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# Sequential Closed-Form Semi-Blind Receiver for Space-Time Coded Multi-Hop Relaying Systems

Walter da C. Freitas Jr., Gérard Favier, André L. F. de Almeida, *Senior Member, IEEE*

**Abstract**—In this letter, we present a sequential closed-form semi-blind receiver for a one-way multi-hop amplify-and-forward (AF) relaying system. Assuming Khatri-Rao space-time (KRST) coding at each relay, it is shown that the system with  $K$  relays can be modeled by means of a generalized nested PARAFAC model. Decomposing this model into  $K+1$  third-order PARAFAC models, we develop a closed-form semi-blind receiver for jointly estimating the information symbols and the individual channels, at the destination node. Each step consists of a Khatri-Rao factorization. Parameter identifiability conditions are given, and simulation results are provided to illustrate the effectiveness of the proposed semi-blind receiver.

**Index Terms**—Multi-hop relaying, sequential closed-form semi-blind receiver, space time (ST) coding, amplify-and-forward, generalized nested PARAFAC model.

## I. INTRODUCTION

Relay nodes will play an important role in 5G communication systems. In such systems, the high-capacity wireless backhaul offers operators an alternative solution to conventional backhauling using multi-hop short-distance communications [1]–[4].

Tensor decompositions have been widely exploited for point-to-point wireless communication systems. The practical motivation for tensor modeling comes from the fact that one can simultaneously benefit from multiple (more than two) signal diversities, like space, time and frequency diversities, for instance. In the context of cooperative wireless communications, few results have been published on tensor-based receivers, and most of them are limited to a single relay scenario. In previous works [5]–[8], the problem of channel estimation for multiple input multiple output (MIMO) relaying systems was considered under a supervised approach. The works [5]–[7] considered one-way two-hop systems, while [8] addressed two-way MIMO relaying systems. In [9] and [10], three-hop relaying systems were studied. Recent works proposed semi-blind receivers for joint symbol and channel estimation. For instance, one can cite the PARAFAC-PARATUCK [11] and the nested PARAFAC receivers [12], [13]. However, the generalization of tensor-based semi-blind receivers to the multi-hop relaying scenario has not been addressed so far to the best of authors' knowledge.

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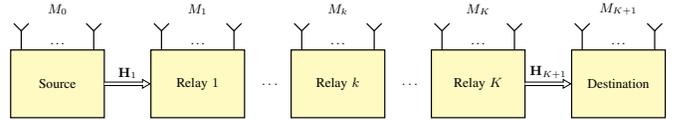


Fig. 1. System model with  $K$  relays.

In this paper, we consider a one-way multi-hop relaying MIMO wireless system composed of  $K$  relays. Using an AF protocol, each relay retransmits the received signals without decoding. Assuming that the signals are encoded with a simplified Khatri-Rao space-time (KRST) coding scheme [14], it is shown that the signals received at destination satisfy a generalized nested PARAFAC model. By decomposing this model into  $K+1$  third-order PARAFAC models, and assuming that the coding matrices are known at the destination node, a closed-form semi-blind receiver based on rank-one matrix approximations is derived for jointly estimating the information symbols and the individual channels.

**Notation:** Scalars, column vectors, matrices and tensors are denoted by lower-case, boldface lower-case, boldface upper-case, and calligraphic letters, e.g.,  $a, \mathbf{a}, \mathbf{A}, \mathcal{A}$ , respectively. The operator  $\text{diag}(\cdot)$  forms a diagonal matrix from its vector argument. The Khatri-Rao and Hadamard products are denoted by  $\diamond$  and  $*$ , respectively. We use the superscripts  $T, *, H, -1, \dagger$  for matrix transposition, complex conjugation, Hermitian transposition, inversion, and Moore-Penrose pseudo inversion, respectively.

Given a third-order tensor  $\mathcal{X} \in \mathbb{C}^{I \times J \times K}$ , with entries  $x_{i,j,k}$ , the matrices  $\mathbf{X}_{JK \times I}$ ,  $\mathbf{X}_{KI \times J}$ , and  $\mathbf{X}_{IJ \times K}$  denote tall mode-1, mode-2 and mode-3 unfoldings, with  $x_{i,j,k} = [\mathbf{X}_{IJ \times K}]_{(i-1)J+j,k} = [\mathbf{X}_{JK \times I}]_{(j-1)K+k,i} = [\mathbf{X}_{KI \times J}]_{(k-1)I+i,j}$ . A PARAFAC decomposition of  $\mathcal{X} \in \mathbb{C}^{I \times J \times K}$ , with rank- $R$  and matrix factors  $(\mathbf{A}, \mathbf{B}, \mathbf{C})$ , will be abbreviated  $\|\mathbf{A}, \mathbf{B}, \mathbf{C}; R\|$ . Tall and flat mode-1 matrix unfoldings of  $\mathcal{X}$  are respectively given by

$$\mathbf{X}_{JK \times I} = (\mathbf{B} \diamond \mathbf{C})\mathbf{A}^T = (\mathbf{X}_{I \times JK})^T. \quad (1)$$

Mode-2 and mode-3 unfoldings are easily deduced by permuting  $(I, J, K)$  and  $(\mathbf{A}, \mathbf{B}, \mathbf{C})$  accordingly.

