Incremental modeling of relaxation of prestressing wires under variable loading and temperature

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INCREMENTAL MODELING OF RELAXATION OF PRESTRESSING WIRES UNDER VARIABLE LOADING AND TEMPERATURE

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HIGHLIGHTS

• The coupling of non-linear effects of temperature and initial load ratio on relaxation is considered.
• The model for relaxation of prestressing bars at high-temperature (20-140°C) is developed.
• The model is validated for relaxation test with a stepwise heating (20°C-100°C-140°C).

ABSTRACT

This paper presents a model of prestressing steel relaxation under various levels of loading and temperature. This incremental model enables the delayed strain of prestressing steel wires to be calculated and takes the non-linear coupling of temperature and loading effects into account. It considers different experimental results that could occur in nuclear containment structures, for which a temperature up to 140°C could be reached for prestressing wires should a loss-of-coolant accident occur. The constitutive law for prestressing steel relaxation is presented, followed by an illustration of the numerical implementation of the proposed model. Finally, the model responses are confronted with experimental results available in the literature supplied by Toumi Ajimi et al. (2017) for the thermo-mechanical conditions between 20 – 140°C in terms of temperature and 70% – 80% of loading ratio.

Key words: Prestressing wire, Relaxation, loading ratio, elevated temperature, incremental model

1. INTRODUCTION

In the context of extending the life of nuclear containment structures, which form the third and last protection barrier of a nuclear reactor building, prestressing steel wires with high elastic limit and high tensile strength are required to improve the tensile strength of the whole concrete structure and to prevent possible leakage of radioactive elements in case of an accident with potential failure of the first two barriers. However, steel wires can undergo a loss of tension (relaxation) when they are subjected to an imposed strain, especially when they are simultaneously subjected to high temperature [1]–[5] as could be the case during an accident [6].

Predicting the stress relaxation of prestressing steel tendons is usually dealt with in the standard codes by using empirical laws determined for a specific and constant initial load and temperature [7] (BPEL annex 2, article 3 & BPEL annex 6, article 4.1).

However, it is observed in the literature that loading stresses have a major influence on the mechanical properties of wires, including their relaxation behavior [8], [9]. Erdelyi [10] obtained values of relaxation varying between 1.8 % and 5.1 % for loading ratios of 60 % and 90 % respectively on a prestressing central wire after 5000 hours of test. The same relaxation behavior versus prestress ratio has been highlighted by Magura et al. [11]. Some models have thus been developed based on the code formulas to take account of the effect of initial prestress on relaxation [12] [11] but they do not consider the effect of temperature, despite its strong influence on relaxation rate as shown by Rostasy et al. [13].

In the context of the present work, which is part of the French national research project MACENA (Control of Nuclear Vessel in Accident Conditions), which aims to study the behavior of a nuclear containment structure during a loss-of-coolant accident, where the rebars and tendons of the prestressed concrete containment structure may be subjected to temperatures up to 140°C. The evaluation of the long-term reliability of these structures exposed to such temperature increases thus requires a numerical model able to predict the relaxation of prestressing wires considering the impacts of variable temperature and initial loading (prestress). Moreover, as the influence of variation in temperature and loading ratio on the relaxation rate tends to be relatively complex, the necessity for an efficient incremental model to simulate the delayed strain of prestressing wires in different thermomechanical conditions is
clear. Adapted from an incremental rheological model of delayed strain in concrete [14], the present model is proposed to take the non-linearity of thermomechanical couplings into account.

The originality of this article lies in a comparison between experimental data supplied by Toumi Ajimi et al. [15] and results predicted by the model. Two laboratories collaborated in this work: IFSTTAR/SMC and LMDC. The first part of the paper briefly recalls the experimental test conditions for relaxation of prestressing central wires at different temperatures. Further details can be found in the work of Toumi Ajimi et al. [15]. Then, in the second part, the constitutive equations of the proposed model are given, followed by a numerical implementation. In this part, the influence of temperature on relaxation rate is pointed out and the non-linear relation between loading ratio and relaxation rate is also modeled, in the same ways as the thermomechanical coupling. In the third part, various relaxation tests are simulated and the numerical predictions are compared to experimental results given by Toumi Ajimi et al. [15]. These tests concern stress relaxation at various loading ratios and temperatures, and stepwise cooling/heating in which prestressing steel wires are subjected to unloading/reloading during the temperature changes.

