



**HAL**  
open science

# Vibroacoustic response of panels under diffuse acoustic field excitation from sensitivity functions and reciprocity principles

Christophe Marchetto, Laurent Maxit, Olivier Robin, Alain Berry

## ► To cite this version:

Christophe Marchetto, Laurent Maxit, Olivier Robin, Alain Berry. Vibroacoustic response of panels under diffuse acoustic field excitation from sensitivity functions and reciprocity principles. *Journal of the Acoustical Society of America*, 2017, 141 (6), pp.4508 - 4521. 10.1121/1.4985126 . hal-01710837

**HAL Id: hal-01710837**

**<https://hal.science/hal-01710837>**

Submitted on 16 Feb 2018

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Vibroacoustic response of panels under diffuse acoustic field excitation from sensitivity functions and reciprocity principles

Christophe Marchetto\* and Laurent Maxit

*Univ Lyon, INSA-Lyon, Laboratoire Vibrations Acoustique, F-69621 Villeurbanne, France*

Olivier Robin and Alain Berry

*Groupe d'Acoustique de l'Université de Sherbrooke,  
Université de Sherbrooke, Sherbrooke, J1K 2R1, Canada*

(Dated: May 11, 2017)

## Abstract

This paper aims at developing an experimental method to characterize the vibroacoustic response of a panel to a diffuse acoustic field excitation with a different laboratory setup than those used in standards (*i.e.*, coupled rooms). The proposed methodology is based on a theoretical model of the diffuse acoustic field and on the measurement of the panel's sensitivity functions which characterize its vibroacoustic response to wall plane waves. These functions can be estimated experimentally using variations of the reciprocity principle which are described in the present paper. These principles can either be applied for characterizing the structural response by exciting the panel with a normal force at the point of interest or for characterizing the acoustic response (radiated pressure, acoustic intensity) by exciting the panel with a monopole and a dipole source. For both applications, the validity of the proposed approach is numerically and experimentally verified on a test case composed of a baffled simply supported plate. An implementation for estimating the sound transmission loss of the plate is finally proposed. The results are discussed and compared with measurements performed in a coupled anechoic-reverberant room facility following standards.

PACS numbers: PACS: 43.40.At, 43.40.Dx

---

\* christophe.marchetto@usherbrooke.ca

## 1 I. INTRODUCTION

2 The experimental vibroacoustic characterization of panels under a diffuse acoustic field  
3 (DAF) excitation is of great interest for the industry. This excitation is commonly used  
4 to determine the sound reduction index of panels as described in several standards using  
5 coupled reverberant-reverberant room [1] [2] or reverberant-anechoic room [3] [4] laboratory  
6 facilities. Theoretically, a DAF is defined as an infinite set of uncorrelated plane waves with  
7 uniformly distributed incidence angles. In standard laboratory measurements, this excita-  
8 tion is reproduced using a reverberant room and only partially corresponds to its theoretical  
9 definition, especially below the Schroeder frequency of the room where the sound field is  
10 dominated by well-defined acoustic cavity modes. Even above the Schroeder frequency, the  
11 pressure field is not perfectly homogeneous and the lack of grazing incidence plane waves  
12 has been pointed out in the literature. Inter-laboratory variations of vibroacoustic measure-  
13 ments in reverberant rooms can be attributed to these phenomena, but other parameters  
14 are involved such as room dimensions, niche effects, panel mounting conditions, aperture  
15 size and measuring protocols [5] [6] [7].

16 In this context, the aim of this study is to investigate an alternative and robust approach  
17 to experimentally characterize a panel's response to a DAF excitation by using only the  
18 theoretical model of this excitation to overcome the limitations of a reverberant room mea-  
19 surement discussed above. Indeed, the mathematical formulation of a panel's vibro-acoustic  
20 response when submitted to random excitations in the wavenumber domain allows estimat-  
21 ing the system's response, at any point on the structure or in the acoustic medium, from  
22 wall-pressure cross spectral density (CSD) functions (characterizing the excitation) and from  
23 so-called 'sensitivity functions', which were introduced in [8] [9] for the analogous problem  
24 of panels excited by a turbulent flow. The latter are defined as the panel's response to wall-  
25 pressure plane waves and characterize the panel's vibroacoustic behavior. The estimation of  
26 the panel's response submitted to a DAF excitation therefore only requires the experimental  
27 measurement of sensitivity functions in the acoustic wavenumber domain.

28 A method is proposed for estimating the sensitivity functions experimentally. Whereas  
29 the direct interpretation of the sensitivity functions would require exciting the panel by sets  
30 of wall plane waves, which is not easy from an experimental point of view, an alternative  
31 method based on a reciprocity principle is proposed. The reciprocity principle states that the

32 sensitivity functions at any point on the structure or in the acoustic medium are equivalent  
 33 to the panel's velocity response expressed in the wavenumber domain when the system is  
 34 excited by a vibration or acoustic source at the same point. From an experimental point  
 35 of view, it is then only necessary to excite the system with a vibration or acoustic source  
 36 and to apply a wavenumber transform to the measured transfer function between the panel  
 37 velocity and the source magnitude to obtain the sensitivity functions for a wide range of  
 38 wavenumbers. Globally, the proposed experimental process consists in exciting the panel  
 39 with the source of given magnitude at the point of interest. The spatial vibratory response of  
 40 the panel is then measured with a scanning laser vibrometer. In a subsequent post-processing  
 41 phase, a discrete 2-D wavenumber transform of the measured vibratory field is performed to  
 42 deduce the sensitivity functions. Finally, using the wall-pressure model of a DAF and the  
 43 previously estimated sensitivity functions, the response when the panel is excited by a DAF  
 44 can be deduced at any point on the structure or in the acoustic medium. To evaluate the  
 45 sound transmission loss, this process is repeated for a series of points belonging to a surface  
 46 surrounding the panel to estimate the acoustic intensity at these points when the panel is  
 47 excited by a DAF.

48 The remainder of the paper is organized as follows: the considered vibro-acoustic problem  
 49 and the quantities characterizing a panel under a DAF are described in Sec. II. Then, the  
 50 mathematical formulation of the vibro-acoustic problem is presented in Sec. III where the  
 51 sensitivity functions appearing in the formulation are defined. An alternative interpretation  
 52 of these functions based on the reciprocity principle is proposed in Sec. IV. This inter-  
 53 pretation suggests a simple implementation for measuring the sensitivity functions. The  
 54 proposed methodology for characterizing the panel response under a diffuse acoustic field  
 55 is summarized in Sec. V. Numerical and experimental validations are provided in Sec. VI.  
 56 Finally, a comparison with measurements performed following standards [4] is proposed in  
 57 Sec. VII.

## 58 **II. VIBROACOUSTIC CHARACTERIZATION OF PANELS UNDER DIFFUSE** 59 **ACOUSTIC FIELD**

60 Let us consider a baffled panel of surface  $\Sigma_p$  with arbitrary boundary conditions separating  
 61 two semi-infinite acoustic domains. Each of these acoustic domains is characterized by a

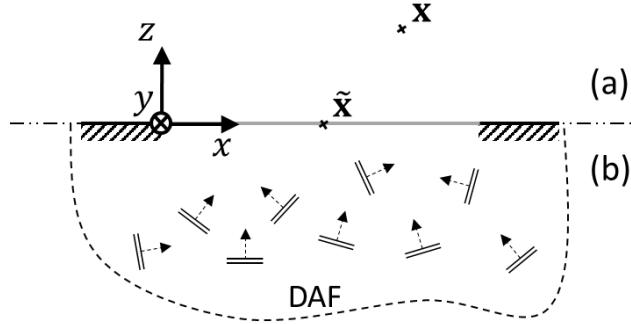


FIG. 1. Panel (gray line) and coordinate system. (a) receiving side: semi-infinite domain. (b) source side: theoretical DAF.

62 mass density  $\rho_0$  and a sound velocity  $c_0$ . As shown in Fig. 1, one supposes that a DAF is  
 63 generated on one side of the panel. We define  $\mathbf{x} = (x, y, z)$  the observation point in the  
 64 receiving half space  $z > 0$  or on the panel  $z = 0$  and  $\tilde{\mathbf{x}} = (\tilde{x}, \tilde{y})$  the excitation point on the  
 65 panel surface. Both points are defined in the Cartesian coordinate system  $(x, y, z)$  shown in  
 66 Fig. 1.

67 To experimentally characterize the vibroacoustic behavior of this panel, two quantities  
 68 are considered:

- 69 - the one-sided normal velocity frequency response  $v$  at a given point on the panel,  
 70 which for random excitations is given by the auto spectral density (ASD) function of  
 71 the normal velocity  $G_{vv}(\mathbf{x}, f)$ ,
- 72 - the transmission loss (TL) defined by

$$TL(f) = 10 \log_{10} \left( \frac{\Pi_{inc}(f)}{\Pi_{rad}(f)} \right), \quad (1)$$

73 where  $f$  is the frequency and is considered positive.

74 The incident acoustic power and the radiated acoustic power are denoted  $\Pi_{inc}$  and  $\Pi_{rad}$ , re-  
 75 spectively. For a DAF exciting a panel of area  $\Sigma_p$ , the incident acoustic power is theoretically  
 76 given by [10]

$$\Pi_{inc}(f) = \frac{G_{p_b p_b}(f) \Sigma_p}{8 \rho_0 c_0}, \quad (2)$$

77 where  $G_{p_b p_b}(f)$  is the one-sided frequency wall-pressure ASD function and the factor 8  
 78 accounts for pressure doubling at the panel surface [11]. The radiated power of the panel

79 into the receiving medium is obtained by integrating the normal active sound intensity flow  
 80 passing through a virtual surface  $\Sigma_v$  surrounding the panel

$$\Pi_{rad}(f) = \iint_{\Sigma_v} \mathbf{I}_{act}(\mathbf{x}, f) \mathbf{n} d\mathbf{x}, \quad (3)$$

81  $\mathbf{n}$  being the unit exterior vector normal to  $\Sigma_v$ ,  $d\mathbf{x}$  the surface element and  $\mathbf{I}_{act}(\mathbf{x}, f)$  the  
 82 active sound intensity at point  $\mathbf{x}$ . The latter is directly related to the one-sided frequency  
 83 CSD function  $G_{pv_0}(\mathbf{x}, f)$  between the sound pressure  $p$  and the particle velocity  $v_0$  at point  
 84  $\mathbf{x}$  with [12]

$$\mathbf{I}_{act}(\mathbf{x}, f) = \text{Re}\{G_{pv_0}(\mathbf{x}, f)\}. \quad (4)$$

85 Theoretically, the radiated power is obtained by solving the formal integral in Eq. (3).  
 86 For numerical and experimental applications, the integral of Eq. (3) may be approximated  
 87 by discretizing the surface  $\Sigma_v$  and using the rectangular integration rule:

$$\Pi_{rad}(f) \approx \sum_{\mathbf{x} \in \sigma_v} \mathbf{I}_{act}(\mathbf{x}, f) \mathbf{n} \delta\mathbf{x}, \quad (5)$$

88 where  $\sigma_v$  represents the set of points defined on  $\Sigma_v$  and  $\delta\mathbf{x}$  is the elementary point area.

89 To characterize the vibration response of a panel to a DAF, it is then necessary to  
 90 evaluate  $G_{vv}$  at the considered point on the panel while the evaluation of  $G_{pv_0}$  for the set  
 91 of points  $\sigma_v$  is required to estimate the TL. An approach for evaluating these quantities  
 92 based on deterministic transfer functions and using a reciprocity principle is presented in  
 93 the following sections.

