Application of Cyclo-Non-Stationary Indicators for Bearing Monitoring Under Varying Operating Conditions

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ABSTRACT

Condition monitoring assesses the operational health of rotating machinery, in order to provide early and accurate warning of potential failures such that preventative maintenance actions may be taken. To achieve this target, manufacturers start taking on the responsibilities of engine condition monitoring, by embedding health monitoring systems within each engine unit and prompting maintenance actions when necessary. Several types of condition monitoring are used including oil debris monitoring, temperature monitoring and vibration monitoring. Among them, vibration monitoring is the most widely used technique. Machine vibro-acoustic signatures contain pivotal information about its state of health. The current work focuses on one part of the diagnosis stage of condition monitoring for engine bearing health monitoring as bearings are critical components in rotating machinery. A plethora of signal processing tools and methods applied at the time domain, the frequency domain, the time-frequency domain and the time-scale domain have been presented in order to extract valuable information by proposing different diagnostic features. Among others, an emerging interest has been reported on modeling rotating machinery signals as cyclostationary, which is a particular class of non-stationary stochastic processes. A process x(t) is said to be nth-order cyclostationary with period T if its nth-order moments exist and are periodic with period T. Several tools, such as the Spectral Correlation Density (SCD) and the Cyclic Modulation Spectrum (CMS) can be used in order to extract interesting information concerning the cyclic behavior of cyclostationary signals. In order to measure the cyclostationarity from order 1 to 4, concise and global indicators have been proposed. However, in a number of applications such as aircraft engines and wind turbines the characteristic vibroacoustic signatures of rotating machinery depend on the operating conditions of the rotational speed and/or the load. During the last decades fault diagnostics of rotating machinery under variable speed/load has attracted a lot of interest. The classical cyclostationary tools can be used under the assumption that the speed of machinery is constant or nearly constant, otherwise the vibroacoustic signal becomes cyclo-non-stationary. In order to overcome this limitation a generalization of both SCD and CMS functions have been proposed displaying cyclic Order versus Frequency. The goal of this paper is to propose a novel approach for the analysis of cyclo-nonstationary signals based on the generalization of indicators of cyclostationarity in order to cover the speed varying conditions. The proposed indicators of cyclo-non-stationarity (ICNS) are expected to summarize the information at various statistical orders and at lower computational cost compared to the Order-Frequency SCD or CMS. This generalization is realized by introducing a new speed-dependent angle averaging operator. The effectiveness of the approach is evaluated on an acceleration signal captured on

the casing of an aircraft engine gearbox, provided by SAFRAN, in the frames of SAFRAN contest which took place at the Surveillance 8 International Conference.

INTRODUCTION

Condition monitoring of rotating machinery is increasingly applied in the industry including the energy and aerospace sectors as it appears able to increase the reliability, the availability, the accuracy and the productivity of the operations while it contributes to the reduction of the number and the duration of unexpected machine breakdowns, which can lead to loss of life, loss of production and environmental pollution. Rolling element bearings can be classified as the most widely used industrial machine elements. A plethora of fault detection and diagnosis methods and approaches have been proposed in order to detect and assess fast, accurately and on time the generation, existence and further development of fault during machinery operation, including the High Frequency Resonance Technique [1], the Hilbert Transform [2], the Complex Shifted Morlet Wavelets [3] and various techniques applied in the time domain, in the frequency domain or the time scale domain. During the last decades, an emerging interest has been reported on modelling vibration signals, captured on rotating machinery, as cyclostationary. Several tools such as the Spectral Correlation Density (SCD) and the Cyclic Modulation Spectrum (CMS) [4] have been proposed as diagnostic tools, assuming constant or almost constant operating conditions (speed/load). In a number of applications, the operating conditions vary continuously leading to a demand for powerful advanced diagnostic methodologies which can cover all the operating range and not only specific operating set points. The goal of this paper is to generalize the indicators of cyclostationarity in order to cover the speed varying operating conditions by introducing a new averaging operator. The effectiveness of the methodology is demonstrated on an acceleration signal captured on an aircraft engine gearbox casing. The rest of the paper is organized as follows. Firstly the theory of the generalized indicators of cyclostationarity is presented and analyzed. Moreover a case study of an aircraft gearbox is presented and the vibration signal is processed. Furthermore the proposed indicators are applied on the signal and some first results are presented. The paper closes with some basic conclusions.