## II. SYSTEM MODEL

We consider a one-way multi-hop MIMO relaying system, composed of  $K$  relays, as illustrated in Figure 1, where  $M_k$  denotes the number of antennas at node  $k$ , with  $k = 0, \dots, K+1$ . The nodes indexed by  $k=0$  and  $k=K+1$  represent the source and the destination, respectively. The matrix  $\mathbf{H}_{k+1} \in \mathbb{C}^{M_{k+1} \times M_k}$ ,  $k = 0, \dots, K$ , denotes the channel between nodes  $k$  and  $k+1$ , which is assumed as flat-fading and quasi-static during the total transmission time.

Let us define the symbol matrix  $\mathbf{S} \in \mathbb{C}^{N \times M_0}$  containing  $N$  data-streams composed of  $M_0$  symbols that are multiplexed by  $M_0$  transmit antennas at the source. Source and relays encode the signals to be transmitted with a KRST coding matrix  $\mathbf{G}_k \in \mathbb{C}^{P_k \times M_k}$ , chosen as a truncated discrete Fourier transform (DFT) matrix [12] implying  $\mathbf{G}_k^T \mathbf{G}_k^* = \mathbf{I}_{M_k}$  for  $k = 0, \dots, K$ . The processing at the relay nodes follows the AF protocol. Let  $\tilde{\mathcal{X}}^{(k)} = \mathcal{X}^{(k)} + \mathcal{N}^{(k)}$  be the noisy received signal tensor at node  $k$ , the entries of the noise tensor  $\mathcal{N}^{(k)}$  being zero-mean circularly-symmetric complex-valued Gaussian random variables.

#### A. Case with two relays

In this section, we consider the particular case with two relays ( $K = 2$ ). In the next section, we present a generalization to an arbitrary number  $K$  of relays. The signals received at relay-1 define a third-order tensor  $\tilde{\mathcal{X}}^{(1)} \in \mathbb{C}^{M_1 \times P_0 \times N}$  satisfying a PARAFAC model  $\|\mathbf{H}_1, \mathbf{G}_0, \mathbf{S}; M_0\|$ , a flat mode-1 unfolding of which is given by

$$\tilde{\mathbf{X}}_{M_1 \times P_0 N}^{(1)} = \mathbf{H}_1 (\mathbf{G}_0 \diamond \mathbf{S})^T + \mathbf{N}_{M_1 \times P_0 N}^{(1)}. \quad (2)$$

Relay-1 re-encodes the received signals using a KRST code  $\mathbf{G}_1$ . The signals received at relay-2 define a fourth-order tensor  $\tilde{\mathcal{X}}^{(2)} \in \mathbb{C}^{M_2 \times P_1 \times P_0 \times N}$  which satisfies the following equations

$$\tilde{\mathbf{X}}_{M_2 \times P_1 P_0 N}^{(2)} = \mathbf{H}_2 \left( \mathbf{G}_1 \diamond \tilde{\mathbf{X}}_{P_0 N \times M_1}^{(1)} \right)^T + \mathbf{N}_{M_2 \times P_1 P_0 N}^{(2)}, \quad (3)$$

$$= \mathbf{H}_2 \left( \mathbf{G}_1 \diamond (\mathbf{G}_0 \diamond \mathbf{S}) \mathbf{H}_1^T \right)^T \quad (4)$$

$$+ \mathbf{H}_2 \left( \mathbf{G}_1 \diamond \mathbf{N}_{P_0 N \times M_1}^{(1)} \right)^T + \mathbf{N}_{M_2 \times P_1 P_0 N}^{(2)}.$$

The information part in (4) corresponds to a nested PARAFAC model [15], whereas Eq. (3) is a PARAFAC model  $\|\mathbf{H}_2, \mathbf{G}_1, \tilde{\mathbf{X}}_{P_0 N \times M_1}^{(1)}; M_1\|$  for the contracted form of  $\tilde{\mathcal{X}}^{(2)}$  resulting from a combination of its last two modes  $(p_0, n)$ .