2. EXPERIMENTS

An experimental program was designed on T15.2 prestressing wires at different temperature levels and under different loading ratios. The tests were performed on central steel wires at constant temperature, using a maximum value of 20°C, 40°C, 100°C or 140°C. Both tensile and stress relaxation tests were carried out by Toumi Ajimi et al. [15]. The results regarding the evolution of tensile behavior with temperature are given in Table 1.

<table>
<thead>
<tr>
<th>Maximum temperature, °C</th>
<th>$f_y$, MPa</th>
<th>$E$, GPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>1867 +/- 3</td>
<td>226.1 +/- 5.0</td>
</tr>
<tr>
<td>40</td>
<td>1854 +/- 15</td>
<td>226.3 +/- 0.9</td>
</tr>
<tr>
<td>100</td>
<td>1793 +/- 4</td>
<td>224.5 +/- 3.4</td>
</tr>
<tr>
<td>140</td>
<td>1726 +/- 14</td>
<td>213.4 +/- 1.1</td>
</tr>
</tbody>
</table>

Table 1: Experimental mechanical properties of prestressing wires (tensile tests) [15]
In stress relaxation investigations, two levels of initial stress ratio (0.7 and 0.8) were studied for each temperature level. The experiments of Toumi Ajimi et al. [15] showed that the relaxation was more affected at higher temperature and higher stress ratio. In addition, at 20°C, two other initial stress ratios were investigated: 0.6 and 0.9. More details can be found in [15].

It is important to note that, in both cases, the temperature state, although stationary, was not constant along the extensometer (Figure 1). However, for each configuration, the average temperature was assessed using temperature sensors. For tensile tests, the average temperatures were 30°C, 79°C and 110°C for ultimate temperatures of 40°C, 100°C and 140°C, respectively. For stress relaxation investigations, they were assessed to be respectively 35°C, 76°C and 100°C.

![Figure 1: Scheme of relaxation sample with the heating device on the prestressing wire (a) and temperature evolution along the heated section (b) [15]](https://doi.org/10.1016/j.conbuildmat.2017.12.123)

In addition, stepwise heating and cooling were performed on the same type of wires under a stress ratio of 0.8. The first transient test started at 20°C, rising to 100°C and finally 140°C. Second one started at 140°C, followed by 100°C and finally 20°C. For these two tests, a specific loading/unloading procedure was adopted to exclude thermal expansion effects from the results. For each step, the wire was unloaded before its temperature state was modified, and was then loaded again with the remaining load obtained from the preceding step. Results are given in Figure 5 and Figure 6, where they are compared with the model predictions.
3. CONSTITUTIVE EQUATIONS

3.1. Model principles

The relaxation model is based on the rheological model summarized by an idealized scheme in Figure 2. It consists of three levels, an elastic part to model instantaneous behavior, a Kelvin module to model viscoelastic strain, and a nonlinear Maxwell module to model permanent strain. This model is formulated to be usable in nonlinear finite element codes able to consider incremental evolution of thermomechanical conditions, so it does not use relaxation functions but stores the material state only through state variables acting on instantaneous characteristics of material. These state variables are the strain of each level presented in Figure 2, the applied stress and the maximum loading rate. To consider the progressive reduction of the relaxation kinetics, the Maxwell level is nonlinear, its viscosity depending on the strain of the Maxwell level (\( \varepsilon^M \) in Figure 2). A nonlinear Maxwell module has already been used successfully, and for the same reason, to model the decrease of concrete creep kinetics [14].