GENERALIZED INDICATORS OF CYCLO-STATIONARITY

Short introduction to cyclostationary indicators

Nowadays it is widely accepted that, under stationary regime, faulty rolling element bearings produce mechanical signatures which are perfectly characterized within the theory of cyclostationary processes [1]. Generally speaking, a process is cyclostationary if its statistics are periodic. Different orders of cyclostationarity are defined depending on the type of statistics that is considered. A signal whose the first-order statistics (i.e. the ensemble average) is periodic is said to contain first-order cyclostationarity. This is actually equivalent to recognizing that the signal comprises a periodic component. A signal whose second-order statistics (i.e. as returned by the instantaneous autocorrelation function) is periodic is said to contain second-order cyclostationarity, which reflects the existence of a periodic flow of its energy. This is typically the case of broadband processes which are periodically modulated, either in amplitude or in phase. More generally, a signal is said to contain n-th order cyclostationarity if its n-th order statistics evidences periodicity. For practical reasons, higher-orders beyond orders 3 or 4 are rarely considered. The third-order cyclostationarity reflects the tendency of the signal to have a periodic asymmetry whereas the fourth-order cyclostationarity reflects periodicity in its impulsive behavior.

As demonstrated in Ref. [6], the assessment of cyclostationarity up to order 4 may provide a valuable tool for the health monitoring of mechanical components such as gears. The different orders allow for a more accurate diagnosis of the system being monitored and components which are characterized by different cyclic frequencies can be controlled individually. One important objective of Ref. [6] was to propose scalar (rather than multidimensional) indicators of n-th order cyclostationarity in order to comply with constraints of simplicity.

Namely, given a signal $x(t)$ and a cyclic frequency α that denotes a suspected fault frequency in the system, the indicators of cyclostationarity can be constructed as follows. On the first-order (n=1), all components in the signal which are periodic with a fundamental frequency $a = 1/T$ are extracted, up to a given number of harmonics, by means of the P-operator, thus resulting in the T-periodic contribution (the so called first-order cumulant):

$$C_{1x}(t) = P\{x(t)\} \tag{1}$$

By construction, $C_{1x}(t) = C_{1x}(t+T)$, and it contains no other component than those being synchronous with the fault frequency. The exact form of the P-operator is given in Ref. [6] and further discussed in Ref. [7]. An estimate of the first order is actually returned by the Synchronous Average (SA). Similarly, on the "pure" second-order (n=2), the P-operator is used to extract the T-periodic contribution of the energy flow as:

$$C_{2x}(t) = P\{|x_c(t)|^2\} \tag{2}$$

where $x_c(t) = x(t) - P\{x(t)\}$ is the residual part of the signal which is purposely considered here to remove the first-order cyclostationary contribution. This is related to the so-called second-order cumulant. A direct generalization provides the third-order cumulant:

$$C_{3x}(t) = P\{|x_c(t)|^3\} \tag{3}$$

In order to comply with the mathematical definition of cumulants, the fourth-order quantity is strictly defined as

$$C_{4x}(t) = P\{|x_c(t)|^4\} - 3|C_{2x}(t)|^2 \quad (4)$$

which carefully removes all possible contributions from orders 1 and 2 (it can be shown that the third-order does not participate to the fourth-order when the residual signal $x_c(t)$ is considered). Next, since periodic phenomena (with non-zero fundamental frequency) are of concern, it makes sense to concentrate on the cyclic part only of the so-defined cumulants by subtracting the time average (DC part) $\langle C_{nx}(t)\rangle$. Based on these quantities, the indicator of cyclostationarity (ICS) of order n, ICS_{nx} is defined as the measure of the intensity of the n-th order contribution to the signal power, i.e.

$$ICS_{nx} = \frac{\langle |C_{nx}(t) - \langle C_{nx}(t)\rangle|^2\rangle}{\langle |x(t)|^2\rangle^n} \quad (5)$$

where operator $\langle \cdots \rangle$ stands for the time average.

As demonstrated in Ref. [6], the computation of the four scalar indicators ICS_{nx}, n=1,…,4, provides a simple and efficient solution to the condition monitoring of rotating components such as gears. The object of this work is to extend these ideas to the health monitoring of rolling element bearings under non stationary regimes. Formally speaking, this addresses the case of cyclo-non-stationary processes on different orders.

Definition of indicators of cyclo-non-stationarity

Cyclo-non-stationarity generalizes the concept of cyclostationarity to nonstationary operating regimes. Briefly stated, it consists in conditioning the statistics to the rotation speed ω of the component of interest. Having said this, the indicator of cyclo-non-stationarity (ICNS) of order n is readily noted as $ICNS_{nx}(\omega)$ with an explicit dependence on the rotation speed. Its exact definition is however not that straightforward. Firstly, it requires the signal to be considered in the angular domain rather than in the time domain in order to keep constant the values of the cyclic frequencies α that are synchronous with the rotation speed ω whatever the value of ω is. Secondly, it requires a generalization of the P-operator conditioned to ω that extracts all periodic components in a signal (or exponentiated versions of it) with fundamental period $1/\alpha$. Specifically, upon introducing the angle variable θ such that $d\theta/dt = \omega$, the P-operator is substituted for its generalized version in Eqs. (1) to (4) to extract the speed-conditioned (angular) cumulants $C_{nx}(\theta;\omega)$, n=1,…,4. By construction, $C_{nx}(\theta;\omega) = C_{nx}(\theta + 1/\alpha\,;\omega)$ when ω is held constant; however, in general $C_{nx}(\theta;\omega_1) \neq C_{nx}(\theta + 1/\alpha\,;\omega_2)$ when $\omega_1 \neq \omega_2$. This is directly related to the new Generalized Synchronous Average (GSA) which was recently introduced in Ref. [8]. Finally, the ICNS's are defined as

