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Revisiting Cent-Fonts Fluviokarst Hydrological Properties with Conservative Temperature Approximation

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Abstract: We assess the errors produced by considering temperature as a conservative tracer in fluviokarst studies. Heat transfer that occurs between karstic Conduit System (CS) and Porous Fractured Matrix (PFM) is the reason why one should be careful in making this assumption without caution. We consider the karstic aquifer as an Open Thermodynamic System (OTS), which boundaries are permeable to thermal energy and water. The first principle of thermodynamics allows considering the enthalpy balance between the input and output flows. Combined with a continuity equation this leads to a two-equation system involving flows and temperatures. Steady conditions are approached during the recession period or during particular phases of pumping test experiments. After a theoretical study of the error induced by the conservative assumption in karst, we have applied the method to revisit the data collected during a complete campaign of pumping test. The method, restricted to selected data allowed retrieving values of base flow, mixing of flow, intrusions of streams, and aquifer answer to drawdown. The applicability of the method has been assessed in terms of propagation of the temporal fluctuations trough the solving but also in terms of conservative assumption itself. Our results allow retrieving the main hydrological properties of the karst as observed on field (timed volumetric samplings, geochemical analyses, step pumping test and allogenic intrusion of streams). This consistency argues in favor of the applicability of the conservative temperature method to investigating fluviokarst systems under controlled conditions.

Keywords: groundwater hydrology; karstic hydrology; energy budget; water energy interactions; water resource; modeling; conservative tracer; temperature; mixing

1. Introduction and Presentation of Cent-Fonts Fluviokarst

This paper addresses the issue of using the water temperature as a conservative tracer for karstic functioning studies. Whereas costly investments are often necessary to evaluate the potential of karstic aquifers, temperature records may bring cheap complementary method for obtaining information. However underground flows undergo heat transfers with the embedding rocks by advection and by conduction. Despite of this difficulty several studies used temperature as a mixing tracer. Several works [1,2] study the water exchanges between underground flows and surface stream. Others [3] retrieved aquifer recharges solving heat transport equation constrained by measurements of vertical temperature. Slow and rapid equilibrations between ground water and aquifer rocks have been analyzed [4] to study the dynamics of exchanges in fractured carbonate systems. Genthon et al. [5] determined the deep preferential path of rainfall water in caves from annual temperature variations of spring. This author also used temperature to determine limestone drainage in lagoon by removing the...
tidal component and emphasizing its poor correlation with rain temperature [6]. This short list is far to be exhaustive.

As mentioned in the following theoretical part of this work, the formal applicability of the conservative temperature approximation depends on the existence steady conditions that are never perfectly verified in nature. The question consists of estimating the outcome of this assumption. Indeed, our work is not the first to address this problem. In spite of disturbances due to meteorological conditions, Karanjac and Altug [7] used temperature records to characterize the recharge area, transmissivity and hydraulic regime of karst. Following Stonestrom and Constantz [8], O’Driscoll and DeWalle [9] studied the stream-ground water temperature interactions from stream-air temperature fluctuations. They used weekly averaging and equilibrium temperature concepts [10–12]. All these studies rely on damping of diurnal or seasonal variations of temperature records. This is done either by time averaging [13] or by natural damping of temperature fluctuations by depth [14,15]. It is clear that the method presented in this work is not adapted to transient flow or rapidly varying water conditions. Conversely, many situations as recession in karst undergo naturally damping of the temporal fluctuations both for temperatures and flows.

These preliminary considerations justify the interest to study the karst systems as an Open Thermodynamic Systems (OTS), that is to say a “black box” that exchanges water and heat energy with its surrounding environment. However, let us try to describe the thread of this paper. The following part of the present Section 1 recalls the hydrological description of the Cent-Fonts site and its consistency with the White’s fluviokarst model. Thus, we benefit of an exceptional set of high quality data recorded during the 2005 Cent-Font pumping tests and preliminary studies. This will provide a fair opportunity to test the method. Section 2 displays theory and resulting equations on a reduced form of the White’s model considered as an OTS. Section 3 describes the available data that will be revisited and Section 4 illustrates the application of the method to these selected data. The results are discussed in the “Summary and discussion” Section 5.

The Cents-Fonts resurgence is the only free-flowing of an underground drainage basin that covers 40 km$^2$ in the median part of Hérault River (Figure 1). The surface of the watershed reaches 60 km$^2$ considering the Buèges stream. It stretches in outcrops of thick calcareous and dolomite massifs (Middle and Upper Jurassic). Since several decades, many geological, geochemical and hydrological studies focused on the potential spring water production of this site (i.e., [16–27]).

The Buèges stream and the Cévennes fault mark the boundaries of the watershed to the north and northwest, while the altitude of the Hérault River matches the karstic base level to the southeast (Figure 1). Morphologically, the area is a plateau, uplifted of 200 to 500 m during the late Quaternary and eroded during Oligocene. The watershed drains rainfalls through an epikarstic transition zone that supplies the Cent-Fonts resurgence base flow [22]. To the north, Buèges stream flows permanently on Triassic low permeability outcrops however it disappears à few kilometers after Saint-Jean de Buèges where the course crosses a Bathonian, calcareous dolomite swallow zone. After this area, the Buèges stream course follows a valley that remains dry most of the year except after major rainfalls. Then a surface course catches up with Hérault River at a confluence point north of Lamalou stream (Figure 1). Tracing experiments have confirmed the underground path of Buèges stream intrusion from swallow zone to the Cent-Fonts resurgence [17,20].

Bathonian, dolomitic layers of Middle Jurassic between 150 and 300 m thick embed the saturated zone of the Cent Fonts fluviokarst near the resurgence. The aquifer also possibly meets an underlying Aalenian-Bajocien layer. The near spring, Conduit System (CS) of the resurgence (Figure 2) has been explored thank to speleological diving that reached 107 m below the base level (Hérault River) [28]. The terminal part of the CS displays an outlet cave, roughly “Y” shaped, with two sub-horizontal branches joining above a deep sub-vertical chimney. The two ends of the “Y” branches open outside. These caves are seldom active except during peak flows. Most of the time, the Cent-Fonts resurgence flows into Hérault River through a shallow network of springs that gush out a few tens of centimeters above the base level. A spring also arises directly through the bottom of the Hérault River bed [20].
Figure 1. Simplified geological map of the Cent-Fonts karst, redrawn after Petelet et al. [16]. Note the location of the P7 borehole where far field temperature ($T_\infty$) has been recorded.

Figure 2. Unrolled 3-D speleological map of the Cent-Fonts CS near the resurgence. Altitudes are in m NGF (Nivellement Général de la France). Note the locations of the Cge, Reco and F3 boreholes.
The Cent-Fonts karst matches precisely the fluviokarst idea depicted by Smart [29]. Following his definition, fluviokarst consist: in “karst landscapes where the dominant landforms are valleys cut by surface rivers. Such original surface flow may relate either to low initial permeability before caves (and hence underground drains) had developed, or to reduced permeability due to ground freezing in a periglacial environment. In both cases the valleys become dry as karstification improves underground drainage” [30]. White [31,32] proposed a model where the hydrological behaviors are forced by the external boundary conditions. The various recharges in the CS are Allogenic Intrusions, internal Run-off and Porous Fractured Matrix flows. Intrusions of neighbor streams cause the first category of flows. The second results from sporadically floods that occur after heavy rainfalls. The third category gathers percolation flows through soils and epikarstic layer that reach the CS as a Diffuse Infiltration through the fractured and porous rocks of the aquitard (Figure 3a). In the local context of the Cent-Font watershed, the permanent course of Buèges stream matches the Upper Allogenic Stream. Later, at the swallow zone, the Buèges splits into a Surface Stream and an Upper Allogenic Intrusion. The Surface stream forms a non-perennial flow that joins the Hérault River north of Lamalou confluence. The Upper Allogenic Intrusion, $Q_{UA}$, joins the CS after an underground journey. Table 1 recalls the notations and acronyms of this article. This model also considers the Diffuse Infiltration that percolates through the CS wall. The only output flow of the fluviokarst is the spring, $Q_S$, that falls into the Hérault River at the karstic base level.

