{align*}
\]  

(1)

With
The trajectory of the robot are given by the following system (15). The differential equations representing the desired brief controller is based on the limit cycle methods [14], [15]. The control law is expressed as follows: where

\[ v = v_{\text{max}} e^{-\frac{1}{2} \cos(\tilde{\theta})} \quad (a) \]
\[ \omega = \omega_r + k_1 \tilde{\theta} \quad (b) \]

The proposed control law is expressed as follows:

\[ V = \frac{1}{2} \tilde{\theta}^2 \]

The control law is asymptotically stable if \( \dot{V} < 0 \).

\[ \dot{V} = \tilde{\theta} \dot{\tilde{\theta}} \]

By replacing (6) in (8.b), we get

\[ \dot{\tilde{\theta}} = -k_1 \tilde{\theta} \]

and \( \dot{V} \) becomes

\[ \dot{\tilde{\theta}} = -k_1 \tilde{\theta} \]
\[ \dot{V} = -k_1 \tilde{\theta}^2 < 0 \quad (10) \]

and \( \dot{V} \) becomes for every \( \tilde{\theta} \neq 0 \).

The controller is then asymptotically stable. Once each basic task and the control law are defined, the proposed architecture of control which coordinates them is given in next section.

### III. THE PROPOSED ARCHITECTURE OF CONTROL

Even if each controller is individually stable, it is important to constrain switch between them to avoid instability of the overall system, see [10]. Here, it is proposed to generalize the Lyapunov function previously defined (cf. Section II-C) for the overall control system. Indeed, it was proved (cf. Section II-C) that this function is strictly decreasing. However, the problem arises (as for all hybrid systems) at switching moments where the set point is discontinuous. This means that there is an unavoidable jump of the error \( \tilde{\theta} \) at these moments. This naturally leads to jumps in the Lyapunov function after the switch and this jump may lead to increasing it.

Hence, it is proposed to adjust the gain \( k_1 \) of the control law (cf. Equation 8) at the switch moments so that even if the value of the Lyapunov function increases during the switch, it returns to its value before switch \( V(t_{bs}) \) in a finite time \( T_{max} \). \( t_{bs} \) is the moment just before switch.

In addition, the robot should not navigate more than a distance \( d_{max} \) when \( (V(t) > V(t_{bs})) \) in order to insure stability criterion as soon as possible. Also, when the robot performs the obstacle avoidance task, it is necessary that \( (d_{max} < \epsilon) \) (cf. Section II-B) to avoid collision with the obstacle. Notice that \( \epsilon \) is the minimal distance separating the robot from the obstacle once this one is detected (cf. Section II-B).

\[ V(t_{bs} + T_{max}) \leq V(t_{bs}) \quad (11) \]

Where \( t_s \) is the switch moment.

The resolution of the differential equation (9) gives the orientation error with respect to time \( \tilde{\theta} \)

\[ \dot{\tilde{\theta}}(t) = \tilde{\theta}(t_{bs})e^{-k_1(t-t_s)} \quad (12) \]

Equation (12) allows to easily deduce the Lyapunov function:

\[ \dot{V}(t) = \frac{\tilde{\theta}^2(t)}{\epsilon}e^{-2k_1(t-t_s)} \quad (a) \]
\[ V(t) = V(t_{bs})e^{-2k_1(t-t_s)} \quad (b) \]
\[ V(t) = V(t_{bs})e^{-2k_1(t-t_s)} \quad (c) \]

Thus, \( k_1 \) is expressed as

\[ k_1 = \frac{\ln(V(t)/V(t_{bs}))}{-2(T_{max}-t_s)} \quad (14) \]

Note that \( k_1 \) is always positive. Indeed, \( V(t) \leq V(t_{bs}) \) (cf. Equation 13) and then \( \ln(V(t)/V(t_{bs})) \leq 0 \).

