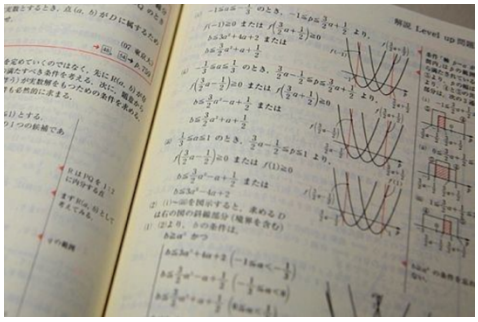
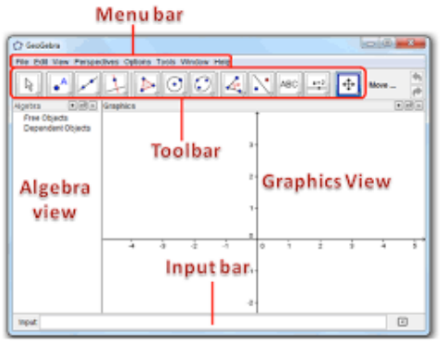
Mathematics teachers resources in a time of transitions. The MaTRiTT report

Luc Trouche

The MaTRiTT report

Mathematics Teachers Resources in a Time of Transitions





Edited by Luc TROUCHE (ENS de Lyon, France)

The MaTRiTT team

Coordinators: Luc TROUCHE (ENS de Lyon), Jiansheng BAO (ECNU)
Participants: Sophie SOURY-LAVERGNE (ENS de Lyon), Binyan XU (ECNU)
PhD students co-supervised: Luxizi ZHANG, Fangchun ZHU, Chongyang WANG
Guest researchers: Verônica GITIRANA (UFPE, Brazil), Takeshi MIYAKAWA (Joetsu University, Japan)

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Mathematics Teachers Resources in a Time of Transitions

Table of contents

Introduction

The MaTRiTT project is an answer to the 2017 Xu Guangqi call of the French Ambassy in Beijing. It is a new step for the collaboration between ENS de Lyon and ECNU (Shanghai). This collaboration developed in the frame of JoRISS (ECNU, ENS de Lyon and CNRS), particularly since January 2017, in the frame of an educational research platform hosted by JoRISS.

In the field of mathematics education, this collaboration gave matter to previous projects (see the CORE-M project here, picture right side), previous seminars (for example here) and common papers (for example Pepin et *al.* 2016).

The current project involved two researchers in IFÉ (Luc Trouche and Sophie Soury-Lavergne), two researchers in ECNU (Jianshen Bao and Binyan Xu) and three PhD. students (Chongyang Wang, Luxizi Zhang, and Fangchun Zhu), co-supervised by these researchers. MaTRiTT means *Mathematics Teachers Resources in a Time of Transitions*. Transitions means both curricular transitions (Gueudet *et al.*, 2017), and digital transitions (Monaghan & Trouche, 2016).

These transitions imply a metamorphosis of mathematics teachers' work, that the MaTRiTT project analyses throughout their interactions with their resources and task design (Mackrell et al., 2013). Situated in the frame of the *documentational approach to didactics* (Gueudet & Trouche 2009), MaTRiTT is particularly interested in teachers' resource systems (Pepin *et al.,* 2016), and in teachers' collective work (Miyakawa & Winslow, 2017).

These issues are complex, but for addressing them, MaTRiTT benefited of interactions with other projects and of the expertise of the researchers involved in the project.

Luc Trouche is involved in two projects:

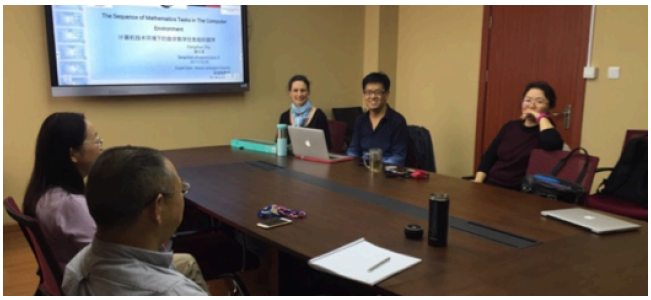
- The French national project ReVEA (2014-2018), meaning *Living resources for learning and teaching*, studying the evolutions of teachers' work with resources in four disciplines: mathematics, english, physics-chemistry (taught by the same teacher in France) and technology;
- A project developing in the region of Lyon, PREMaTT (meaning *Thinking the resources of mathematics teachers in a time of transition*), which can be considered as a twin project of MaTRiTT

Sophie Soury-Lavergne coordinates the OCINAEE project (Connected objects and digital interfaces for learning in primary schools), whose aim is to explore, in mathematics, the didactical potential of a new learning layout bridging the tangible world and the digital world thanks to manipulatives such cards, dices and games connected to a platform via a robot.

Binyan Xu and Jiansheng Bao are coordinating the design of mathematics textbooks in the region of Shanghai and the follow-up of their usages.

MaTRiTT analyses the metamorphosis of teachers' work with resources in opening two windows on Chinese and French realities. The project itself (3 000 euros) has only funded a one week stay for two students in Shanghai, but it benefited of a collaboration over a long time. Three issues are structuring the MaTRiTT project, and are addressed by three PhD students, each of them co-supervised by a Chinese and a French researcher. Their MaTRiTT contribution corresponds to their own interests of research, and is a part of their PhD:

- Luxizi Zhang (co-supervised by Pr. Bao and Pr. Trouche) analyzes the structure and use of textbooks in China and France; a preliminary contribution has been given in ECNU last September (see picture below, left side, in a working session with students in ENS de Lyon);
- Fangchun Zhu (co-supervised by Pr. Xu and Pr. Soury-Lavergne), analyzes the use of dynamic geometry software in China and France (a preliminary contribution has been given in ECNU last October, see picture below, right side);
- Chongyang Wang (co-supervised by Pr. Xu and Pr. Trouche) analyzes teachers' collective documentation work in China and France.



MaTRiTT was structured by three seminars, the first and the second ones held in ECNU, the last one in IFÉ (ENS de Lyon). This seminar (see picture p. 2) constitutes the final event for the project. It gathers, thanks to videoconferencing, the two teams, in Shanghai and Lyon, and benefits of the participation of a researcher from Japan, Takeshi Miyakawa and a researcher from Brazil, Verônica Gitirana, currently present in Lyon. The video recording of the seminar, and the slides supporting the contributions are available here.

MaTRiTT constitutes finally a double bridge (figure below):

- A bridge between researches in China and France focusing on mathematics teachers' work on resources;
- A bridge between the work of three PhD students, starting their doctoral studies at three different moments (Wang in 2015, Zhu in 2016, Zhang in 2017).

A better understanding of the metamorphosis at stake is hoped to be gained from this confronting point of view.

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Variation problems in teaching functions: Contrasting Chinese and French mathematics textbooks

Luxizi Zhang, ENS de Lyon & ECNU
http://eductice.ens-lyon.fr/EducTice/equipe/membres/doctorants/luxizi_zhang

Abstract: Teaching and learning mathematics through variation is a widespread idea in China as reflected in the old Chinese maxim, “Only by comparing can one distinguish”(有比□才有□别) (Gu et al. 2017). In this part of this report, the author examined and contrasted the tasks of the notion of function in mathematics textbooks through variation between China and France with the background of curriculum and textbooks. The results implied that textbooks in China and France both contain variation problems, and focus on setting scaffolding activities and presenting different situations for one mathematical topic. However, contrasted with Chinese textbooks, the organization of tasks in French textbooks lacks the process of letting students themselves to “think” and to “compare” the variant and invariant elements among these tasks, which is a good learning opportunity for students to deepen and consolidate conceptual understanding.

Key words: variation problems, didactical variable, textbooks, changes of registers, function

1. Teaching with variation--a Chinese way of mathematics teaching

There is a stereotype in western countries for a long time that Chinese mathematical teaching is a typical type of "passive indoctrination" and "mechanical training". They also described "Chinese learners" as "rote-learners". However, at the same time, Chinese students have achieved excellent results in international test (IEA, TIMSS, PISA, etc.). This paradox makes researchers have to think about the reasons (Bao *et al.* 2003). Besides the limitations of external observers and cultural prejudices, Marton states that some western researchers describe "Chinese learners" as "rote-learners" because they often regard memory and understanding as two aspects of opposites, while putting repetitive learning and mechanical learning at the same place. In fact, repetitive learning with variation can be a meaningful learning (Bao *et al.* 2003).

The author thinks that the different understanding of teaching with variation may be a key to explaining the different mathematics teaching between China and western countries (especially France in this article). The reflection and discussion on the teaching and the experience of teaching with variation will help us to interpret the "paradox of Chinese learners".

In this article, the author contrasts the “variation” in China and France by studying the “variation” proposed in the curriculum standard and the variation problems of function in textbooks. The author hopes to reveal the essential characteristics of Chinese mathematics teaching through this contrast and at the same time, further explore and construct the theoretical framework of teaching through variation.

2. Theoretical considerations

“BIANSHI” and the variation theory of learning

Teaching and learning through variation has been practiced in China for a long time (Gu *et al.* 2017). As a traditional and typical teaching method of Chinese mathematics, teaching with variance in China has not only extensive experience foundation but also has been tested by practice. The majority of Chinese teachers teach consciously or unconsciously through variation problems. Based on this phenomenon in China, Gu Lingyuan (1981) conducted a systematic and in-depth experimental study and theoretical analysis on “BIANSHI jiaoxue” (teaching and learning with variation). This research mainly involves two aspects of work (see figure 1): one is to systematically restore and arrange "concept variation" in traditional teaching; the other is to extend "conceptual variation" to "process variation", so as to make teaching with variation applies to both the mastery of mathematical concepts and the growth of mathematical activity experience (Bao et al. 2003).

According to Gu Lingyuan, conceptual variation refers to the strategies that are used to discern essential features of a concept and to experience connotation of the concept by exploring varying embodiments of the concept (i.e., instances, contexts), and procedural variation refers to creating variation problems or situations for students to explore in order to find solutions to problems or develop connections among different concepts step by step or from multiple approaches (Gu *et al.* 2017). Compared to conceptual variation, procedural variation is a procedural dynamic process.

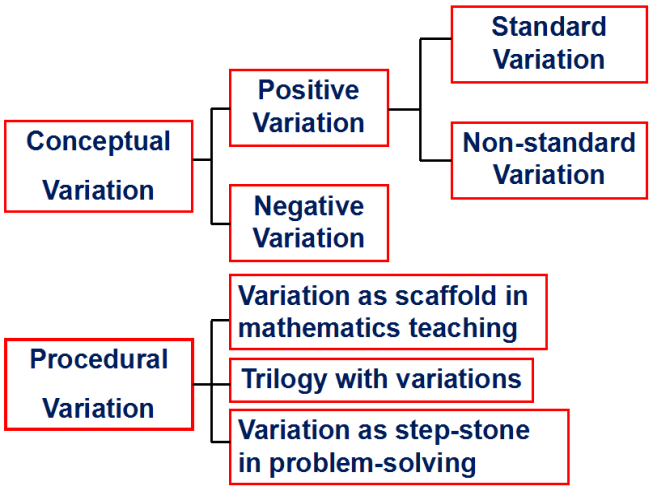


Fig 1: Different variation in mathematics teaching

In fact, we can also find the relevant theoretical foundation and basis of these two kinds of variation above from the western teaching theory. For example, Marton's group has developed the variation theory of learning, which also highlights the important role of invariance (similarity) in affording the discernment of differences (Pang et al. 2017). In France, Brousseau (1986) gives a definition of *didactical variables* as follows: A variable of a situation or a problem is called a didactical variable, when it can be modified by the teacher, and whose modifications (even slight ones) can significantly influence students' behavior and provoke different procedures or types of responses. In fact, the notion of didactic variables reflects the necessity to distinguish, classify and model the situations from a didactical perspective.