**Figure 3.** (a) Conceptual fluviokarst redrawn from White [31,32] but restricted to recession period (neither runoff nor surface flow). The karstic aquifer is embedded in a saturated PFM drained by a CS that gathers inflows. Outflow discharges at the base level of the neighbor stream through a spring (or pump). The hydrological system includes an upper allogenic stream, which flow joins the CS through a swallow zone. The CS also receives PFM, diffuse flow through an epikarstic layer; (b) An OTS is surrounded by pervious boundaries bounding the CV. Water and thermal energy inputs come from the allogenic streams and PFM. Outputs leave the CS through a spring (or pump). During recession no reverse flow occurs from CS to PFM.
Table 1. Chronology of pumping test phases and periods.

<table>
<thead>
<tr>
<th>Periods</th>
<th>Pumping Operation</th>
<th>Beginning mm/dd (hh:mm)</th>
<th>Ending</th>
<th>Revisited Periods</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Pre-pumping phase</td>
<td>1992</td>
<td>07/27 (07:25)</td>
<td>07/14 (00:00)–07/18 (23:55)</td>
</tr>
<tr>
<td>b</td>
<td>Step drawdown</td>
<td>07/27 (07:30)</td>
<td>07/30 (12:40)</td>
<td>-</td>
</tr>
<tr>
<td>c</td>
<td>Constant pumping</td>
<td>08/01 (07:10)</td>
<td>08/09 (07:20)</td>
<td>-</td>
</tr>
<tr>
<td>d</td>
<td>Recovering test</td>
<td>08/09 (07:20)</td>
<td>08/09 (13:00)</td>
<td>-</td>
</tr>
<tr>
<td>e</td>
<td>Constant pumping</td>
<td>08/09 (13:05)</td>
<td>09/02 (07:15)</td>
<td>08/13 (00:00)–09/31 (23:55)</td>
</tr>
<tr>
<td>f</td>
<td>Recovering</td>
<td>09/02 (07:20)</td>
<td>09/03 (07:40)</td>
<td>-</td>
</tr>
<tr>
<td>g</td>
<td>Equilibrium-pumping</td>
<td>09/03 (07:45)</td>
<td>09/06 (06:00)</td>
<td>09/04 (12:00)–09/05 (11:55)</td>
</tr>
</tbody>
</table>

As mentioned above, the Cent-Fonts resurgence has received much attention since decades (see Ladouche et al. [23] for a review). In September 1992, Cge and the city of Montpellier organized a series of pumping tests but heavy rainfalls and resulting runoff forced to stop the experiments after only 16 days. This short time prevents reaching significant aquifer drawdowns despite the high rate of pumping (0.5 m³/s). Thus, this experimentation failed to assess the hydrological properties of the karst and let believe in a high water production potential [33].

Many field observations, such as gauging data of the spring, of the Hérault River and of the Buèges stream have been recorded for several years before the summer 2005 pumping test experiments. New boreholes were drilled. New records of temperatures and discharges were collected in Buèges and Hérault. Temperatures and hydraulic heads were also recorded in the “P7” (Figure 1), “Reco”, “Cge” and “F3” boreholes (Figure 2). The 2005 pumping test campaign started with a sequence of step-drawdown tests. Heavy pumping sequences followed with a high constant rate pumping and a drawdown constant pumping (see Table 1 for accurate chronology). The drawdown induced during these heavy pumping allowed speleological explorations of the Cent-Fonts chimney. It also opened the possibility of in situ timed volumetric gauging and geochemical sampling of Hérault River intrusions in the CS. This rich set of data has been extensively analyzed already [26,34]. The present work aims to revisit part of these data to assess the accuracy of conservative tracer assumption for water temperature. In the following, we will consider that two error sources mainly affect the applicability of the method: (1) The natural temporal instabilities that affect the data series; (2) the effect of conservative temperature assumption itself according to our previous work [35,36].

The next section of the paper recalls the theoretical context of Open Thermodynamic System for fluvio-karst. Section 3 describes the data chosen from the 2005 pumping test campaign, Section 4 presents their analysis and accuracy assessments. The results are summarized and discussed in Section 5.

2. Theoretical Context

An Open Thermodynamic System (OTS) does not require a precise knowledge of the locations and shapes of its boundaries to study the balance of the fluxes that enter or quit its Control Volume (CV). In that sense, CS of fluvio-karsts are similar to CV of OTS (Figure 3b). We will benefit of this analogy to study the balance of fluxes and energy in the Cent-Fonts fluvio-karst CV despite the inaccurate knowledge of its boundaries [35].

In our previous work [36] we assessed the error resulting, in OTS, of conservative temperature assumption within the theoretical context of White’s fluvio-karst model. This analysis relied on two successive numerical solving of thermal behavior. The first considered the temperature as a conservative tracer, while the second, following works by Covington et al. [37,38] solved the complete set of energy equations including convective, diffusive and dispersive terms. The amplitude of differences between both results shows the first order of the error due to the conservative assumption. We drew abacus curves of this error versus thermal diffusivity ratio and the Péclet numbers, Prandtl numbers and Reynolds numbers of the CS. The error remains less than 1 percent in the dimensionless space and converges to zero for the most extremes values of karst features. This dimensionless error needs to be rescaled for each singular fluvio-karst to calculate its physical amplitude.
In our study, we consider water as a Boussinesq fluid with constant thermal capacity, thermal expansion and density. As a result, water motion forms a zero divergence velocity field in the PFM and in the CS (Equation (1)).

\[ \text{div}(\mathbf{v}) = 0 \]  

(1)

It is possible to convert the volume integrals over fluviokarst CS in flux integrals over the PFM boundaries and hydraulic sections of conduits entering or leaving the CS. Then, the terms of Equation (2) are equal to the input flows into the CS (positive algebraic values, \(q_i\)) and to the flows escaping from the CS (negative algebraic values, \(q_o\)). Therefore, the continuity equation leads to the mass conservation equation that links the various flows.

\[ \sum_{\text{input}} q_i + \sum_{\text{output}} q_o = 0 \]  

(2)

The first law of thermodynamics stipulates that the internal energy change of the CV (Equation (3)) corresponds to the balance of the energy differences between all the incoming and outgoing flows (index \(j\)) and considering the work done. Thus, the internal energy change (\(\delta E\)) is equal to the summation of: the enthalpy by unit of mass (\(h_j\)), the potential energy (\(e_{\text{pot}} j\)), the kinetic energy (\(e_{\text{cin}} j\)), the external heat transfer (\(\delta \Phi j\)) and the work exchanges (\(\delta W j\)) with the surrounding [39,40].

\[ \delta E = \sum_j (h_j + e_{\text{pot}} j + e_{\text{cin}} j) dm_j + \sum_j \delta \Phi j + \sum_j \delta W j \]  

(3)

In the following, we will consider no chemical contribution to enthalpy and flow transfers with negligible exchange between heat and work. We also will consider steady flux conditions in the CV (we discuss this point later). These hypotheses cancel the internal energy change but also the potential energy and kinetic energy changes. Then, Equation (3) becomes a balance between the specific enthalpies by unit of time of the flows entering (\(h_i\)) and escaping (\(h_o\)) the CS (Equation (4)).

\[ \sum_{i} h_i \rho_i q_i + \sum_{o} h_o \rho_o q_o = 0 \]  

(4)

Now, specific enthalpy depends only on thermal capacity (\(C_p\)) and temperature (referred to an arbitrary value \(T_a\)) (Equation (5)).

\[ h_j = C_p j (T_j - T_a) \]  

(5)

Combination of Equations (4) and (5) leads to the enthalpy balance (Equation (6)).