$$ICNS_{nx}(\omega) = \frac{\langle |C_{nx}(\theta;\omega) - \langle C_{nx}(\theta;\omega)\rangle|^2\rangle}{\sigma(\omega)^{2n}} \quad (6)$$

where the operator $\langle \cdots \rangle$ stands for the angle average and $\sigma(\omega)^{2n}$ for the signal power at speed ω.

It is important to remark that the cyclic frequency α used in the definition does not necessarily equal the rotation speed ω in general, yet it is assumed that the ratio $2\pi\alpha/\omega$ is a constant – known as "order" in rotor dynamics (this notion of "speed order" is not to be confused with the notion of "cyclo(non)stationary order" introduced before even though they share the same terminology).

The generalized P-operator

Recently, the "Generalized Synchronous Average" (GSA) has been introduced in Ref. [8] to estimate the synchronous deterministic part of cyclo-non-stationary signals. It is able to accommodate with the amplitude and phase modulation induced by the speed variation. However, it requires the cyclic order set to be a multiple of some fundamental order. In this work it is aimed to propose a generalized P-operator to extent the GSA in order to be able to extract the synchronous deterministic part of any cyclic order set.

Since the deterministic part is concentrated in the order domain around the harmonics related to the signal cyclic order set, it can be thus reasonably estimated by narrow bandpass filtering around these harmonics. The filter bandwidth must cover the leakage induced by the speed variations. Building on this, the generalized P-operator of a signal $x(\theta)$ is defined for a cyclic order set $\alpha_i \in$ A:

$$\mathrm{P'}\{\mathrm{x}(\theta)\} = \mathrm{x}(\theta) \otimes \sum_{\alpha_i \in \mathrm{A}} \mathrm{f}_{\alpha_i}(\theta; \varDelta\alpha) \quad (7)$$

where $\mathrm{f}_{\alpha_i}(\theta;\varDelta\alpha)$ is a narrow bandpass filter of central order α_i and bandwidth $\varDelta\alpha$.

Monitoring strategy for rolling element bearings

The indicators of cyclo non stationarity ICNS are proposed in this work to be used as diagnostic indicators focusing to the health monitoring of rolling element bearings. In this case the ICNS should be estimated for all cyclic frequencies α that correspond to possible characteristic fault frequencies, e.g. the Ball-Pass Frequency on the Inner Race (BPFI), the Ball-Pass Frequency on the Outer Race (BPFO), the Spin Frequency (SF), etc. As the signals are resampled in the angular domain the corresponding characteristic fault orders are used in the place of frequencies e.g. the Ball-Pass Order on the Inner Race (BPOI), the Ball-Pass Order on the Outer Race (BPOO), the Spin Order (SO), etc. when considering the ratio $2\pi\alpha/\omega$. Note also that because of the absence of physical phase locking mechanism in bearings, the fault orders are likely to experience slight shifts in time especially when the rotation speed is varying [9]. Therefore, the ICNS's may have to be computed for a set of possible values in a small band around the expected fault frequencies. Furthermore the number of the fault harmonics/orders which will be taken into account as well as the band of the filters are parameters which will be optimized in future work.

The ICNS's corresponding to all possible fault frequencies should be compared as functions of the rotation speed ω. It is expected that faulty rolling element bearings will exhibit an increase of the ICNS's corresponding to the incriminated fault frequency, in particular

- on order n=1 when the fault frequency and its first harmonics occur in a band of high signal-to-noise-ratio
- on order n=2 when the fault is carried by components so high in frequency that they are of a nearly random nature
- on order n=4 due to the impulsive nature of bearing faults in their early stage (localized defects)
- at some critical values of the speed ω which are more favorable to the appearance of the bearing faults.

It should be emphasized that the diagnostic indicators are proposed in order to cover industrial cases where the shaft rotation speed of the machinery under monitoring does not simply vary in specific speed levels/regimes but may vary continuously in time such as in the case of wind turbines and aircraft engines. The main idea is that the proposed diagnostic indicators can be estimated in regular time intervals and their values can be trended in real time focusing towards online monitoring systems. Additionally the trending of such fault indicators can be further expanded and used in order to perform prognostics and estimate the remaining useful life (RUL) of the monitored bearings.