![Idealized rheological scheme for relaxation model](https://doi.org/10.1016/j.conbuildmat.2017.12.123)

3.2. Nonlinear Maxwell level strain

Permanent relaxation corresponds to the Maxwell strain, \( \varepsilon^M \), of the idealized rheological scheme (see Figure 2). The constitutive law of viscous strain, \( \varepsilon^M \), for reinforcement is analogous to the one used for concrete by Sellier et al. [14]. The viscous strain is proportional to the current elastic strain, \( \varepsilon^E \), and inversely proportional to a characteristic time, \( \tau^M \), through Eq. (1). The characteristic time is then affected by a nonlinear function called a “consolidation coefficient”, \( Cc \), depending on the current viscous strain.
\[
\frac{\partial \varepsilon^M}{\partial t} = \frac{\varepsilon^E}{\tau^M C_c}
\]  

(1)

The consolidation coefficient depends on the current viscous strain and is expressed by Eq. (2).

\[
C_c = \frac{1}{k} \exp \left( \frac{1}{k} \frac{\varepsilon^M}{\varepsilon^E} \right)
\]  

(2)

with \( k \) a **relaxation amplitude coefficient** for the current temperature and loading ratio (Eq.(3)).

\[
k = k_{\text{ref}} \cdot k^T \cdot k^M
\]  

(3)

\( k \) considers the coupled effects of temperature and mechanical nonlinearity involved in the relaxation process. The physical conditions (temperature (T) and loading ratio (M)) affect the relaxation velocity through two functions, \( k^M \) and \( k^T \), which will be detailed in 3. 2. 1 and 3. 2. 2, and \( k_{\text{ref}} \) is a constant proportional to the relaxation capability. This material constant, \( k_{\text{ref}} \), called the "**reference relaxation amplitude coefficient**" is the ratio of a characteristic strain \( \varepsilon^K_{\text{ref}} \) and a reference elastic strain resulting from a **conventional stress level** \( \sigma_{\text{ref}} \) corresponding to the loading rate used to fit \( \varepsilon^K_{\text{ref}} \).

The constant \( k_{\text{ref}} \) is linked to the previous parameters as follows:

\[
k_{\text{ref}} = E \frac{\varepsilon^K_{\text{ref}}}{\sigma_{\text{ref}}}
\]  

(4)

In Eq. (4), the fitting parameter \( \varepsilon^K_{\text{ref}} \) is called the **reference relaxation potential** because the relaxation amplitude is proportional to \( \varepsilon^K_{\text{ref}} \). \( E \) is the elastic modulus of the reinforcement.
3. 2. 1. Nonlinear effect of loading ratio

In Eq. (3), $k^M$ considers the nonlinear amplification of the relaxation rate with the mechanical loading level. Therefore, this coefficient starts from 1 for weak loading level and diverges when the loading level reaches a critical value causing tertiary delayed strains. $k^M$ is provided by Eq. (5):

$$k^M = \frac{\mu^{cr}}{\mu^{cr} - \mu}$$  \hspace{1cm} (5)

with $\mu$ the loading level defined by Eq. (6), in which $f_y$ is the elastic limit:

$$\mu = \left| \frac{\sigma}{f_y} \right|$$  \hspace{1cm} (6)

In Eq. (5), $\mu^{cr}$ is a critical loading level depending on the tendency of the material to increase its relaxation rate nonlinearly when the load increases; it is linked to a characteristic data $\chi$ called the nonlinearity relaxation factor, defined such that the relaxation remains linear if $\chi = 1$ and becomes non-linear when $\chi > 1$, as illustrated in Figure 3:

$$\mu^{cr} = \frac{2}{3} \frac{\chi}{\chi - 1}$$  \hspace{1cm} (7)

![Figure 3: Nonlinear amplification function $k^M$ versus loading level](https://doi.org/10.1016/j.conbuildmat.2017.12.123)
3.2.2. Temperature effect