Changes of registers

Duval (2006) listed several different kinds of registers of semiotic representation: the natural language, numerical expressions, algebraic expressions and graphic representations, figures or diagrams. Various researches have shown that working with a single register does not allow the students to fully access the concept. One representative of a concept, in one register, can only show part of the concept properties. Therefore, it is necessary for teachers to change the representations of registers of one concept in teaching, which is helpful to comprehend the characteristics of the concept. This theory is accordance with multi-representation and different variation in the concept understanding which Gu (1981) has proposed (see figure 2).

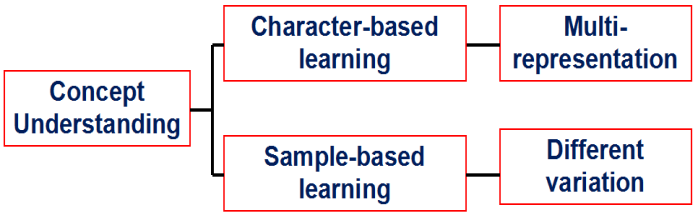


Fig 2: Conceptual variation

3. Variation in the curriculum standard

Both China and France have national curriculum standard. As curriculum standards guide the compiling of teaching materials and reflect the country's expectations of the implementation of the curriculum, the author has chosen the French curriculum standard of cycle 4 and grade 10, and the Chinese curriculum standard of high school studied to have a general view of mathematical variation in these two countries. The author has divided the variation into four types: various solutions, various representations, various situations and various teaching methods (see table 1).

As we can see from the table, French curriculum pays more attention to the various representations and various situations while Chinese curriculum has mentioned all types of variation and the variation of teaching methods is not in French curriculum. This also implies that teaching with variation is a typical and unique characteristic of Chinese mathematics teaching.

Types of variation	Contents in the curriculum
Various solutions	Must enable students to become aware of the richness and variety of the mathematical process (French grade 10) Test, try various resolution tracks (French cycle 4) Teachers can encourage students to explore how to determine linear regression lines in a variety of ways (Chinese high school)
Various representations	Students develop their intuition by moving from one mode of representation to another: numeric, graphic, algebraic, geometric, etc. These register changes are favored by the use of versatile software such as spreadsheets or dynamic geometry software (French cycle 4) Realize the various representations of the curve (Chinese high school)
Various situations	The program provides tools for modeling various situations in the form of mathematized problems (French cycle 4) ·The situations proposed in this context come from very different fields: plane or space geometry, biology, economy, physics, news etc. (French grade 10) ·This topic should be started with a variety of examples (Chinese high school)
Various teaching methods	Use a variety of teaching methods and tools (Chinese high school)

Table 1: Variation in the curriculum standard of China and France

4. Variation of function in textbooks of high school

Function and textbooks in China and France

At present, a new round of curriculum reform of high school has been conducting both in China and France. Textbook, as the important media between intended and implemented curriculum, generally plays a role in transiting education policy and curriculum standard into practical teaching. Variation pedagogy that focuses on providing deliberate mathematical task selection and implementation in classroom instruction (Gu *et al.* 2004) has direct implications for compiling textbooks (Zhang *et al.* 2017). In China, mathematics textbooks based on unified national "mathematics curriculum standards" (MOE 2011) are major and core resources for teachers' teaching and students' learning (Zhang *et al.* 2017). In mathematics learning, the examples of textbook are important approaches for students to understand the concept of mathematical knowledge and comprehend mathematical thoughts and methods.

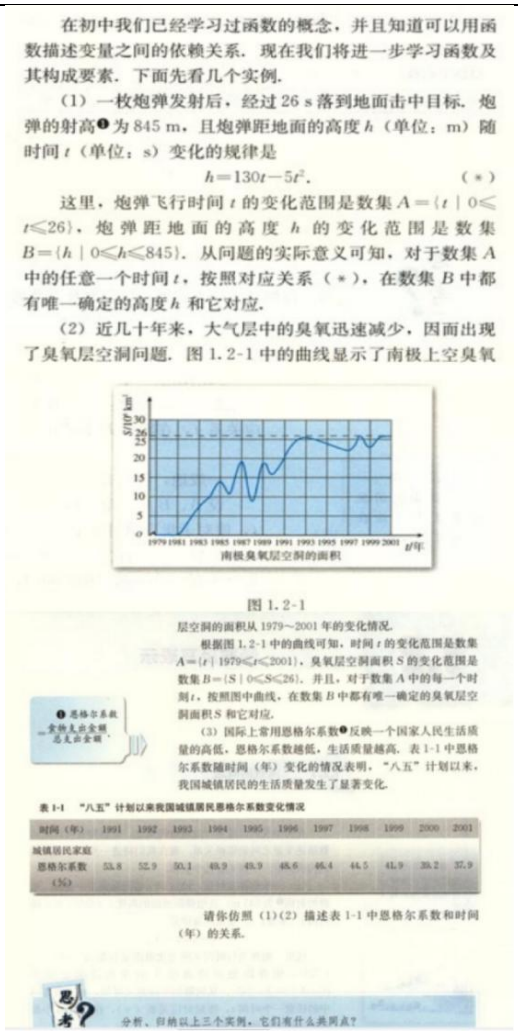
Function is an important mathematical model to describe the changing law of the objective world and the mathematics thoughts and methods of function will run through the high school mathematics curriculum (teachers' guidebook, PEP p. 9). In the French curriculum of grade10 of function, it is said "The situations proposed in this context come from very different fields: plane or space geometry, biology, economy, physics, news etc. Software made it available to students (spreadsheet, plotter, dynamic geometry software, numerical computation, formal calculus, etc.) can be exploited usefully. This is consistent with the idea of Chinese teachers' guidebook which says "through ample actual examples, students can further understand that function is an important mathematical model to describe the dependent relationship between variables". Both of curriculum points out the necessity of presenting abundant actual examples, which let the students realize function exists everywhere in real life, motivate the students to learn and help the students to understand the characteristics of function.

In this section, the author has chosen two typical examples (see figure 3 and figure 4, cf. translation in Appendix) of functions respectively from Chinese (People's education press) and French textbook (Sésamath) to study the variation problems in textbooks.

Conceptual variation: understand the concepts from different perspectives

As we all know, a basic feature of mathematical concepts is abstract, but many mathematical concepts come directly from the specific emotional experience, therefore, the key to the introduction of concepts into teaching is to establish the connection between perceptual experience and abstract concepts (Bao *et al.* 2003).

Both the Chinese and French textbooks create several problem situations for students to make a preparation to learn the concept of function. The range of independent variables of these example functions in two textbooks is limited. In fact, for most of the real-world problems of functions, the range of independent variables is limited and we can let students have a direct mathematical perception through these examples and their activities.



在初中我们已经学习过函数的概念，并且知道可以用函数描述变量之间的依赖关系．现在我们将进一步学习函数及其构成要素．下面先看几个实例．

（1）一枚炮弹发射后，经过 26 s 落到地面击中目标．炮弹的射高❶为 845 m，且炮弹距地面的高度 h（单位：m）随时间 t（单位：s）变化的规律是

$$h=130t-5t^2. \quad (*)$$

这里，炮弹飞行时间 t 的变化范围是数集 $A=\{t \mid 0\leqslant t\leqslant 26\}$，炮弹距地面的高度 h 的变化范围是数集 $B=\{h \mid 0\leqslant h\leqslant 845\}$．从问题的实际意义可知，对于数集 A 中的任意一个时间 t，按照对应关系（*），在数集 B 中都有唯一确定的高度 h 和它对应．

（2）近几十年来，大气层中的臭氧迅速减少，因而出现了臭氧层空洞问题．图 1.2-1 中的曲线显示了南极上空臭氧层空洞的面积从 1979～2001 年的变化情况．

南极臭氧层空洞的面积

图 1.2-1

根据图 1.2-1 中的曲线可知，时间 t 的变化范围是数集 $A=\{t \mid 1979\leqslant t\leqslant 2001\}$，臭氧层空洞面积 S 的变化范围是数集 $B=\{S \mid 0\leqslant S\leqslant 26\}$．并且，对于数集 A 中的每一个时刻 t，按照图中曲线，在数集 B 中都有唯一确定的臭氧层空洞面积 S 和它对应．

❶ 思格尔系数 = 食物支出金额 / 总支出金额

（3）国际上常用恩格尔系数❶反映一个国家人民生活质量的高低，恩格尔系数越低，生活质量越高．表 1-1 中恩格尔系数随时间（年）变化的情况表明，“八五”计划以来，我国城镇居民的生活质量发生了显著变化．

表 1-1 “八五”计划以来我国城镇居民恩格尔系数变化情况

时间（年）	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001
城镇居民家庭恩格尔系数（%）	53.8	52.9	50.1	49.9	49.9	48.6	46.4	44.5	41.9	39.2	37.9

请你仿照（1）（2）描述表 1-1 中恩格尔系数和时间（年）的关系．

思考？ 分析、归纳以上三个实例，它们有什么共同点？

Fig. 3: The example in Chinese textbook

Translation

Example 1: After a shell was fired, it fell down to the ground and hit the target after 26s. The height of the shell's projectile is 845 m, and the height h (unit: m) of the shell from the ground varies with time t (unit: s):

$$h = 130t - 5t^2. (*)$$

Here, the variation range of the shell's flight time t is the number set A = {t | 0 <t < 26}, and the variation range of the height *h* of the shell from the ground is the set B = {h | 0 <h <845}. According to the practical significance of the problem, we know that for any time t in the set A, according to the corresponding relation (*), there is a uniquely determined height h in the set B, which corresponds to it.

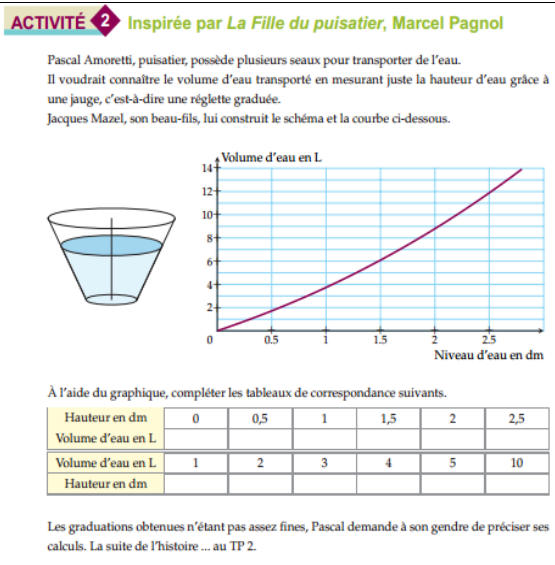
Example 2: In recent decades, the ozone in the atmosphere has decreased rapidly due to the problem of ozone hole. The curve in Figure 1.2-1 shows the change of the area of the ozone hole over Antarctica from 1979 to 2001.

According to the curve in Figure 1.2-1, the range of variation of time t is the number set A = {t | 1979 <t <2001}, and the variation range of the ozone hole area S is the number set B = {S | 0 <S <26}. And for each time t in the set A, according to the curve in the figure, there is a uniquely determined ozone hole area S in the number set B corresponding to it.

Example 3: Internationally, we commonly use Engel's coefficient to reflect people's quality of life in one country. The lower the Engel's coefficient, the higher the quality of life. The change of the Engel's coefficient over time (year) in Table 1-1 shows that since the "Eighth Five-Year " plan, the quality of life of residents in our country has changed significantly.

Please describe the relationship between the Engel's coefficient and time (year) in table 1-1 by following the example of (1) and (2).

Thinking: Analyze, summarize the three examples above, what do they have in common?



ACTIVITÉ 2 Inspirée par *La Fille du puisatier*, Marcel Pagnol

Pascal Amoretti, puisatier, possède plusieurs seaux pour transporter de l'eau.
Il voudrait connaître le volume d'eau transporté en mesurant juste la hauteur d'eau grâce à une jauge, c'est-à-dire une réglette graduée.
Jacques Mazel, son beau-fils, lui construit le schéma et la courbe ci-dessous.

Volume d'eau en L

Niveau d'eau en dm

À l'aide du graphique, compléter les tableaux de correspondance suivants.

Hauteur en dm	0	0,5	1	1,5	2	2,5
Volume d'eau en L						

Volume d'eau en L	1	2	3	4	5	10
Hauteur en dm						

Les graduations obtenues n'étant pas assez fines, Pascal demande à son gendre de préciser ses calculs. La suite de l'histoire ... au TP 2.