APPLICATION ON AN AIRCRAFT ENGINE GEARBOX

Analysis of the aircraft engine signal

The proposed methodology allows for the calculation of the CNS indicators exploiting the information provided by a tachometer (or an encoder). In case a tachometer is not available the instantaneous angular speed can be estimated by the vibration signals using advanced signal processing tools such as the Multi-order probabilistic approach-MOPA [11]. In order to validate its effectiveness, the methodology is applied on a vibration signal of the SAFRAN Contest [10]. The SAFRAN contest was organized in the frames of the International Conference Surveillance 8 which took place in Roanne in France on the 20th and 21st October 2015. SAFRAN provided three vibration signals (acceleration signals without units) mounted on the gearbox casing of an aircraft engine and a tachometer signal. The kinematics as well as the characteristic frequencies of the gearbox (referenced to the HP shaft rotating speed N2) are presented at Figure 14 and at Table 1, respectively. The aim of the contest was the extraction of the nonstationary rotation speed of the HP shaft (N2) and the detection of a fault in one or more of the three bearings supporting the shafts L1, L4 and L5 from vibration signals captured by an accelerometer mounted on the gearbox casing. In this application the tachometer signal captured by a tachometer mounted on the shaft L4 was used as well as one of the vibration signals, captured on the casing of the gearbox. The resolution of the tachometer is 44 pulses per revolution. The speed signal and the vibration signal have been recorder simultaneously with a sampling frequency Fs equal to 50 KHz while the record duration is 3.4 minutes. The instantaneous speed of the shaft L4 was estimated using the tachometer signal and is presented in Figure 1. The aircraft engine is operating in stationary condition during approximately the 45 first seconds and then speeds up. During the last 40 seconds the engine operates again at steady speed. A part of the vibration signal is presented at Figure 2.

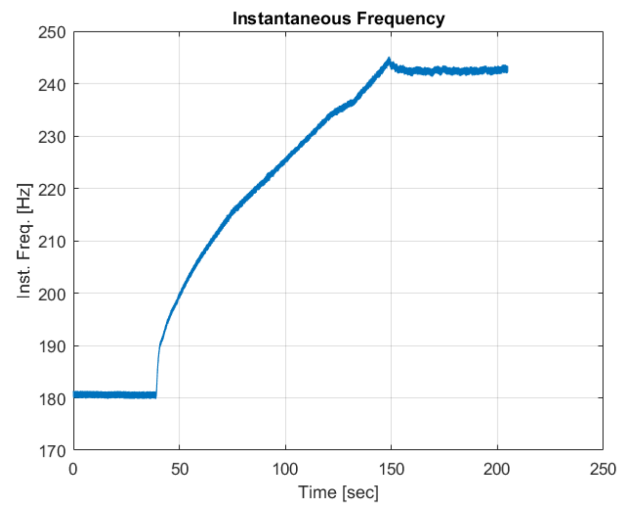

Figure 1: Instantaneous rotating frequency of the shaft L4

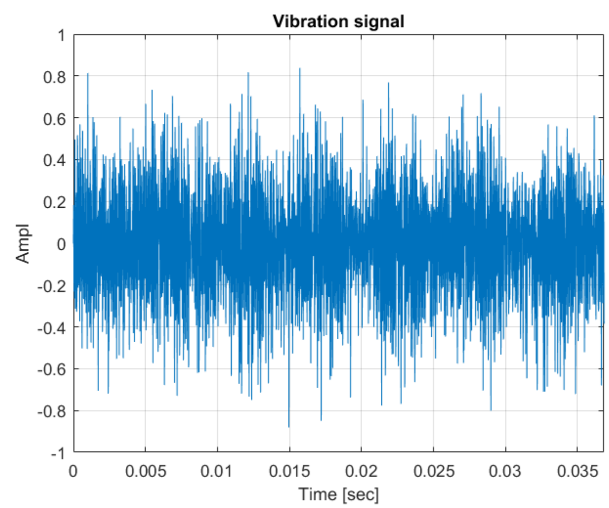

Figure 2: Vibration signal

The instantaneous speed of the shafts L5 and L1 are estimated using the gearbox kinematics: $L5 = \frac{62}{61} L4$ and $L1 = \frac{32}{31}\frac{62}{47} L4$. The instantaneous rotating speed of shaft L5 is further used in order to resample the vibration signal in the angular domain using piecewise cubic spline interpolation. The order spectrum is presented in Figure 3. A number of high amplitude orders are visible at the order spectrum. The signal was further filtered using a 8th order band pass Butterworth filter with a lower cutoff order OL equal to 80 and a higher cutoff order OH equal to 130. Finally the Hilbert transform was applied on the filtered signal in order to extract the envelop of the filtered signal. The envelop order spectrum of the filtered signal is presented at Figure 4. A fault at the outer race of the bearing supporting the shaft L5 is clearly identified in the envelop order spectrum as the first 4-5 harmonics of the BPFO (around 7.759) order are present (red arrows). Furthermore the BPFO is modulated by a cage fault frequency (0.431 order) which is also visible at low orders