After KRST coding at relay-2 and transmission through the channel  $\mathbf{H}_3$ , the signals received at the destination define a fifth-order tensor  $\tilde{\mathcal{X}}^{(3)} \in \mathbb{C}^{M_3 \times P_2 \times P_1 \times P_0 \times N}$ , which can be written as

$$\tilde{\mathbf{X}}_{M_3 \times P_2 P_1 P_0 N}^{(3)} = \mathbf{H}_3 \left( \mathbf{G}_2 \diamond \tilde{\mathbf{X}}_{P_1 P_0 N \times M_2}^{(2)} \right)^T + \mathbf{N}_{M_3 \times P_2 P_1 P_0 N}^{(3)}. \quad (5)$$

Replacing  $\tilde{\mathbf{X}}_{P_1 P_0 N \times M_2}^{(2)}$ , and  $\tilde{\mathbf{X}}_{P_0 N \times M_1}^{(1)}$  by their expressions (3) and (2), we obtain the following matrix unfolding for  $\tilde{\mathcal{X}}^{(3)}$

$$\begin{aligned} \tilde{\mathbf{X}}_{M_3 \times P_2 P_1 P_0 N}^{(3)} &= \mathbf{H}_3 \left[ \mathbf{G}_2 \diamond \left( \mathbf{G}_1 \diamond (\mathbf{G}_0 \diamond \mathbf{S}) \mathbf{H}_1^T \right) \mathbf{H}_2^T \right]^T \\ &\quad + \underbrace{\mathbf{H}_3 \left( \mathbf{G}_2 \diamond \left( \mathbf{G}_1 \diamond \mathbf{N}_{P_0 N \times M_1}^{(1)} \right) \mathbf{H}_2^T \right)^T}_{\mathbf{N}_{M_3 \times P_2 P_1 P_0 N}^{(1 \rightarrow 3)}} \quad (6) \\ &\quad + \underbrace{\mathbf{H}_3 \left( \mathbf{G}_2 \diamond \mathbf{N}_{P_1 P_0 N \times M_2}^{(2)} \right)^T}_{\mathbf{N}_{M_3 \times P_2 P_1 P_0 N}^{(2 \rightarrow 3)}} + \mathbf{N}_{M_3 \times P_2 P_1 P_0 N}^{(3)}. \end{aligned}$$

The information part in (6) corresponds to a (generalized) fifth-order nested PARAFAC model. The terms  $\mathbf{N}_{M_3 \times P_2 P_1 P_0 N}^{(1 \rightarrow 3)}$  and  $\mathbf{N}_{M_3 \times P_2 P_1 P_0 N}^{(2 \rightarrow 3)}$  are the noise contributions from relays 1 and 2 received at the destination. Eq. (5) is a PARAFAC model  $\|\mathbf{H}_3, \mathbf{G}_2, \tilde{\mathbf{X}}_{P_1 P_0 N \times M_2}^{(2)}; M_2\|$  for the contracted form of  $\tilde{\mathcal{X}}^{(3)}$  obtained by combining its last three modes  $(p_1, p_0, n)$ .

From now on, we consider a noiseless formulation due to space limitation. Another unfolding of the PARAFAC model (5) is

$$\mathbf{X}_{M_3 P_2 \times P_1 P_0 N}^{(3)} = (\mathbf{H}_3 \diamond \mathbf{G}_2) \mathbf{X}_{M_2 \times P_1 P_0 N}^{(2)}. \quad (7)$$

Replacing  $\mathbf{X}_{M_2 \times P_1 P_0 N}^{(2)}$  by its expression (3) leads to

$$\mathbf{X}_{M_3 P_2 \times P_1 P_0 N}^{(3)} = (\mathbf{H}_3 \diamond \mathbf{G}_2) \mathbf{H}_2 \left( \mathbf{G}_1 \diamond \mathbf{X}_{P_0 N \times M_1}^{(1)} \right)^T \quad (8)$$

$$= \mathbf{H}_{M_3 P_2 \times M_1}^{(1 \rightarrow 3)} \left( \mathbf{G}_1 \diamond \mathbf{X}_{P_0 N \times M_1}^{(1)} \right)^T, \quad (9)$$

where

$$\mathbf{H}_{M_3 P_2 \times M_1}^{(1 \rightarrow 3)} = (\mathbf{H}_3 \diamond \mathbf{G}_2) \mathbf{H}_2 \quad (10)$$

is a matrix unfolding of the third-order effective channel tensor  $\mathcal{H}^{(1 \rightarrow 3)} \in \mathbb{C}^{M_3 \times P_2 \times M_1}$  linking the relay-1 and the destination (node-3). From (9), we can conclude that the contracted form of  $\mathcal{X}^{(3)}$  resulting from the combination of its first two modes  $(m_3, p_2)$  and its last two modes  $(p_0, n)$ , satisfies a PARAFAC model  $\|\mathbf{H}_{M_3 P_2 \times M_1}^{(1 \rightarrow 3)}, \mathbf{G}_1, \mathbf{X}_{P_0 N \times M_1}^{(1)}; M_1\|$ .