Fig. 4: The example in French textbook

Translation

Activity 2: Inspired by The Girl of the Pummeler, Marcel Pagnol.

Pascal Amoretti, a sinker, has several buckets to carry water. He would like to know the volume of transported water only by measuring the height of water level through a gauge, which is called a graduated ruler.

Jacques Mazel, his son-in-law, builds the diagram and curve below.

By using the graph, complete the following correspondence tables.

The obtained graduations are not fine enough, Pascal asks his son-in-law to specify his calculations. The rest of the story ... at TP 2.

In Chinese textbook, example (1) and (2) is described in detail with the set and the corresponding language. For learning each example with background, students are supposed to discuss their commonalities, such as each example involves two number sets and there is a definite correspondence between these two number sets etc. As for example (3), a requirement is raised with the expectation that with a certain guide above, students can try their own to describe by using the set and corresponding language, and then prepare for answering the questions in "Thinking" below. The most obvious difference between Chinese and French textbook is that French textbook lacks the part of “thinking” which in fact is the most important part for students develop the general concept of function by their own.

What’s more, in Chinese textbook, there is another “thinking”(see figure 5) for students to compare the definition of function in middle school and in high school. In China, function in high school is regarded not only as a dependent relationship between variables, but is also depicted by set and corresponding language. It illustrate the necessity of introducing the new definition of function and improve students understanding of the function.

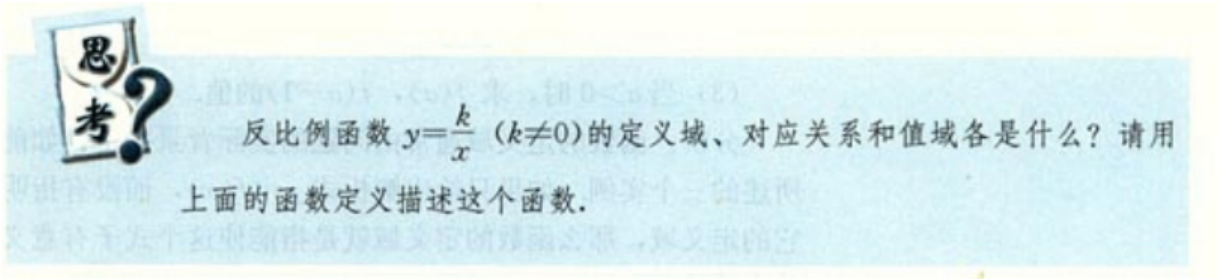


Fig. 5: Example of “thinking” in Chinese textbook (translate: what’s the domain of function, the corresponding relation and the range of the inverse proportional function $y=\frac{k}{x}, k\neq 0$, please describe this function by using definition of function above)

Different ways of registers

In Chinese textbook, it presents three different examples and give the chance to students to compare, think and finally generalize the concept of function. The aim is to let the students experience a special relationship between the number of sets (that is, the function) through these specific actual examples. The three examples are chosen from different fields: movement, nature and the economic life, which can not only make students feel that the function is widely used in many aspects, but also make the students realize that the corresponding relationship can not only be a clear analytic formula (Example 1) but also an intuitive curve (Example 2) or table (Example 3). Similarly, in the French textbook, it also shows different representations of function in one example: the intuitive curve and the table.

5. Results: From the two typical examples of textbooks perspective

As the contrast shown above, both the Chinese and French textbooks present several registers (China: table, algebraic expressions, graphic representations and natural language; France: figure, table, graphic representations). Both these two textbooks pay attention to setting scaffolding activities and presenting different situations for one mathematics topic. However, contrasted with Chinese textbooks, the organization of tasks in French textbooks lacks the process of letting students themselves to “think”, to “contrast” and to “generalize” the variant and invariant elements among these tasks which is the essential characteristics of variation.

6. Further studies

This article is only a start point of authors’ study. The research can be further conducted in studying the textbooks’ structure (number and categories of exercises), the process of teachers developing variation throughout their documentation work (Gueudet & Trouche 2009), such as textbooks, online resources, working in TRG etc., and also in observing how teachers apply the variation in class.

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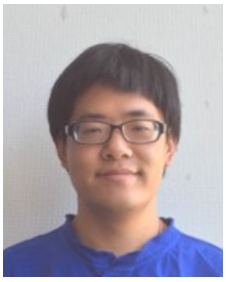
Interactions between Takeshi Miyakawa and Luxizi Zhang during the final MaTRiTT seminar

Sequencing Mathematics Tasks in a Dynamic Geometry Environment: Contrasting French and Chinese cases



Fangchun Zhu, ENS de Lyon & ECNU
http://eductice.ens-lyon.fr/EducTice/equipe/membres/doctorants/fangchun_zhu

Abstract: Dynamic geometry software affects teachers' choice and decision of mathematics tasks and their organization. But there is the deep gap between real status in teaching and the results of didactical research. To solve this gap, this study focus two critical elements: teachers and their knowledge shown in their choice of different types of tasks, which is related to different ways to use dynamic geometry, and their sequence, which is related teachers' orchestration of instrument. Two examples from China and France were chosen to contrast the tasks and possible orchestration of different teachers. In Chinese example, software is used to generate new tasks and suggests a teacher-centered way to use the software. The role software played in French tasks is more like an amplifier of paper-pencil activities, supporting a more student-centered organization of the class.

Key words: dynamic geometry, instrumental orchestration, task, sequence

1. The reality and requirements of dynamic geometry software used in mathematics lessons

With traditional teaching methods, teachers (even experienced teachers) feel difficult to teach geometry only with paper, pencil, ruler, compass or other traditional tools, especially teaching dynamic geometry like moving points. Only with paper-pencil, moving process cannot be shown easily (). Although teachers can construct geometry objects accurately, for students, it is still difficult to imagine abstract geometry properties both in plane geometry and solid geometry, like the angle made by two planes. Dynamic geometry provides a new way or a much easier way to help them master the geometry knowledge. In teachers' professional education programs, in China, the formations about geometry software are far from real classroom teaching (Fu *et al*, 2012), which means teachers are told how to use the software without enough examples in mathematics to show how to integrate it into lessons. If we want teachers to use dynamic geometry in their classes, we need to know what the teachers can do with their existing knowledge and then we can promote them. So this research tries to describe what Chinese and French teachers can do with dynamic geometry and then analyze their knowledge shown in their teaching practice.

In both countries, with the transition of curriculum, information technology, like dynamic geometry software, plays more and more critical role in mathematics education. In France, from primary schools (grade 1; age 6), the use of technological tools is required in two topics: "numbers and calculation" and "space and geometry." Concerning "space and geometry," software allowing programming the movement of a robot or a character on a screen and dynamic geometry software (DGS) are introduced to teachers. Although DGS is common used in lower secondary school, in primary school we cannot see much use of DGS in mathematics teaching (Soury-Lavergne & Maschietto, 2015).

In China, in curriculum level, although DGS is not always shown in the syllabus, we can also see great efforts for integrating other technologies into mathematics teaching. For example, in Shanghai mathematics curriculum syllabus, from the third grade in primary school, the calculator is encouraged to be used in all grades of mathematics learning and makes it known as a useful instrument for students learning (Shanghai Education Commission, 2004). Not like France syllabus, no many suggestions or requirements are given to teachers to integrate dynamic geometry software into teaching geometry. But this means teachers are more free to choose whether they need to use software in their class or not.

Although we do not see many details in the requirements in syllabus level, Chinese teachers organized many informal research groups spontaneously. They use communication tools like "QQ", "Wechat" to make close relationships with other colleagues. Some informal group are established for teachers discussing their teaching experience in using dynamic geometry software, one example would be introduced in following part.

Both in China and France, much dynamic geometry software is developed for mathematics teaching and learning. For example, in China, "Z+Z Super Sketchpad" is one famous software for teachers in geometry teaching. It is developed by Zhang Jingzhong and his team. It is not only suitable for middle school mathematics and physical teachers, but also for students to start independent exploration,

cooperation and exchange of geometry. Also in France, Cabri is one important example in mathematics (especially geometry) teaching. Many French mathematics teachers use it in their classroom. Other software like GeoGebra, geometry sketchpad is also very popular in both of the countries.

As we can see, with the broad use of technology like dynamic geometry software in teaching and learning mathematics, mathematics education paid more emphasis on the effects computer technology brought into mathematics learning process. Since the 1980s, the impact of ICT on learning and teaching of mathematics has taken a major place in the corporate, pedagogical and research literature for at least three reasons (Trouche, 2003):

- The potential of new tools;
- The evolution of students' equipment;
- Institutional injunctions.

In the research done by Laborde (2000), she made a reaction to four papers which discuss the question: "Is proof activity in danger with the use of dynamic geometry systems (DGS)". These papers report about various teaching sequences based on the use of such DGS and analyze the possible roles of DGS in both the teaching and learning of proof. He wanted to make a global discussion about the role played DGS in learning proof for students by addressing following four points: the variety of possible contexts for proof in a DGS, the dual nature of proof (cognitive and social) as reflected in the 'milieu' constructed around the use of a DGS, from observing to proving, and the overcoming of the opposition between doing and proving. In his report, one paper made by Hadas, Hershkowitz and Schwarz (2000) designed two sequences of tasks in which the order of the tasks led students to develop expectations which turned out to be obviously wrong when they checked them in the dynamic geometry environment.

In this paper, I want to solve the following questions: What is the difference in the mathematics tasks teachers choose and in their sequences based on dynamic geometry software?

2. The role DGS played in mathematics tasks

Mathematics tasks in dynamic geometry environment can be classified into the following types (Laborde, 2001): 1, DGS is used mainly as facilitating material aspects of the task while not changing it conceptually: e.g. constructing the midpoints of a triangle and then its medians to get a nice diagram, to print or to project. 2, DGS is supposed to facilitate the mathematical task and produce easily accurate diagrams and many cases: DGS is here used as a visual amplifier for identifying properties, e.g. three medians intersecting in one point. In the first two types, dynamic geometry software plays as an amplifier of the paper and pencil tasks (Soury-Lavergne, 2017). 3, DGS enables to modify the solving strategies due to available tools and the possibility to select them, e.g., creating a parallelogram without using parallel line tools. 4, DGS enables to design new tasks, that have no meaning or "raison d'être" without DGS, e.g. black boxes tasks reproducing a given dynamic diagram. In the last two types, dynamics geometry software is the genesis of new constraints, affordances and tasks (Soury-Lavergne, 2017).

3. Instrumental orchestration and its relation with dynamic geometry and mathematics tasks

Another important element in integrating dynamic geometry into teaching is how teachers organize their teaching process with the help of dynamic geometry. Especially in this research, I focus on two important artefacts in geometry teaching: task and dynamic geometry software. So the question can be modified as the following: how teachers organize the tasks they design or choose before class in dynamic geometry environment. Instrumental orchestration (Trouche, 2004; Drijvers, 2010) can help us analyze the way teachers organize their teaching practice with plenty of artefacts:

An instrumental orchestration is defined as how teachers want to organize the class teaching with different kinds of artefacts available in learning environment (Trouche, 2004; Drijver, 2010). We can distinguish instrumental orchestration into three different elements: a didactic configuration, an exploitation mode and a didactical performance.

In this study, I focus on the two first elements of teachers' orchestrations as shown in the following table.

Instrumental orchestration		Focus in this research
Didactic configuration	The way the teacher distributes spatially and temporally the artefacts, arranges the teaching setting, and how students have access to the artefacts.	The process of teacher's choice and use of mathematical tasks and DGS and how to configure them in the classroom.
Exploitation mode	The way the teacher decides to exploit a didactical configuration for the benefit of his or her didactical intentions.	The sequence of tasks and DGS teacher presents to the students and the role DGS plays in the tasks.