\[ \sum_i C_p i (T - T_a) \rho_i q_i + \sum_o C_p o (T_a - T) \rho_o q_o = 0 \]  

(6)

Finally, with constant density and thermal capacity Equation (6) reduces to a classical mixing equation (Equation (7)) that links temperatures (°K or °C) and mass transfers in the CS.

\[ \sum_{\text{input}} q_i T_i + \sum_{\text{output}} q_o T_o = 0 \]  

(7)

Equations (2) and (7) form the basis of a linear system able to discover two unknown flows in the CS (so-called \(Q_k\) and \(Q_l\) in Equation (8)).

\[ q_k T_k + q_l T_l = - \sum_{o \neq k} q_o T_o - \sum_{i \neq k} q_i T_i \]

(8)
Thus, these the two unknown flows are (Equation (9)).

\[
q_k = \frac{1}{(T_k - T_l)} \left[ - \sum_{o \neq k} q_o T_o - \sum_{i \neq k} q_i T_i + T_l \right] = \frac{1}{(T_k - T_l)} \left[ - \sum_{o \neq k} q_o (T_o - T_l) - \sum_{i \neq k} q_i (T_l - T_i) \right],
\]

\[
q_l = - \sum_{o \neq k} q_o - \sum_{i \neq k} q_i - q_k.
\]

As mentioned above, we will use the theoretical results of Machetel and Yuen [36] to quantify the sensitivity of the model to the conservative enthalpy assumption. However since we assumed that no other sources of heat are present (neither chemical heat, nor work conversion to heat), enthalpy conservation comes down on to temperature conservation. We will also use the differential form (Equation (10)) of Equation (9) to assess the effects of temperature and discharge uncertainties solving Equation (9).

\[
\delta q_k = \frac{1}{(T_k - T_l)} \left[ - \sum_{o \neq k} \left[ \delta q_o (T_o - T_l) - q_o \left( (\delta T_o - \delta T_l) - \frac{(\delta T_k - \delta T_l)(T_o - T_l)}{(T_k - T_l)} \right) \right] \right]
\]

\[
\delta q_l = - \sum_{i \neq k} \delta q_i - \sum_{i \neq k} \delta q_l - \delta q_k.
\]

The applicability of the method developed above depends on a “reasonably” steady CS. This is never the case in nature where temperature and flow variations due to human activities or diurnal or meteorological cycles disrupt steadiness. This is a recurring problem for hydrological studies. It can be significantly with careful choice of working periods and a 24-h moving averaging of data impacted by diurnal effects [10–12]. The thermal inertia of soil also damps the meteorological or seasonal effects with deepening [14,15].

However, we also have to trust on the common sense of operators and analysts to “instinctively” avoid the most unstable periods for data collection. Thus, the conservative temperature assumption is unsuitable for runoff flow studies or all other kinds of events that imply transient and unstable thermal or water fluxes in the CS. This is why, the present work use a restriction of the White’s model to the recession period, with no run-off flow, and with a complete loss of the Upper Allogenic Stream (no surface stream). The 2005 Cent-Fonts pumping tests take place during the summer season when rainfalls are scarce on the watershed. However, even during that time, we focused the method on “steadiest” periods for water temperatures and flows. This is also why, despite available data until November, we stopped our analyses on 6 September (16:55) when a runoff due to a heavy thunderstorm flooded the boreholes (Figure 4).

3. Presentation of the Cent-Fonts Pumping Test Data

3.1. Period (a): Data Collected Prior the Beginning of the Pumping Test

Several years of flows and water temperatures have been recorded at the karst spring and in Buèges since 1997 to study the recession of the base flows during dry periods. This knowledge is
essential to understand the answers of the aquifer during the pumping tests. The base flow \( (Q_S) \) gathers the upper allogenic intrusion of Buèges stream \( (Q_{UAI}) \) and the diffuse infiltration \( (Q_{PFMB}) \). The study of the karst recession conducted from 1997 to 2001 used a modified Mangin method to distinguish the parts played by \( Q_{PFMB} \) and \( Q_{UAI} \) to the base flow [23]. \( Q_S \) is described thank to three terms calculated from a Maillet homographic function [41]. Hence, two recession coefficients appear that characterizes the respective recession evolutions of \( Q_{PFMB} \) or \( Q_{UAI} \). Ladouche et al. [23] calculated recession coefficients of 0.0080 (1998); 0.0088 (2000) and 0.0088 (day⁻¹) (2001) for the \( Q_{PFMB} \) contribution to the base flow [23] (p. 64).

Complementary flows and water temperatures have been recorded in the weeks preceding the step-drawdown sequence on 27 July (see Table 1 for an accurate chronology of the pumping tests). Figure 4 presents the far field \( (T_\infty) \), the Upper Allogenic Intrusion \( (T_{UAI}, \text{Buèges}) \), the Neighbor Allogenic Intrusion \( (T_{NAI}, \text{Hérault}) \) and the CS \( (T_{CS}, \text{in Cge borehole}) \) temperatures recorded from 1 July to 6 September. During the pumping tests, Cge’s devices provided CS hydraulic heads and temperatures until their disconnection by drawdown. \( T_{CS} \) temperature was also recorded in the F3 borehole at the output of the pump (see Figures 2 and 4).

![Temperature series](image)

**Figure 4.** The temperature series used in the study are \( T_{UAI} \) (recorded at the Buèges losses), \( T_{NAI} \) (Hérault), \( T_{CS} \) (recorded in CGE borehole until disconnection on 13 August 12h15 and at pump output), and \( T_\infty \) (recorded in the P7 borehole).

### 3.2. Period (b): Data Collected during the Step-Drawdown Sequence (27 to 30 July)

Step-drawdown sequences are one of the most often performed pumping test to find out the behavior of wells and aquifer features. For the Cent-Fonts pumping test campaign, the pump has been placed directly inside a large CS conduit. The hydraulic heads recorded in F3, Reco and Cge will display such close curves they will be undistinguishable at the Figure 5 scale despite the bottleneck between Cge and F3 boreholes (Figures 2 and 5). This superposition of curves reveals the hydraulic connectivity in this final part of the CS. Thus, the step-drawdown sequence will efficiently find out the resurgence yield by overestimating or underestimating the rate of pumping drying the spring.

Table 2 recalls the step-drawdown chronology, the pumping rates and the drying effects on griffons. \( Q_D \) of 0.2 and 0.3 m³/s did not achieve the completed drying of griffons while it was reached for 0.4 and 0.5 m³/s [26] (pp. 55–59). These results allow inferring that between 27 July and 30 July the base flow of the resurgence spring (that is \( Q_{PFMB} + Q_{UAI} \)) was ranging between 0.3 and 0.4 m³/s.

Four “coma-shaped” events (due to sudden deepening followed by recovering) occurred on the hydraulic head for each pumping of the step-drawdown sequence (Figure 5, Period (b)). During that time, the CS temperature remains constant to a few tens of degree while, on the opposite, \( T_{UAI} \) (Buèges) and \( T_{NAI} \) (Hérault) display diurnal temperature oscillations that reach 1 to 3 degrees (Figure 4). These diurnal temperature variations oscillate over 4 degrees, of meteorological trend that affect both \( T_{UAI} \) and \( T_{NAI} \) between 11 July and 30 July. The meteorological trend has the same amplitude on \( T_{UAI} \) and \( T_{NAI} \). The amplitude of \( T_{UAI} \) diurnal oscillations remains lower than those of \( T_{NAI} \) because of
shortness Buèges course and its low emergence temperature (12.5 °C). The stability of \( T_{CS} \) despite these oscillations results from the damping effects of soils and 10 days underground transfer from the losses area to the Cent-Font resurgence [20].