Permuting the matrix factors in (9) gives the following unfolding of  $\mathcal{X}^{(3)}$

$$\mathbf{X}_{M_3 P_2 P_1 \times P_0 N}^{(3)} = \left( \mathbf{H}_{M_3 P_2 \times M_1}^{(1 \rightarrow 3)} \diamond \mathbf{G}_1 \right) \mathbf{X}_{M_1 \times P_0 N}^{(1)}. \quad (11)$$

By replacing  $\mathbf{X}_{M_1 \times P_0 N}^{(1)}$  by its expression (2), we get

$$\begin{aligned} \mathbf{X}_{M_3 P_2 P_1 \times P_0 N}^{(3)} &= \left( \mathbf{H}_{M_3 P_2 \times M_1}^{(1 \rightarrow 3)} \diamond \mathbf{G}_1 \right) \mathbf{H}_1 (\mathbf{G}_0 \diamond \mathbf{S})^T, \\ &= \mathbf{H}_{M_3 P_2 P_1 \times M_0}^{(0 \rightarrow 3)} (\mathbf{G}_0 \diamond \mathbf{S})^T, \end{aligned} \quad (12)$$

where

$$\mathbf{H}_{M_3 P_2 P_1 \times M_0}^{(0 \rightarrow 3)} = \left( \mathbf{H}_{M_3 P_2 \times M_1}^{(1 \rightarrow 3)} \diamond \mathbf{G}_1 \right) \mathbf{H}_1 \quad (13)$$

is an unfolding of the fourth-order effective channel tensor  $\mathcal{H}^{(0 \rightarrow 3)} \in \mathbb{C}^{M_3 \times P_2 \times P_1 \times M_0}$  linking the source and the destination. Eq. (12) corresponds to the PARAFAC model  $\|\mathbf{H}_{M_3 P_2 P_1 \times M_0}^{(0 \rightarrow 3)}, \mathbf{G}_0, \mathbf{S}; M_0\|$  of a contracted form of  $\mathcal{X}^{(3)}$  obtained by combining its modes  $(m_3, p_2, p_1)$ .

Permuting the matrix factors in (12), (13), and (10) gives the following unfoldings of  $\mathcal{X}^{(3)}$ ,  $\mathcal{H}^{(0 \rightarrow 3)}$ , and  $\mathcal{H}^{(1 \rightarrow 3)}$

$$\mathbf{X}_{N M_3 P_2 P_1 \times P_0}^{(3)} = \left( \mathbf{S} \diamond \mathbf{H}_{M_3 P_2 P_1 \times M_0}^{(0 \rightarrow 3)} \right) \mathbf{G}_0^T = \mathbf{R} \mathbf{G}_0^T, \quad (14)$$

$$\mathbf{H}_{M_3 P_2 M_0 \times P_1}^{(0 \rightarrow 3)} = \left( \mathbf{H}_{M_3 P_2 \times M_1}^{(1 \rightarrow 3)} \diamond \mathbf{H}_1^T \right) \mathbf{G}_1^T = \mathbf{Q} \mathbf{G}_1^T, \quad (15)$$

$$\mathbf{H}_{M_3 M_1 \times P_2}^{(1 \rightarrow 3)} = (\mathbf{H}_3 \diamond \mathbf{H}_2^T) \mathbf{G}_2^T = \mathbf{Z} \mathbf{G}_2^T. \quad (16)$$

We assume that the coding matrices  $\mathbf{G}_k$ , for  $k = 0, 1, 2$ , are known at the destination node.

Decomposing the fifth-order nested PARAFAC model (6) into three third-order PARAFAC models represented by Eq. (14)-(16) allows us to derive a three-step closed-form semi-blind receiver to estimate the symbol matrix  $\mathbf{S}$  and the channel matrices  $\mathbf{H}_k$ ,  $k = 1, 2, 3$ . Indeed, using the column orthonormality of the code matrices  $\mathbf{G}_k$ ,  $k = 0, 1, 2$ , the least squares (LS) estimates of  $(\mathbf{R}, \mathbf{Q}, \mathbf{Z})$  can be successively calculated as

$$\hat{\mathbf{R}} = \tilde{\mathbf{X}}_{N M_3 P_2 P_1 \times P_0}^{(3)} \mathbf{G}_0^*, \quad (17)$$

$$\hat{\mathbf{Q}} = \hat{\mathbf{H}}_{M_3 P_2 M_0 \times P_1}^{(0 \rightarrow 3)} \mathbf{G}_1^*, \quad (18)$$

$$\hat{\mathbf{Z}} = \hat{\mathbf{H}}_{M_3 M_1 \times P_2}^{(1 \rightarrow 3)} \mathbf{G}_2^*, \quad (19)$$

where  $\tilde{\mathbf{X}}_{N M_3 P_2 P_1 \times P_0}^{(3)}$  is a noisy version of  $\mathbf{X}_{N M_3 P_2 P_1 \times P_0}^{(3)}$ .