Table 1: Instrumental orchestration and its relations with mathematics tasks and dynamic geometry software

4. Analysis of one example video in China

Because of the popular use of communication tools like "QQ", "Wechat". Teachers can share their resources or experience and get some advices from other teachers. Here, I introduce one of these groups named "Firefly mathematics studio", in China. Its object is to teach teachers how to use geometry sketchpad and integrate it into mathematics lessons. The leader of the group uploaded some short videos about how to use geometry sketchpad. In these videos, he introduced the software and its function. There are also some short teaching videos to show one teacher use sketchpad to analyze some mathematics tasks both in geometry and function. Other teachers can learn from these videos and also discuss with their colleagues to promote their teaching process.

In this report, I chose one video to analyze the tasks and its organization with dynamic geometry. The example I chosen is one special video in China. It is not a record of real class teaching. It did not contain real students, so we cannot determine if the process shown in the video real happened in class. But we still can find out many important features from it, like the types of tasks, their sequences and one of the possible way to use dynamic geometry software in class.

Following part showed the analysis of the geometry tasks in this video based on the categories mentioned above. The short video in geometry continued about 9 minutes. It contained 5 different but related tasks. These tasks are about finding the locus of a moving point. In these tasks, students can see the following constraints on the screen: A right triangle ABC. D and E are the vertices of a triangle DEB with AB = BC = 2. Point D moved on [AC]. They need to find the locus of point E and its length. four other tasks are given following, in which this teacher changed the types of polygon with points D, E and B like figure 2.

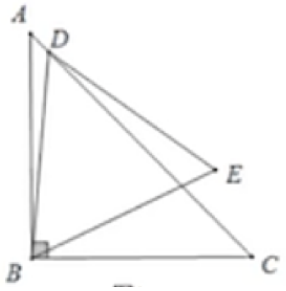


Figure 1: First task in Chinese teaching video

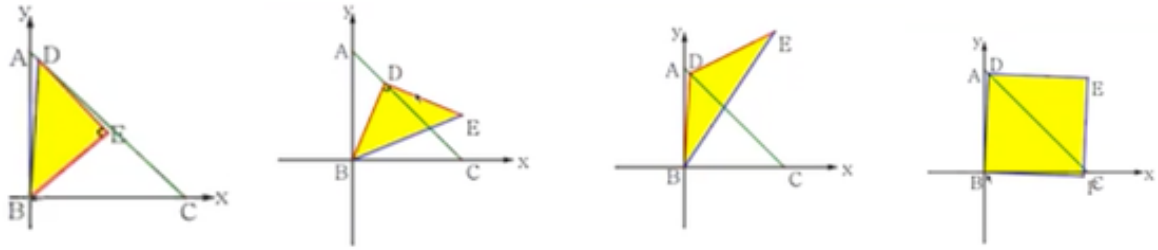


Figure 2: Other four tasks in the Chinese video with Geometry Sketchpad

All of these tasks, we can find they all contain one moving point D, students need to trace another point (point E) which moves linked with point D. During the teaching process, this teacher controlled the software all the time. Through dynamic geometry, she showed the moving process like figure 3. Without the help of the dynamic geometry, it is very difficult for students imagine the moving process which is very abstract for them. These tasks are more like to the last two categories of Laborde, in which dynamic geometry played as genesis of problems. Without dynamic geometry these problems are not exist or hard to solve. With the help of sketchpad, students can find more easier strategy to solve this task, they do not need to draw too many figure to show how point E move like they do in paper-pencil environment.

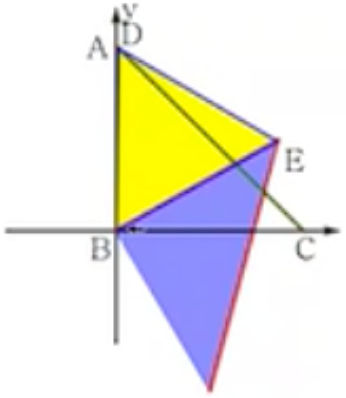


Figure 3: The locus of point E (red segment) shown with dynamic geometry software

From the sequence of the tasks, this teacher changed the type of polygon with point D, E and B and made the problem a little more complexity. The way she organized and modified the tasks is like the traditional Chinese teaching method: “Variant Teaching”. She purposefully changed one or some of the constraints to make students rethink of their solving strategies and master geometry knowledge.

Although from the video we cannot see the learning process of the students, it suggests a possible way to integrate dynamic geometry into mathematics class which involves projection, means teachers can use dynamic geometry software to project tasks or knowledge or solving process on the screen. This way is more like a traditional mathematics lesson in which teacher controlled software and show all the elements he wanted students to know., we can also imagine that these students may not be given much time to explore the solving strategies with software. This lesson was similar to “teacher centered” style. Teacher controlled all the teaching and learning process. Students are more passive during the lesson.

5. Analysis of French tasks with dynamic geometry software

The figure below showed the sample activity in France, two tasks designed with GeoGebra.

I. Construction d’un triangle dont on connaît les longueurs de ces côtés
Construction of a triangle of which the lengths of these sides are known

Ouvrez un document Geogebra. Effectuez toutes les étapes ci-dessous pour construire le triangle ABC tel que AB = 8 cm, AC = 5 cm et BC = 6 cm.

Open a document Geogebra. Follow all the steps below to construct the triangle ABC such that AB = 8 cm, AC = 5 cm and BC = 6 cm

Ouvrez un nouveau document Geogebra et tracer le triangle suivant avec la même méthode que dans le I. :

Open a new Geogebra document and draw the following triangle with the same method as shown in I.

Le triangle EFG tel que EF = 10 cm, EG = 7,6 cm et FG = 6,5 cm.

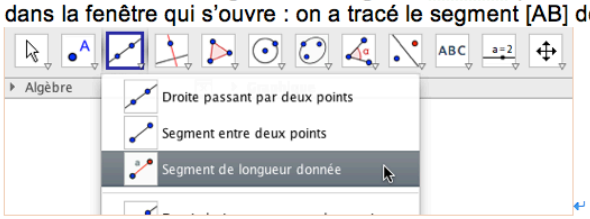
The triangle EFG such that EF = 10 cm, EG = 7.6 cm and FG = 6.5 cm.

Figure 4: Tasks shown in French list of activities

These two tasks required students to construct triangles. The length of its three sides were given. Students need to solve the tasks with GeoGebra. Because these students never use this software to solve geometry problems. So in order to help students find suitable strategy, this teacher showed the way how to operate the software to construct triangles. She added some pictures from GeoGebra, like

figure 5, to explained the steps students needed to follow, such as “Use the button “segment of given length”, then click the point A and import 8 in the window that opens: the segment [AB] of 8 cm has been plotted”. So students need not to discuss and explore their strategies, what they need to do is following the guide in the list.

2) Utiliser le bouton « segment de longueur donné», puis cliquer sur le point A et taper 8 dans la fenêtre qui s'ouvre : on a tracé le segment [AB] de 8 cm.



Use the button “segment of given length”, then click the point A and import 8 in the window that opens: the segment [AB] of 8 cm has been plotted.

Figure 5: Steps about how to solve the tasks shown in French document

The second task is similar to the first one, but without any steps of operations. Teacher only change the length of three sides. These changes of the task did not promote students’ problem solving skills because there was no change in their solving strategies getting before.

From the activity list, this teacher wanted to use technology in his lessons, and he designed some activities to integrate the software into mathematics learning process. But due to the limit knowledge of students in using technology, teachers had difficulties in designing effective tasks to create suitable environment for students learning. As shown in the document above, they needed to show all the process of solving problems, it made students have no chance to explore they only need to follow the list step by step and then repeat in next problem. Dynamic geometry software is used, in these two tasks, as an amplifier like the first two types of tasks. And from the document, it also suggests a possible orchestration with a computer for one or two students and another traditional way to use DGS: a technology oriented and controlled students’ activities.

6. The two examples, from China and France, show different types of tasks and in both sequences, the task type stays the same.

The role dynamic geometry played in Chinese tasks are more like the genesis of new constraints, affordances and tasks according Laborde’s types of tasks with dynamic geometry which means without dynamic geometry, these tasks are difficult or even cannot to be solved. these tasks contained two moving points which is one of the important reason why this teacher decided to use software. And these tasks have close relations, teacher used them like “Variant Teaching”. She purposefully and systematically changes the non-essential features of the task, changes the conditions or conclusions in the problem, changes the content and form of the problem, configure various environments for practical application. So these tasks can form a system to help students master the mathematical contents and use them to solve problems effectively.

And in this video, it suggested one way to use dynamic geometry. It seems like traditional and teacher-centered. This teacher is familiar with this kind of teaching environment, she controlled the software and gave lectures to her students. And the students sat and keep notes in their textbooks. So students still have limited time to develop their own strategies to solve mathematical problems. All the learning processes are controlled by this teacher and students are more passive in the classroom.

In the French activity list, we can find only a first level use of dynamic geometry, software in the French tasks is most like an amplifier such as the first two types in Laborde, which means the use of dynamic geometry is more technology oriented. Because of limited by the environment, almost all the class only have one computer so students cannot have enough chance to operate and explore the software or mathematics knowledge. Although some of the French schools have one or more computer laboratory, mathematics teachers have few chances to have their lessons in it. Generally, only one or two students might have time to explore with computer. Maybe some mathematics teachers could integrate technology into their lessons. But only limited computer room did not meet all the students’ needs. She needed to divide the whole class into two parts, one part could learn in computer room and another in traditional class and the next lesson she made exchange.

From the two sequences with mathematics tasks, both Chinese and French teachers do not change the types of tasks and their orchestration during the teaching practice. In all the tasks, the function of dynamic geometry software is all the same.

In the future, we need to focus on the following questions: the link between the types of mathematics tasks and teachers' instrumental orchestrations. Like the two examples above, was it because the Chinese tasks have more complexity than the French ones, so teachers thought it was easier to use teacher-centered way to have lesson? Teachers need to achieve the pre-set objectives in limited time, so teacher-centered way to teach maybe one effective way to help students master mathematics knowledge in a short time. And looking for more examples of sequences of tasks with evolution of the type of use of DGS and different orchestrations.

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Fangchun's seminar in ECNU, Shanghai October 2017. On his side, Sophie Soury-Lavergne and Yan Zhu (associated professor in ECNU). On the other side, Binyan Xu and Jiansheng Bao.

Teachers' resources choices for collective lesson preparations: two contrasted cases in China and France

Chongyang Wang, ENS de Lyon & ECNU
http://eductice.ens-lyon.fr/EducTice/equipe/membres/doctorants/chongyang_wang

Abstract: With an aim of exploring the collective aspect of teachers' resources work in the two contrasting contexts, this study took two cases (two teachers in China and two teachers in France) and analyzed the collective lesson preparation in each case: the French teachers have to prepare a new topic ("algorithmic") without sufficient ready resources, and the Chinese teachers have to prepare an open class ("the properties of in-equality") without enough teaching experiences for the new teachers. Focusing on two main teachers (Gao in the Chinese case and Anna in the French case), their behaviors and ideas will be followed from two dimensions: the resources integrated and their interactions with their documentation-working mates. Elements of documentation expertise are hoped to be found through this study.

Key words: documentation work, mathematics teacher, documentation-work mate, collectives.

In a time of transitions in both curriculum and digital technology, teachers have to react to and fulfill the new challenges/opportunities with their available resources. Cooperation among teachers is considered as a crucial dimension for teacher's professional development (Hargreaves, 1995; Rao & Zhang, 2007), especially for their long-term professional development (Lavie, 2006). In the framework of MaTRiTT (mathematics teachers resources in a time of transitions), this study is situated in a time of change, which demands the teachers working with resources intensively: a new middle school curriculum reform in France had started since September 2016, and one of the big changes is "algorithmic", which appeared in middle school stage for the first time (Trouche & Wang, 2015); while in China, although without any education reforms in curriculum, it was the time (March to May each year) for the novice teachers to prepare open classes with the instructions of the experienced teachers. Among the various dimensions, this study aims to explore teachers' resources from an aspect of "collective": What are the influences from collective work on teachers' resources system development? How do the influences help teachers in facing the challenges from the changes in curriculum?

With an aspect of micro-collective lesson preparation in two contrasting cases, one in China (and experienced teacher Gao and her apprentice Yao), one in France (two experienced teachers Anna and Cindy). The two lesson preparations happened both in a time of transition; but also, they happened naturally as part of teachers' regular work. Taking into account the different contexts of collective working and resources environments, the analysis on the collective lesson preparations will be combined with the states of teachers' resource systems. From the contrasting cases analysis, it is hoped to deepen the notion of documentation expertise, which is the key concept of the author's PhD.