In Eq. (3), \( k^T \) accounts for the temperature effect on the relaxation rate. The relaxation phenomenon is accelerated by the temperature increase and coupling with the loading level also exists since an amplification of the temperature effect is observed for loading rates greater than 0.7. To consider the effect of temperature on relaxation, the model uses the thermal activation multiplication coefficient \( (k^T) \) of the delayed strain potential (Eq. (8)), and to consider that the effect of temperature is amplified in case of high loading rates, coefficient \( A^T \) depends on the loading rate (Eq. (9)). Several forms of \( k^T \) and \( A^T \) were tested, and equations (8) and (9) correspond to the best fit found in the next section. Firstly, an Arrhenius law was tested, as the steel wires were thermally activated, but the curvature of the curve was not sufficient to obtain the delayed strain potential at 100°C. Therefore, \( k^T \) was finally expressed as an exponential function depending on the thermal activation coefficient \( A^T \):

\[
k^T = \begin{cases} 
\exp \left[ A^T \left( T - T_{ref} \right)^n \right] & \text{if } T > T_{ref} \\
\exp \left[ -A^T \left( T_{ref} - T \right)^n \right] & \text{if } T \leq T_{ref}
\end{cases}
\]

with \( T \) the temperature, \( T_{ref} \) the reference temperature for which the relaxation characteristic time, \( \tau^M \), (Eq. 1) was fitted, and \( n \) a fitted parameter. The thermal activation coefficient is given by Eq. (9):

\[
A^T = A^T_{ref} \exp \left( \gamma \cdot \mu^{TM} \right)
\]

In Eq. (9), \( A^T_{ref} \) is a fitted constant to account for the effect of temperature on the relaxation rate, \( \gamma \) is a material constant, and \( \mu^{TM} \) is a loading rate able to change the thermal activation, defined as follows:

\[
\mu^{TM} (t + dt) = \max \left( \mu^{TM} (t), \mu (t + dt), \mu^{thr} \right)
\]

where \( \mu^{thr} \approx 0.7 \) is the minimum loading level able to change the thermal activation, and \( \mu \) is the loading level given by Eq. (6).
3. 3. Kelvin level strain

Based on experimental results, a rapid delayed strain of the prestressing wires was detected in the tests comprising unloading and reloading stages. The experiments also showed that this rapid strain, like permanent strain, was influenced by the temperature. In the model, the rapid relaxation is modelled by a Kelvin module (see Figure 2 and Eq. (11)):

$$\frac{\partial \varepsilon^K}{\partial t} = \frac{1}{\tau^K} \left( \varepsilon^K - \psi^K - \varepsilon^K \right)$$

with $\tau^K$ the characteristic time for the Kelvin strain and $\psi^K$ the rate of this relaxation relative to the elastic strain. To limit the number of fitting parameters, these two variables depend on the temperature through the same coefficient $k^T$ (Eq. (8)) used for the Maxwell element presented above and are given by Eqs. (12) and (13) respectively.

$$\tau^K = \frac{\tau^K_{\text{ref}}}{k^T}$$

$$\psi^K = \frac{\psi^K_{\text{ref}}}{k^T}$$

with $\tau^K_{\text{ref}}$ and $\psi^K_{\text{ref}}$ the characteristic time and the rate of reversible relaxation fitted at reference temperature $T_{\text{ref}}$. Ideally, to fit these two parameters, a relaxation test followed by an unloading phase is needed, but in the present work the recovery stage was not recorded; only the reloading stage for the test at stepwise decreasing temperatures allowed the short term delayed strain modelled by the Kelvin level to be captured.

3. 4. Numerical implementation

The previous equations can be implemented in any finite element code based on a displacement formulation. Here, the model was implemented in the finite element code CAST3M [16]. The model is resolved by solving the differential equations (1) and (11) using the time discretization with a finite-difference time-domain method ($\theta$ method). A set of equations involving the elastic strain, Kelvin strain and Maxwell strain is formed, to be solved at each time increment $\Delta t$ (Eq. (14)).