### 94 **III. MATHEMATICAL FORMULATION OF THE VIBROACOUSTIC RESPONSE** 95 **OF PANELS UNDER DIFFUSE ACOUSTIC FIELD**

96 Let us consider the blocked wall-pressure field  $p_b(\tilde{\mathbf{x}}, t)$  exerted on the panel by a DAF  
 97 excitation at point  $\tilde{\mathbf{x}}$  as a function of time. The response of the panel at point  $\mathbf{x}$  when the  
 98 panel is excited by  $p_b(\tilde{\mathbf{x}}, t)$  is denoted  $\alpha(\mathbf{x}, t)$ . If  $\mathbf{x}$  is on the panel,  $\alpha$  stands for  $v$  whereas  
 99 it stands for  $p$  or  $v_0$  if  $\mathbf{x}$  is in the acoustic domain. This response can be expressed by the

100 convolution product [13]

$$\alpha(\mathbf{x}, t) = \iint_{\Sigma_p} \int_{-\infty}^{\infty} h_{\alpha/F_n}(\mathbf{x}, \tilde{\mathbf{x}}, t - \tau) p_b(\tilde{\mathbf{x}}, \tau) d\tau d\tilde{\mathbf{x}}, \quad (6)$$

101 where  $h_{\alpha/F_n}(\mathbf{x}, \tilde{\mathbf{x}}, t)$  is the impulse response (structural velocity, acoustic pressure or particle  
102 velocity-wise) at point  $\mathbf{x}$  for a normal unit force applied at point  $\tilde{\mathbf{x}}$ . Assuming that the  
103 random process is ergodic, the cross-correlation function  $R_{\alpha\alpha'}(\mathbf{x}, t)$  is defined by

$$R_{\alpha\alpha'}(\mathbf{x}, t) = \int_{-\infty}^{\infty} \alpha(\mathbf{x}, \tau) \alpha'(\mathbf{x}, t + \tau) d\tau. \quad (7)$$

104 where  $\alpha'$  also designates  $v$ ,  $p$  or  $v_0$ . Introducing Eq. (6) in Eq. (7) and performing a time  
105 Fourier transform of the resulting expression of the cross-correlation function gives the space-  
106 frequency spectrum  $S_{\alpha\alpha'}(\mathbf{x}, \omega)$ , which after some manipulations (see [13] for details) can be  
107 written as

$$S_{\alpha\alpha'}(\mathbf{x}, \omega) = \iint_{\Sigma_p} \iint_{\Sigma_p} H_{\alpha/F_n}(\mathbf{x}, \tilde{\mathbf{x}}, \omega) H_{\alpha'/F_n}^*(\mathbf{x}, \tilde{\mathbf{x}}, \omega) S_{p_b p_b}(\tilde{\mathbf{x}}, \tilde{\mathbf{x}}, \omega) d\tilde{\mathbf{x}} d\tilde{\mathbf{x}}, \quad (8)$$

108 where  $H_{\alpha/F_n}(\mathbf{x}, \tilde{\mathbf{x}}, \omega)$  is the time Fourier transform of  $h_{\alpha/F_n}(\mathbf{x}, \tilde{\mathbf{x}}, t)$  and corresponds to the  
109 panel frequency response function (velocity, pressure or particle velocity-wise) at point  $\mathbf{x}$   
110 when it is excited by a normal force  $F_n$  applied at point  $\tilde{\mathbf{x}}$ ;  $S_{p_b p_b}(\tilde{\mathbf{x}}, \tilde{\mathbf{x}}, \omega)$  is the time  
111 Fourier transform of the cross-correlation function of the blocked wall-pressure; finally \* is  
112 the complex conjugate. Defining the wavenumber-frequency spectrum of the wall-pressure  
113  $S_{p_b p_b}(\mathbf{k}, \omega)$  as the wavenumber transform of the space-frequency spectrum  $S_{p_b p_b}(\tilde{\mathbf{x}}, \tilde{\mathbf{x}}, \omega)$ ,  
114 one has

$$S_{p_b p_b}(\tilde{\mathbf{x}}, \tilde{\mathbf{x}}, \omega) = \frac{1}{4\pi^2} \iint_{-\infty}^{\infty} S_{p_b p_b}(\mathbf{k}, \omega) e^{j\mathbf{k}(\tilde{\mathbf{x}} - \tilde{\mathbf{x}})} d\mathbf{k}. \quad (9)$$

115 where  $\mathbf{k} = (k_x, k_y)$  is the wavevector defined in the plane  $(x, y)$  and  $d\mathbf{k}$  is the two-dimensional  
116 wavenumber element. By introducing Eq. (9) in Eq. (8) and rearranging the terms, one  
117 obtains

$$S_{\alpha\alpha'}(\mathbf{x}, \omega) = \frac{1}{4\pi^2} \iint_{-\infty}^{\infty} H_{\alpha}(\mathbf{x}, \mathbf{k}, \omega) H_{\alpha'}^*(\mathbf{x}, \mathbf{k}, \omega) S_{p_b p_b}(\mathbf{k}, \omega) d\mathbf{k}, \quad (10)$$

118 where

$$H_{\alpha}(\mathbf{x}, \mathbf{k}, \omega) = \iint_{\Sigma_p} H_{\alpha/F_n}(\mathbf{x}, \tilde{\mathbf{x}}, \omega) e^{-j\mathbf{k}\tilde{\mathbf{x}}} d\tilde{\mathbf{x}}. \quad (11)$$

119 The  $H_\alpha(\mathbf{x}, \mathbf{k}, \omega)$  functions are called the sensitivity functions [14] and characterize the vi-  
 120 broacoustic behavior of the panel. The function  $H_{\alpha/F_n}(\mathbf{x}, \tilde{\mathbf{x}}, \omega)$  is the time Fourier transform  
 121  $h_{\alpha/F_n}(\mathbf{x}, \tilde{\mathbf{x}}, t)$  and, therefore, corresponds to the transfer function between the panel velocity  
 122 frequency response and the frequency spectrum of the applied effort.

123 The wall-pressure CSD function in the space-frequency domain of a DAF can be expressed  
 124 by [15]

$$S_{p_b p_b}(r, \omega) = S_{p_b p_b}(\omega) \frac{\sin(k_0 r)}{k_0 r}, \quad (12)$$

125 where  $r = |\tilde{\mathbf{x}} - \tilde{\tilde{\mathbf{x}}}|$ ,  $k_0 = \omega/c_0$  is the acoustic wavenumber and  $S_{p_b p_b}(\omega)$  is the wall-pressure  
 126 ASD function. The space-wavenumber transform of Eq. (12) gives the wall-pressure CSD  
 127 function in the wavenumber-frequency space

$$S_{p_b p_b}(\mathbf{k}, \omega) = S_{p_b p_b}(\omega) \Phi_{p_b p_b}(\mathbf{k}, \omega), \quad (13)$$

128 where

$$\Phi_{p_b p_b}(\mathbf{k}, \omega) = \begin{cases} \frac{2\pi}{k_0} \frac{1}{\sqrt{k_0^2 - |\mathbf{k}|^2}} & \text{if } |\mathbf{k}| < k_0 \\ 0 & \text{if } |\mathbf{k}| \geq k_0 \end{cases}. \quad (14)$$

129 As the wall-pressure CSD function of a DAF is null for wavenumbers larger than the acoustic  
 130 wavenumber, the integration domain involved in Eq. (10) can be restricted to the wavenum-  
 131 bers contained in the acoustic domain (*i.e.*,  $|\mathbf{k}| < k_0$ ). Moreover, in practice, this integral  
 132 is approximated considering a set of wavevectors in the acoustic domain  $\Omega_{\mathbf{k}}$  and using the  
 133 rectangular integration rule. It should be stressed here that  $S_{\alpha\alpha'}(\mathbf{x}, \omega)$  is a two-sided spec-  
 134 trum as a function of the angular frequency. It can be related to the one-sided spectrum as  
 135 a function of the frequency  $G_{\alpha\alpha'}(\mathbf{x}, f)$  by

$$G_{\alpha\alpha'}(\mathbf{x}, f) = 4\pi S_{\alpha\alpha'}(\mathbf{x}, \omega). \quad (15)$$

136 For the sake of coherence with experiments, one-sided frequency spectra will be considered  
 137 in the remainder of the article.

138 According to Eq. (10) and (15), the one-sided frequency ASD function of the velocity  $v$



139 at point  $\mathbf{x}$  ( $z = 0$ ) of a panel excited by a DAF can be estimated with

$$G_{vv}(\mathbf{x}, f) \approx \frac{1}{4\pi^2} \sum_{\mathbf{k} \in \Omega_{\mathbf{k}}} |H_v(\mathbf{x}, \mathbf{k}, \omega)|^2 G_{p_b p_b}(f) \Phi_{p_b p_b}(\mathbf{k}, \omega) \delta^2 \mathbf{k}, \quad (16)$$

140 whereas the one-sided frequency CSD function between the pressure  $p$  and the particle  
141 velocity  $v_0$  at a given point  $\mathbf{x}$  into the acoustic domain can be estimated with

$$G_{pv_0}(\mathbf{x}, f) \approx \frac{1}{4\pi^2} \sum_{\mathbf{k} \in \Omega_{\mathbf{k}}} H_p(\mathbf{x}, \mathbf{k}, \omega) H_{v_0}^*(\mathbf{x}, \mathbf{k}, \omega) G_{p_b p_b}(f) \Phi_{p_b p_b}(\mathbf{k}, \omega) \delta^2 \mathbf{k}, \quad (17)$$

142 where  $\delta \mathbf{k}$  represents the wavenumber resolution and  $G_{p_b p_b}(f) = 4\pi S_{p_b p_b}(\omega)$  is the one-sided  
143 frequency ASD function of the blocked wall-pressure.

144 To evaluate these two quantities, the sensitivity functions  $H_v$ ,  $H_p$  and  $H_{v_0}$  for wavenum-  
145 bers belonging to  $\Omega_{\mathbf{k}}$  are thus to be determined. A direct interpretation of these sensitivity  
146 functions can be deduced from Eq. (11). Since  $H_{\alpha/F_n}(\mathbf{x}, \tilde{\mathbf{x}}, \omega)$  is the response  $\alpha$  at point  $\mathbf{x}$   
147 for a unit normal force at point  $\tilde{\mathbf{x}}$ ,  $H_{\alpha}$  represents the frequency response  $\alpha$  at point  $\mathbf{x}$  due  
148 to a wall-pressure plane wave of wavevector  $-\mathbf{k}$  (*i.e.*, due to the pressure field  $e^{-j\mathbf{k}\tilde{\mathbf{x}}}$ ). This  
149 direct interpretation is depicted in Figs. 2(a), 2(c) and 2(e) for  $H_v$ ,  $H_p$  and  $H_{v_0}$ , respectively.

150 The sensitivity functions must, therefore, be estimated only at the point of interest  $\mathbf{x}$   
151 and for the set of wavevectors  $\Omega_{\mathbf{k}}$ . A large number of waves should be considered to entirely  
152 cover the acoustic wavenumber domain. Moreover, from an experimental point of view,  
153 wall-pressure plane waves cannot be easily reproduced. To circumvent these issues, another  
154 interpretation of these sensitivity functions based on the reciprocity principle is given in the  
155 next section.

#### 156 IV. ALTERNATIVE INTERPRETATION OF THE SENSITIVITY FUNCTIONS

157 In order to propose another interpretation of the sensitivity functions, let us consider the  
158 standard reciprocity principle which states that the response of a system is invariant with  
159 respect to the exchange of points of excitation and observed response [16]. Following the  
160 previous notation, it can be translated into

$$H_{\alpha/F_n}(\mathbf{x}, \tilde{\mathbf{x}}, \omega) = H_{v/\tilde{\alpha}}(\tilde{\mathbf{x}}, \mathbf{x}, \omega), \quad (18)$$

161 where  $H_{v/\bar{\alpha}}(\tilde{\mathbf{x}}, \mathbf{x}, \omega)$  is the frequency response function between the panel velocity and a  
 162 source  $\bar{\alpha}$ , dual of  $\alpha$ . As the normal force  $F_n$  is applied on the panel, the exchanged observa-  
 163 tion point is also on the panel which explains why the right-hand side of Eq. (18) remains  
 164 the velocity response of the panel, regardless of  $\alpha$ . However the type of excitation source  $\bar{\alpha}$   
 165 depends on  $\alpha$  and it will be detailed below for each quantity considered for  $\alpha$ .

166 Sticking to the general case, by introducing Eq. (18) in Eq. (11) one obtains

$$H_{\alpha}(\mathbf{x}, \mathbf{k}, \omega) = \iint_{\Sigma_p} H_{v/\bar{\alpha}}(\tilde{\mathbf{x}}, \mathbf{x}, \omega) e^{-j\mathbf{k}\tilde{\mathbf{x}}} d\tilde{\mathbf{x}}. \quad (19)$$

167 The right hand side of Eq. (19) can be interpreted as the space-wavenumber transform of  
 168  $H_{v/\bar{\alpha}}(\tilde{\mathbf{x}}, \mathbf{x}, \omega)$  with respect to the spatial variable  $\tilde{\mathbf{x}}$ . The points  $\tilde{\mathbf{x}}$  become observation points  
 169 on the panel surface  $\Sigma_p$ , which means that the space-wavenumber transform is performed  
 170 over the vibration velocity field of the panel. To sum up, the sensitivity function  $H_{\alpha}(\mathbf{x}, \mathbf{k}, \omega)$   
 171 may be obtained by exciting the panel with a source  $\bar{\alpha}$  at point  $\mathbf{x}$  and by calculating the  
 172 space-wavenumber transform of the panel velocity frequency response normalized by the  
 173 source frequency spectrum. This second interpretation of the sensitivity functions is now  
 174 detailed for the three cases involved in the evaluation of the panel response excited by a  
 175 DAF.