![Figure 5. Hydraulic head recorded in the CGE, F3 and Reco boreholes. Letters (a to g) refer to periods of Table 1. Note the four “coma-shaped” events (due to sudden deepening followed by recovering) that occurred on the hydraulic head for each of the pumping of the step-drawdown sequence (period b); the almost constant rate of hydraulic head increase during the constant high rate pumping (periods c and e) and the rapid stabilization of the hydraulic head during the equilibrium pumping (period g).](image)

**Table 2.** Step drawdown pumping, \( Q_{PFMB} \) from Equation (13), \( Q_{ULAI} \) recorded at Buèges, total recovering.

<table>
<thead>
<tr>
<th>Step-Drawdown Phase</th>
<th>Mean ( Q_p ) (m³/s)</th>
<th>Outlets Drying [26]</th>
<th>Mean ( Q_{PFMB} ) (m³/s)</th>
<th>Mean ( Q_{ULAI} ) (m³/s)</th>
<th>( Q_{PFMB} + Q_{ULAI} ) (m³/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>07/27 (07:30–12:20)</td>
<td>0.203</td>
<td>Partial drying outlet n°'4</td>
<td>0.305</td>
<td>0.042</td>
<td>0.347</td>
</tr>
<tr>
<td>07/26 (06:35–12:00)</td>
<td>0.301</td>
<td>No drying of other outlets</td>
<td>0.303</td>
<td>0.034</td>
<td>0.337</td>
</tr>
<tr>
<td>07/29 (06:20–12:20)</td>
<td>0.500</td>
<td>Drying of outlet n°'4 (partial drying)</td>
<td>0.300</td>
<td>0.031</td>
<td>0.331</td>
</tr>
<tr>
<td>07/30 (06:30–12:35)</td>
<td>0.402</td>
<td>Drying of all outlets</td>
<td>0.297</td>
<td>0.033</td>
<td>0.330</td>
</tr>
</tbody>
</table>

3.3. Period (c): Constant High Rate Pumping (1 to 9 August)

Long high rate pumping is used to assess the answer of the aquifer to drawdown. From 1 August to 9 August, the hydraulic head in the CS increases linearly with time (note that to avoid concave curves in Figure 5, the data are not plotted versus the log of time as usual). However, the drawdown induces a reversal of the hydraulic head that triggers the intrusion of Hérault in the CS and a new contribution that adds to the base flow \( Q_{PFMB} \) coming from the PFM. Two new flows, \( Q_{NAI} \) and \( Q_{PFMD} \) are added to \( Q_{PFMB} \) and \( Q_{ULAI} \) while spring drying let \( Q_p \) be the only discharge of the CS. During that time, \( T_{ULAI} \) and \( T_{NAI} \) display diurnal and meteorological variations while \( T_{∞} \), recorded 25 m below the surface in the P7 borehole, remains remarkably constant (Figure 4, Period (c)).

\( T_{CS} \) is recorded both in the Cge and F3 boreholes. The first is measured close to the arrival of the neighbor allogetic intrusion of Hérault (Figure 2). The second \( T_{CS} \) is recorded deeper, close to the arrivals of \( Q_{ULAI} \) and \( Q_{PFMB} \). These two flows carry temperatures \( T_{ULAI} \) and \( T_{∞} \) lower than this of \( T_{NAI} \). Thus, the values of the two \( T_{CS} \) series diverge rapidly as soon as the drawdown triggers arriving of hot intrusive Hérault water in the CS branch near the Cge borehole. \( T_{CS} \) (Cge recorded) increases rapidly a few hours after the starting of the high rate constant pumping from its 13.7 °C constant value since 2 August, 12h05. After a few days of transient evolutions, the temperatures in Cge and F3 boreholes stabilize respectively around 20 and 15 °C. Their temporal variations are correlated with the meteorological and diurnal trends observed for \( T_{NAI} \) and \( T_{ULAI} \) (Figure 4, Period (c)).

The second series of \( T_{CS} \) records (F3) stabilizes rapidly around 15 °C. It displays diurnal oscillating changes that are clearly due to the mixing of hot \( Q_{NAI} \) with the cold \( Q_{PFMB} \), \( Q_{PFMD} \) and \( Q_{ULAI} \).
in the vicinity of the pump. This stabilization of the $T_{CS}$ increase indicates that $Q_{NAI}$ acts like an almost constant vadose flow despite the increase of the hydraulic head between the water table and Hérault [26] (p. 191).

According to Maréchal et al. [34] we will considerer in the following the dewatering of the conduit network as a supplementary outgoing flow from the CS, $Q_{CS}$.

3.4. Period (d): Recovering Test (9 August)

The 6 h pump stop of 9 August allowed a recovering of 3.43 m (Figure 5, Period (d), and [26] (p. 65). During the interruption, $Q_{NAI}$ brought warm water of Hérault that accumulated in the chimney above the pump. After pump re-starting, the temperature, $T_p$ displayed a short peak 9 August, 13h05 consecutive to the rapid extraction of this warm water. Similar phenomena occurred at the restarting 22 August, 13h30 and 3 September, 07h45 (Figure 4, Periods e and g).

3.5. Period (e): Constant High Rate Pumping (9 August to 2 September)

After 6 h of recovering, high rate pumping has been restarted from 9 August until 2 September. During Period (e), the hydraulic head decreased almost linearly with a slope similar to the one of Period (c). On 13 August at 12h15, the drawdown reached the level of the temperature probe in Cge borehole (51.6 m NGF), causing its disconnection and the loss of the $T_{CS}$ signal recorded there. At the end of Period (e), the drawdown approached the level of the pump in F3 that caused its stopping (Table 1).

As mentioned above for Period (c), two diurnal and meteorological trends are noticeable in the $T_{NAI}$ and $T_{UAI}$ temperature records. These variations are also present in the last part of $T_{CS}$ (Cge recorded) and in $T_{CS}$ (F3 recorded) (Figure 4, Period (e)).

From the beginning of August, the drawdown allowed speleological explorations of the resurgence branches (Figure 6). Surprisingly, while significant $Q_{PFMD}$ infiltrations were expected in CS, no water was apparently percolating through the chimney wall (Figure 6a–c). However, several cascading vadose flows were observed between marks 657–658 of the lifeline (Figure 6d) and mark 670 (Figure 6e). Partial catchments of these Hérault intrusions at point 657 collect 0.045 to 0.050 m$^3$/s [26] (p. 40). This value gives a rough lower bound value of $Q_{NAI}$ since other hidden or deeper entries were probably active. These speleological explorations also opened the opportunity to collect geochemical samples to quantify the mixing of Hérault intrusion in the CS.

Figure 6. Speleological views of CS chimney. (a) CS wall at lifeline mark n° 658 (8 August); (b) lifeline mark 658; (c) “dry” CS wall at mark n° 658; (d) Cascading Hérault intrusion at mark 658 (CS1 in Table 3); (e) Second Hérault intrusion at lifeline mark 670 (1 September) (CS2 in Table 3).
Water samples were collected 1 September at the above intrusion points. We used PP® bottles previously washed with chlorhydric acid, then bromhydric acids, to prevent contamination [42]. Bottles and corks have been rinsed four times on site. The solutions have been filtered, acidified and prepared in two dilutions for analysis. Complementary samples were collected the same day at the Buèges spring, in Hérault and at the pump.

**Table 3. CS1 and CS2: Hérault intrusions in CS, HR: Hérault, BS: Buèges, PO: Pump output.**

<table>
<thead>
<tr>
<th></th>
<th>CS1</th>
<th>CS2</th>
<th>PO</th>
<th>BS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rb</td>
<td>1.054</td>
<td>1.131</td>
<td>1.582</td>
<td>0.460</td>
</tr>
<tr>
<td>Sr</td>
<td>76.854</td>
<td>82.443</td>
<td>91.054</td>
<td>58.334</td>
</tr>
<tr>
<td>Ba</td>
<td>52.231</td>
<td>52.577</td>
<td>94.786</td>
<td>12.723</td>
</tr>
</tbody>
</table>

The samples were analyzed for the Rb, Sr, and Ba on the VG Plasmaquad II turbo ICPMS of Montpellier 2 University (Table 3 and Figure 7). Sr, Rb, and Ba have been chosen as field tracers for water circulation and mixing [4] [42]. Figure 7 shows the alignment of the samples in a (Ba/Sr) vs. (Rb/Sr) graph. The alignment of point denotes a mixing between two poles [45].