1. Resources environments in China and France

China and France are both countries with "centralization in education": the governments hold the control of the unified national curriculum program, influencing the teaching resources environment for the teachers.

In China, there is a system named "three-level management of school curriculum", issued by the Ministry of Education in July 2001, as a result, the national school curriculum is consisted by national curriculum (representing the national education value), local curriculum (expressing the local practical needs), and school-based curriculum (allowing the school-running characters) (Zhong, Q., Cui, Y., & Zhang, H., 2001, P.354). However, it does not mean that the three levels are isolated, national curriculum and local curriculum are prescribed by upper level regulating education authorities, and schools and teachers just have to follow, but schools and teachers are given power and space to develop their own curriculum through the utilization of the curriculum resources in schools and the communities. The school-based curriculum is independently developed and managed by individual schools according to national guidelines, the interests and demands of the students, and the visions of the schools. In this way, there are few versions of textbooks (almost unified for teachers in specific cities) but very rich learning-aid materials market. While in France, due to the liberty of the press of textbook, there are various choices for the French teachers.

2. Collective work context in France and China

Collective working could be more developed in China, due to the culture and institutional reasons (will be presented with the reprehensive case of Teaching Research Group), but in France, with the new demanding of cooperation among teachers (such as inter-disciplinary teaching practice) from the curriculum program, French teachers are also required to adapt themselves to collective work. Working as the contexts of the two teachers' collective work, TRG and AeP will be presented in this section from their cultural backgrounds and historical developments.

With top performances in international tests, such as PISA, TRG is considered as an important factor for Chinese teachers' professional development (Wang, 2013). While remaining a limited network of schools associated to IFE in France, AeP is now a new typical exploration in teachers' collective work in France. They are both school-level collectives with strong institutional supports, closely linking with research institutions, which make them serve as a hub for connecting some professional collectives (both teaching collectives and researching collectives) as well as the resources circulating in the collective activities.

2.1 Collective culture and TRG in China

Working collectively, in China, is considered as essential since Confucius. "Whenever walking in a company of several persons, among them must be someone worth learning from (三人行，必有我□)". From the view of culture, the school-level working culture in China has been described as *collective* in Yang's study (2010). Research on teacher education in China shows that Chinese teachers are benefiting from some efficient school-based means (Li & Huang, 2008): they gain a deep understanding of basic mathematics and adequate pedagogical expertise through the activities of TRG. The word "TRG" firstly appeared in Chinese Education Ministry regulation in 1952, aiming to "study and improve the way of teaching". In 1957, the property and tasks of TRG were emphasized again and more clearly stated (Wang, 2013). Since the 1990s, TRG undertook the work of carrying out post-1990 curriculum reform. From 2001, encouraged to participate in education experiments, TRG slowly gained research components. Now the TRG has become a basic unit for teachers' collective work in each school, a main platform where resources are generated and shared through the regular collective activities.

Generally, a TRG consists of teachers from the same discipline, such as mathematics TRG, or English TRG. And each TRG is composed by several different Lesson Preparation Groups (LPG) based on grade, like a mathematics LPG in Grade 6. In most of the Chinese schools, teachers work full time with their own office or office desk, and generally speaking, teachers from a same LPG are arranged to share the same office (sometimes with other discipline teachers, it depends on the scale and conditions of the school), so that they could communicate with each other conveniently. LPG also works as the basic unit of teaching research activities. Sharing a same office, teachers from the same LPG work collectively very often.

The working modes of TRG could be sorted into "task-based activity" and "operation mode of diagnose-based activity" (Hu & Wang, 2014). The former is represented by collective design of resources, such as school-based exercise books, and the latter could be embodied through Chinese open class with several rounds of adjustments during the lesson preparation. Thus, the daily activities in TRG mainly focus on issues about design or implementation of teaching, for but not only for resources.

2.2 Collective experiences and AeP in France

AeP is a network of schools linked to IFÉ (French Institute of Education, see http://ife.ens-lyon.fr/lea/lea-english-version), ENS de Lyon. The first network, comprising 12 schools, was set up in 2011; up to 2015, there are 34 schools in this network, aiming to go beyond the boundaries between basic, clinical and technological research. To be a member of AeP, the school needs to be strongly supported by their administrative staff and to meet the interest of research of a research team in IFÉ. Then the school and the research team will co-propose to IFÉ a joint AeP project which builds on questions emerging from the actor's concerns (Chabanne *et al.,* 2015). Once a school, such as middle school A, becomes a member of AeP, its name will be changed into "AeP A" by other members. AeP builds an explicit association between schools and research institutes, to gain resources and better understanding from interactions between teachers and researchers, or teachers themselves. The short history of AeP does not mean that the collective work among teachers in France is recent. Actually, before AeP, IREM (Institute of Research in Mathematics Education), which gathers teachers and researchers, has existed since 1968, and the origin of teachers' collective work can be traced back to 1900, as the French Dictionary of Pedagogy (Buisson, 1911) saying, "Teaching is collaborating".

Different from TRG in China, AeP is not a compulsory choice for neither teachers, nor schools, that is

why at the beginning part of this session AeP was introduced as a limited network of schools. However, the teachers who join AeP have compulsory cooperation with the researchers in IFE, because each member of AeP needs to sign a contract based on a common research project, which generally lasts for three years, and could be renewed if it is agreed by both sides. In this way, activities of AeP provide the opportunities for teachers' collective work with both their colleagues inside their school and the researchers outside.

3. Theoretical framework

This study is based on two frameworks: *Documentational Approach to Didactics* (Gueudet & Trouche, 2009) for analyzing teachers' resource work as well as the elements of the expertise shown in their resource work; and *Activity Theory* (Engeström, 2001) for tracing the influences from collective work. This section will start from the definition of resource, then the two theoretical frameworks, and a preliminary definition of expertise in teachers' work with resources will be given based on the literature review.

Regarding the notion of resource, Adler (2000) described the resource as the verb "re-source"; a resource could be anything with the potential to *re-source* a teacher's activity. This study kept the idea of Adler's definition, but different from the resources that being discussed in the other two projects of Luxizi Zhang and Fangchun Zhu, the scale of resources being concerned in this study is broader, includes all resources used by the teachers, such as textbooks, teaching guidance or learning aid books, no matter digital ones, material ones, or personal ones, etc.

3.1 Documentational Approach to Didactics (DAD)

According to Documentational Approach to Didactics (DAD), the interactions between teachers and resources, including restricting, selecting, implementing, modifying, adapting, saving and sharing, were defined as *documentation work*: a document is composed of a resource and schemes of utilization. The documents of a teacher are articulated in a structured documentation system; correspondingly, the resource system constitutes the "resource" part of the documentation system without the scheme part of the documents. A scheme in Vergnaud's study (2009) was defined as the invariant organization of activity for a given class of situations, comprising the goal(s) of the activity, the rules to generate activity, the operational invariants for picking up and selecting the relevant information, and the possibilities of inference. The frameworks of DAD provided a view to see teachers' expertise in resource work by analyzing the elements of scheme in using resources: goal(s), rules of generating activity (how to do), the operational invariants (why to do), and the possibilities of inference.

3.2 Activity Theory (AT)

Teachers belong to institutions (Chevallard, 2006), their work is neither isolated nor individual, but part of society, their documentation work is connected to others, and culturally and socially situated (Gueudet, Pepin & Trouche, 2013). Hence, the *Activity Theory* (Engeström, 1987; 2001) is adapted as the second framework. Engeström expanded a mediational triangle as "activity system" from Vygotsky (1978) and Leont'ev (1978), with six elements: subject, mediating artifacts, object, rules, community and division of labor, emphasizing that activity should be situated into a cultural and historical background with five principles: (1) the activity system as a whole as the unit of analysis; (2) multi-voicedness, "multiple points of view, traditions and interests"; (3) historicity, "activity systems take shape and get transformed over lengthy periods of time"; (4) contradictions, "as sources of change and development"; (5) the possibility of expansive transformations (ibid, pp.136-137). Inspired from Activity Theory, teachers' resource work could be situated into an activity system with the corresponding elements: teacher (subject), resource (mediating artifacts), object, rules, collective (community), and division of labor. While the activity system is taken as the unit of analyzing teacher's activity, the resource system of the teacher also gets developed (re-organized or enriched) along with the teacher's resource mobilization from her resource system to achieve the goal of the activity. It is believed in this study that the resource that works as the mediating artifact in the activity system comes from the teacher's resource system. This way, teachers' resource system could be understood by observing how the teacher adapts it in specific activities, while teachers' activities could be analyzed through a lens of her resource system.

Activity Theory provided a framework to trace the influences from the collective, (1) the history and culture of the collective where the teachers work in will be considered when analyzing their collective work, (2) the points of view from other colleagues in this collective will also be paid attention to when following the targeted teachers, (3) the contradictions between the targeted teachers and other colleagues (like the conflicts in using resources), as well as the collective or the environment (such as new tasks or challenges) will be studied carefully to see the "source of change and development". The

specific tools inspired from DAD and Activity Theory will be introduced at length in the methodology section.

The expertise in teachers' documentation work is defined as *Documentation Expertise (DE)* in this study. According to Berliner (1988), expertise "is *specific to a domain*, and to particular contexts in domains, which is developed over hundreds and thousands of hours". Key elements of expertise are linked with teaching problems solving efficiently and creatively with a wide range of knowledge and experiences (Sternberg & Horvath, 1995), or to be more precise, teachers with "adaptive expertise" were proposed as "specialists in retrieving, organizing, utilizing, and reconsidering their professional knowledge and beliefs" (Avalos, 2011). Drawing from the definitions of expertise and documentation work, DE is defined preliminarily as the schemes while interacting (retrieving, selecting, organizing, modifying, adapting, creating and sharing off) with resources. DE is considered as a developing state of teacher expertise, offering a resource aspect to explore the way of teacher's professional development (Pepin et al., 2016).

To summarize, DE could be defined as a series of structured schemes in resources retrieving, selecting, organizing, modifying, creating and sharing off, with an aim of achieving some teaching goals or solving some teaching problems.

4. Methodology and tools

The methodology and tools inspired by DAD (such as the reflective investigation), and Activity Theory (such as the documentation-working mate) will be presented.

4.1 Reflective investigation and the tools developed

Reflective investigation involves the teachers as part of the study throughout the whole data collection, with four principles: long-term follow-up; in- and out-of-class follow-up; broad collection of the material resources used and produced throughout the follow-up; and reflective follow-up of the documentation work (Gueudet et al., 2013, p. 27).

To know how the teacher organizes and represents her available resources, and in line with our pilot study (see Pepin et al., 2016), we adapted the tools of "*Reflective Mapping of Resource System (RMRS)*", in which a teacher is asked to draw a map to present her resources in a structured way based on her own reflection, and *"Inferred Mapping of Resource System (IMRS)"*, in which the researcher completes some information on the RMRS derived from combining the interview and observation. It should be noticed that the RMRS and IMRS are not final, but will be improved, complemented, and reorganized continuously during the long-term follow up, along with the development of teachers' reflections on their resource systems.

Some other tools were also developed. An online *"Reflective Investigation Box (RI Box)"* was built and shared between the researcher and the targeted teacher, in which the teacher could share her resources used in her activity (such as lesson plans, screenshots of blackboard writing etc.), and respond to the questions (either about resources in RI Box, or any other questions) from the researcher regularly. The software chosen for supporting RI Box depends on teachers' using habits in different contexts, for example, RI Box supported by Dropbox[1] in France, and Wechat[2] in China.

Besides field notes of teachers' activity, observation and school visiting from the researcher were also adapted. The combination of field notes and RI Box provides the possibility of a long-term follow-up of the teachers' resources, for example, during the activity, what resources are integrated, where these resources come from, and how they are integrated.