176 **Case of plate velocity** ( $\alpha = v$ ): The reciprocity principle states [17] that the ratio of  
 177 the normal velocity of the panel at point  $\mathbf{x}$  over the applied normal force at point  $\tilde{\mathbf{x}}$  is equal  
 178 to the ratio of the normal velocity of the panel at point  $\tilde{\mathbf{x}}$  over the normal force applied at  
 179 point  $\mathbf{x}$ . Eq. (18) becomes

$$H_{v/F_n}(\mathbf{x}, \tilde{\mathbf{x}}, \omega) = H_{v/F_n}(\tilde{\mathbf{x}}, \mathbf{x}, \omega). \quad (20)$$

180 In this case,  $\bar{\alpha}$  is a normal force and thus, the sensitivity function  $H_v(\mathbf{x}, \mathbf{k}, \omega)$  is obtained  
 181 by exciting the panel with a normal force at point  $\mathbf{x}$  and by performing a space-wavenumber  
 182 transform of the transfer function between the panel vibration velocity response and the  
 183 force frequency spectrum, as illustrated in Fig. 2(b).

184 **Case of radiated pressure** ( $\alpha = p$ ): Lyamshev reciprocity relations for elastic struc-  
 185 tures excited by point forces [16] indicate that the ratio of the pressure at point  $\mathbf{x}$  over the  
 186 applied normal force at point  $\tilde{\mathbf{x}}$  is equal to the ratio of the normal velocity of the panel at

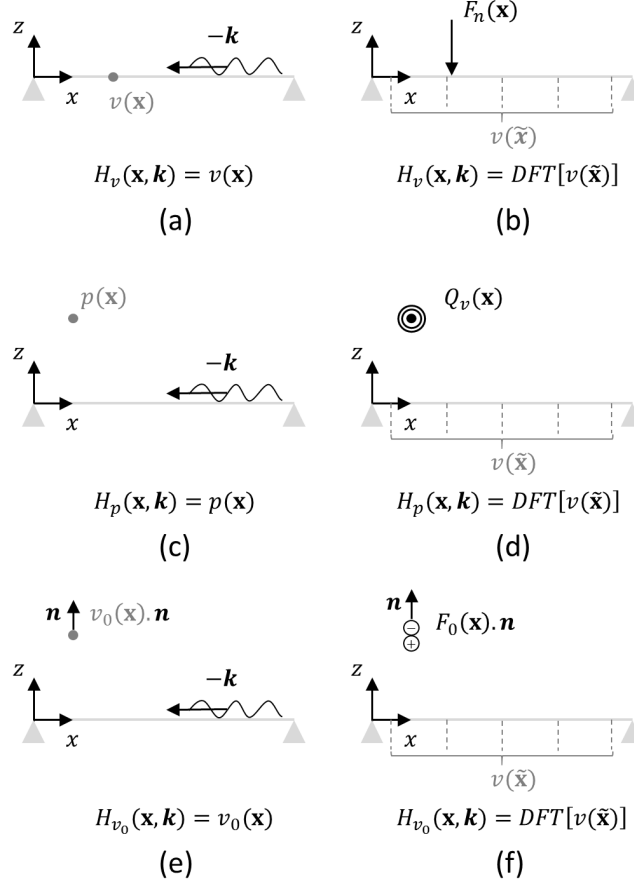


FIG. 2. Direct interpretation of the sensitivity functions: (a)  $H_v$ , (c)  $H_p$ , (e)  $H_{v_0}$  and corresponding reciprocal interpretation (b)  $H_v$ , (d)  $H_p$ , (f)  $H_{v_0}$ . (d) and (f) see appendix for demonstration.

187 point  $\tilde{\mathbf{x}}$  over the volume velocity  $Q_v$  of a monopole source placed at point  $\mathbf{x}$ . The demon-  
 188 stration of this particular reciprocity relation according to Lyamshev [18] is provided in  
 189 Appendix A 1 (this classical demonstration is useful to introduce reciprocity in terms of  
 190 particle velocity and dipole source strength, which is demonstrated in appendix A 2). In  
 191 this case  $\bar{\alpha}$  is a monopole source of volume velocity  $Q_v$  and Eq. (18) becomes

$$H_{p/F_n}(\mathbf{x}, \tilde{\mathbf{x}}, \omega) = H_{v/Q_v}(\tilde{\mathbf{x}}, \mathbf{x}, \omega). \quad (21)$$

192 The sensitivity function  $H_p(\mathbf{x}, \mathbf{k}, \omega)$  is therefore obtained by exciting the panel with a  
 193 monopole source at point  $\mathbf{x}$  and by performing a space-wavenumber transform of the panel  
 194 vibration velocity response normalized by the frequency spectrum of the monopole's volume  
 195 velocity, as illustrated in Fig. 2(d).

196 **Case of particle velocity** ( $\alpha = v_0$ ): Fahy stated that the reciprocity relationship could  
 197 be extended to acoustic dipoles and particle velocities [16], however no demonstration could  
 198 be found in the literature. As a point dipole can be represented by a point force injected in  
 199 the fluid, the ratio of the particle velocity at point  $\mathbf{x}$  over the applied normal force at point  
 200  $\tilde{\mathbf{x}}$  is equal to the ratio of the normal velocity of the panel at point  $\tilde{\mathbf{x}}$  over the force injected  
 201 in the fluid  $F_0$  at point  $\mathbf{x}$ . This latter reciprocity relation is demonstrated in Appendix A 2.  
 202 The dual source  $\bar{\alpha}$  being a dipole source of force  $F_0$  injected in the fluid, Eq. (18) now  
 203 becomes

$$H_{v_0/F_n}(\mathbf{x}, \tilde{\mathbf{x}}, \omega) = H_{v/F_0}(\tilde{\mathbf{x}}, \mathbf{x}, \omega) \quad (22)$$

204 As demonstrated in Appendix A 2, the force  $F_0$  is injected in the same direction  $\mathbf{n}$  as  
 205 the desired direction of the particle velocity  $v_0$ . One can obtain the sensitivity function  
 206  $H_{v_0}(\mathbf{x}, \mathbf{k}, \omega)$  by exciting the panel with a dipole source at point  $\mathbf{x}$  and by performing a  
 207 space-wavenumber transform of the panel vibration velocity response normalized by the  
 208 frequency spectrum of the force injected by the dipole source in the fluid, as illustrated in  
 209 Fig. 2(f).

210 To sum up, the sensitivity functions can be obtained by exciting the system at the point  
 211 of interest  $\mathbf{x}$  and by performing a space-wavenumber transform of the panel velocity field.  
 212 In practice, the vibratory field has to be measured on a regular grid of points denoted  
 213  $\Gamma_{\tilde{\mathbf{x}}}$ , using a scanning laser vibrometer, for example. The space-wavenumber transform is  
 214 therefore approximated by a discrete Fourier transform. In order to avoid aliasing effects,  
 215 the spatial resolution  $\delta\tilde{\mathbf{x}}$  over  $\Gamma_{\tilde{\mathbf{x}}}$  should be determined so that the spatial variations of  
 216 the vibratory field can be correctly represented by the grid of points. For a homogeneous  
 217 isotropic thin panel,  $\delta\tilde{\mathbf{x}}$  should be less than or equal to a quarter of the natural flexural  
 218 wavelength of the panel  $\lambda_f$  at the highest frequency of interest. For a more complex panel,  
 219 a preliminary study should be carried out to define this parameter (for instance, by using a  
 220 numerical model of the panel or by using a trial and error procedure).

## 221 V. DESCRIPTION OF THE PROPOSED METHODOLOGY

222 A methodology for experimentally estimating the vibroacoustic response of a panel excited  
 223 by a DAF is now presented. This methodology is based on Eqs. (16) and (17), and the second  
 224 interpretation of the sensitivity functions, as described in the previous section.

225 **Vibration response of the panel:** The methodology for evaluating the velocity ASD  
 226 function  $G_{vv}$  at a given point  $\mathbf{x}$  of the panel ( $z = 0$ ) can be summarized as follows:

- 227 - Excite the panel with a normal mechanical force at point  $\mathbf{x}$  (for instance by using a  
 228 shaker) and measure the normal velocity response of the panel at points  $\tilde{\mathbf{x}} \in \Gamma_{\tilde{\mathbf{x}}}$  to  
 229 determine  $H_{v/F_n}(\tilde{\mathbf{x}}, \mathbf{x}, \omega)$ ,
- 230 - Perform a discrete Fourier transform of the panel velocity response  $H_{v/F_n}(\tilde{\mathbf{x}}, \mathbf{x}, \omega)$   
 231 (with respect to  $\tilde{\mathbf{x}}$ ) to obtain the sensitivity functions  $H_v(\mathbf{x}, \mathbf{k}, \omega)$  at point  $\mathbf{x}$  for  $\mathbf{k} \in \Omega_{\mathbf{k}}$ ,
- 232 - Use Eqs. (16) and (14) to estimate the velocity ASD function  $G_{vv}$  at point  $\mathbf{x}$  under an  
 233 ideal DAF excitation.

234 **Acoustic response of the panel:** The acoustic response of the panel is characterized  
 235 by the TL as described in Sec. II. It can be obtained by following the next five steps:

- 236 - Excite the panel with a monopole source at a given point of interest  $\mathbf{x}$  and measure the  
 237 normal velocity response of the panel at points  $\tilde{\mathbf{x}} \in \Gamma_{\tilde{\mathbf{x}}}$  to determine  $H_{v/Q_v}(\tilde{\mathbf{x}}, \mathbf{x}, \omega)$ ,
- 238 - Excite the panel with a dipole source at a given point of interest  $\mathbf{x}$  and measure the  
 239 normal velocity response of the panel at points  $\tilde{\mathbf{x}} \in \Gamma_{\tilde{\mathbf{x}}}$  to determine  $H_{v/F_0}(\tilde{\mathbf{x}}, \mathbf{x}, \omega)$ ,
- 240 - Perform a discrete Fourier transform of the panel velocity responses obtained for  
 241 both monopole and dipole cases to estimate the sensitivity functions  $H_p(\mathbf{x}, \mathbf{k}, \omega)$  and  
 242  $H_{v_0}(\mathbf{x}, \mathbf{k}, \omega)$  at point  $\mathbf{x}$ ,
- 243 - Calculate the pressure – particle velocity CSD function at point  $\mathbf{x}$  using Eqs. (17)  
 244 and (14),
- 245 - Calculate the active sound intensity at point  $\mathbf{x}$  using Eq. (4).

246 The five previous steps are repeated for points  $\mathbf{x} \in \sigma_v$  (discretizing the whole virtual  
 247 surface  $\Sigma_v$  surrounding the panel) to calculate the radiated power using Eq. (5). The TL is  
 248 finally deduced using Eq. (1) while the incident acoustic power is evaluated with Eq. (2).

249 **VI. NUMERICAL AND EXPERIMENTAL VALIDATION OF THE PROPOSED**  
 250 **APPROACH**

251 **A. Test case description**

252 For numerical and experimental validation purposes a test case is considered, which  
 253 consists in a rectangular thin aluminum plate, simply supported on its four edges, baffled,  
 254 and submitted to a DAF excitation on one side. This baffled plate separates two semi-infinite  
 255 domains filled with air ( $\rho_0 = 1.3 \text{ kg.m}^{-3}$  and  $c_0 = 343 \text{ m.s}^{-1}$ ). The plate's geometrical and  
 256 mechanical properties are detailed in Table I. The structural loss factor  $\eta_{mn}$  of the  $(m, n)$   
 257 mode has been experimentally estimated using the -3 dB bandwidth method on the first  
 258 few resonances of the plate and is taken into account in the numerical simulations. A mean  
 259 value of  $\eta = 0.005$  has been measured. Simply-supported boundary conditions have been  
 260 chosen because they lead to a simple analytical solution of the plate equation of motion.  
 261 In addition, the experimental setup proposed by Robin et al. [19] for reproducing these  
 262 boundary conditions has already been validated.

263 The frequency range of interest is [170, 2000 Hz] with a frequency resolution of 0.625 Hz.  
 264 The low frequency limit is set according to the frequency response of the monopole source  
 265 and the high frequency limit has been chosen arbitrarily. This frequency range is below the  
 266 critical frequency of the panel,  $f_c$ , given by

$$f_c = \frac{c_0^2}{2\pi} \sqrt{\frac{\rho h}{D}}, \quad (23)$$

267 where  $D = \frac{Eh^3}{12(1-\nu^2)}$  is the flexural stiffness. For the considered case,  $f_c = 3867 \text{ Hz}$ .