According to the regression curves and the rate of pumping \( Q \), \( Q_{NAI} \) ranges from 0.039 to 0.048 m\(^3\)/s for Ba, from 0.060 to 0.083 m\(^3\)/s for Sr, and from 0.077 to 0.087 m\(^3\)/s for Rb. However, the lower values of the Ba/Sr ratio may reflect a sorption effect onto mineral-water interfaces [44]. The averaged value of these six measures gives \( Q_{NAI} = 0.066 \) m\(^3\)/s.

![Figure 7. Plot of Ba/Sr vs. Rb/Sr (see Table 3). The alignments of points are characteristic of a two-pole mixing between Buèges and water Hérault. Two dilutions have been applied on the samples before ICPMS analyses (open squares: dilution by a factor two, open circles: no dilution). Assuming that Buèges water composition is characteristic of the far field water composition, we can write, after Vidal [45]:](image)

\[
Q_{NAI} = Q_P \left( c_P - c_{UAI} \right) / \left( c_{NAI} - c_{UAI} \right),
\]

where \( c_P, c_{UAI} \) and \( c_{NAI} \) respectively stand for mass concentrations of Rb, Ba and Sr in \( Q_P, Q_{UAI} \) and \( Q_{NAI} \) (Table 3).

### 3.6. Period (f): Recovering Test (2 September to 3 September)

A second stop of the pump lasts 24 hours and 25 minutes, from 2 September to 3 September, that induced a 13.49 m recovering in the chimney (Figure 5).

### 3.7. Period (g): Drawdown Constant Pumping (3 September to 6 September)

The equilibrium-pumping (Period (g)) followed one month of constant pumping at constant rate that resulted in an important drawdown. After the recovering of Period (f), the equilibrium-pumping aimed to assess the dynamics of aquifer answer to hydraulic head. It started on 3 September at 7h45.
with \( Q_p = 0.324 \, \text{m}^3/\text{s} \). The pumping rate has been dropped to \( Q_p = 0.304-0.305 \, \text{m}^3/\text{s} \) a few hours later to stabilize the hydraulic head at 35.0 ± 0.1 m NGF (Figure 5).

Since, this value is around 40 m deeper than the karst base level the Hérault intrusion remained active (the first intrusion of Hérault occurred for 76.7 m NGF on 1 August at 07h10).

4. Revisiting the 2005 Pumping Test Data

The challenge of studying temperature as a conservative tracer also faces the natural complexity of karsts, which CS mixes water coming from low resistance conduits and low permeability PFM. A few decades ago, studies were often considering medium where CS was not disrupting the aquifer but continuous models were unsatisfactorily. Since a few years, improved analytical models take better into account for the different behavior of the two types of reservoirs [34] or even three reservoirs [45]. However, despite the importance of calibration for the models, temperature has not been used probably because of the non-conservative character of this signal [26,34]. In the following we will take benefit of particular periods of the pumping test to get new constraints for the models.

Our process consists, firstly, to revisit the data of Period (a) to recalculate the base flow \( Q_{PFMB} \) that forms the background over which the assessment of \( Q_{PFMD} \) is possible. Secondly, we benefit of the base flow knowledge to reevaluate the intrusion of Hérault, \( Q_{NAI} \), on Period (g). Thirdly, relying on \( Q_{PFMB} \) and \( Q_{NAI} \), we re-assess \( Q_{PFMD} \), the answer of the aquifer to drawdown, during the constant pumping (Periods (c) and (e)).

4.1. Revisiting \( Q_{PFMB} \) from Period (a)

4.1.1. Revisiting the Data

Unlike Maréchal et al. [34], but following the results of [26] (p. 64), we consider that the recession of the Cent-Fonts base flow is better described separating the Buèges contribution \( Q_{UAI} \). Then, the recession of \( Q_{PFMB} \) is calculated using Equation (11) in which we need to set the amplitude coefficient \( Q_{PFMB}(t_0) \).

\[
Q_{PFMB}(t) = Q_{PFMB}(t_0) e^{-0.0088(t-t_0)}
\]  

(11)

Without pumping, the only flow leaving the CS is the one of the spring. In July 2005, as the surface course of Buèges fully disappears, and since no pumping affects the resurgence, no drawdown occurs in the CS. Under these natural conditions \( Q_S \) gathers \( Q_{UAI} \) and \( Q_{PFMB} \) and the water table in CS sits on top of the base level by a few centimeters. This weak hydraulic head is enough to prevent Hérault to intrude.

Then, replacing \( q_k = Q_{PFMB}, T_k = T_\infty, q_l = -Q_S, T_l = T_{CS}, q_i = Q_{UAI} \) and \( T_i = T_{UAI} \) in Equation (8) leads to the Equation (12) below where the spring discharge \( Q_S \) and the base flow \( Q_{PFMB} \) are unknowns.

\[
Q_{PFMB} = \frac{Q_{UAI}(T_{UAI}-T_{CS})}{(T_{CS}-T_\infty)}
\]

\[
Q_S = Q_{PFMB} + Q_{UAI}
\]  

(12)

It is, therefore, possible to fix \( Q_{PFMB}(t_0) \) in the recession curve with the values \( Q_{UAI}, T_{UAI}, T_{CS} \) and \( T_\infty \) recorded for several weeks. Figure 8 displays an enlargement of these records from 14 July (00:00) to 19 July (23:55). \( T_{UAI} \) displays diurnal oscillations of 0.2 to 0.4 °C around its 24-h moving averaging and a small meteorological increasing trend of a few degrees (Figure 8, top). In bottom panel of Figure 8, \( Q_{UAI} \) also displays diurnal oscillations due to the water catchments upstream of the swallow zone. Concurrently, \( T_\infty \) and \( T_{CS} \) remain remarkably stable (Figure 8, top). The \( Q_{PFMB} \) curve in the bottom panel of Figure 8 displays the results of Equation (12) solving. We calculated its mean values (0.336 m³/s) and affected it at the median time to = 16 July (12:00) to extrapolate the PFM base flow for the remainder of our study (Equation (13)).

\[
Q_{PFMB}(t) = 0.336e^{-0.0088(t-t_0)}
\]  

(13)
Figure 8. (a) Temperatures and (b) flows recorded or calculated during the pre-pumping period. The conduit system temperature $T_{CS}$ recorded in the CGS borehole and the matrix-conduit flow temperature $T_{\infty}$ are almost constant over this period while the upstream allogenic intrusion temperature $T_{UAI}$ displays both diurnal and meteorological trends that are smoothed by the 24-h averaging. The upstream allogenic intrusion, $Q_{UAI}$, also displays a diurnal behavior due to water catchments upstream of the swallow zone. $Q_{PFM}$ is solved with Equation (14).

4.1.2. Assessment of Error Due to Data Variability

Two kinds of inaccuracy may affect the measures and, therefore, the use of $Q_{UAI}$, $T_{UAI}$, $T_{CS}$, and $T_{\infty}$ for the method described in this article. The first are the precisions of the measures while the second relate to the difference between the steady state and the physical conditions in the CV. In the following, we will consider that modern thermometers result in negligible errors (less than 0.1 °C) in front of those resulting of unsteady behaviors. For Buèges gauging, the level error $\delta Q_{UAI}$ at the swallow zone is not explicitly mentioned [26]. However, it is reduced by the total swallowing that limits it to the one of the zone entry. It is also reduced by the low flow context that allows more accurate gauging [46]. Therefore, without better assessment of this error, it seems reasonable to consider that it remains lower than a few liters by second. This level matches the one of standard errors induced by the diurnal variations (Table 4).