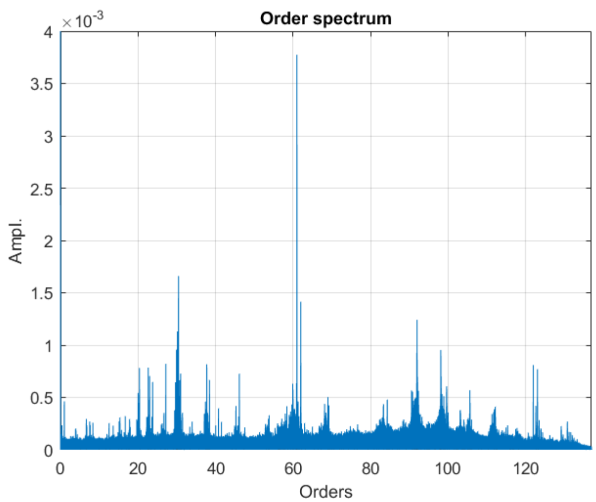

Figure 3: Order Spectrum

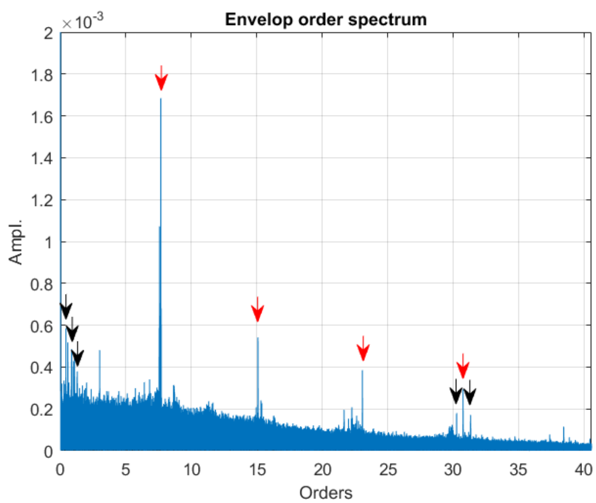

Figure 4: Envelop order spectrum of the filtered signal

Estimation of the cyclo non stationary indicators

The vibration signal is further processed and the indicators of the cyclo non stationarity of order 2, 3 and 4 are estimated. The vibration signal is cut in 10 equal parts. The indicators are estimated for the four different fault orders of the three bearings which support the shafts L1, L4 and L5: the Ball pass order on the Outer Race (OR), the Ball pass order on the Inner Race (IR), the Ball pass order of the Rolling Element (RE) and the Ball pass order of the Cage (CA). Based on the four fault orders, four different numbers of samples per revolution are selected. Each signal part is angular resampled based on the number of samples per revolution in order to pass to the angular domain. Five harmonics for each fault order are used for the calculations while the order domain filter width is selected equal to 3%. The indicators or order 2, 3 and 4 are estimated for each of the four bearing fault order and for each part of the signals. The indicator values and the mean value of the shaft rotation speed of each signal part are firstly sorted and the results for the shaft L5 are presented in Figure 5, Figure 6 and Figure 7. The indicators seem to increase with the speed. The indicators which correspond to the fault at the outer race present higher values in a number of speeds and as a result can be considered as potential damage indicators. The corresponding indicators for shafts L1 and L4 are presented at Figure 8 - Figure 13. The indicators of L1 and L4 present values which are lower than the indicators of L5 and in general do not demonstrate a significant increase.

In contrast to the gears where the kinematics and the gear mesh are phase locked, the signals emitted by defected bearings may experience a shift at the fault orders especially if the speed operating conditions are changing. For the aforementioned reason the authors are currently working on proposing criteria for the automated selection of a band around the expected fault orders where the indicators should be calculated.