TABLE I  
CLOSED-FORM SEMI-BLIND KRF RECEIVER FOR  $K$  RELAYS.

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1) A priori information: Measurements tensor $\tilde{\mathcal{X}}^{(K+1)}$ , and code matrices $\mathbf{G}_k$ , $k = 0, \dots, K$ .
2) Symbol estimation: $\mathbf{S}$ .
(2.1) Compute the LS estimate $\hat{\mathbf{R}} = \tilde{\mathbf{X}}_{NM_{K+1}P_K \dots P_1 \times P_0}^{(K+1)} \mathbf{G}_0^*$ .
(2.2) Apply the KRF algorithm to estimate the matrix factors $\mathbf{S}$ and $\mathbf{H}_{M_{K+1}P_K \dots P_1 \times M_0}^{(0 \rightarrow K+1)}$ from $\hat{\mathbf{R}}$ .
(2.3) Remove the scaling ambiguities from $\hat{\mathbf{S}}$ and $\hat{\mathbf{H}}_{M_{K+1}P_K \dots P_1 \times M_0}^{(0 \rightarrow K+1)}$ using (23) and (24), respectively.
(2.4) Project the estimated symbols onto the symbol alphabet.
3) Channels estimation: $\mathbf{H}_{k+1}$ , $k = 0, \dots, K$ .
Given $\hat{\mathbf{H}}_{M_{K+1}P_K \dots P_1 \times M_0}^{(0 \rightarrow K+1)}$ estimated at step 1.2.
For $k = 0$ up to $K - 1$ , repeat
(3.1) Reshape the estimate $\hat{\mathbf{H}}_{M_{K+1}P_K \dots P_{k+1} \times M_k}^{(k \rightarrow K+1)}$ as $\hat{\mathbf{H}}_{M_{K+1}P_K \dots P_{k+2}M_k \times P_{k+1}}^{(k \rightarrow K+1)}$ , written as in (22), and compute the LS estimate $\hat{\mathbf{Q}} = \hat{\mathbf{H}}_{M_{K+1}P_K \dots P_{k+2}M_k \times P_{k+1}}^{(k \rightarrow K+1)} \mathbf{G}_{k+1}^*$ .
(3.2) Apply the KRF algorithm to estimate $\mathbf{H}_{k+1}$ and $\mathbf{H}_{M_{K+1}P_K \dots P_{k+2} \times M_{k+1}}^{(k+1 \rightarrow K+1)}$ from $\hat{\mathbf{Q}}$ .
(3.3) Remove the scaling ambiguities from $\hat{\mathbf{H}}_{k+1}$ and $\hat{\mathbf{H}}_{M_{K+1}P_K \dots P_{k+2} \times M_{k+1}}^{(k+1 \rightarrow K+1)}$ using (25) and (26), respectively.

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From the LS estimate  $\hat{\mathbf{R}}$  given by Eq. (17), the factors  $\mathbf{S}$  and  $\mathbf{H}_{M_3P_2P_1 \times M_0}^{(0 \rightarrow 3)}$  are found by the Khatri-Rao product factorization (KRF) algorithm proposed in [16]. Then, the estimate  $\hat{\mathbf{H}}_{M_3P_2P_1 \times M_0}^{(0 \rightarrow 3)}$  is reshaped as  $\hat{\mathbf{H}}_{M_3P_2M_0 \times P_1}^{(0 \rightarrow 3)}$  to compute  $\hat{\mathbf{Q}}$  by means of Eq. (18), from which the factors  $\mathbf{H}_1$  and  $\mathbf{H}_{M_3P_2 \times M_1}^{(1 \rightarrow 3)}$  are extracted by applying the KRF algorithm. Finally,  $\hat{\mathbf{H}}_{M_3P_2 \times M_1}^{(1 \rightarrow 3)}$  is reshaped as  $\hat{\mathbf{H}}_{M_3M_1 \times P_2}^{(1 \rightarrow 3)}$  to compute  $\hat{\mathbf{Z}}$  by means of Eq. (19), from which the channels ( $\mathbf{H}_2, \mathbf{H}_3$ ) are extracted using again the KRF algorithm.