4.2 Documentation working Mate

Following the principle of "multi-voicedness" of Activity Theory, a new notion of *documentation-working mate* was proposed here as someone who works closely with the targeted teacher, with mutual influences on their documentation work and DE development. *Mate* in Oxford Dictionary is defined as infers "a fellow member of joint occupant of a specific thing, like table-mate", with an "origin related to meat (the underlying concept being that of eating together)[3]". The reason to choose "mate" and but not "peer" as in "peer education" (Turner & Shepherd, 1999) is that "mate" breaks the boundary of age and education/professional background. For a given teacher, her *documentation working mate* could

[1] Dropbox is a file hosting service, which offers cloud storage, file synchronization, personal cloud, and client software. See more information on: https://www.dropbox.com

[2] Wechat is a Chinese social media, with the functions like instant messaging (text and voice), hold-to-talk messaging, broadcast (one-to-many) messaging, video conferencing, group chatting, official accounts and moments etc. See more information on: https://web.wechat.com

[3] See in https://en.oxforddictionaries.com/definition/mate

be a colleague with similar working experiences in her school, or someone from a totally different working context as an university or research institute etc. In each case of this study, a *documentation working mate* will be selected according to the targeted teacher: they form the smallest but closest collective, and the documentation working mate will be followed in the same way as the targeted teacher.

4.3 Research design

Designed as a case study focusing on DE from different contexts, two mathematics teachers were selected from two middle schools, one (named Gao) from Shanghai in China, and the other (named Anna) from Lyon in France.

The two teachers' work is situated in different collectives, TRG in China, and AeP in France. TRG is chosen because it is a widely spread collective schoolwork unit for Chinese teachers since 1952, and AeP is selected because it is an association of French schools linked to the French Institute of Education (IFÉ) which started in 2013 and expanded rapidly but with rapid expansion.

The cities where the schools locate, Shanghai and Lyon, are both developed cities, and the middle schools selected were both located in the city center and they all have close cooperation with the research institutions where the author works in, one is an affiliated school of ECNU and the other is a member of AeP. From the level of students' performance and teaching technology equipment, they were both ordinary schools, neither top nor bottom.

The two teachers were selected because of (1) their willingness to participate in the research; (2) administrative support from their schools; (3) active participation in collective work; (4) rich working experiences and good technology operation skills.

5. Analysis and results

The analysis will be presented in two steps: (1) the evolution of the different versions of lesson plans of each case, as well as the resources integrated along with the collective work process; (2) the documentation expertise (the schemes) shown during the collective lesson preparation.

5.1 Evolution of the lesson plans and DE found in the Chines case

Within a process of MOKE activity (a continuous and intense collective lesson preparation work on one topic), there were four times of collective lesson preparation work in the Chinese case, and each time, Yao modified the lesson plan with the suggestions from her mentor, Gao. At the beginning, Yao used to plan to teach the three properties of in-equality in one lesson, but Gao suggested her to choose only one: "it is too much for them, and if you teach too much, it will influence the depth of this lesson" (a view on the students). Yao kept Gao's suggestion, and then in the first version of the lesson plan, Gao pointed that the exercises and examples selected by Yao were not typical and profound enough, she gave a learning-aid exercise book to Yao for reference (a view on teaching content and recommended resources). Yao enriched her lesson plan largely and implemented in front of Gao. In their discussion after the lesson, Gao suggested to delete some exercises and add some activities because "the reactions of the students and classroom teaching atmosphere were not very good" (a view on the students), she proposed an activity of "using a balance to introduce the necessity of in-equality", but also mentioned "do not use too much, otherwise it will be a physic lesson not a mathematics lesson". Yao took this advice and implemented in her third lesson plan. Gao rejected the activity of the "balance activity" after she observed the third open class, "it was too small when you were operating the balance in front of the classroom, the students sitting behind cannot see it" (a view on the students), and she gave a suggestion to use mini-video in the summarization part for saving more time, and the specific website to search for the videos (a view on the lesson skill and resources). In the last version of the lesson plan, Gao asked Yao to add some exercises of equality, which was disagreed by several teachers, "lessons should be never isolated, it should always keep the links with their previous knowledge." (a view on students and mathematics content).

Seeing through the whole trajectory of suggestions from Gao, there is a flow in her lesson preparing work:

(1) Finding the balance point of "content" and the "interest of the student", for Gao, is a process of trial and error. She enriched the content of the lesson as much as possible, then cut down and add the necessary activities according to the reactions of the students. This showed a student-centered idea of Gao. In an interview with Gao, she shared a "tip" to attract students' interest in learning mathematics:

Q: "Why do your students always get the best performance in the whole grade?"

A: "To be honest, I know how to go along with them, and let the students like me. They like me first, then they like learning what I teach: mathematics."

(2) Integrating the available resources (what Gao gave to Yao) is a process of complementing the details ignored in her resource system. In Gao's resource system, the resources were categorized mainly according to the sources of her resources: the exercises from various learning-aid exercises books, the feedbacks from students' homework (the points that students often make mistakes), the experiences from the discussions with other teachers (the points that students have difficulties to understand), the new resources obtained in the lectures (the new ideas on the examinations or the new exercises types etc.) However, the specific process of using these resources is hard to be seen only through the responds from the interview and mappings of resource system. Gao said she seldom use online resources, but proposed a website with mini videos to Yao, can be considered as a paradox that even ignored by herself, or a complementation for her real resource system.

5.2 Lesson plan generating and DE shown in the French case

Different from the MOKE activity of Gao and Yao, in the French case, Anna and Cindy did not repeat the process of refining the lesson plan. During one hour of this collective work, they hold a process from discussing on the program (because this was their first time of preparing this topic), then to the textbooks (it was the time to decide a textbook from various choices), then integrating the available resources that can be used in designing the activities for students. In this one collective lesson preparation, it was a process in interpreting the program: reading and proposing their understanding about the program by selecting the resources. Seeing the words and actions from Anna, some schemes of resource work can be found:

(1) Starting points of a new task can be traced back to the personal initial understanding on the topic. For Anna, at the beginning, both she and Cindy mentioned the difficulties (lack of teaching/learning/training experiences in teaching algorithmic), but in the following decisions, Anna showed her strong personal idea towards "what should be taught to students in algorithmic": "Teaching algorithmic is not teaching a software".

(2) Taking the curriculum program as a basic for making decisions of resources, which is a common idea hold by both Anna and Cindy. Anna made some preparation work before this formal collective lesson preparation: she copied the texts of algorithmic from the program and pasted it into a word document.

5.3 A summarization of the two contrasting cases in a collective aspect

The collective, as well as the resources obtained from it, provides a common base for teachers' collective work. The position of "collective" in providing resources is more obvious in the French case, because several resources (such as the activities in IREM) come from the common collectives they participated, and the collectives appeared in their dialogues were more specific and high-qualified with very professional and well-organized websites (running by the teams of academic researchers). While the French case, the influences from the collective were shown more in the instruction process from Gao to Yao. Combining with the interview with Gao, she learned almost in the same way from her mentor, like what Yao was learning from her now. The mentor-apprentice instructions donates more opportunities for the novice teachers to learn the experiences with specific and real problems, from general questions of "how much contents should be taught" to details like "how to organize the order of the exercises".

However, there are also constrains from the collective. In the Chinese case, Yao followed most of Gao's suggestions, and Gao implemented her ideas through instructing Yao's work, which could be considered as a double-beneficiation mode for teachers' professional development. In the case of Anna and Cindy, as two more equal-position teachers, Anna and Cindy conducted their collective work in a way of true "cooperation": donate the resources and ideas to each other. However, the collective also brings constrains: Yao grows up in the limit of Gao's experiences, while Anna and Cindy continue their dialogues based on their common collectives. Out of the collectives they are participating, views may change when the contexts changes.

6. Discussions and Perspective

Drawing from the results, two issues, the carrier of resources and the collective, as well as the consideration on our following work, will be discussed in this section.

6.1 A discussion on carrier of resources

The carrier of information (can be also called media) could be a factor worth being discussed further. When the teachers are presenting their resource system, one structure will be decided, according the medias (online or in on papers), the locations (in the library or in the computer), or the sources (from website or from people). The carrier of information has its limits with both advantages and disadvantages, for example, books in the library could be considered as some resources with a series

of serious quality filter work by authors, editors, publishing stores and the managers of the library, while the online resources could be less critically examined on the content and quality, but more flexibility on the richness. Another example, MOOC used to bring a tornado during the first years after it was born, with an ideal of spreading the best education free charge to ordinary being, and building the links between the knowledge, the knowledge pursuer and the teachers. However, decades later, in the time of information overload, problems remain to be there, not a problem of overload, but maybe filter failure.

Seeing in this aspect, although there are differences in the resources preferences in the two cases, it does not mean that using digital/online resources is more developed than keeping the habit of using paper materials. Knowledge and experiences behind such preferences can be understood as a kind of interpretation, and interpretation is a social being, with specific context and without superiority or inferiority. This is the value of making our contrasting case studies: understanding more.

6.2 A discussion on the influences of collective

The richness of resources along with the booming of Internet brings an issue of the source/information credibility. A resource user can also be a resource producer. In the huge room with unlimited information and resources, two mechanisms can be considered in searching for and selecting resources: algorithm tool and social communication tool. On one hand, you can search with keywords, then the search engine can react instantly with the various and rich results through its developed algorithm technology. Also, you can keep a resources following up in a specific social circle, online or offline, this is the value of collective: a common interest can be a natural filter and provider for the resources. From this aspect, the collectives that the teachers are participating can be paid more attention on the openness (providing more opportunities for teachers in talking with others, such as the IREM and AeP network in France, the regular lectures for teachers in China) and the quality (enhancing the resources and information quality).

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Chongyang Wang, defining a MOKE in a Prosfer master class, in ECNU, November 2015

Some comments and suggestions on the research

Jiansheng Bao (Department of Mathematics, East China Normal University)
http://faculty.ecnu.edu.cn/s/76/t/155/main.jspy

The MaTRiTT project is focused on the mathematics teachers' resources in a time of transition. Around 10 year ago, most Chinese mathematics teachers were mainly depended on paper-based teaching researches (such as textbooks, instructional guides) during their lesson planning, but now there are more and more different kinds of teaching resources, especially e-resources, are available and easy to use in mathematics classrooms. It is really the time for transition and research. So I think, the mathematics teachers in both countries will benefit from the MaTRiTT project.

1. Xizi's work on variation problems

Existing studies (e.g., Gu, Huang, & Marton, 2004; Wong, 2008; Pang, Marton, Bao et al., 2016) have documented that Bianshi (i.e. teaching with variation) is a commonly used and effective approach in mathematics teaching and learning in China. According to Jones and Herbst (2012), teaching with variation is a well-established strategy among Chinese mathematics teachers. The variations found in the classroom teaching of mathematics have been interpreted and categorized differently. Some scholars (e.g., Wong et al., 2009; Sun, 2011) consider the use of 'spiral Bianshi/variation' in different aspects of mathematics teaching to be different methods or stages in mathematization. Others (e.g., Watson & Mason, 2005; Mason & Johnston-Wilder, 2006) have highlighted the importance of distinction-making by students, through their discernment of change and invariance in the course of making sense of mathematical situations. Meanwhile, researchers in the West have also emphasized the use of variation to improve mathematics teaching and learning, such as deliberately varying mathematical tasks to facilitate students' learning of mathematics (Rowland, 2008; Watson & Mason, 2008).

Although the teaching with variations (or Bianshi) is a popular way for mathematics teaching in China, but many mathematics teachers still feel difficult to find suitable and good variations to help students' conceptual understanding and problem-solving. Xizi's work is to find the role and qualities of variation problems in mathematics textbooks in China and France. In this study, we can study more about how teachers vary in their lessons. There are two main categories of variation in the mathematics literature in China: conceptual variation and process variation. Maybe we can add more details into these two categories. And another important questions need to be focus is how mathematics teachers use the variation to have lessons. Also we need to pay attention to why teachers task designing or way to teach new contents or learning style.

2. Fangchun's work on dynamic geometry environment

As Fangchun mentioned in his reports, with the transition of curriculum, information technology, like dynamic geometry software, plays more and more critical role in mathematics education. In China, in curriculum level, especially in the new *Mathematics Curriculum Standar4ds for High School* which published in 2018, it is encouraged to integrate information technologies into mathematics textbooks, instruction and assessments.