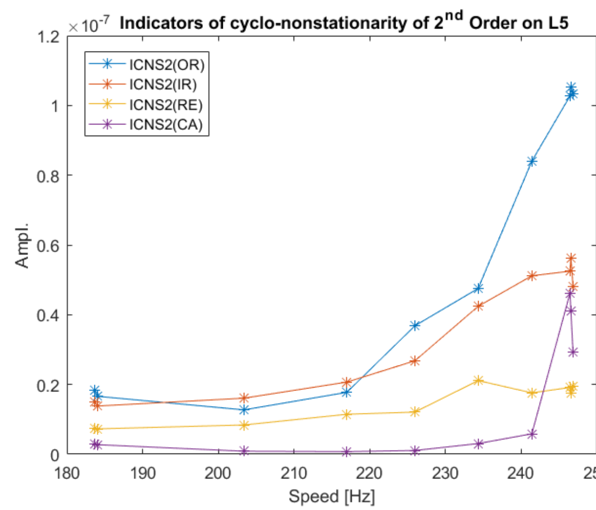

Figure 5: Indicators of cyclo-nonstationarity of 2nd Order on L5

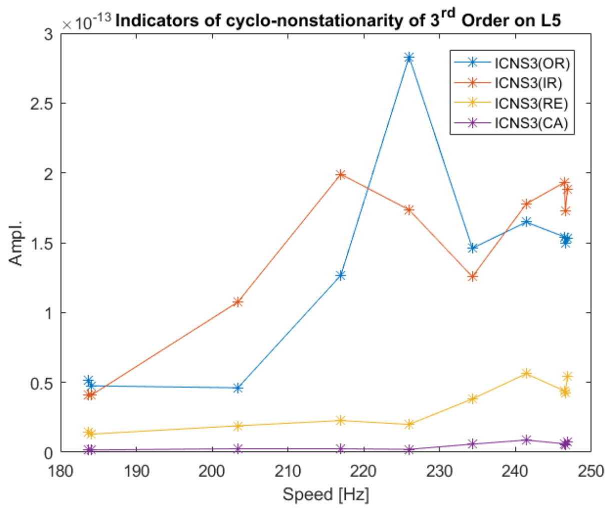

Figure 6: Indicators of cyclo-nonstationarity of 3rd Order on L5

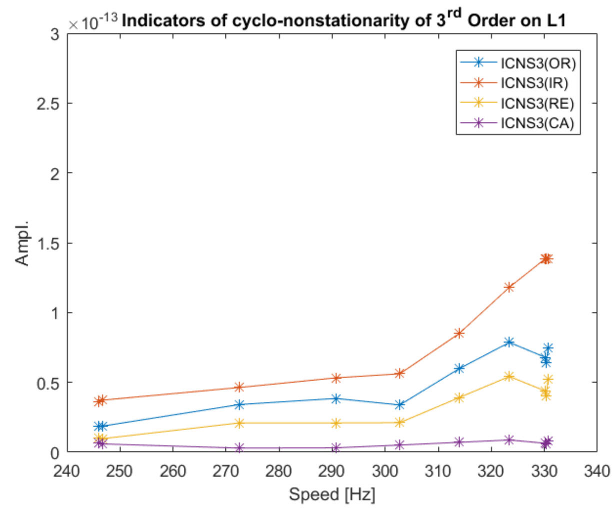

Figure 9: Indicators of cyclo-nonstationarity of 3rd Order on L1

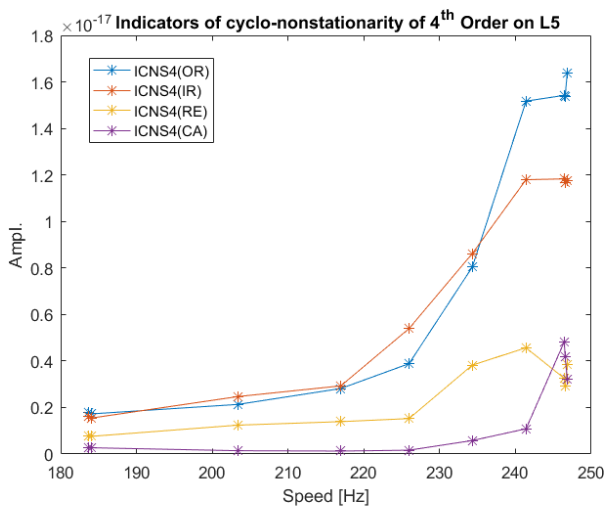

Figure 7: Indicators of cyclo-nonstationarity of 4th Order on L5

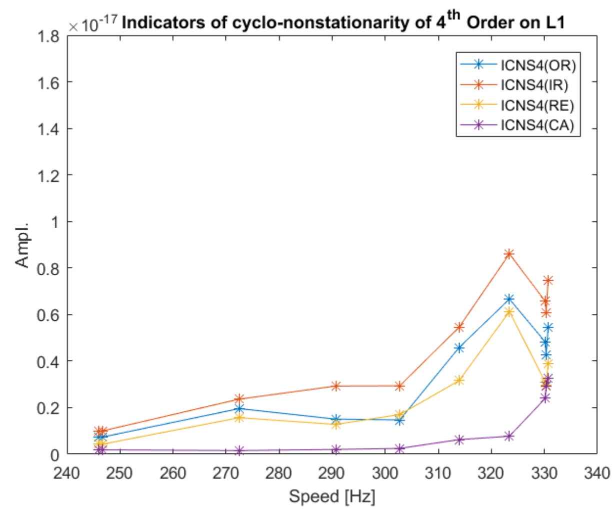

Figure 10: Indicators of cyclo-nonstationarity of 4th Order on L1

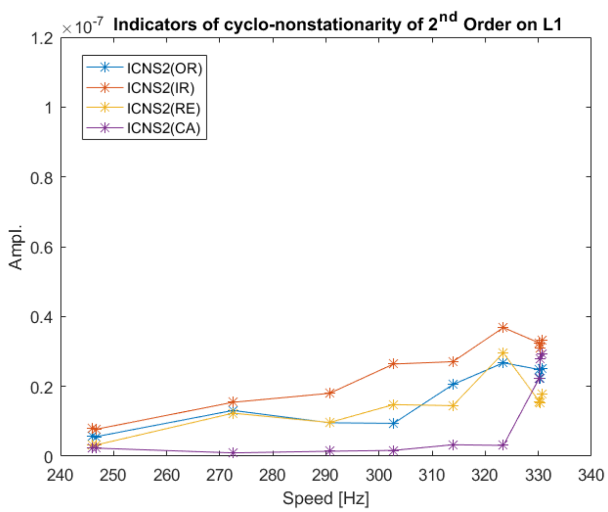

Figure 8: Indicators of cyclo-nonstationarity of 2nd Order on L1

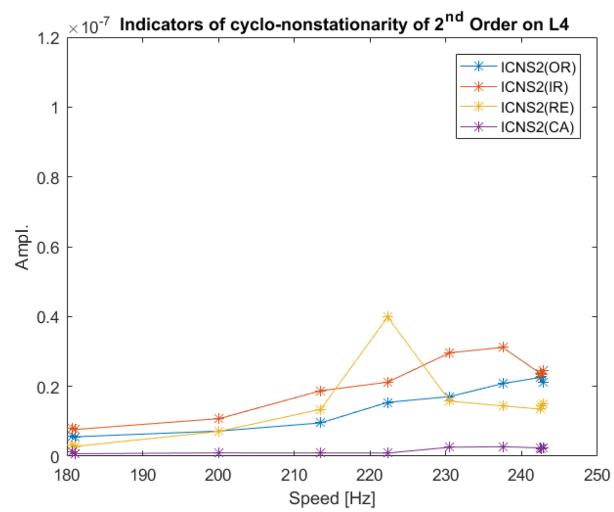

Figure 11: Indicators of cyclo-nonstationarity of 2nd Order on L4

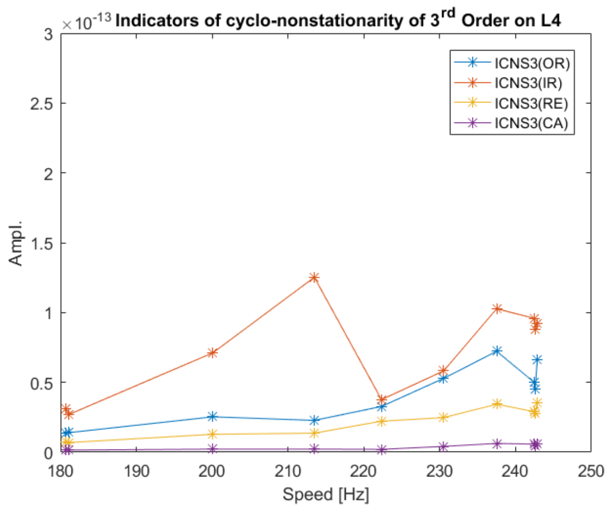

Figure 12: Indicators of cyclo-nonstationarity of 3rd Order on L4

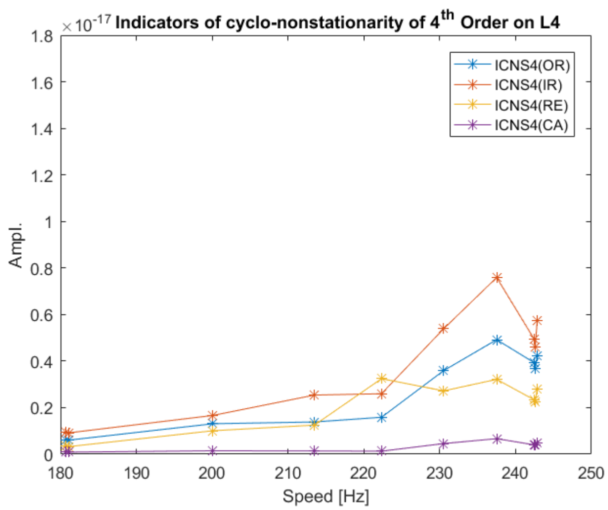

Figure 13: Indicators of cyclo-nonstationarity of 4th Order on L4

CONCLUSION

In this paper a novel approach for the analysis of cyclo-non-stationary signals based on the generalization of indicators of cyclostationarity is proposed focusing towards the health monitoring of rotating machinery operating under continuously varying operating conditions, such as wind turbines and aircraft engines. The generalization is realized by the introduction of a new speed-dependent averaging operator. The proposed indicators of cyclo-non-stationarity (ICNS) summarize the information at various statistical orders. The performance of the indicators is demonstrated using a vibration signal captured on an aircraft engine gearbox under a run up operating condition. The authors are currently working towards the optimization and automatization of the selection of the parameters of the method, such as the width of the bandpass filter and the number of the harmonics used.