Equation (10) provides a powerful tool to assess how errors spread through the solving. According our preceding comments, and considering the stability of $T_{CS}$ and $T_{\infty}$, we have reported $\delta T_{\infty} = 0$ and $\delta T_{CS} = 0$ in Equation (10). The differential form of Equation (9) becomes Equation (14), for which the numerical coefficients have been calculated by using the mean values and the Standard Errors of temperatures and flows.
\[ \delta Q_{PFMB} = \frac{1}{T_{CS} - T_{\infty}} \left[ (T_{UAI} - T_{\infty})\delta Q_{UAI} + Q_{UAI}\delta T_{UAI} \right] \]
\[ \delta Q_s = \delta Q_{PFMB} + \delta Q_{UAI} \]
\[ \delta Q_{PFMB} = 6.21 \delta Q_{UAI} + 0.038 \delta T_{UAI} \tag{14} \]

More realistic physical conditions have been searched for in the CS by operating 24-h moving averaging on the diurnal variations of raw data (Table 4, columns 6 and 8). Such numerical process accounts for the natural thermal and kinetic inertia acting along underground water flows and allows damping the error that could arise from their neglecting. Thus, over a few days, a meteorological trend increases the temperature recorded at the Buèges swallow zone by one to two °C (Figure 8). However, the final standard error remain limited to 0.64 °C on raw data (Table 4, column 6) and to 0.55 °C after 24-h averaging of the data (Table 4, column 8). The natural damping of temperature fluctuations that affects \( Q_{UAI} \) during its underground travel to the CS prompts to consider the standard error as an upper bound.

Equation (14) shows that 1 °C of error on \( T_{UAI} \) induces 38 L/s (liters by second) of error on \( Q_{PFMB} \); and that one L/s of error on \( Q_{UAI} \) induces 7 L/s on the final result. We can therefore expect that \( Q_{PFMB} \) is obtained to a few tens of L/s (around 20%) (Table 4, columns 7 and 9). This is consistent with the step drawdown tests that revealed a spring flow \( Q_{PFMB} + Q_{UAI} \) comprised between 0.3 and 0.4 m³/s.

4.1.3. Assessment of Error due the Conservative Temperature Assumption

Another way to explore the effects of non-stationarity on the solutions consists in confronting karst numerical models that consider (or not) temperature as a conservative tracer. This has been done in a previous study where several numerical models have been calculated over very broad ranges of karst morphological and hydrological parameters. According to Equation (15) of [36] a first order of the error induced by the conservative temperature assumption is reached by the following Equation (15):

<table>
<thead>
<tr>
<th>Periods</th>
<th>Parameter</th>
<th>Status</th>
<th>Figure</th>
<th>Mean</th>
<th>SE</th>
<th>Error</th>
<th>24mSE</th>
<th>24mError</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>( T_{CS} )</td>
<td>M</td>
<td>4, 8</td>
<td>13.6</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>( Q_{UAI} )</td>
<td>M</td>
<td>8</td>
<td>0.053</td>
<td>0.010</td>
<td>-</td>
<td>0.005</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>( T_{UAI} )</td>
<td>M</td>
<td>8</td>
<td>22.3</td>
<td>0.64</td>
<td>-</td>
<td>0.55</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>( T_{\infty} )</td>
<td>M</td>
<td>4, 8</td>
<td>12.2</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>( Q_{PFMB} )</td>
<td>C</td>
<td>8</td>
<td>0.336</td>
<td>-</td>
<td>0.086%–25.6%</td>
<td>-</td>
<td>0.052%–15.5%</td>
</tr>
<tr>
<td></td>
<td>( Q_s )</td>
<td>C</td>
<td>8</td>
<td>0.391</td>
<td>-</td>
<td>0.096%–24.7%</td>
<td>-</td>
<td>0.057%–14.7%</td>
</tr>
<tr>
<td>(g)</td>
<td>( Q_P )</td>
<td>M</td>
<td>9</td>
<td>0.304</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>( T_P )</td>
<td>M</td>
<td>9</td>
<td>15.73</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>( T_{\infty} )</td>
<td>M</td>
<td>4, 9</td>
<td>12.2</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>( Q_{UAI} )</td>
<td>M</td>
<td>9</td>
<td>0.015</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>( T_{UAI} )</td>
<td>M</td>
<td>9</td>
<td>22.0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>( T_{\infty} )</td>
<td>M</td>
<td>9</td>
<td>25.1</td>
<td>0.52</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>( Q_{PMB} )</td>
<td>C</td>
<td>9</td>
<td>0.070</td>
<td>-</td>
<td>0.004%–5.7%</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( Q_{PFM} )</td>
<td>C</td>
<td>9</td>
<td>0.219</td>
<td>-</td>
<td>0.003%–1.4%</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>(e)</td>
<td>( Q_P )</td>
<td>M</td>
<td>10</td>
<td>0.396</td>
<td>0.008</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>( T_P )</td>
<td>M</td>
<td>10</td>
<td>15.16</td>
<td>0.015</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>( T_{\infty} )</td>
<td>M</td>
<td>4, 10</td>
<td>12.2</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>( Q_{UAI} )</td>
<td>M</td>
<td>10</td>
<td>0.0184</td>
<td>0.006</td>
<td>-</td>
<td>0.002</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>( T_{UAI} )</td>
<td>M</td>
<td>10</td>
<td>20.1</td>
<td>0.89</td>
<td>-</td>
<td>0.84</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>( Q_{NAI} )</td>
<td>E</td>
<td>9, 10</td>
<td>0.070</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>( T_{NAI} )</td>
<td>M</td>
<td>10</td>
<td>23.2</td>
<td>1.21</td>
<td>-</td>
<td>0.97</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>( T_{CS} )</td>
<td>E</td>
<td>4</td>
<td>( T_{P} )</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( Q_{PMB} )</td>
<td>E</td>
<td>Equation (13)</td>
<td>var.</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( Q_{PFM} )</td>
<td>C</td>
<td>10</td>
<td>0.248</td>
<td>-</td>
<td>0.016%–5.7%</td>
<td>0.014%–5%</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( Q_s )</td>
<td>C</td>
<td>10</td>
<td>0.278</td>
<td>-</td>
<td>0.029%–97%</td>
<td>0.021%–70%</td>
<td></td>
</tr>
</tbody>
</table>
\[ \delta T = \varepsilon' \left( T_{CS} + \frac{T_{\infty}}{T_{\text{UAI}} - T_{\infty}} \right) \]  

(15)

The error, \( \varepsilon' \), is calculated versus the hydrological and morphological properties as thermal diffusivity ratio (9.93), Conduit Peclet number (1.5 \times 10^6), Prandtl number (6.99) and Conduit Reynolds number (4.29 \times 10^4). Tables 1 and 5 recall the form and the physical values compatible with the Cent-Fonts karst. Then, the abacus curves of Figure 4 of [36] indicate \( \varepsilon' = 0.00613 \) at the exit of the CS.

With \( \varepsilon' = 0.00613 \), \( T_{CS} = 286.75 \) K, \( T_{\text{UAI}} = 295.45 \) K and \( T_{\infty} = 285.35 \) K (Table 4, Period (a)) the first order of the error \( \delta T = 1.93 ^\circ C \). Coming back to Equation (14), we can see that it may induce 0.073 m^3/s of error for \( Q_{\text{PFMB}} \). This result is higher but remains consistent with the previous estimate and the results of the step drawdown tests.

Table 5. Assessment of error due the conservative temperature assumption. Period, characteristic length, hydraulic radius, characteristic velocity, Dimensionless thermal diffusivity, Peclet number, Prandtl number, Conduit Reynolds number, dimensionless error, rescaled thermal error.