268 In this section, we will focus on:

- 269 - the velocity sensitivity functions  $H_v$  at point  $\mathbf{x}_M$  of coordinates  $(x = 0.06 \text{ m}, y = 0.3 \text{ m}, z = 0 \text{ m})$   
 270 on the plate,
- 271 - the pressure and particle velocity sensitivity functions  $H_p$  and  $H_{v_0}$  at point  $\mathbf{x}_N$  of  
 272 coordinates  $(x = 0.06 \text{ m}, y = 0.3 \text{ m}, z = 0.1 \text{ m})$  into the acoustic medium. The particle  
 273 velocity  $v_0$  will be determined in direction  $z$ ,
- 274 - the frequency response of the velocity response at point  $\mathbf{x}_M$  and of the active intensity  
 275 at point  $\mathbf{x}_N$  in the direction  $z$ .

276 To apply the methodology described in Sec. V, the panel velocity field has to be measured  
 277 or calculated on a grid of points  $\Gamma_{\tilde{\mathbf{x}}}$ . In the following, a uniform mesh of  $15 \times 13$  points is  
 278 considered in directions  $x$  and  $y$  respectively and a gap of 30 mm along the edges is left for  
 279 practical reasons. This leads to a spatial resolution of  $\delta_x = \delta_y = 30$  mm and ensures at least  
 280 4 points per flexural wavelength for all frequencies of interest. The highest wavenumbers  
 281  $k_x^{max}$  and  $k_y^{max}$  that can be resolved in directions  $x$  and  $y$ , respectively, are given by

$$k_x^{max} = k_y^{max} = \frac{\pi}{\delta_x} = \frac{\pi}{\delta_y} \simeq 105 \text{ m}^{-1}. \quad (24)$$

282 These wavenumbers are well above twice the acoustic wavenumber (related to the Shan-  
 283 non criterion) at the highest frequency of interest (*i.e.*,  $k_0 = 37 \text{ m}^{-1}$  at 2000 Hz). As  
 284 a consequence, the considered grid of points provides correct estimation of the sensitivity  
 285 functions in the acoustic wavenumber domain  $\Omega_{\mathbf{k}}$ . The wavenumber resolutions  $\delta k_x$  and  $\delta k_y$   
 286 in directions  $x$  and  $y$  respectively, are given by

$$\delta k_x = \frac{2\pi}{L_x} \simeq 13 \text{ m}^{-1}; \quad \delta k_y = \frac{2\pi}{L_y} \simeq 15 \text{ m}^{-1}. \quad (25)$$

287 These wavenumber resolutions are relatively large because of the small dimensions of the  
 288 panel. In order to improve the wavenumber resolution, zero-padding is used to obtain a  
 289 wavenumber resolution of  $0.5 \text{ m}^{-1}$  along  $k_x$  and  $k_y$ .

290 In order to assess the accuracy of the reciprocity approach for evaluating the panel sensi-  
 291 tivity functions, the results obtained with this approach are compared with those obtained  
 292 by considering the direct interpretation of these functions (as described in Sec. III). This  
 293 comparison will allow validating the uniform mesh of  $15 \times 13$  points used for the discrete  
 294 spatial Fourier transform of the panel velocity field. The numerical model used for this study  
 295 is described in Appendix B.

## 296 B. Experimental set-up

297 Fig. 3 shows the experimental setup used to measure the sensitivity functions. The  
 298 plate was glued on thin blades and fixed on a frame as described in [19] to reproduce simply

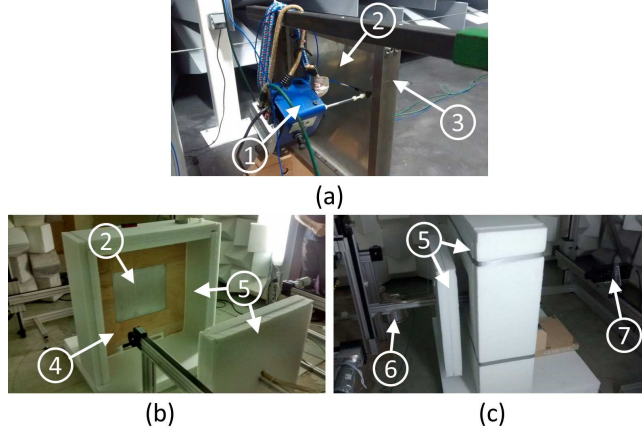


FIG. 3. Experimental setup. (a) plate excited by a shaker to determine  $H_v$ . (b) and (c) baffled plate excited by a monopole source to determine  $H_p$  and  $H_{v_0}$ . 1 - shaker with impedance head. 2 - plate. 3 - frame. 4 - baffle. 5 - sound absorbing foam. 6 - monopole source mounted on 3-axis robot. 7 - single-point laser vibrometer mounted on 2-axis robot.

299 supported boundary conditions. To determine the velocity sensitivity functions  $H_v$ , the plate  
 300 was excited by a normal force at point  $\mathbf{x}_M$  of coordinates ( $x = 0.06$  m,  $y = 0.3$  m,  $z = 0$  m).  
 301 This force was applied using a TMS SmartShaker K2007E01 with integrated amplifier, which  
 302 was fed with a swept sine over the considered frequency range and the force was measured  
 303 using an impedance head PCB288D01 (as shown in Fig. 3(a)). An adapter was used between  
 304 the impedance head and the plate reducing the area of mechanical coupling to approximately  
 305 a 5 mm diameter circle.

306 For acoustic applications, the plate was baffled in a  $1 \times 1$  m<sup>2</sup> plywood panel of 2 cm  
 307 thickness. The experiment was performed in a hemi-anechoic room and 10 cm thick sound  
 308 absorbing foam (Decibel France Polyphone 63 T) was placed on the ground and around the  
 309 plate (see Figs. 3(b) and 3(c)) to avoid potential reflections and possible influence of the  
 310 background noise generated by the robot used to estimate the TL (see Figs. 3(c) and section  
 311 VII B). This allowed approaching fully anechoic conditions and ideal monopole and dipole  
 312 excitations.

313 To estimate the pressure sensitivity functions at point  $\mathbf{x}_N$  of coordinates ( $x = 0.06$  m,  $y =$   
 314  $0.3$  m,  $z = 0.1$  m), the plate was excited by a Microflown Mid-High frequency monopole-  
 315 HFM source at point  $\mathbf{x}_N$  fed with a white noise signal on the considered frequency range.  
 316 The monopole source consists of a high impedance loudspeaker connected to a socket by a  
 317 tube with an inner diameter of 15 mm. The frequency range over which the source is effective



318 and acts like a monopole is [100, 7000 Hz]. The calibration of the source (volume velocity  
319  $Q_v$  per unit input voltage  $U$ ) was obtained by measuring the radiated sound pressure  $p$  at a  
320 given distance  $r$  in anechoic conditions for a given input voltage  $U$  and using the theoretical  
321 model of a monopole in free field for the relation between  $p$  and  $Q_v$ . The effect of the tube  
322 on the frequency response has thereby been accounted for.

323 To estimate the particle velocity sensitivity functions at point  $\mathbf{x}_N$  in the direction  $z$ ,  
324 the response of the plate to a dipole source has been reconstructed by exciting the plate  
325 with the same monopole source, still fed with a white signal and moved from the previous  
326 position by a distance  $d = 3$  cm in the direction of the injected force (in this case  $z$ ).  
327 Two monopoles close to each other and out of phase have been thereby reconstructed by  
328 subtracting the measured transfer functions  $H_v/Q_v$ . A preliminary experimental study in free  
329 field conditions and using the plate was performed to determine an appropriate separation  $d$ .  
330 It showed that below a separation of 0.5 cm, the vibration fields induced for both positions  
331 of the monopole source were not sufficiently different to be noticeably measured. On the  
332 other hand, above a separation of 5 cm, the directivity of the reconstructed dipole did not  
333 match that of a theoretical dipole. A value of 3 cm for  $d$  appeared to be an optimal value  
334 for the present case. It should be noted that the condition  $k_0 d \ll 1$  does not hold at the  
335 highest frequencies. However, the induced vibrations were in accordance with the response  
336 of a plate to a theoretical dipole. Since the numerical and experimental sensitivity functions  
337 and pressure - particle velocity CSD functions are in good agreement (see next section),  
338 these results validate the experimental method for reconstructing a dipole source.

339 According to the methodology described in Sec. V, the vibratory response of the panel  
340 has been measured on the grid of  $15 \times 13$  points with a single point Polytec laser vibrometer  
341 for each case of the excitation (*i.e.*, force, monopole, dipole). Also, in each case, the time  
342 Fourier transform was directly performed in the post-processing software with ten linear  
343 averages.

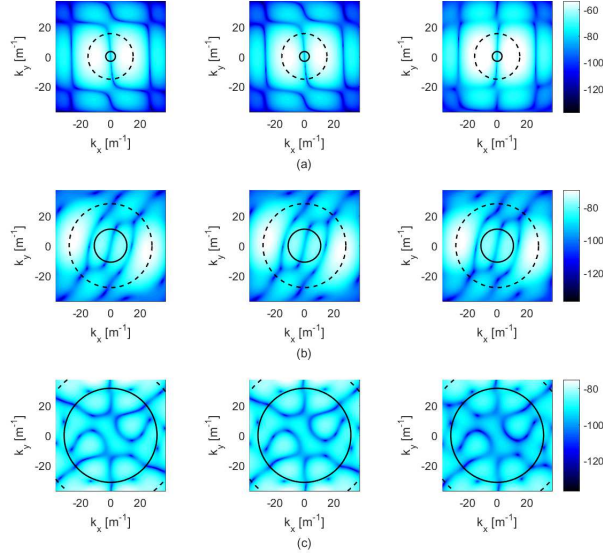


FIG. 4. Velocity sensitivity functions at point  $\mathbf{x}_M$ ,  $|H_v|^2$  (dB, ref.  $1 \text{ m}^2 \cdot \text{s}^{-2}$ ): direct calculation (left), numerical reciprocity approach (middle), experimental reciprocity approach (right). (a)  $f = 178 \text{ Hz}$ . (b)  $f = 600 \text{ Hz}$ . (c)  $f = 1710 \text{ Hz}$ . - - -, circle of radius  $k_f$ . —, circle of radius  $k_0$ .

### 344 C. Comparison between numerical and experimental results

#### 345 1. Sensitivity functions

346 Fig. 4 shows the velocity sensitivity functions  $H_v$  obtained with the direct calculation  
 347 and the reciprocal approach using numerical and experimental data, respectively. They are  
 348 provided for three different frequencies, the lowest corresponding to the (2,1) vibration mode  
 349 frequency (Fig. 4(a)) and the two others being off-resonance cases (Figs. 4(b) and 4(c)).  
 350 The product of sensitivity functions  $H_p$  and  $H_{v_0}^*$ , which is involved in the expression of  
 351  $G_{pv_0}$  (Eq. (17)), is shown in Fig. 5 at the same frequencies. The circles of radius  $k_0$  and  
 352  $k_f = \sqrt{\frac{2\pi f c}{c_0}} k_0$ , corresponding to the acoustic and flexural natural wavenumbers respectively,  
 353 are also indicated in Figs. 4 and 5.

354 In Fig. 4 and 5, results obtained by simulating numerically the direct and the reciprocity  
 355 methods match perfectly. This validates the grid of points considered on the panel and  
 356 the use of zero-padding to improve the wavenumber resolution without affecting the results.  
 357 It also validates the method described above to reconstruct a dipole from two monopole  
 358 sources.

359 In Fig. 4 and 5, the numerical and experimental results are generally in good agreement.

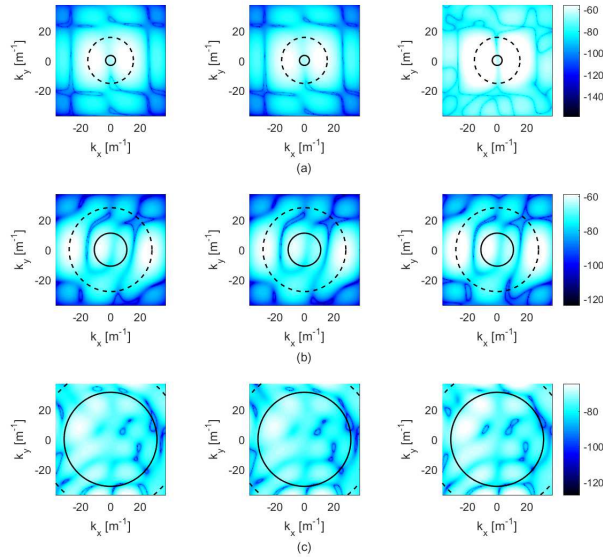


FIG. 5. Product of sensitivity functions at point  $\mathbf{x}_N$ ,  $|\text{Re}\{H_p \times H_{v_0}^*\}|$  (dB, ref.  $1 \text{ Pa}\cdot\text{m}\cdot\text{s}^{-1}$ ): direct calculation (left), numerical reciprocity approach (middle), experimental reciprocity approach (right). (a)  $f = 178$  Hz. (b)  $f = 600$  Hz. (c)  $f = 1710$  Hz. - - -, circle of radius  $k_f$ . —, circle of radius  $k_0$ .