\[
\begin{align*}
\Delta \varepsilon^\text{tot} &= \Delta \varepsilon^E + \Delta \varepsilon^K + \Delta \varepsilon^M \\
\psi^K \left( \varepsilon^K + \left( \theta + \frac{\tau^K}{\Delta t} \right) \Delta \varepsilon^K \right) &= \varepsilon^E + \theta \Delta \varepsilon^E \\
\frac{\tau^M C_C}{\Delta t} \Delta \varepsilon^M &= \varepsilon^E + \theta \Delta \varepsilon^E
\end{align*}
\] (14)

where $\theta$ is the semi implicit coefficient, taken equal to $1/2$ in order to obtain a mid-point method. In these equations, strains $(\varepsilon^E, \varepsilon^K, \varepsilon^M)$ are known at the beginning of the time increment (see Eq. (15)), while strain increments $(\Delta \varepsilon^E, \Delta \varepsilon^K, \Delta \varepsilon^M)$ are unknowns for the current time step $\Delta t$, and $\Delta \varepsilon^\text{tot}$ is the total strain increment imposed. For a material never loaded before, the initial conditions are given by equation (15)

\[
(t = 0) \begin{cases}
\varepsilon^E = \mu \frac{f_y}{E} \\
\varepsilon^K = 0 \\
\varepsilon^M = 0
\end{cases}
\] (15)

Variations of $\tau^K, \psi^K,  \tau^M$ and $C_C$ due to the variations in thermomechanical conditions are considered explicitly. For instance, the consolidation coefficient $C_C$ (Eq. (2)) at the end of the time step is actualized with $\varepsilon^M$, ready for the next step. This approximation is admissible as long as the time step stays relatively small compared to the characteristic time. Alternatively, the consolidation coefficient could be linearized to improve the convergence rate, but this improvement was not necessary for the material studied below. Note that, if the time interval used by the finite element code user is too large, it can be subdivided (in the subroutine dedicated to this model) in order to avoid inaccuracy. For instance, the following studies were performed with $\Delta t$ limited to 10% of the characteristic times $(\tau^K, \tau^M C_C)$. The method of optimizing the time sub-step has already been used in concrete creep modelling by Sellier et al. [14].
4. APPLICATIONS

This part presents the applications of the model to the experimental relaxation tests briefly presented in Section 2 and in greater detail by Toumi Ajimi et al. [15]. The investigations conducted at constant temperature allowed fitting parameters to be obtained and the ability of the model to simulate the prestress relaxation to be validated in different configurations. In the present section, the comparisons of experimental results for relaxation (points) with numerical results (curve) are presented by the evolution of prestress loss with time. The prestress loss is the ratio of strain at the current time to the initial strain.

4.1. Calibration methodology

The relaxation experiments used to identify the model parameters reported in Table 2 can be classified in two series of tests. The first series assessed the stress relaxation of steel wires at different temperatures (T=20°C, 40°C, 100°C and 140°C) and different loading ratios (μ=0.7 and 0.8), each kept constant during the test (Figure 4). It is of interest to note that, as observed in the literature, the experiments show that relaxation is more affected by higher temperature and higher stress ratio. The other test used to calibrate the model was a test performed at 80% loading rate but with a temperature decreasing by steps during the relaxation test (steel subjected to 140°C, 100°C and 20°C). During the temperature change, samples were unloaded and then heated to reach the next desired temperature before reloading. This was intended to suppress the effects due to thermal expansion of the steel wire.