<table>
<thead>
<tr>
<th>Periods</th>
<th>L (m)</th>
<th>r (m)</th>
<th>V(m/s)</th>
<th>D</th>
<th>Pe</th>
<th>Pr</th>
<th>Red</th>
<th>( \varepsilon )</th>
<th>( \delta T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>5 \times 10^3</td>
<td>5</td>
<td>4.29 \times 10^{-3}</td>
<td>9.93</td>
<td>1.5 \times 10^6</td>
<td>6.99</td>
<td>4.29 \times 10^4</td>
<td>0.00613</td>
<td>1.93</td>
</tr>
<tr>
<td>(c, e, g)</td>
<td>2 \times 10^2</td>
<td>5</td>
<td>2.78 \times 10^{-3}</td>
<td>9.93</td>
<td>3.9 \times 10^6</td>
<td>6.99</td>
<td>2.78 \times 10^4</td>
<td>0.002</td>
<td>0.62</td>
</tr>
</tbody>
</table>

4.2. Revisiting \( Q_{\text{NAI}} \) from Périod (g)

4.2.1. Revisiting the Data

As recalled above, the equilibrium-pumping that started on 3 September (7h45) and ended on 6 September (6h00) followed one month of constant pumping and a one day recovering. After a few hours, the initial rate \( Q_P = 0.324 \) m^3/s has been lowered to \( Q_P = 0.304-0.305 \) m^3/s that stabilized the hydraulic head at 35.0 ± 0.1 m NGF (Figure 5). This hydraulic head is around 40 m deeper than the base level Hérault. Consequently, the Hérault intrusion \( Q_{\text{NAI}} \) remained fully active during all the equilibrium-pumping.

During Period (g), the incoming flows in the CS are \( Q_{\text{UAI}}, Q_{\text{NAI}} \) and both basic and drawdown induced PFM contribution \( Q_{\text{PFM}} = Q_{\text{PFMB}} + Q_{\text{PFMD}} \). These input flows equilibrate the output flow \( Q_P \) while the stabilization of the hydraulic implies that the dewatering of the CS stops (\( Q_{\text{CS}} = 0 \)). Hence, Equation (9) can be rewritten as Equation (16) to calculate \( Q_{\text{NAI}} \) and \( Q_{\text{PFM}} \).

\[
Q_{\text{NAI}} = Q_P (T_{\text{P}} - T_{\infty}) - Q_{\text{UAI}} (T_{\text{UAI}} - T_{\infty}) \frac{(T_{\text{NAI}} - T_{\infty})}{(T_{\text{NAI}} - T_{\text{P}})}
\]

\[
Q_{\text{PFM}} = Q_P (T_{\text{P}} - T_{\infty}) - Q_{\text{UAI}} (T_{\text{UAI}} - T_{\text{P}}) \frac{(T_{\text{NAI}} - T_{\infty})}{(T_{\text{NAI}} - T_{\text{P}})}
\]

We focus on a remarkably stable 24-h data range from 4 September (12:00) to 5 September (12:00) (Figure 9). Indeed, the sinusoidal shapes of \( T_{\text{UAI}} \) and \( T_{\text{NAI}} \) are fully damped by the 24-h moving averaging, while \( T_{\infty}, T_{\text{P}}, \) and \( Q_{\text{UAI}} \) remain rather constant.

The bottom panel of Figure 9 displays the results obtained for \( Q_{\text{NAI}} \) and \( Q_{\text{PFM}} \). They lead to mean values of \( Q_{\text{NAI}} = 0.070 \) m^3/s and \( Q_{\text{PFM}} = 0.219 \) m^3/s. The first agrees well with the geochemical results presented in Section 3 while the second is only a few L/s higher than the base flow \( Q_{\text{PFMB}} = 0.216 \) m^3/s obtained from Equation (11) for \( t = 5 \) September (00:00). This seems indicate that, a few hours after stabilization of the hydraulic head, the PFM contribution due to drawdown, \( Q_{\text{PFMD}} \), is very low.
Equation (17) shows that 1 °C of error on $T_{NAI}$ induces 5.6 L/s of error on $Q_{NAI}$ or $Q_{PFM}$; that of 1 L/s on $Q_{UAI}$ results in 0.760 L/s; and that 1 °C on $T_{UAI}$ induces 1.2 L/s. When the standard error of Table 4 are introduced in Equation (17), the error falls to a few L/s (a few %) (Table 4, Period (g)).
4.2.3. Assessment of Error Due the Conservative Temperature Assumption

During the equilibrium pumping, but also during constant pumping, most of the CS mixing occurs near the pump, in the top part of the chimney just beside Hérault (Figure 2). This proximity causes the diurnal oscillation of TP that occur with a delay of 20 h (Figure 4). The drawdown changes significantly the configuration of the CV with a mixing zone close to Hérault. The distance between the river and the chimney that contains the pump is less than 200 m. We have to take these consequences of the drawdown to assess the error due to the conservative temperature approximation during heavy pumping. Therefore, using these delay and distance to estimate the properties of the CV, the CS Peclet number and the Conduit Reynolds number fall respectively to $Pe = 3.9 \times 10^6$ and $Re = 2.78 \times 10^4$ (Table 5). The ratio of (PFM to Water) thermal diffusivities and the Prandtl number remain unchanged. We will use this new dimensionless configuration revisiting the data on Periods (e) and (g). (With these new values the abacus curves of Figure 4 of [36] tell that $\varepsilon'$ reaches around 0.002 at the exit of the CS.

$$\delta T = \varepsilon' \left( T_p + \frac{T_\infty}{(T_{\text{NAI}} - T_\infty)} \right)$$  

(18)

With $\varepsilon' = 0.002$, $T_p = 288.88$ K, $T_{\text{NAI}} = 298.25$ K and $T_\infty = 285.35$ K (Table 4, Period (g)) $\delta T$ reaches 0.62 °C. Through Equation (17), it induces a 0.003 m$^3$/s error for $Q_{\text{PFM}}$. Similarly to the comparison of error on Period (a), both methods of error assessment lead to consistent results.

4.3. Revisiting $Q_{\text{PFMD}}$ from Period (e)

4.3.1. Revisiting the Data

The knowledge of $Q_{\text{PFMB}}$ and $Q_{\text{NAI}}$ makes possible a backward analysis of the data recorded during Periods (c) and (e). Indeed, $Q_{\text{PFMB}}$ forms the background over which it is possible to assess $Q_{\text{PFMD}}$. On the other hand, the vadose character of the Hérault intrusion results in a constant amplitude despite an increasing drawdown [26] (p. 191). These two previous results bring two “corner stones” situations separated by the constant pumping sequence. This allows us calculating two unknown flows: $Q_{\text{PFM}}$ that includes the drawdown induced contribution $Q_{\text{PFMD}}$ and the dewatering of the CS ($Q_{CS}$). The linear increase of the hydraulic head with time over Periods (c) and (e) (Figure 5) suggests a constant dewatering rate, bolstering us to assume a near steady CV situation. The input flows are $Q_{\text{LAI}}, Q_{\text{NAI}}, Q_{\text{PFMB}}, Q_{\text{PFMD}}$ and the output flows are $Q_{CS}$ and $Q_p$.

In order to maintain these “constant” conditions at best, our data processing skipped the data 24 h before and after the stopping. This allowed avoiding the transient phenomena observed at the restarting of the pump for temperature (Figure 4) and discharges. Consequently, we focused our analysis from 10 August (00:00) to 1 September (19:10). Within the context, Equation (9) can be rewritten as Equation (17).

$$Q_{\text{PFM}} = \frac{Q_p(T_p - T_{CS}) - Q_{\text{LAI}}(T_{\text{LAI}} - T_{\text{CS}}) - Q_{\text{NAI}}(T_{\text{NAI}} - T_{\text{CS}})}{(T_\infty - T_{CS})}$$

$$Q_{CS} = \frac{Q_p(T_\infty - T_p) - Q_{\text{LAI}}(T_{\text{LAI}} - T_{\text{CS}}) - Q_{\text{NAI}}(T_{\text{NAI}} - T_{\text{CS}})}{(T_\infty - T_{CS})}$$  

(19)

The brutal increase of the temperature records in the Cge borehole shows that, as soon as Hérault intrudes through the horizontal shallow branch of the CS, $T_{CS}(\text{Cge})$ is less representative of the dewatering temperature $T_{CS}$. Therefore, we will alternatively consider that the temperature recorded at the pump may represents another assessment $T_{CS}(f3)$. This assumption seems reasonable since $T_{CS}(f3)$ is only a few degrees higher than the $T_{CS}$ temperature obtained on Period (a) before the mixing with the hot Hérault water.