Comparing to other subjects in school, mathematics is more difficult for students to understand. Many mathematical concepts are very abstract, formal and symbolic. Visualization is a useful tool for mathematics teaching and learning. Dynamic geometry software may help students to construct geometric models for some abstract mathematics concepts, find strategies for mathematical problem-solving, and form mathematical intuition.

However, in the real mathematics classrooms, there are still very few teachers to use dynamic geometry software in their teaching. This is not because many teachers are nor familiar with the technologies, but also because they have not find the advantages of dynamic geometry software and still rely on the traditional teaching styles. I think, Fangchun's case study could provide some good sample tasks and lessons for other teachers.

3. Chongyang's work on teachers' collective work

In the research field of teacher education, researchers usually pay more attentions on teachers' professional knowledge, such as PCK and MKT. But in the Chinese tradition, teachers' professional development are focused teachers' working abilities. This is a kind of Practice-based Knowledge and

Competency (PKC), which is usually developed by teachers' collective work, such as collective textbooks analysis, task design, and open lessons. So I think Chongyang's research is useful for the constructing of the Chinese style theories and practices in teacher professional development. This research pays attention to the documentation expertise in collective work, in teachers' practices, especially in collective work; human relationship is one important element in analyzing teachers' behavior with other colleagues.

In Chongyang's report, she gives reasons why there is a gap between teachers' resources and dynamical software (why teachers don't use technology?): teachers don't want to change too much in their teaching and teachers want to let the students see the process of drawing. I added some reasons here: We now use textbooks in a traditional way, and textbooks were not technology-oriented designed before. But I think this phenomenon will change as we are now designing new textbooks of Shanghai and we are planning to put many tasks with technology in the new textbooks.

There are still many research questions for further study:

- What kinds of resources teachers use most? (Teachers' development resources (task design, pedagogy, exercises); teachers'' teaching resources)
- What is the way teachers use/construct the resources?
- How do the teachers choose the resources? (Which one is necessary for teaching?)
- Can be more clearly about the two aspects: how to collect the resources and how to use the resources? (Agreed with pro. Xu)

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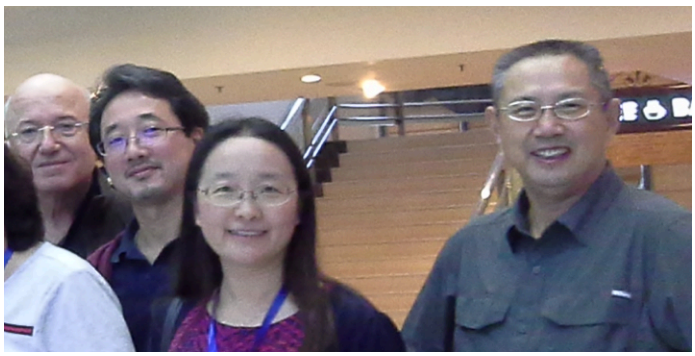


From left to right, Luc Trouche, Takeshi Miyakawa, Binyan Xu and Jiansheng Bao at the first meeting of the International Program Committee (IPC) of the 14th International conference on mathematical education (ICME-14).

Meeting held in September 2017, ECNU, Shanghai.

Comments and suggestions

Binyan Xu (College of Teacher Education, East China Normal University)
http://faculty.ecnu.edu.cn/s/1469/t/16871/main.jspy

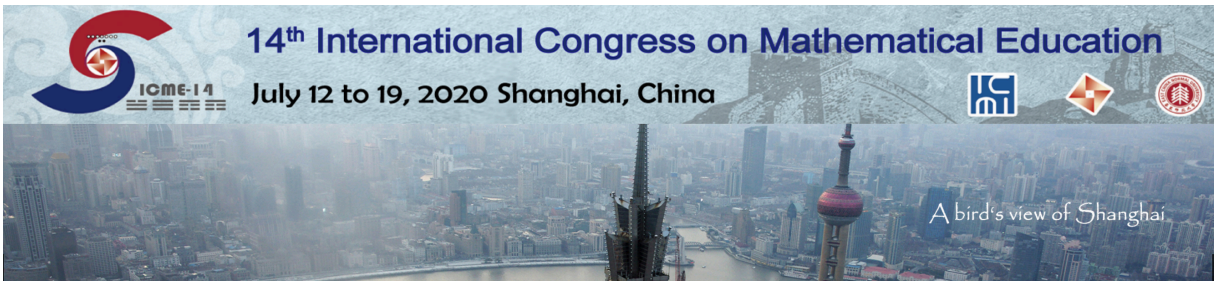
MaTRiTT Project enhanced the research collaborations between ENS de Lyon and ECNU (Shanghai). As Prof. Luc Trouche in the introduction remarked, the both groups from ENS de Lyon and ECNU focus on mathematics teachers resources in a time of transitions and have recognized several related research considerations which include investigation of teachers' documentation work, analysis of mathematics textbooks, or exploration of usage of digital resources in mathematics classroom. Not only the researchers worked cooperatively, but also PhD. students engage in this research area. In the final seminar the three PhD. Students reported their preliminary research and showed us some interesting and considerable results.

The presentation of Luxizi Zhang reported the characteristics of Chinese and French mathematics textbooks from the variation viewpoint. She analyzed the mathematics topic – function within the textbooks and proposed that the Chinese mathematics textbooks paid attention to developing tasks /mathematics content in different situations and using various scaffolding activities, while the French mathematics textbooks attached importance to constructing different learning activities which encourage students to inquiry and explore directly. The development of mathematics textbooks often reflects particular didactic considerations. I suggested that Xizi should clarify the meaning of variation perspectives in different context, and review more documents about curriculum development in different cultural setting, in order to illustrate the nature of mathematics textbooks.

The presentation of Fangchun Zhu referred to the role of geometric software (one digital resource) in mathematics classrooms. Based on classroom observation, he found that one Chinese mathematics teacher demonstrated mathematics content using geometric software proficiently; students had no chance to experience such software. But, one French teacher designed a learning task that students needed to use geometric software to inquiry. It seemed that two teachers had different didactic considerations while implementing mathematics teaching. In order to understand and analyze such teaching characteristics, I suggested that Fangchun should collect additional data, including teachers' knowledge of geometric software, teachers' understanding of role of geometric software, students' viewpoint of using geometric software in mathematics learning.

The presentation of Chongyang Wang focused on development of teachers' expertise of documentation work while teachers working collectively. This is a very important research issue related to mathematics teacher resources. Chongyang collected different kind of data from French and Chinese teachers based on a reasonable theoretical framework. She explained how teachers worked collectively in different cultural background and what kinds of patterns of teachers' documentation work have been developed. The patterns, which Chongyang established based on collected data, could exhibit intuitively how Chinese or French mathematics teachers dealt with all sorts of teaching resource and why they manipulated resources in that ways. The preliminary findings are inspired. So I suggested that Chongyang could begin with the next question, namely, which roles the teacher's collective work play in order to improve teachers' expertise of documentation work.

MaTRiTT Project supported the PhD. students very much. On the one hand the students could travel to Shanghai and collect data, on the other hand the French and Chinese supervisors could organize several joint seminars and discuss with students intensively. As Chinese supervisor and researcher, I am grateful to the French supervisors and researchers, Prof. Luc Trouche and Sophie Soury-Lavergne for achieving this MaTRiTT Project. This is excellent platform for our collective research works. As one of the co-chair of LOC of ICME 14, I encourage the students to present their research works at ICME 14, 2020.



Short comments and questions on the research from three different perspectives

Takeshi Miyakawa, Joetsu University of Education, Japan,
http://www.juen.ac.jp/math/miyakawa/index-en.html

During my one-year stay in ENS de Lyon as a visiting scholar, I had a chance to know PhD research works (in progress) of mathematics education by three Chinese students, and to take part in the final seminar of MaTRiTT Project. The theme of project "Contrasting teachers work with resources in a time of transitions, in France and China" interested me deeply, because I had been also working on the comparison between France and Japan (Miyakawa, 2017). The presentations given by the PhD students in this seminar in addition to their daily works allow me first to reaffirm the commonality among east Asian countries and at the same time their distance from European countries (France in particular), and then to ask several questions which would be interesting to investigate in the future studies. In what follows, I would like to make some little contributions to this project by providing some comments on the theme of project and each work of PhD students.

The first general comment is concerning the international comparison or contrasting. The objective of such studies as a scientific research, we probably share, is not to evaluate the performances of each country, but to identify and understand the phenomena of mathematics education in order to obtain some implications for the educational practices. In such a perspective, it is expected to go further beyond identifying the differences between different countries, and to develop the theories to better understand educational phenomena and what makes such differences. The differences very often provide some clues in this respect.

1. Zhang's work on variation problems

Zhang's work focuses on the textbook, which is the most important resource for teaching mathematics in most of countries. The three principal elements in her work, which are 1) variation; 2) textbook; and 3) teacher's work, could be analyzed separately, in order to show the contrasts between China and France in each aspect. But it will be more interesting to investigate how these three elements are intertwined and what kinds of mathematics learning are produced. Such study will advance our understanding on the phenomena related to the teachers work with resources beyond the differences between countries. One of key questions for now would be what kind of theoretical framework allows us to bring forward this study.

Another aspect, which seems important to deeply investigate, is the mathematical knowledge at stake: what kind of mathematics is really dealt with in each country. I think the difference between European countries, especially France, and Asian countries like China and Japan is much bigger than what we imagine. The point is not only whether some specific concepts are taught or not, but their nature. For example, the proof in geometry is commonly an object to be taught in lower secondary schools in France and in Japan. However, not only its form but also its roles in geometry are different (Miyakawa, 2017). It is necessary therefore to make clear in the first place the nature of mathematical knowledge at stake in each country. Such study would allow us to identify the phenomena of teachers work which are specific to the mathematics teaching.

2. Zhu's work on dynamic geometry environment

Dynamic geometry software (DGS) is rarely used in Japanese lower secondary school. Almost no teacher uses it for students to explore geometrical tasks, and only a very few teachers use it for the purpose of showing dynamic movement of geometric figures, even in the ordinary classroom with students who have iPad at hand, and even in the classroom with interactive electric board with e-textbook. There are several reasons for such avoidance of DGS. Teachers might not be ready for teaching new stuffs; there are not enough tasks or resources; etc. I think one of the aspects that will be very interesting to investigate is the body of mathematical knowledge they want to teach as geometry in lower secondary school. What if it is only the axiomatic structure of geometry by reorganizing the geometric properties, which have been already known to students? What if it is only the proof and proving as a mathematical method? At least, the kinds of geometry, which are to be taught in France and China, would be very different, as I mentioned above, and affect the way of teaching.

3. Wang's work on teachers' collective work

Wang's work focuses on teachers' collective work. It is very interesting to investigate the questions like, how is the nature of collective work different in these two countries? What makes such differences? As long as I know, these questions have not been fully investigated so far in mathematics education research. I think the difference is big enough, due to the mode of interpersonal relationship, the way to perceive the teacher's work, the mathematical contents to teach, the structure of school and educational institutions, and so on. In order to understand the mechanism that makes differences in teachers' collective work, it would be helpful to refer to the ecological perspective of the anthropological theory of the didactic (ATD) (e.g., Bosch & Gascón, 2014) which allows us to pay attention to the conditions that make viable or promote such collective work in a given place (or institution) and to the constraints that hinder it. It is the system of these conditions and constraints that shape teacher's collective work. This theory also provides a tool to classify the conditions and constraints that come from different levels.

Another question I came up with in Wang's work is related to the main issues, the characterization of the notion of documentation expertise: how the documentation expertise relates to the expertise in the classroom or in the organization of lesson? Moreover, how would the expertise be different in China and in France in the circumstance where the organization of mathematics teaching is different as well.

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Jiansheng Bao, Takeshi Miyakawa, Luc Trouche and Binyan Xu are part of the International Program Committee (IPC) of the 14th International conference on mathematical education (ICME-14), to be held in ECNU, Shanghai. A picture of the first IPC meeting below.