360 One can observe that the sensitivity functions are slightly overestimated experimentally at  
 361 the (2,1) vibration mode frequency compared to the numerical results. This can be explained  
 362 by the fact that the modal damping loss factor has been estimated from the response of the  
 363 plate to a shaker excitation. The added mass from the shaker possibly had an influence on  
 364 the evaluation of the damping of the (2,1) mode.

365 A good agreement is particularly noticed within the acoustic wavenumber circle (delin-  
 366 eated by a full line). Again, only values in the acoustic wavenumber domain contribute  
 367 to the plate's vibroacoustic response to a DAF. However, the sensitivity functions are also  
 368 correctly estimated experimentally for wavenumbers higher than the acoustic wavenumber.

## 369 2. Plate velocity ASD function

370 The velocity ASD function  $G_{vv}(\mathbf{x}, f)$  at point  $\mathbf{x}_M$  of the panel excited by a DAF with a  
 371 unit wall-pressure ASD function ( $G_{p_b p_b}(f) = 1 \text{ Pa}^2\cdot\text{Hz}^{-1}$ ) has been estimated using Eq. (16)  
 372 and the three previously described sensitivity functions. Fig. 6(a) compares the results  
 373 obtained with the direct calculation and the numerical reciprocity approach. The two curves  
 374 are perfectly superimposed, showing that the sensitivity functions obtained with the two

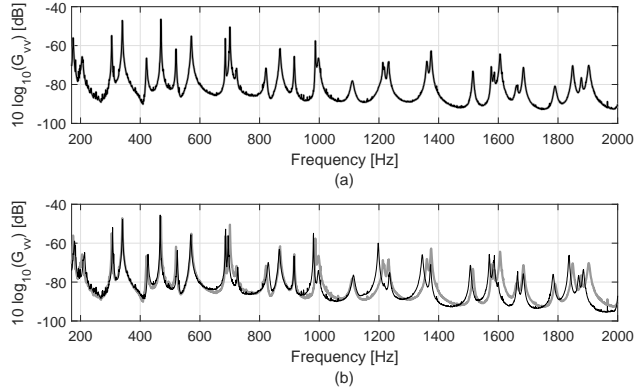


FIG. 6. Velocity ASD functions  $G_{vv}$  (dB, ref.  $1 \text{ m}^2 \cdot \text{s}^{-2} \cdot \text{Hz}^{-1}$ ). (a) direct calculation (bold gray line) vs. numerical reciprocity approach (light black line). (b) numerical reciprocity approach (bold gray line) vs. experimental reciprocity approach (light black line).

375 approaches are essentially identical in the acoustic domain and on the whole considered  
 376 frequency range. The small noticeable peaks between resonance frequencies of the plate on  
 377 both curves are attributable to the wavenumber sampling in Eq. (16) which induces errors  
 378 especially as  $|\mathbf{k}|$  approaches  $k_0$ , in which case the DAF wall-pressure CSD goes to infinity  
 379 (see Eq. (14)).

380 Fig. 6(b) compares the results obtained numerically and experimentally while considering  
 381 the reciprocity method to estimate the sensitivity functions. A good agreement is observed  
 382 between the two results, which experimentally validates the proposed methodology for the  
 383 considered test case. Slight shifts of the resonance peaks in the high frequency range are  
 384 noticed. They can be explained by small differences between the experimental and the  
 385 theoretical boundary conditions of the panel or more likely by the added mass from the  
 386 shaker.

### 387 3. Pressure – particle velocity CSD function

388 Fig. 7(a) shows the real part of the pressure – particle velocity CSD function  $G_{pv_0}(\mathbf{x}, f)$   
 389 at point  $\mathbf{x}_N$  when the plate is excited by a DAF ( $G_{p_b p_b}(f) = 1 \text{ Pa}^2 \cdot \text{Hz}^{-1}$ ). These results have  
 390 been obtained using the direct calculation and the reciprocal approach for the sensitivity  
 391 functions. Again, the two curves are in good agreement, which shows that the sensitivity  
 392 functions  $H_p$  and  $H_{v_0}$  are properly determined on the entire frequency range using the

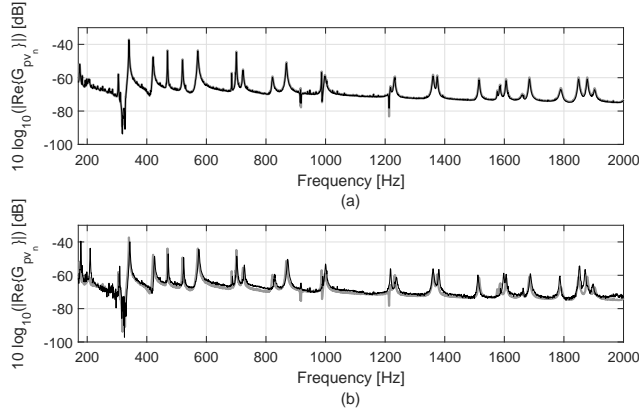


FIG. 7. Pressure – particle velocity CSD functions  $\text{Re}\{G_{pv_0}\}$  (dB, ref.  $1 \text{ W}^2 \cdot \text{m}^{-2} \cdot \text{Hz}^{-1}$ ). (a) direct calculation (bold gray line) vs. numerical reciprocity approach (light black line). (b) numerical reciprocity approach (bold gray line) vs. experimental reciprocity approach (light black line).

393 reciprocity principle.

394 The experimental and theoretical pressure – particle velocity CSD functions are com-  
 395 pared in Fig. 7(b). Their good agreement shows that the active sound intensity is correctly  
 396 estimated in this situation using the reciprocity approach. Furthermore, the experimental  
 397 monopole and dipole excitations correctly reproduce the theoretical conditions. The reso-  
 398 nance peaks are better estimated compared to those obtained to estimate the plate’s velocity  
 399 ASD function in Fig. 6(b), particularly at high frequencies. This can be possibly explained  
 400 by the accuracy in positioning the source which, for the experiment with the shaker was done  
 401 manually (subject to more errors) whereas for the acoustic applications, it was controlled  
 402 with a robot allowing a higher accuracy. A more likely explanation is the dynamic influence  
 403 of the mass added with the shaker, which explains the slight shifts of the resonance peaks  
 404 in Fig. 6(b).

## 405 VII. COMPARISON WITH REVERBERANT ROOM MEASUREMENTS

406 The proposed approach is finally compared with measurements performed at the Univer-  
 407 sity of Sherbrooke transmission loss facility (coupled reverberant-anechoic rooms) using a  
 408 plate similar to the one used in the previous section (similar dimensions, material and bound-  
 409 ary conditions) and following test standard ASTM E2249-02 (2016) [4]. The reverberant  
 410 room has a volume of approximately  $140 \text{ m}^3$  ( $7.5 \times 6.2 \times 3 \text{ m}^3$ ), and the Schroeder frequency

411 of the room, above which the sound field can be considered diffuse, is approximately 410 Hz.  
 412 The plate was mounted in an existing niche between the coupled anechoic-reverberant rooms  
 413 (the panel being flush mounted on the reverberant room side). A double-wall structure with  
 414 mechanical decoupling was then built around the plate to prevent acoustic leaks and flank-  
 415 ing paths, as described in [19]. A loudspeaker fed with a white noise signal excited the  
 416 reverberant chamber.

#### 417 **A. Panel velocity response**

418 A first experiment in the transmission loss facility was carried out to evaluate the vibra-  
 419 tory response of the panel under a DAF. A Polytec scanning laser vibrometer placed on the  
 420 anechoic side was used to measure the plate velocity ASD function. A  $9 \times 9$  microphones  
 421 array (1/4 inch B&K 4957) separated by 10 cm in directions  $x$  and  $y$  was centered to the  
 422 plate and used to directly measure wall-pressure fluctuations 1 cm away from the plate. An  
 423 average sound pressure level over all 81 microphones was then calculated to evaluate a mean  
 424 wall-pressure ASD function  $\bar{G}_{p_b p_b}(f)$  on the reverberant side.

425 The sensitivity functions of this second plate were estimated experimentally using the  
 426 reciprocity method. Some differences with the sensitivity functions of the plate considered  
 427 in Sec. VI (not shown here) indicate that the positioning of the force applied with the shaker  
 428 is not exactly at the considered point  $\mathbf{x}_M$ . Indeed, the position has a significant influence  
 429 on the vibratory response, particularly at high frequencies where the mode shapes get more  
 430 complex. The mounting base of the force sensors also has a finite diameter of approximately  
 431 5 mm, which makes the applied force not perfectly punctual. However, the modal frequencies  
 432 correspond to the theory for both plates. The velocity ASD function was calculated using  
 433 Eq. (16) and (14) whilst including the measured wall-pressure ASD function  $\bar{G}_{p_b p_b}(f)$ .

434 The plate velocity ASD function measured in the reverberant room at point  $\mathbf{x}_M$  is com-  
 435 pared to the result obtained with the proposed method in Fig. 8. The two obtained responses  
 436 are in good agreement up to 800 Hz. Above this frequency, differences are noticeable and  
 437 can be explained with:

- 438 - the inaccuracy in the positioning of point  $\mathbf{x}_M$  where the velocity ASD function is
- 439 measured in the reverberant chamber (corresponding to the point force position in the
- 440 reciprocity method) whose influence increases with the frequency,

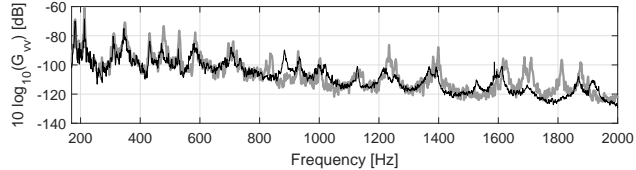


FIG. 8. Velocity ASD functions  $G_{vv}$  (dB, ref.  $1 \text{ m}^2 \cdot \text{s}^{-2} \cdot \text{Hz}^{-1}$ ): reverberant room measurements (bold gray line) vs. experimental reciprocity approach (light black line).

441 - the deviation of the pressure field in reverberant room to an ideal DAF. An analysis  
 442 of the pressure field measured with the microphone array shows spatial variations in  
 443 contradiction with the assumptions of a perfect DAF. Moreover, it is well known that  
 444 a reverberant chamber has difficulty creating grazing incidence waves. The absence  
 445 of grazing incidence waves can hardly be quantified as it varies from one reverberant  
 446 room to another. Some authors suggest corrections on the theoretical model of the  
 447 excitation to better represent the actual excitation in a reverberant room [20] [21].

## 448 B. Sound transmission loss

449 A second experiment in the transmission loss facility was conducted for estimating the  
 450 plate TL. A 1/2 inch Bruel & Kjaer rotating microphone was used to measure the spatially-  
 451 averaged sound pressure level  $L_p$  in far radiation field. The average sound intensity level  $L_I$   
 452 was measured on the anechoic side by using a Bruel & Kjaer sound intensity probe composed  
 453 of two 1/2 inch microphones with a 12 mm spacing. The sound intensity probe was manually  
 454 moved 5 cm away from the plate to scan over a parallel surface identical to the plate area,  
 455 as described in [4] for the case of a plate flush mounted on the source side. The transmission  
 456 loss of the structure is given by  $TL = L_p - L_I - 6$  [22]. An illustration of this experiment  
 457 and the considered virtual surface is given in Fig. 9(a).