The value of \( k_1 \) allowing to reach \( V(t_{bs}) \) in a finite time \( T_{max} \) is

\[ k_1 = \frac{\ln(V(t_{bs})/V(t_{bs}))}{-2T_{max}} \quad (15) \]

Note that the restriction on \( T_{max} \) is necessary especially in the case of obstacle avoidance. Indeed, the stability criterion of hybrid systems (cf. Equation 11) must be satisfied in minimal time. Moreover, the distance achieved during \( T_{max} \) has to be \( (d_{max} \leq \epsilon) \) (cf. Section II-B) to avoid collision with the obstacle. It is easy to see that the minimum necessary time to achieve this distance is

\[ t_{min} = \frac{\epsilon}{v_{max}} \]

corresponding to a straight robot navigation to the obstacle center with its maximum linear velocity. (15) becomes then

\[ k_1 = \frac{\ln(V(t_{bs})/V(t_{bs}))}{-2t_{min}} \quad (16) \]

Note that \( k_1 \) is not defined if \( V(t_{bs}) = 0 \). The notion of weak stability [16] allows to define a threshold \( V_{min} \) such that if \( V(t) < V_{min} \), then the system is (weakly) stable without comparing \( V(t) \) to \( V(t_{bs}) \). It means that

\[ k_1 = \frac{\ln(V_{min}/V(t_{bs}))}{-2t_{min}} \quad (17) \]

Thus, \( k_1 \) is recalculated in this way and replaced in (8b). We can then summarize the proposed control architecture as in figure (Fig. 3).

### B. The mechanism of the architecture of control

The block AS (for Action Selection) selects the suitable controller to apply to the robot according to the environment: if no obstacle is detected, Attraction to the target task is accomplished. If there is a discrete event (switching from one controller to another, transition from attraction to repulsive phase, etc.), the block transition phase prevents the control from affecting the robot’s actuators, until the block adaptation gain recalculates the gain \( k_1 \) as previously highlighted.
To estimate the relevance of the proposed control architecture, it is proposed to simulate a mobile robot navigation to reach a target in presence of obstacles. Simulation is made twice. In the first case, the used control law has a constant gain during all the navigation ($k_1 = 1$) (there is no gain adjustment in the switch moments). Switching control indicating the active controller can be seen in figure (Fig. 4).

In the second case, the proposed control architecture is implemented on the robot. In the two cases, the robot reaches its target while avoiding obstacles. However, by comparing $T_{\text{max}1}, T_{\text{max}2}$ which are convergence times for obstacle avoidance controller in figures (Fig. 4) and (Fig. 6), it is noticed that the Lyapunov function of the proposed architecture of control converges faster than the architecture with a constant gain. Evolution of the gain $k_1$ is given in the same figure (Fig. 6). Note that attraction to the target controller converges fastly in the two cases even if in the proposed architecture, we can see that it is slightly faster.

IV. SIMULATION RESULTS

To estimate the relevance of the proposed control architecture, it is proposed to simulate a mobile robot navigation to reach a target in presence of obstacles. Simulation is made twice. In the first case, the used control law has a constant gain during all the navigation ($k_1 = 1$) (there is no gain adjustment in the switch moments). Switching control indicating the active controller can be seen in figure (Fig. 4).

In the second case, the proposed control architecture is implemented on the robot. In the two cases, the robot reaches its target while avoiding obstacles. However, by comparing $T_{\text{max}1}, T_{\text{max}2}$ which are convergence times for obstacle avoidance controller in figures (Fig. 4) and (Fig. 6), it is noticed that the Lyapunov function of the proposed architecture of control converges faster than the architecture with a constant gain. Evolution of the gain $k_1$ is given in the same figure (Fig. 6). Note that attraction to the target controller converges fastly in the two cases even if in the proposed architecture, we can see that it is slightly faster.

V. CONCLUSION

A control architecture based on hybrid systems has been proposed. With these systems, it is possible to divide the control architecture into a set of elementary controllers to examine each controller separately. Even if each individual controller is stable, global stability is not necessarily guaranteed. In this paper, the overall stability was established thanks to a single Lyapunov function. The proposed idea is to adjust the gain of the control law in order to accelerate convergence of the Common Lyapunov Function CLF after each switch. The simulation results have confirmed the theoretical study. In future works, it is proposed to introduce the gain $k_1$ as a dynamical gain. Thus, once the lyapunov function converges, it returns to its nominal value without disturbing the control.

REFERENCES