### B. General Case

Let us now consider the general case of  $K \geq 2$  relays. The signals received at the destination are given by

$$\tilde{\mathbf{X}}_{M_{K+1} \times P_K P_{K-1} \dots P_0 N}^{(K+1)} = \mathbf{H}_{K+1} \left( \mathbf{G}_K \diamond \tilde{\mathbf{X}}_{P_{K-1} \dots P_0 N \times M_K}^{(K)} \right)^T + \mathbf{N}_{M_{K+1} \times P_K P_{K-1} \dots P_0 N}^{(K+1)}, \quad (20)$$

which defines a  $(K+3)$ th-order tensor  $\tilde{\mathcal{X}}^{(K+1)} \in \mathbb{C}^{M_{K+1} \times P_K \times P_{K-1} \times \dots \times P_0 \times N}$ .

Disregarding the noise terms, let us replace recursively  $\mathbf{X}_{P_{K-1} \dots P_0 N \times M_K}^{(K)}$  as a function of  $\mathbf{X}_{P_{K-2} \dots P_0 N \times M_{K-1}}^{(K-1)}$ , then as a function of  $\mathbf{X}_{P_{K-3} \dots P_0 N \times M_{K-2}}^{(K-2)}$ , and so on, up to  $\mathbf{S}$ . We obtain the following equivalent third-order PARAFAC model  $\|\mathbf{H}_{M_{K+1}P_K \dots P_1 \times M_0}^{(0 \rightarrow K+1)}, \mathbf{G}_0, \mathbf{S}; M_0\|$  for  $\mathcal{X}^{(K+1)}$

$$\mathbf{X}_{NM_{K+1}P_K \dots P_1 \times P_0}^{(K+1)} = \left( \mathbf{S} \diamond \mathbf{H}_{M_{K+1}P_K \dots P_1 \times M_0}^{(0 \rightarrow K+1)} \right) \mathbf{G}_0^T, \quad (21)$$

where  $\mathbf{H}_{M_{K+1}P_K \dots P_1 \times M_0}^{(0 \rightarrow K+1)}$  is a matrix unfolding of the  $(K+2)$ -th order effective channel tensor  $\mathcal{H}^{(0 \rightarrow K+1)} \in \mathbb{C}^{M_{K+1} \times P_K \times \dots \times P_1 \times M_0}$  linking the source and the destination, which satisfies the following recurrence relation, for  $k =$

$0, 1, \dots, K - 1$

$$\mathbf{H}_{M_{K+1}P_K \dots P_{k+2}M_k \times P_{k+1}}^{(k \rightarrow K+1)} = \left( \mathbf{H}_{M_{K+1}P_K \dots P_{k+2} \times M_{k+1}}^{(k+1 \rightarrow K+1)} \diamond \mathbf{H}_{k+1}^T \right) \mathbf{G}_{k+1}^T, \quad (22)$$

This is a third-order PARAFAC model  $\|\mathbf{H}_{M_{K+1}P_K \dots P_{k+2} \times M_{k+1}}^{(k+1 \rightarrow K+1)}, \mathbf{G}_{k+1}, \mathbf{H}_{k+1}^T; M_{k+1}\|$  for the contracted form of the effective channel tensor  $\mathcal{H}^{(k \rightarrow K+1)} \in \mathbb{C}^{M_{K+1} \times P_K \times \dots \times P_{k+1} \times M_k}$  linking the node  $k$  and the destination, obtained by combining its modes  $(m_{K+1}, p_K, \dots, p_{k+2})$ .

The symbol matrix and the channels  $\mathbf{H}_k$ ,  $k = 1, \dots, K+1$ , can be estimated in  $K+1$  steps. After the computation of the LS estimate  $\hat{\mathbf{R}} = \tilde{\mathbf{X}}_{NM_{K+1}P_K \dots P_1 \times P_0}^{(K+1)} \mathbf{G}_0^*$  deduced from Eq. (21), the symbol matrix  $\mathbf{S}$  and the total effective channel  $\mathbf{H}_{M_{K+1}P_K \dots P_1 \times M_0}^{(0 \rightarrow K+1)}$  are determined by applying the KRF algorithm to  $\hat{\mathbf{R}}$ . Then, the channels are estimated in a sequential way by exploiting the recurrence equation (22), initialized with the total effective channel estimate obtained in the first step, and reshaped as  $\hat{\mathbf{H}}_{M_{K+1}P_K \dots P_2 M_0 \times P_1}^{(0 \rightarrow K+1)}$ . The resulting closed-form semi-blind receiver for  $K$  relays is summarized in Table I. Note that  $\mathbf{H}^{(K \rightarrow K+1)} = \mathbf{H}_{K+1}$ . Moreover, in the case  $K = 2$ , the algorithm described in Table I is equivalent to computing the LS estimates (17)-(19), and then applying the KRF algorithm for extracting the matrix factors from the estimated Khatri-Rao products.