458 In addition, the proposed methodology described in Sec. V for estimating the TL was  
 459 applied. A numerical study on the definition of the virtual surface  $\Sigma_v$  (which is not detailed  
 460 in this paper) showed that considering a virtual surface equal to the plate area and positioned  
 461 5 cm away from it would result in a slight overestimation of the TL, particularly at high  
 462 frequencies. This overestimation is due to sound intensity levels outside the virtual surface  
 463  $\Sigma_v$  that are thus not taken into account in the calculation of the radiated power. The

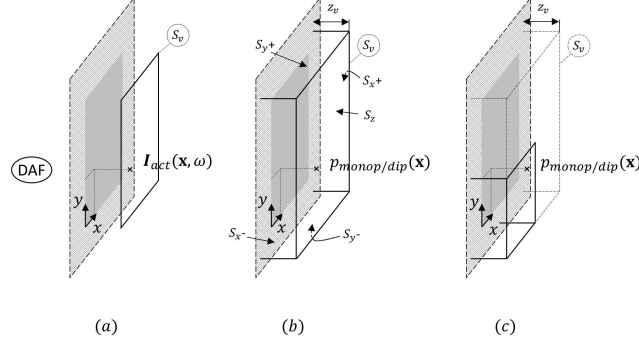


FIG. 9. Illustration of the virtual surface  $\Sigma_v$  considered for estimating the radiated power. (a) in the transmission loss facility. (b) with the reciprocal approach. (c) with the reciprocal approach considering the symmetry properties of the system.

464 virtual surface  $\Sigma_v$  over which the active sound intensity should be estimated to obtain the  
 465 radiated power should enclose the plate entirely. Therefore, the considered surface  $\Sigma_v$  was  
 466 decomposed into 5 surfaces: (a), the surface  $S_z$  which is directly in the front of the plate,  
 467 of dimensions  $0.66 \text{ m} \times 0.6 \text{ m}$  and positioned at  $z_v = 0.05 \text{ m}$ ; (b), the four lateral surfaces  
 468 (denoted  $S_{x\pm}$ ,  $S_{y\pm}$  as shown in Fig. 9(b)) to enclose the whole panel. The active sound  
 469 intensity on  $S_z$  was calculated on a grid of  $12 \times 10$  points uniformly distributed along  $x$   
 470 and  $y$ , respectively. The sound intensity was only calculated on 10 aligned points on  $S_{x\pm}$   
 471 uniformly distributed along  $y$  and 12 aligned points on  $S_{y\pm}$  uniformly distributed along  $x$ .  
 472 In both case, the points were positioned at  $z_v/2$ . Note that in the reciprocal approach, the  
 473 direction of the active sound intensity is defined by the direction of the force injected by  
 474 the dipole (see Fig. 2(f)). To determine the active sound intensity at point  $\mathbf{x}$  using the  
 475 reciprocity principle, the plate was excited successively by a monopole and dipole source at  
 476 point  $\mathbf{x}$ .

477 To reduce the number of excitation points and the measurement time, the symmetries  
 478 of the system (with respect to  $x = L_x/2$  and  $y = L_y/2$ ) were considered. Only the points  
 479 belonging to a fourth of  $\Sigma_v$ , as illustrated in Fig. 9(c), were considered, leading to a total of  
 480 40 positions of excitation as compared to 160 in Fig. 9(b). The experiment was performed  
 481 in an anechoic room, using two translating robots to automate the process (see Figs. 3(b)  
 482 and 3(c)). One robot was used to move the monopole source over each point  $\sigma_v$  defining  
 483 the discretized surfaces (considering the above symmetry) and the other robot was used to  
 484 move a Polytec single-point laser vibrometer measuring the panel velocity on a  $15 \times 13$  point



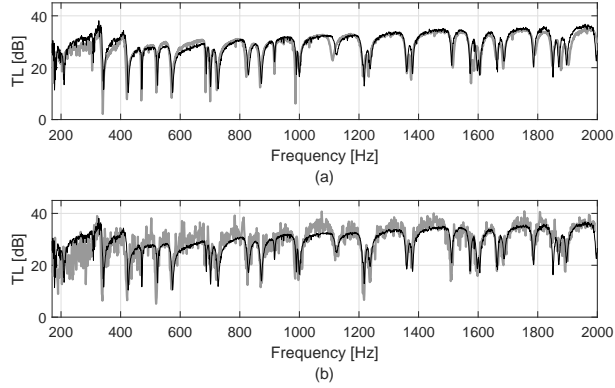


FIG. 10. Transmission loss (dB). (a) numerical approach (bold gray line) vs. experimental reciprocity approach (light black line). (b) transmission loss facility (bold gray line) vs. experimental reciprocity approach (light black line).

485 mesh.

486 As a reference to compare the following numerical and experimental results, a numerical  
 487 model has been established to accurately evaluate the radiated power from wall pressure  
 488 and wall velocity values. At the wall, the particle velocity sensitivity function  $H_{v_0}$  normal  
 489 to the plate is equal to the plate velocity sensitivity function  $H_v$ . Expressing it in the  
 490 wavenumber domain allows using the wavenumber relation between the acoustic pressure  
 491 and the particle velocity [23] and thus determining  $H_p$  at the wall. Finally, by expressing  
 492 Eqs. (4), (5) and (17) in the wavenumber domain and using  $H_p$  and  $H_{v_0}$  determined at the  
 493 wall, one obtains the radiated power directly at the wall.

494 Fig. 10(a) shows experimental results obtained based on the reciprocity method, while  
 495 considering the virtual surface  $\Sigma_v$  described at the beginning of this section and illustrated  
 496 in Figs. 9(b) and 9(c), versus simulation results for which the radiated power was evaluated  
 497 at the panel surface. The curves match very well on the whole frequency range. This  
 498 demonstrates that the proposed reciprocity approach accurately reproduces the theoretical  
 499 TL for the considered test case. Moreover, it validates the definition of the virtual surface  
 500  $\Sigma_v$  considered in the experiment.

501 The experimental results obtained with the reciprocity principle are compared in Fig. 10(b)  
 502 to the experimental results obtained in the transmission loss facility. The transmission loss  
 503 facility results are noisier than those derived from the reciprocity principle. A good agree-  
 504 ment is however noticed in general. Relatively small differences are noticeable above the

505 Schroeder frequency (410 Hz). These could be explained by the non-perfectly diffuse char-  
506 acter of the sound field in the reverberant room. On the other hand, below 410 Hz the TL  
507 measured in the transmission loss facility is on average 6.5 dB lower than the one obtained  
508 with the reciprocal approach. This is explained by the modal behavior of the reverber-  
509 ant room below the Schroeder frequency, which enhances the non diffuse character of the  
510 incident sound field.

## 511 **VIII. CONCLUSION**

512 In this paper, a methodology for characterizing the response of flat panels to a diffuse  
513 acoustic field excitation without using a reverberant room was proposed. This approach is  
514 based on the mathematical formulation of the random excitation problem in the wavenum-  
515 ber domain. This formulation indicates that the panel's response at point  $\mathbf{x}$  (on the panel  
516 or in the acoustic medium) to a random field depends on two quantities in the wavenumber  
517 domain: the wall-pressure cross spectral density function of the excitation and on so-called  
518 'sensitivity functions' at point  $\mathbf{x}$  which characterize the panel. Using the reciprocity princi-  
519 ple, it has been shown that these functions can be determined from the panel velocity field  
520 in the wavenumber domain when the system is excited by a source of unit amplitude at the  
521 point of interest  $\mathbf{x}$ . The sensitivity functions can be estimated easily by experiment based  
522 on the reciprocal interpretation.

523 The proposed approach avoids the use of a reverberant room to determine the sound  
524 transmission loss factor and vibration response of plane panels under a diffuse field excitation.  
525 As the excitation is represented by an analytical model, this approach can be applied to  
526 experimentally characterize the vibroacoustic response of a panel to an ideal diffuse acoustic  
527 field. It should however be underlined that the main limitations of the proposed approach  
528 rely on the assumptions of the mathematical formulation of the problem: the system should  
529 be linear (*i.e.*, elastic material, small deformations) and time invariant, and the condition  
530 of a baffled panel in an anechoic environment should be verified (particularly for acoustic  
531 applications). It offers however a large field of applications.

532 From a practical point of view, the vibration response of a panel to a diffuse field excitation  
533 can be easily estimated using a mechanical source of effort and a vibration measuring device  
534 to determine the vibratory response of the panel (in this study, a shaker and a scanning laser

535 vibrometer were used). In this particular case, the use of a baffle is not crucial, because no  
536 acoustic excitation are considered.

537 For acoustic applications, the condition of baffled panel in an anechoic environment is  
538 fundamental to agree with the assumption of blocked pressure. Monopole and dipole sources  
539 are required to determine the radiated pressure and the particle velocity, respectively. It can  
540 be quite challenging to experimentally reproduce those sources. In this study, a monopole-  
541 like source was used and moved from a certain distance to represent two monopoles close  
542 to each other and out of phase. A dipole source could thereby be reproduced. For the  
543 determination of the sound transmission loss factor, the monopole and dipole excitations  
544 have to be applied on several points discretizing a virtual surface, which encloses the whole  
545 panel. For each position of the excitation, the vibration response of the entire panel has to  
546 be measured. In this study, two translating robots were used to automate the process. As  
547 opposed to measurements in a transmission loss facility, this experiment was highly time-  
548 consuming. However, with the recently developed vibration measuring techniques (such as  
549 optical measurement), the time of experiment could be largely reduced.

550 To conclude, the method has been validated numerically and experimentally for the  
551 considered test case. Comparisons of numerical and experimental results have shown that  
552 the sensitivity functions have been well estimated both inside and outside the acoustic circle  
553 in the wavenumber domain. A good agreement between numerical and experimental results  
554 has also been obtained whether for the velocity spectrum at a point on the panel or for the  
555 sound intensity spectrum at a point in the acoustic domain. An application of the proposed  
556 methodology for estimating the sound transmission loss of the plate has been presented and  
557 the results have been compared with standard measurements in a coupled room facility. In  
558 the near future, the method will be extended to the characterization of panels excited by a  
559 turbulent boundary layer.

## 560 **ACKNOWLEDGMENTS**

561 This work was supported by the Labex CeLyA of Université de Lyon, operated by the  
562 French National Research Agency (ANR-10-LABX-0060/ANR-11-IDEX-0007).

563 Special thanks must go to Mr Patrick Blachier for his considerable contribution to all

564 experimental setups required to perform this study.

---

- 565 [1] ISO 10140-2:2010 Acoustics – Laboratory measurement of sound insulation of building ele-  
566 ments – Part 2: Measurement of airborne sound insulation (International Standard Organi-  
567 zation, Geneva, Switzerland, 2010).
- 568 [2] ASTM E90-09 Standard Test Method for Laboratory Measurement of Airborne Sound Trans-  
569 mission Loss of Building Partitions and Elements (ASTM International, West Conshohocken,  
570 PA, 2009).
- 571 [3] ISO 15186-1:2000 Acoustics – Measurement of sound insulation in buildings and of building  
572 elements using sound intensity – Part 1: Laboratory measurements (International Standard  
573 Organization, Geneva, Switzerland, 2000).
- 574 [4] ASTM E2249-02 (2016) Standard Test Method for Laboratory Measurement of Airborne  
575 Sound Transmission Loss of Building Partitions and Elements Using Sound Intensity (ASTM  
576 International, West Conshohocken, PA, 2016).
- 577 [5] T. Bravo, S. J. Elliott, “Variability of low frequency sound transmission measurements”, *J.*  
578 *Acoust. Soc. Am.* **115**(6), 2986-2997 (2004).
- 579 [6] A. Dijckmans, C. Vermeir, “Numerical investigation of the repeatability and reproductibility  
580 of laboratory sound insulation measurements”, *Acta Acust. United Ac.* **99**, 421-432 (2013).
- 581 [7] N. Garg, L. Gandhi, A. Kumar, P. Kumar, P. K. Saini, “Measurement uncertainty in air-  
582 borne sound insulation and single-number quantities using sound pressure and sound intensity  
583 approaches”, *Noise Control Eng. J.* **64**(2), 153-169 (2016).
- 584 [8] W. K. Bonness, D. E. Capone, S. A. Hambric, “Low-wavenumber turbulent boundary layer  
585 wall-pressure measurements from vibration data on a cylinder in pipe flow”, *J. Sound Vib.*  
586 **329**, 4166–4180 (2010).
- 587 [9] S. A. Hambric, Y. F. Hwang, W. K. Bonness, “Vibrations of plates with clamped and free  
588 edges excited by low-speed turbulent boundary layer flow”, *J. Fluid Struct.* **19**, 93–110 (2004).
- 589 [10] O. Robin, A. Berry, S. Moreau, “Experimental vibroacoustic testing of plane panels using  
590 synthesized random pressure fields”, *J. Acoust. Soc. Am.* **135**(6), 3434-3445 (2014).
- 591 [11] J-D. Chazot, O. Robin, J-L. Guyader, N. Atalla, “Diffuse Acoustic Field Produced in Re-  
592 verberant Rooms: A Boundary Diffuse Field Index”, *Acta Acust. United Ac.* **102**, 503-516