Comments and questions from a Brazilian researcher

Verônica Gitirana, Universidade Federal de Pernambuco
http://ufpe.academia.edu/VeronicaGitirana

The MaTRiTT Project brings to the research community of Mathematics Education and on Teachers' Resource important issues about the mathematics teachers' work with resources as well as about the resource itself, such as the textbooks. Throughout theoretical lenses, Documental Approach to Didactics (Gueudet & Trouche, 2009) and of the Teachers' Collective Work (Miyakawa & Winslow, 2017), the two French and Chinese realities are studied by three Chinese PhD students co-supervised by a pair of researchers, one Chinese and one French. I was kindly invited to the last seminar of the Project, when other researchers and research students and I had opportunity to see some results of the research that are outcomes of this methodological structure of undertaking *contrasting* researches, which allows us to understand one reality by observing aspects of the other.

My contributions to the discussion of the researches undertaken in the project include some different aspects: first, I would like to emphasize how, in spite of investigating a small part of Chinese and French education, their results demark clearly the cultural characteristics of each country and how they are related to teachers' practice with resource; second, how the results reveal the need to integrate the two theoretical bases, Instrumental Orchestration and Documental Approach to didactics, to achieve a better understanding of teachers practice with software; finally, pointing out to more investigation on how and which characteristics of resources can aid teachers' development, in particular to use the idea of didactic variable while using textbooks.

1. Wang's work on teachers' collective work

Chongyang Wang, co-supervised by Pr. Xu and Pr. Trouche, brings to us the analysis of the collective documentation work of a pair of teachers from each country, in China and France.

The large amount of data analysis she brought is fantastic! The comparative study undertaken, even starting from a pair of teacher of each country, her study of teachers' documental expertise reveals very clearly the cultural differences. In my viewpoint, it was very important to observe how their culture affect the way they take information, an essential component of the schemes (Vergnaud & Récopé, 2000), for lesson planning. While in China, the expertise of the teacher with more time of experience plays a central role in younger teacher source; in France, the collective of researcher plays the most important role in teachers' source. Differences on where or with who teachers trust to "take the relevant information" are deeply demarcated by culture. In fact, if we consider also the *Anthropological Theory of Didactics* (Chevallard & Cirade, 2010), her results can give hints on how cultural organization affects the access to and use of knowledge of institutions, of different levels, the teachers use to "take their information". She also discusses on how the roles assumed by teachers in the two different cultures affect their documental expertise.

2. Zhu's work on dynamic geometry environment

As regard to Zhu' study, co-supervised by Pr. Xu and Pr. Soury-Lavergne, he takes his interest on the use of dynamic geometry software. He brings into discussion teachers' instrumental orchestration within dynamic geometry software in the two countries.

Zhu took into discussion some interesting results on identifying and classifying teachers' instrumental orchestration on using Dynamic Geometry Software (DGS) in class. He classifies different orchestrations teachers create while using dynamic geometry software. One of the interesting discussion, raised by Pr. Xu during the seminar, is the need to understand why teachers' use DGS in a traditional (as presentation) way, maintaining the same methodology. I will here take into account something I read a long time ago in the book – "Software Goes to School" (Perkins, 1997). In a moment of transforming, mainly on transforming to include digital technology in classroom, teachers have to deal with new technology, new methodology as well as new contents to integrate software in class in the way we defend. Nonetheless, many times teachers do only part of this transformation. In my point of view, even when teachers use the software without changing their methodological practice, it is the starting point of a great transformation in their practice. Transforming all, resource, teaching methodology and contents is a great challenge for their documental system; after a study of their orchestration, approach their development can help us to better understand the evolution of their re-source. In my practice of teachers' trainer, I have observed that when teacher first meet DGS, it

represents a moment when they stop – for a moment – to learn more about geometry. And so, as they are in teacher training course, they integrate the software, in a first time, to improve their own knowledge of Geometry. The transition to digital technology implies a metamorphosis of mathematics teachers' work, nonetheless, a metamorphosis of their work involves a metamorphosis of their own knowledge. The transformation of an artefact into an instrument involves a transformation of the user (teacher) (Rabardel, 2001; Lucena & Gitirana, 2016). One of the first steps in their transformation is the transformation of their own knowledge of the content.

Zhu characterize teachers' orchestration with the dynamic geometry software, and brings interesting insights on how to discuss their schemes of use of the software. Nonetheless, teachers' reason to use, sometimes in a traditional way, can be investigated taking into consideration the *Documental Approach to Didactics* to understand these orchestrations in a more complete process of incorporating resource in their professional valise. This work led us to consider, even more, the need to emphasize the benefits of the integration between the two approaches. I argue that integrating two theoretical approaches "Instrumental orchestration" and "Documental Approach to Didactics" has a potential to reveal much about it.

3. Zhang's work on variation problems

Zhang, co-supervised by Pr. Bao and Pr. Trouche, analyzes the structure and use of textbooks in China and France. In this seminar Zhang brought a comparative analysis of two textbooks, one of each country.

She brings an interesting study on how textbooks take into account function variation as well as didactic variables. Based on the theory of the didactic situations (Brousseau, 2006), in her presentation, she brings the idea of *didactical variable* to investigate it in French and Chinese textbooks. She has some first results showing elements that are varying from different parts of the textbooks, different activities, variety of representations, variety of number fields, etc. This is a very interesting way of analyzing the textbook by searching characteristics on them, that can help teacher development in considering the didactical variables for adapting the exercises to their classes. Nonetheless, these variations in different activities can take only a small bit of the sense of didactical variable; because, even integrating activities that vary on representations, context does not enable (due to the necessary training) teachers to vary any aspect of an activity whenever it is necessary to promote students' learning.

4. A Brazilian experience

Based on my experience on Brazilian National Program of Textbook Evaluation (PNLD) in Mathematics, as textbook assessor as well as coordinator of the process, since PNLD 2002 till PNLD 2018 (Brasil, 2001, 2017), I make some points on Zhang's presentation. In all the textbooks, I have already read (I think more them 120 collections from primary schools to upper secondary schools). I have read very few examples in which the teachers' guide discusses a bit what kind of variation teachers could (or should) do for each kind of students' situation. In Brazilian Textbook Evaluation Program (PNLD), we had been promoting that by adding some requirements to its public notice that invites publisher to submit the textbooks. We added requirements demanding that all textbooks, in the Teachers' guide must orient the use of the activities discussing different strategies of resolution, possible students' errors and possible changes on each proposed activity. The changes are not satisfactory yet, but some efforts are already done.

So, Zhang's study while incorporating the idea of didactic variable into the analysis of the textbook is very fruitful for the discussion of this resource. It deals with the viewpoint of the research on the characteristic a resource should have to promote development of teachers' practice.

Finally, my contributions point out to important issues brought by: the methodological way the co-supervision of the three PhD studies has been undertaken; their results which aid us to see how cultural differences emerges on teachers' relation with resource in their practice, showing how important is to use different lens and understand different cultures; the need of new investigations that we can already see by their investigations.

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Verônica Gitirana, with her team (Edumatec), at Universidade Federal de Pernambuco, Recife (Brazil) during the stay of Luc Trouche for a master class in 2016.

Conclusions and perspectives

Sophie Soury-Lavergne, French Institute of Education, ENS de Lyon
http://eductice.ens-lyon.fr/EducTice/equipe/membres/permanents/sophie-soury-lavergne

This MaTRiTT project has been a very rich opportunity to gather different points of view on the ongoing PhD research works jointly conducted by ENCU and ENS. With the outside critical views of Pr. Miyakawa and Pr. Gitirana, we have reach some first agreements and identified principles, which may ground our French-Chinese collective of researchers.

To begin, we share a common view on what could be achieved by looking at the teachers' resources from both French and Chinese perspectives. We are not looking for differences and means to compare our both countries. We are rather looking for examples that enable us to contrast the situations and to reveal implicit phenomena. This methodology, based on contrasted examples, works like a revelator of each local situation, which puts lights on the phenomena provoked by the evolution of teachers' resources and collective work. Contrasting is a way to open "*a windows on resources*" to cite the famous book of Noss and Hoyles (1996). A second point of agreement is the attention dedicated to the mathematical content. The mathematics at stake, in the students works, in teachers' orchestration, in the work of teachers' collectives and in any resources must always be explicit in our analysis and worked out in the results we produce. It is a condition to provide results that may be useful to our mathematics education community.

We have also identified pathways for our future works.

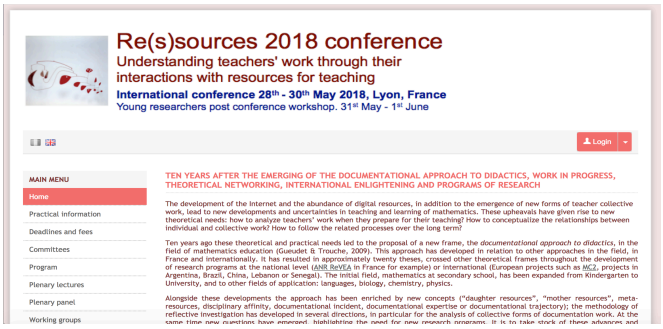
We have to solve some methodological issues when dealing with teachers, resources and collectives. Our methodological needs are critical, when it comes to teachers' practices that may appear to be rare, like a routinized use of dynamic geometry by students in class or a routinized collaboration work between teachers. We have underestimated the difficulties to involved teachers coming from each of our country with some of these specific practices. Moreover, the involvement of teachers in our research must results in a fruitful collaboration both for teachers and researchers. Therefore, the way we organize collaboration with teachers must be improved, in line with the already established methodology of reflective investigation (Gueudet & Trouche, 2012). In this perspective, the French example given by the LéA "Lieux d'Education Associés", field organizations associated to the IFE actions of research, may be helpful.

Beyond methodological issues, as Pr. Gitirana has remarked, we have to further develop theoretical frameworks, connecting the instrumental orchestration (Trouche, 2004); (Drijvers, Doorman, Boon, Reed, & Gravemeijer, 2010) and the Documentational Approach of Didactics (Gueudet & Trouche, 2009) in order to take more deeply into account the relationship between the teachers work outside and inside the classroom. We also have to introduce the institutional dimension in our analysis by referring to the Anthropological Theory of Didactics framework (Bosch & Gascón, 2014).

We also have to improve the intertwining of PhD works. In this perspective, the notion of variation used to analyze the textbooks may be part of the analysis of dynamic geometry tasks, pointing out the possible relationship between variation in the type of dynamic geometry tasks and teachers' knowledge development. The Documentational Expertise elaborated to study teachers' professional development along collective work may also be connected with already developed ideas like variation or dynamic geometry type of tasks. And even if mathematical textbook are not technology oriented, the a priori analysis related to the introduction of technology into some of the tasks provided by the textbook may create new perspectives for studying variation.

The future of this research and its collective of researchers is insured by some milestones, which will give us good opportunities to meet and develop the ongoing work. The first rendez-vous will be the Re(s)sources2018 conference, held in Lyon in May 2018. Pr. Trouche chairs its scientific committee and Pr. Soury-Lavergne supervises its organization.

http://resources-2018.sciencesconf.org/



Waiting for over a hundred of participants, from more than 20 different countries all over the world, this conference will be a unique occasion to debate about the last developments of the Documentational Approach to Didactics. The MaTTRiT project will constitute an important contribution to the conference, through Pr. Xu's plenary lecture "*Teachers collective work inside and outside school as an essential spring of mathematics teachers' documentation: Japanese and Chinese experiences"* and three oral presentations from our PhD students.

The next occasions to meet will be the PhD defenses, in 2018, 2020 and 2021, and the International Congress ICME 2020, in Shanghai. This worldwide meeting for the mathematics education community will also be the place for the MaTTRiT contributors to interact, because Pr. Miyakawa, Pr. Xu, Pr. Bao and Pr. Trouche are members of the Program committee and Pr. Soury-Lavergne as already been invited to give a regular lecture.

In conclusion, there is no doubt that the work developed with the support of Xu Guangqi program will continue to progress for the benefit of the collaboration between France and China and new advances in mathematics education.

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Sophie Soury-Lavergne, during her lecture in ECNU, Shanghai, in October 2017