### III. IDENTIFIABILITY AND SCALING AMBIGUITY

In the case  $K = 2$ , the system parameter identifiability is linked to the uniqueness of the least square (LS) estimates of the Khatri-Rao products  $\mathbf{R}$ ,  $\mathbf{Q}$  and  $\mathbf{Z}$ , i.e. the right-invertibility of  $\mathbf{G}_k^T$ , for  $k = 0, 1, 2$ , in (14)-(16). In the general case ( $K \geq 2$ ), the uniqueness conditions consist in the right-invertibility of  $\mathbf{G}_0^T$  in (21) and of  $\mathbf{G}_{k+1}^T$ , for  $k = 0, \dots, K-1$ , in (22), i.e. the full column rank property for  $\mathbf{G}_k$ , for  $k = 0, \dots, K$ . This implies the necessary conditions  $P_k \geq M_k$ , for  $k = 0, \dots, K$ .

The unknown matrices ( $\mathbf{S}, \mathbf{H}_k$ ,  $k = 1, \dots, K+1$ ) estimated from  $K+1$  Khatri-Rao products are affected by scaling ambiguities, which can be eliminated by assuming the knowledge of the first row of one factor of each Khatri-Rao product. Herein, we assume the first row of  $\mathbf{S}$  and the first column of  $\mathbf{H}_k$ , for  $k = 1, \dots, K+1$ , are known and equal to the all-one vector. In practice, this a priori knowledge can be determined using short training sequences generated at the relays, as pointed out in [13]. In the case  $K \geq 2$ , we have the following ambiguity relations

$$\hat{\mathbf{S}} \leftarrow \hat{\mathbf{S}} \mathbf{\Lambda}_S, \quad (23)$$

$$\hat{\mathbf{H}}_{M_{K+1}P_K \dots P_1 \times M_0}^{(0 \rightarrow K+1)} \leftarrow \hat{\mathbf{H}}_{M_{K+1}P_K \dots P_1 \times M_0}^{(0 \rightarrow K+1)} (\mathbf{\Lambda}_S)^{-1}, \quad (24)$$

$$\hat{\mathbf{H}}_{k+1}^T \leftarrow \hat{\mathbf{H}}_{k+1}^T \mathbf{\Lambda}_{\mathbf{H}_{k+1}}^T, \quad (25)$$

$$\hat{\mathbf{H}}_{M_{K+1}P_K \dots P_{k+2} \times M_{k+1}}^{(k+1 \rightarrow K+1)} \leftarrow \hat{\mathbf{H}}_{M_{K+1}P_K \dots P_{k+2} \times M_{k+1}}^{(k+1 \rightarrow K+1)} (\mathbf{\Lambda}_{\mathbf{H}_{k+1}}^T)^{-1}, \quad (26)$$

where  $\mathbf{\Lambda}_S = \text{diag}^{-1}(\hat{\mathbf{S}}_1)$  and  $\mathbf{\Lambda}_{\mathbf{H}_{k+1}}^T = \text{diag}^{-1}((\hat{\mathbf{H}}_{k+1})_1)$ .

### IV. SIMULATION RESULTS

In this section, simulation results are provided to evaluate the performance of the proposed semi-blind receiver. The

$$\begin{aligned}
\left(\hat{\mathbf{H}}_{M_{K+1}P_K \cdots P_1 \times M_0}^{(0 \rightarrow K+1)}\right)^T &= \left((\mathbf{G}_0^H \mathbf{G}_0) * (\mathbf{S}^H \mathbf{S})\right)^{-1} (\mathbf{G}_0 \diamond \mathbf{S})^H \tilde{\mathbf{X}}_{P_0 N \times M_{K+1} P_K \cdots P_1}^{(K+1)}, \\
&= \text{diag}^{-1}(\rho_1, \dots, \rho_{M_0}) (\mathbf{G}_0 \diamond \mathbf{S})^H \tilde{\mathbf{X}}_{P_0 N \times M_{K+1} P_K \cdots P_1}^{(K+1)}.
\end{aligned} \tag{27}$$

relays are assumed to be uniformly distributed between the source and the destination. The considered performance criteria are the symbol error rate (SER) and the normalized mean square error (NMSE) of the estimated channels. Each SER curve represents an average over  $4 \times 10^4$  Monte Carlo runs. Each run corresponds to a realization of all channel and symbol matrices, and noise tensors. The transmitted symbols are randomly drawn from a unit energy quadrature amplitude modulation (QAM) symbol alphabet. The SER curves are plotted as a function of the symbol energy to noise spectral density ratio ( $E_s/N_0$ ). At each run, the variances of the additive noise terms at the relays and the destination are equal. We assume  $\mathbf{H}_{k+1}$ ,  $k = 0, \dots, K$ , have i.i.d. zero-mean circularly-symmetric complex Gaussian entries with variances given by  $1/\eta^\beta M_k$ , where  $\eta = d/d_0 = 1/(K+1)$  and  $\beta = 3$ ,  $d$  denoting the distance between two consecutive nodes, and  $d_0$  the distance between the source and the destination.