- 593 (2016).
- 594 [12] F. Fahy, "Sound Intensity", pp. 96, Elsevier Applied Science, London (1989).
- 595 [13] C. Maury, P. Gardonio, S. J. Elliott, "A wavenumber approach to modelling the response of  
596 a randomly excited panel, part 1: general theory", J. Sound Vib. **252**(1), 83-113 (2002).
- 597 [14] Y. K. Lin, "Probabilistic theory of structural dynamics", pp. 207, McGraw-Hill, New York  
598 (1967).
- 599 [15] R. K. Cook, R. V. Waterhouse, R. D. Berendt, S. Edelman, M. C. Thompson Jr, "Measurement  
600 of correlation coefficients in reverberant sound fields", J. Acoust. Soc. Am. **27**(6), 1072-1077  
601 (1955).
- 602 [16] F. J. Fahy, "Some Applications of the Reciprocity Principle in Experimental Vibroacoustics",  
603 Acoustical Physics **49**(2), 217-229 (2003).
- 604 [17] L. Maxit, V. Denis, "Prediction of flow induced sound and vibration of periodically stiffened  
605 plates", J. Acoust. Soc. Am. **133**(1), 146-160 (2013).
- 606 [18] L. M. Lyamshev, "A method for solving the proble; of sound radiation by thin elastic shells  
607 and plates", Sov. Phys. Acoust. **102**(5), 122-124 (1958).
- 608 [19] O. Robin, J-D. Chazot, R. Boulandet, M. Michau, A. Berry, N. Atalla, "A plane and thin  
609 panel with representative simply supported boundary conditions for laboratory vibroacoustic  
610 test", Acta Acust. United Ac. **102**(1), 170-182 (2016).
- 611 [20] H. -J. Kang, J. -G. Ih, H. -S. Kim, J. -S. Kim "An experimental investigation on the directional  
612 distribution of incident energy for the prediction of sound transmission loss", Applied Acoustics  
613 **63**, 283-294 (2002).
- 614 [21] N. H. Schiller, A. R. Allen, "Assessment of analytical predictions for diffuse field sound trans-  
615 mission loss", *In proceedings of the 44th InterNoise Congress*, Paper No. IN15.257, pp. 5971-  
616 5982, San Francisco, California, USA (2015).
- 617 [22] R. W. Guy, A. De Mey, "Measurement of sound transmission loss by sound intensity", Cana-  
618 dian Acoustics **13**(2), 25-44 (1985).
- 619 [23] E. G. Williams, J. D. Maynard, "Numerical evaluation of the Rayleigh integral for planar  
620 radiators using the FFT", J. Acoust. Soc. Am. **72**(6), 2020-2030 (1982).
- 621 [24] A. D. Pierce, "Acoustics: An Introduction to Its Physical Principles and Applications", pp.  
622 165-167, Acoustical Society of America, New York (1989).

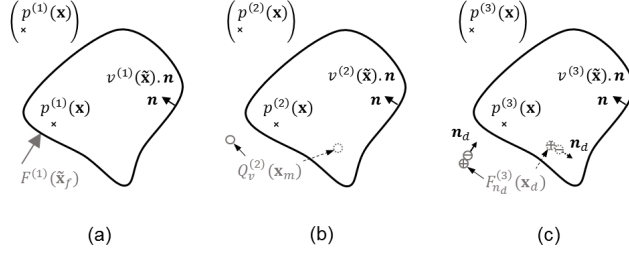


FIG. 11. Illustration of the three considered vibroacoustic problems of an elastic structure excited by: (a) a normal force at point  $\tilde{\mathbf{x}}_f$ , (b) a monopole source at point  $\mathbf{x}_m$  (internal or external to the structure), (c) a dipole source at point  $\mathbf{x}_d$  (internal or external to the structure).

623 **Appendix A: Acoustic reciprocity principles: mathematical formulation of the dif-**  
624 **ferent vibroacoustic problems**

625 Let's consider a thin elastic structure in an acoustic medium of fluid density  $\rho_0$  where  
626 sound waves propagate at a certain speed  $c_0$ . The vibroacoustic response of a structure at  
627 any given point  $\mathbf{x} = (x, y, z)$  (belonging to the acoustic medium or the structure) is studied  
628 for three separate loadings (as illustrated in Fig. 11):

- 629 1. a normal point force at point  $\tilde{\mathbf{x}}_f$ ,
- 630 2. a monopole source at point  $\mathbf{x}_m$ ,
- 631 3. a dipole source at point  $\mathbf{x}_d$ ,

632 where points belonging to the structure are denoted  $\tilde{\mathbf{x}}$ .

633 Any kind of thin elastic structure can be considered; and the excitation and observation  
634 point  $(p^{(i)}(\mathbf{x}), i = 1, 2 \text{ or } 3)$  can either be internal or external to the structure. It is also  
635 assumed that there is no other acoustic loading on the other side of the structure. The  
636 monopole source can be introduced in the Helmholtz equation by a Dirac function at point  
637  $\mathbf{x}_m$ . As a dipole source is defined by two monopoles separated by a distance  $d$  (supposedly  
638 small compared to the acoustic wavelength) and out of phase, a dipole source is introduced  
639 by the gradient of a Dirac at point  $\mathbf{x}_d$  in direction  $\mathbf{n}_d$ . Sticking to the above numbering for  
640 each load case, the Helmholtz equations are given by

$$\Delta p^{(1)}(\mathbf{x}) + k_0^2 p^{(1)}(\mathbf{x}) = 0, \quad (\text{A1})$$

$$\Delta p^{(2)}(\mathbf{x}) + k_0^2 p^{(2)}(\mathbf{x}) = -j\omega\rho_0 Q_v^{(2)} \delta(\mathbf{x} - \mathbf{x}_m), \quad (\text{A2})$$

$$\Delta p^{(3)}(\mathbf{x}) + k_0^2 p^{(3)}(\mathbf{x}) = -F_{n_d}^{(3)} \frac{\partial}{\partial \mathbf{n}_d} \delta(\mathbf{x} - \mathbf{x}_d) [24], \quad (\text{A3})$$

641 where  $\omega$  is the angular frequency,  $Q_v$  the volume velocity flow of the monopole source,  
 642  $F_{n_d}^{(3)} = j\omega\rho_0 dQ_v^{(3)}$  the dipole source strength (or dipole force) and  $k_0 = \omega/c_0$  the acoustic  
 643 wavenumber. The equilibrium equation for the structure in each case is

$$Lv^{(1)}(\tilde{\mathbf{x}}) = F^{(1)} \delta(\tilde{\mathbf{x}} - \tilde{\mathbf{x}}_f) - p^{(1)}(\tilde{\mathbf{x}}), \quad (\text{A4})$$

$$Lv^{(2)}(\tilde{\mathbf{x}}) = -p^{(2)}(\tilde{\mathbf{x}}), \quad (\text{A5})$$

$$Lv^{(3)}(\tilde{\mathbf{x}}) = -p^{(3)}(\tilde{\mathbf{x}}), \quad (\text{A6})$$

644 where  $L$  is the self-adjoint operator of the structure and  $v$  is the structure velocity in direction  
 645  $\mathbf{n}$  normal to the structure. Euler's formula provides a relation between the pressure gradient  
 646 in the fluid and the velocity along the normal external to the fluid. On the surface of the  
 647 structure, the normal external to the fluid corresponds to  $-\mathbf{n}$ . In this particular case, Euler's  
 648 formula becomes

$$\frac{\partial p}{\partial \mathbf{n}}(\tilde{\mathbf{x}}) = j\omega\rho_0 v(\tilde{\mathbf{x}}). \quad (\text{A7})$$

### 649 1. Reciprocity principle for the radiated pressure (monopole source)

650 Multiplying Eq. (A1) by  $p^{(2)}(\mathbf{x})$  and Eq. (A2) by  $-p^{(1)}(\mathbf{x})$ , adding them and integrating  
 651 them over the entire acoustic domain, one obtains

$$\int_V [\Delta p^{(1)}(\mathbf{x}) p^{(2)}(\mathbf{x}) - \Delta p^{(2)}(\mathbf{x}) p^{(1)}(\mathbf{x})] d\mathbf{x} = j\omega\rho_0 Q_v^{(2)} \int_V p^{(1)}(\mathbf{x}) \delta(\mathbf{x} - \mathbf{x}_m) d\mathbf{x}. \quad (\text{A8})$$

652 The volume integral on the left-hand side of Eq. (A8) can be transformed to a surface in-  
 653 tegral using Green's theorem. Then, using Euler's formula together with Eqs. (A4) and (A5),

654 one finally obtains [18]

$$F^{(1)} \int_S \frac{\partial p^{(2)}}{\partial \mathbf{n}}(\tilde{\mathbf{x}}) \delta(\tilde{\mathbf{x}} - \tilde{\mathbf{x}}_f) d\tilde{\mathbf{x}} = j\omega\rho_0 Q_v^{(2)} \int_V p^{(1)}(\mathbf{x}) \delta(\mathbf{x} - \mathbf{x}_m) d\mathbf{x}. \quad (\text{A9})$$

655 From the property of the Dirac delta function (*i.e.*,  $\int_V f(\mathbf{x}) \delta(\mathbf{x} - \mathbf{x}_0) d\mathbf{x} = f(\mathbf{x}_0)$  for  
656 any function  $f$  defined on  $V$  and any point  $\mathbf{x}_0 \in V$ ) one has:

$$F^{(1)} \frac{\partial p^{(2)}}{\partial \mathbf{n}}(\tilde{\mathbf{x}}_f) = j\omega\rho_0 Q_v^{(2)} p^{(1)}(\mathbf{x}_m). \quad (\text{A10})$$

657 Using Eq. (A7), Eq. (A10) becomes

$$\frac{p^{(1)}}{F^{(1)}}(\mathbf{x}_m) = \frac{v^{(2)}}{Q_v^{(2)}}(\tilde{\mathbf{x}}_f). \quad (\text{A11})$$

658 Eq. (A11) shows that the pressure radiated by a structure at point  $\mathbf{x}_m$  when it is excited  
659 by a normal unit point force at point  $\tilde{\mathbf{x}}_f$  equals the structure normal velocity at point  $\tilde{\mathbf{x}}_f$   
660 when it is excited by a monopole source of unit volume velocity at point  $\mathbf{x}_m$ .

## 661 2. Reciprocity principle for the particle velocity (dipole source)

662 Multiplying Eq. (A1) by  $p^{(3)}(\mathbf{x})$  and Eq. (A3) by  $-p^{(1)}(\mathbf{x})$ , adding them and integrating  
663 them over the entire acoustic domain, one obtains

$$\int_V [\Delta p^{(1)}(\mathbf{x}) p^{(3)}(\mathbf{x}) - \Delta p^{(3)}(\mathbf{x}) p^{(1)}(\mathbf{x})] d\mathbf{x} = F_{n_d}^{(3)} \int_V p^{(1)}(\mathbf{x}) \frac{\partial}{\partial \mathbf{n}_d} \delta(\mathbf{x} - \mathbf{x}_d) d\mathbf{x}. \quad (\text{A12})$$

664 As previously, the volume integral on the left-hand side of Eq. (A12) can be transformed  
665 to a surface integral using Green's theorem. Then, using Euler's formula together with  
666 Eqs. (A4) and (A6), one finally obtains [18]

$$F^{(1)} \int_S \frac{\partial p^{(3)}}{\partial \mathbf{n}}(\tilde{\mathbf{x}}) \delta(\tilde{\mathbf{x}} - \tilde{\mathbf{x}}_f) d\tilde{\mathbf{x}} = F_{n_d}^{(3)} \int_V p^{(1)}(\mathbf{x}) \frac{\partial}{\partial \mathbf{n}_d} \delta(\mathbf{x} - \mathbf{x}_d) d\mathbf{x}, \quad (\text{A13})$$

667 Using the previously described property of the Dirac delta function and the property  
668 of the distributional derivative of the Dirac delta function (*i.e.*,  $\int_V f(\mathbf{x}) \frac{\partial}{\partial \mathbf{n}} \delta(\mathbf{x} - \mathbf{x}_0) d\mathbf{x} =$   
669  $-\int_V \frac{\partial}{\partial \mathbf{n}} f(\mathbf{x}) \delta(\mathbf{x} - \mathbf{x}_0) d\mathbf{x} = -\frac{\partial}{\partial \mathbf{n}} f(\mathbf{x}_0)$  for any function  $f$  defined on  $V$  and any point



670  $\mathbf{x}_0 \in V$ ) Eq. (A13) becomes

$$F^{(1)} \frac{\partial p^{(3)}}{\partial \mathbf{n}} (\tilde{\mathbf{x}}_f) = -F_{n_d}^{(3)} \frac{\partial p^{(1)}}{\partial \mathbf{n}_d} (\mathbf{x}_d). \quad (\text{A14})$$

671 Using Euler's formula, the normal velocity of the structure at point  $\tilde{\mathbf{x}}_f$  and the particle  
672 velocity  $v_{n_d}$  in direction  $\mathbf{n}_d$  at point  $\mathbf{x}_d$  are introduced in Eq. (A14). One finally obtains

$$\frac{v_{n_d}^{(1)}}{F^{(1)}} (\mathbf{x}_d) = \frac{v^{(3)}}{F_{n_d}^{(3)}} (\tilde{\mathbf{x}}_f). \quad (\text{A15})$$

673 Eq. (A15) shows that the particle velocity in direction  $\mathbf{n}_d$  at point  $\mathbf{x}_d$  in the acoustic  
674 medium when the structure is excited by a normal unit point force at point  $\tilde{\mathbf{x}}_f$  equals the  
675 structure normal velocity at point  $\tilde{\mathbf{x}}_f$  when it is excited by a dipole source at point  $\mathbf{x}_d$   
676 oriented in direction  $\mathbf{n}_d$  and of unit injected force.