ACKNOWLEDGMENTS

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ANNEX A

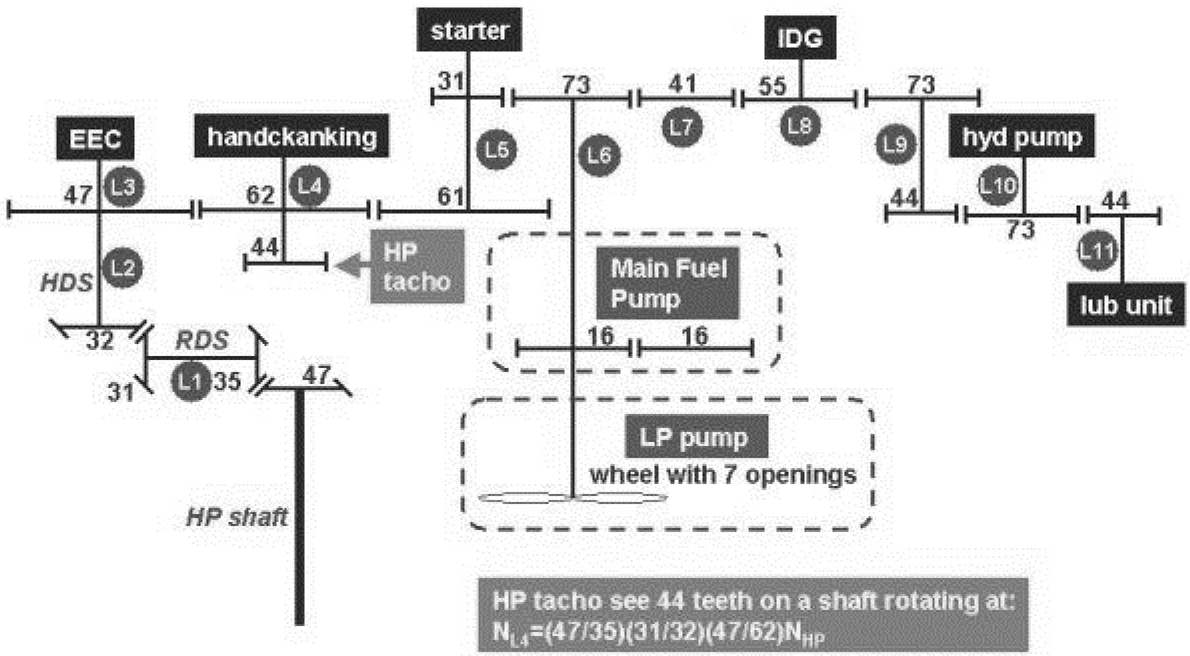

Figure 14: Kinematics of the gearbox

N2 speed (rpm)		line 1		line 2		line 3		N2 tacho visu : 44 teeth	line 4	line 5		line 6		line 7		line 8		line 9		line 10		line 11
10000	HP shaft	RDS		Horizontal Drive Shaft		EEC alternator		handcranking		starter		fuel pump		intermediate		IDG		intermediate		hydro. pump		lubrication unit
	aval	amont	aval	amont	aval	amont	aval	amont	aval	amont	aval	amont	aval	amont	aval	amont	aval	amont	aval	amont	aval	amont
number of teeth	47	35	31	32	47	47	47	62	62	61	31	73	73	41	41	55	55	73	44	73	73	44
rotating speed (rpm)	10000	13429		13009		13009		9862		10023		4256		7579		5649		4256		2566		4256
F 1 tooth (Hz)	167	224	224	217	217	217	217	164	164	167	167	71	71	126	126	94	94	71	71	43	43	71
F 1 tooth (N2)	1.0000	1.3429	1.3429	1.3009	1.3009	1.3009	1.3009	0.9862	0.9862	1.0023	1.0023	0.4256	0.4256	0.7579	0.7579	0.5649	0.5649	0.4256	0.4256	0.2566	0.2566	0.4256
gear mesh frequency (Hz)	7833		6938		10190		10190		10190		5179		5179		5179		5179		3121		3121	
gear mesh (mult N2)	47.0000		41.6286		61.1420		61.1420		61.1420		31.0721		31.0721		31.0721		31.0721		18.7284		18.7284	
Fcoincidence (Hz)	4.8		7.0		216.8		3.5		2.7		2.3		1.7		2.3		1.3		1.0		1.0	

Table 1: Characteristic frequencies of the gearbox referred to HP shaft rotating speed (N2)

	L1	L4	L5
Outer race	4.066	4.03	7.759
Inner race	5.934	5.97	10.241
Rolling element	2.584	2.48	3.556
Cage	0.407	0.403	0.431

Table 2: Characteristic fault frequencies of bearings supporting the shafts L1, L4 and L5 referred to shaft rotations L1, L4 and L5 respectively