Figure 2 compares the SER vs.  $E_s/N_0$  for three different numbers of relays and two system parameter settings, assuming 4-QAM modulation. The results indicate that additional diversity gains are obtained as the number of relays increases from  $K = 1$  to  $K = 3$ . These gains come from the additional space-time spreading introduced by the KRST coding at each relay, which results in lower error rates. Moreover, increasing the values of  $N$ ,  $P_k$  and  $M_k$  from 2 to 4 also leads to better SER performance, as expected.

Figure 3 depicts the NMSE associated with the estimation of the individual channels, assuming  $K = 2$  relays, 4-QAM modulation, and  $P_k = M_k = N = 3$ . As a reference for comparison, we also show the performance of a supervised LS channel estimation scheme, where  $\mathbf{S}$  is a training sequence matrix, with  $\rho_r = \|(\hat{\mathbf{S}})_{\cdot r}\|^2$ ,  $r = 1, \dots, M_0$ . This supervised receiver is based on Eq. (27) with  $K = 2$ , which provides an estimate of the effective channel  $\hat{\mathbf{H}}_{M_3 P_2 P_1 \times M_0}^{(0 \rightarrow 3)}$ , without matrix inversion due to the column orthonormality property of  $\mathbf{G}_0$ . The estimates of  $\mathbf{H}_{M_3 P_2 \times M_1}^{(1 \rightarrow 3)}$  and  $\mathbf{H}_1$  are then calculated from  $\hat{\mathbf{Q}}$  defined in (18), by applying the KRF algorithm. Finally, the individual channels  $\mathbf{H}_2$  and  $\mathbf{H}_1$  are determined from  $\hat{\mathbf{Z}}$  defined in (19) by applying once more the KRF algorithm. From Fig. 3, we can conclude that the NMSEs obtained with the proposed semi-blind receiver are very close to the ones provided by the supervised receiver, while being more spectrally efficient. V. CONCLUSION

We have shown that the signals received at the destination node of a one-way multi-hop amplify-and-forward relaying system employing KRST codes at  $K$  relays define a  $(K+3)$ -th order tensor satisfying a generalized nested PARAFAC model. A sequential closed-form semi-blind receiver for jointly estimating the information symbols and the individual channels has been derived by decomposing this model into  $K+1$  third-order PARAFAC models. Each step of

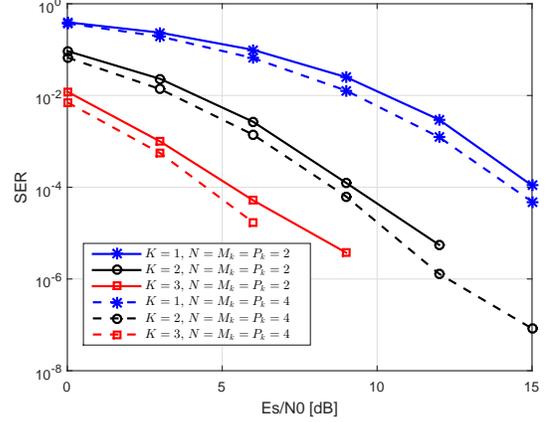


Fig. 2. SER for three different numbers of relays and two system configurations.

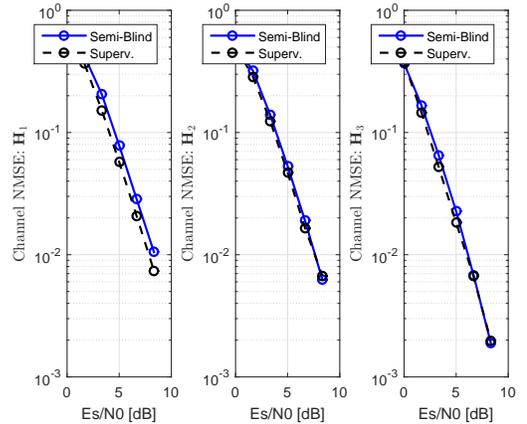


Fig. 3. NMSE of the individual channels with  $K = 2$ .

the proposed algorithm solves a Khatri-Rao factorization problem. Therefore, the dominant complexity of the proposed semi-blind receiver is associated with one singular value decomposition (SVD)-based rank-one approximation for symbols estimation, and  $K$  sequential SVD-based rank-one approximations for individual channels estimation.

The choice of truncated DFT coding matrices ensures system parameter identifiability. Simulation results have demonstrated the effectiveness of the proposed multi-hop relay system. Some extensions of this work include the exploitation of the noise structure to develop a tensor-based receiver using a minimum-mean-square-error (MMSE) algorithm to improve the estimation performance, and an extension to the case of orthogonal frequency division multiplexing (OFDM) relay systems.

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