## 677 Appendix B: Numerical simulations

678 The panel vibratory response should be estimated numerically for the four different  
679 cases of excitation: (a) a wall plane wave for  $H_\alpha(\mathbf{x}, \mathbf{k}, \omega)$ ; (b) a normal point force  
680 for  $H_{v/F_n}(\tilde{\mathbf{x}}, \mathbf{x}, \omega)$ ; (c) a monopole source for  $H_{v/Q_v}(\tilde{\mathbf{x}}, \mathbf{x}, \omega)$ ; (d) a dipole source for  
681  $H_{v/F_0}(\tilde{\mathbf{x}}, \mathbf{x}, \omega)$ . They can be estimated by neglecting the fluid-structure interaction and  
682 considering the modal expansion technique. For a panel that has simply supported bound-  
683 ary conditions on all of its edges, the modal angular frequency  $\omega_{mn}$ , the spatial mode shape  
684  $\phi_{mn}$ , and the modal mass  $M_{mn}$  for the  $(m, n)$  mode are given, respectively, by

$$\omega_{mn} = \left[ \left( \frac{m\pi}{L_x} \right)^2 + \left( \frac{n\pi}{L_y} \right)^2 \right] \sqrt{\frac{D}{\rho h}}, \quad (\text{B1})$$

$$\phi_{mn}(\mathbf{x}) = \sin\left(\frac{m\pi}{L_x}x\right) \sin\left(\frac{n\pi}{L_y}y\right), \quad (\text{B2})$$

$$M_{mn} = \frac{\rho h L_x L_y}{4}, \quad (\text{B3})$$

687 where  $m$  and  $n$  are non-zero strictly positive integers. The normal velocity  $v$  of the plate

688 excited by the pressure distribution  $P(\tilde{\mathbf{x}})$ ,  $\tilde{\mathbf{x}} \in \Sigma_p$  is then obtained with:

$$v(\mathbf{x}, \omega) = j\omega \sum_{m,n} \frac{\mathbf{F}_{mn} \phi_{mn}(\mathbf{x})}{M_{mn} (\omega_{mn}^2 - \omega^2 + j\eta\omega\omega_{mn})}, \quad (\text{B4})$$

689 where the modal force  $\mathbf{F}_{mn}$  is defined by

$$\mathbf{F}_{mn} = \int_{\Sigma_p} P(\tilde{\mathbf{x}}) \phi_{mn}(\tilde{\mathbf{x}}) d\tilde{\mathbf{x}}. \quad (\text{B5})$$

### 690 1. Calculation of the sensitivity functions with the direct interpretation

691 The direct interpretation described in Sec. III and in Fig. 2 indicates that the sensi-  
 692 tivity functions are equal to the system response at point  $\mathbf{x}$  when the panel is excited by  
 693 wall-pressure plane waves of wavenumber  $-\mathbf{k} = (-k_x, -k_y)$ . The modal force is therefore  
 694 calculated by considering the pressure distribution  $P(\tilde{\mathbf{x}}) = e^{-j\mathbf{k}\tilde{\mathbf{x}}}$ . In this case, the analytical  
 695 solution to Eq. (B5) is

$$\mathbf{F}_{mn} = I_m^x I_n^y, \quad (\text{B6})$$

696 where for  $\xi = x$  or  $\xi = y$ ,

$$I_p^\xi = \begin{cases} \left(\frac{p\pi}{L_\xi}\right) \frac{(-1)^p e^{-jk_\xi L_\xi} - 1}{k_\xi^2 - \left(\frac{p\pi}{L_\xi}\right)^2}, & \text{if } |k_\xi| \neq \frac{p\pi}{L_\xi} \\ \frac{1}{2} j L_\xi, & \text{otherwise.} \end{cases} \quad (\text{B7})$$

697 The sensitivity functions  $H_v$  for a point  $\mathbf{x}$  on the panel can be directly estimated using  
 698 Eqs. (B4) to (B7). Based on the velocity response of the plate to a wall-pressure plane  
 699 wave, the radiated pressure at a point  $\mathbf{x}$  in the acoustic domain may be calculated using the  
 700 Rayleigh integral [23] whereas the particle velocity may be deduced from the Euler equation.  
 701 Doing so for a set of wall-pressure plane waves allows calculating the sensitivity functions  
 702  $H_p$  and  $H_{v0}$  at point  $\mathbf{x}$  in the acoustic domain.

### 703 2. Calculation of the sensitivity functions with the reciprocity interpretation

704 The calculation of the velocity sensitivity function  $H_v$  at point  $\mathbf{x}$  using the reciprocity  
 705 principle involves exciting the plate with a normal unit force at point  $\mathbf{x}$ . The modal force is

706 thus simply given by

$$\mathbf{F}_{mn} = \phi_{mn}(\mathbf{x}). \quad (\text{B8})$$

707 According to the process described in Sec. V, the normal velocity of the panel should be  
 708 calculated with Eq. (B4) for points  $\tilde{\mathbf{x}} \in \Gamma_{\tilde{\mathbf{x}}}$ . A discrete Fourier transform is then applied to  
 709 deduce the sensitivity functions  $H_v(\mathbf{x}, \mathbf{k}, \omega)$ .

710 Similarly, the pressure sensitivity function  $H_p$  at point  $\mathbf{x}$  in the acoustic domain is ob-  
 711 tained by exciting the plate by a monopole at point  $\mathbf{x}$ . The particle velocity sensitivity  
 712 function  $H_{v_0}$  at point  $\mathbf{x}$  is obtained analogously using a dipole excitation in the direction  
 713  $\mathbf{n}$  at point  $\mathbf{x}$ . The modal force is then obtained by approximating the integral of Eq. (B5)  
 714 with the rectangular integration rule for a wall-pressure defined by

$$P(\tilde{\mathbf{x}}) = j\omega\rho_0 Q_v \frac{e^{-jk_0 r}}{2\pi r} \quad (\text{B9})$$

715 for the monopole case, with  $r = |\mathbf{x} - \tilde{\mathbf{x}}|$  and  $Q_v = 1 \text{ m}^3 \cdot \text{s}^{-1}$ . For the dipole case,

$$P(\tilde{\mathbf{x}}) = \frac{F_0}{d} \left( \frac{e^{-jk_0 r_1}}{2\pi r_1} - \frac{e^{-jk_0 r_2}}{2\pi r_2} \right). \quad (\text{B10})$$

716 where  $r_1$  and  $r_2$  are two positions of monopoles separated by a distance  $d$  representing a  
 717 dipole at a distance  $r$  [24] and the dipole force will be then considered of unit amplitude  
 718 (*i.e.*,  $F_0 = 1 \text{ N}$ ).

719 Fig. 1. Panel (gray line) and coordinate system. (a) receiving side: semi-infinite  
 720 domain. (b) source side: theoretical DAF.

721 Fig. 2. Direct interpretation of the sensitivity functions: (a)  $H_v$ , (c)  $H_p$ , (e)  $H_{v_0}$   
 722 and corresponding reciprocal interpretation (b)  $H_v$ , (d)  $H_p$ , (f)  $H_{v_0}$ . (d) and (f) see  
 723 appendix for demonstration.

724 Fig. 3. Experimental setup. (a) plate excited by a shaker to determine  $H_v$ . (b) and  
 725 (c) baffled plate excited by a monopole source to determine  $H_p$  and  $H_{v_0}$ . 1 - shaker  
 726 with impedance head. 2 - plate. 3 - frame. 4 - baffle. 5 - sound absorbing foam. 6 -  
 727 monopole source mounted on 3-axis robot. 7 - single-point laser vibrometer mounted  
 728 on 2-axis robot.

729 Fig. 4. Velocity sensitivity functions at point  $\mathbf{x}_M$ ,  $|H_v|^2$  (dB, ref.  $1 \text{ m}^2 \cdot \text{s}^{-2}$ ): direct  
 730 calculation (left), numerical reciprocity approach (middle), experimental reciprocity  
 731 approach (right). (a)  $f = 178 \text{ Hz}$ . (b)  $f = 600 \text{ Hz}$ . (c)  $f = 1710 \text{ Hz}$ . - - -, circle of  
 732 radius  $k_f$ . —, circle of radius  $k_0$ .

733 Fig. 5. Product of sensitivity functions at point  $\mathbf{x}_N$ ,  $|\text{Re}\{H_p \times H_{v_0}^*\}|$  (dB, ref.  
 734  $1 \text{ Pa} \cdot \text{m} \cdot \text{s}^{-1}$ ): direct calculation (left), numerical reciprocity approach (middle), ex-  
 735 perimental reciprocity approach (right). (a)  $f = 178 \text{ Hz}$ . (b)  $f = 600 \text{ Hz}$ . (c)  
 736  $f = 1710 \text{ Hz}$ . - - -, circle of radius  $k_f$ . —, circle of radius  $k_0$ .

737 Fig. 6. Velocity ASD functions  $G_{vv}$  (dB, ref.  $1 \text{ m}^2 \cdot \text{s}^{-2} \cdot \text{Hz}^{-1}$ ). (a) direct calculation  
 738 (bold gray line) vs. numerical reciprocity approach (light black line). (b) numerical  
 739 reciprocity approach (bold gray line) vs. experimental reciprocity approach (light  
 740 black line).

741 Fig. 7. Pressure – particle velocity CSD functions  $\text{Re}\{G_{pv_0}\}$  (dB, ref.  $1 \text{ W}^2 \cdot \text{m}^{-2} \cdot \text{Hz}^{-1}$ ).  
 742 (a) direct calculation (bold gray line) vs. numerical reciprocity approach (light black  
 743 line). (b) numerical reciprocity approach (bold gray line) vs. experimental reciprocity  
 744 approach (light black line).

745 Fig. 8. Velocity ASD functions  $G_{vv}$  (dB, ref.  $1 \text{ m}^2 \cdot \text{s}^{-2} \cdot \text{Hz}^{-1}$ ): reverberant room  
 746 measurements (bold gray line) vs. experimental reciprocity approach (light black  
 747 line).

748 Fig. 9. Illustration of the virtual surface  $\Sigma_v$  considered for estimating the radiated  
749 power. (a) in the transmission loss facility. (b) with the reciprocal approach. (c) with  
750 the reciprocal approach considering the symmetry properties of the system.

751 Fig. 10. Transmission loss (dB). (a) numerical approach (bold gray line) vs. experi-  
752 mental reciprocity approach (light black line). (b) transmission loss facility (bold gray  
753 line) vs. experimental reciprocity approach (light black line).

754 Fig. 11. Illustration of the three considered vibroacoustic problems of an elastic struc-  
755 ture excited by: (a) a normal force at point  $\tilde{\mathbf{x}}_f$ , (b) a monopole source at point  $\mathbf{x}_m$   
756 (internal or external to the structure), (c) a dipole source at point  $\mathbf{x}_d$  (internal or  
757 external to the structure).

TABLE I. Properties of the simply supported aluminum plate.

<b>Parameter (Symbol), Unit</b>	<b>Value</b>
Young's modulus ( $E$ ), GPa	68.9
Poisson's ratio ( $\nu$ )	0.3
Mass density ( $\rho$ ), kg/m <sup>3</sup>	2740
Length ( $L_x$ ), mm	480
Width ( $L_y$ ), mm	420
Thickness ( $h$ ), mm	3.17