Both series of tests (variable temperature with reloading and constant T and loading rate) allowed all the model parameters (Kelvin-Voigt level, Maxwell level and parameters related to non-linearity induced by stress and temperature) to be identified. The identified parameters are reported in Table 2 and the comparisons with tests are plotted in Figure 4 and Figure 5. It should be noted that, due to the dispersion observed on the relaxation at μ=0.8 and 140°C in Figure 4, the model only fits perfectly the results mu_0.8_140°C_exp(2) (diamond points in Figure 4). The results of incremental temperature needed to fit the Kelvin Voigt parameters are only available for mu_0.8_140°C_exp(3) (circular points in Figure 5), for the fitting they were extrapolated to the results of mu_0.8_140°C_exp(2) (diamond empty points in Figure 5) after 130 h.
Table 2: Fitting parameters of model for relaxation of prestressing wires T15.2 ($T_{ref} = 20^\circ C$)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\tau^K_{ref}$ (h)</th>
<th>$\psi^K_{ref}$</th>
<th>$\tau^M$ (h)</th>
<th>$\varepsilon^K$</th>
<th>$\chi$</th>
<th>$\gamma$</th>
<th>$n$</th>
<th>$A^T_{ref}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>10</td>
<td>250</td>
<td>10</td>
<td>6.64 .10^-6</td>
<td>1.45</td>
<td>6.19</td>
<td>1.216</td>
<td>6.52.10^-5</td>
</tr>
</tbody>
</table>

Figure 4: Comparison between model and experimental results of relaxation versus time for different temperatures and loading ratios (0.7 and 0.8)

Figure 5: Comparison of model with relaxation experiment at stepwise cooling (140°C, 100°C and 20°C)
It is noticeable that the consolidation theory used in the Maxwell module of the model efficiently considers the permanent strain history and takes a large increase of consolidation coefficient during cooling into account, which results in a significant reduction of the Maxwell relaxation velocity. This is reflected in Figure 5 as a plateau that is almost horizontal.

4.2. Validation

To evaluate the prediction capability of the model, a third test was used. In this test the temperature was increased from 20°C to 140°C as could occur in the vessel wall during an accident. In order to eliminate the effect of thermal strain in steel (and thus evaluate only the capacity of the model to reproduce the nonlinear effect of stress and temperature variation), the temperature was increased in steps (100°C and 140°C) and samples were unloaded and then heated to reach the desired maximal second temperature before reloading. The comparison between the results obtained with the relaxation model proposed above and the experiments is presented in Figure 6. It shows the prestress loss versus time using the data given in Table 2 (without any additional fitting).

![Figure 6: Comparison of model with test of relaxation using stepwise heating (20°C, 100°C and 140°C)](image)

The results obtained with the modelling were in quite good agreement with experimental results. This also confirms that the loading history may have little impact on stress relaxation results and that thermomechanical effects may be added.
5. CONCLUSION

The delayed strain model developed in this paper is based on the incremental finite difference method. It was proposed to simulate the relaxation of prestressing steel wires under different conditions of initial loading (prestress) and temperature. As pointed out by Toumi Ajimi et al. [15], the existing normative approach from Eurocode 2 does not take temperature into account and does not seem to be consistent with experimental results. Specific experimental investigations with stepwise thermal states were carried out to allow the characteristic parameters of the model to be determined and to be able to validate it with different thermal histories to check that the model does not depend on this issue.

The thermomechanical behavior of prestressing steel wires has two particularities: first, the delayed strain velocity is not simply proportional to the mechanical loading (prestress) but depends on it nonlinearly when the loading ratio is high and, second, the relaxation is sensitive to the temperature, with a sensitivity that varies with the loading ratio. Therefore, a coupling of the temperature and loading effects was considered in the model via a thermal activation coefficient depending on the loading ratio. Moreover, a short-term delayed strain was observed and modelled. Further research on this topic is needed to verify the reliability of this model for other types of prestressing steel wires in different configurations of loading and temperature. The features of this model seem to be suitable to predict the prestress loss during the life of the massive structure of a nuclear containment building and the envisioned loss-of-coolant accident.

For massive structures in prestressed concrete, the prestress loss due to steel relaxation cannot be separated from the analysis of concrete creep and shrinkage effects. Other experimental and numerical programs are currently under way in the MACENA project to quantify the phenomenon of delayed behavior of concrete in this range of temperatures (20°C-140°C). The combination of these research works will allow the thermomechanical behavior of prestressed concrete containment structures to be realistically evaluated during their life and for the scenario of loss-of-coolant accident. These data will then be integrated into the process for deciding whether the operational life of a nuclear power plant should be extended or not.
ACKNOWLEDGEMENTS

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