The results of calculations for $Q_{\text{PFM}}$ and $Q_{CS}$ are presented on the lower diagram of Figure 10. From $Q_{\text{PFM}}$ and $Q_{\text{PFMB}}$ (Equation (13)), it is easy to calculate $Q_{\text{PFMD}}$. For both $T_{CS}(\text{Cge})$ and $T_{CS}(f3)$ assumptions, $Q_{\text{PFMD}}(\text{Cge})$ and $Q_{\text{PFMD}}(f3)$ curves display diurnal oscillations but do not display clear
increasing trends despite of drawdown deepening. Figure 10 shows that $Q_{PFMD}(Cge)$, $Q_{PFMD}(f3)$, $Q_{CS}(Cge)$ and $Q_{CS}(f3)$ are clearly affected by the diurnal and meteorological oscillations on $Q_{NAI}$ and $Q_{NAI}$. Considering the most advantageous situation $T_{CS}(Cge)$, $Q_{PFMD}$ ranges between 0.030 and 0.050 m$^3$/s. However, this drawdown induced contribution is probably overestimated because of a too hot dewatering temperature assumption $T_{CS}$. Indeed, the calculation forces the system to equilibrate on the pump temperature. This will increase the low temperature contribution $Q_{PFM}$ at the expense of the hot dewatering $Q_{PFM}$. On the other hand $Q_{PFMD}(f3)$, computed taking the pump temperature as $T_{CS}$ seems really too low since it would induce a zero or even negative contribution of $Q_{PFM}$ to drawdown. In any case, these results seem consistent with short transient flows coming with the increase of the hydraulic head. These conclusions are consistent with the ones of Maréchal et al. [34], and consistent with the shortness of the flows observed at the beginning of the equilibrium-pumping phase.

![Figure 10](image)

**Figure 10.** (a) Temperatures and (b) flows recorded or calculated during the constant pumping (see Table 1). Top: The records of the conduit system temperature ($T_{CS}$) in CGE borehole have been interrupted after its drawdown disconnection 13 August (12:15). Two extreme hypotheses have been considered. The first extrapolating the last value measured $T_{CS}(CGE) = 19.67$ °C, the second assuming $T_{CS}(F3) = T_{P}$. The temperature recorded at the pump output displays a low amplitude sinusoidal oscillation due to the direct intrusion of the Hérault intrusion. Concurrently, the matrix-conduit flow temperature remains constant. Bottom: Assessment of the matrix-conduit flow $Q_{PFM}$ and of $Q_{CS}$ corresponding to the dewatering of the CS, $Q_{NAI}$ is considered as constant. The matrix-conduit flow $Q_{PFM}$ gathers the base flow of the resurgence ($Q_{PFMB}$) and the supplementary contribution of matrix-conduit flow induced by drawdown $Q_{PFMD}$.

### 4.3.2. Assessment of Error Due to Data Variability

Following the same procedure, the mean values of the parameter over this interval (Table 4—Period (e)) have been introduced into the reduced form of Equation (10) to establish the formal and numerical forms of Equation (20).
The restriction of our model to the quietest part of the recession period cuts off the risk of disruption of the final enthalpy balance. This is the most interesting property of the OTS approach that refers to calculating “corner stones” descriptions of the hydraulic regime between which we extend the results.

4.3.3. Assessment of Error due to the Conservative Temperature Assumption

As mentioned above, we consider that the hydrological configuration induced by drawdown is the same the one of Period (g). Then, the CS Peclet number and the Conduit Reynolds number remain the same and, \( \varepsilon' \) keeps the same value around 0.002 at the exit of the CS.

\[
\delta Q_{PFM} = \frac{1}{(T_{CS} - T_\infty)^2} \left[ \delta T_C(S(Q_P(T_P - T_\infty) - Q_{UAj}(T_{UAj} - T_\infty) - Q_{NAi}(T_{NAi} - T_\infty)) ight] 
- \delta Q_{UAi}(T_{UAi} - T_{CS})(T_\infty - T_{CS}) - \delta T_{UAi}Q_{UAi}(T_\infty - T_{CS}) 
- \delta Q_{NAi}(T_{NAi} - T_{CS})(T_\infty - T_{CS}) - \delta T_{NAi}Q_{NAi}(T_\infty - T_{CS}) 
\]

Therefore, the error on temperature balance due to the conservative approximation \( \delta T \) remains of the order 0.62 °C. Such temperature error would induce several tens liters by second of errors.

5. Summary and Discussion

The heat and matter exchanges occurring through the karstic boundaries impose a consideration of the Open Thermodynamic System (OTS). Within this framework, the first principle of thermodynamics leads to an enthalpy balance between flows entering and leaving the CV. Combining this property with mass conservation leads to systems of two equations involving the flows and temperatures. If formal physical conditions (steady states) are never achieved in nature, they are approached during particular periods as recession or certain phases of the pumping tests. These periods have been used these to calculate “corner stones” descriptions of the hydraulic regime between which we extend the results.

The restriction of our model to the quietest part of the recession period cuts off the risk of disruption of the final enthalpy balance. This is the most interesting property of the OTS approach that refers to calculating “corner stones” descriptions of the hydraulic regime between which we extend the results.

Even though it is never perfectly reached in nature, the steady state approximation is necessary to use the method. While such state is assumed, temperatures equilibrate at the PFM/Conduit interface. However, this situation does not mean cancelling of embedding rocks/water heat transfer at the interface. Indeed, as shown in Machetel and Yuen 2015, PFM temperature gradient is maintained by the advection of cold, far field, water that counteracts the heat diffusion from CS to PFM through the wall. During the recession period, mixing of intrusive flows hotter than far field temperature, results in CS temperature hotter than in PFM. However, when a steady state is reached (or approached), all these local conductive effects (inside PFM and CS but also between PFM and CS) are taken into account by the final enthalpy balance. This is the most interesting property of the OTS approach that refers to the comparison of integrated incoming and leaving external heat sources and not on the local thermal properties inside of the “black box”. It is clear that what we called “conservative temperature assumption” may better be called as “conservative enthalpy approximation”. However since we assumed that no other
sources of heat are present (neither chemical heat, nor work conversion to heat); enthalpy conservation comes down on to temperature conservation.

Thus, the application of the method to the first period (before pumping) of the Cent-Fonts pumping test experiments allowed assessing the basic recession flow of the resurgence. Then, the equilibrium-pumping allowed assessing the Hérault intrusion and the supplementary contribution induced by drawdown. The analysis shows that errors induced by unsteadiness reach a few tens of liters by second. They are of the same order of magnitude than the errors induced by conservative approximation itself.

In conclusion, we have confirmed the validity for using the thermometric method by the field observations that never contradict the results. It would be advantageous; insofar data exists, to check the method on other sites. It could also be interesting and of relatively low additional costs to develop recording and processing of temperature profiting of next pumping experiments. Indeed, as it does not require sophisticated equipment or procedures, the additional costs should remain low compared to drilling and pumping operations.

Our works may open the opportunity of using the steadiest part of the pumping test sequences to calibrate the global operating mode of complex resurgence system. Combining energy equation and mass conservation equations, temperature measurements in surface waters and boreholes may allow assessing the flow properties in borehole or mixing in karst CS. It seems therefore constitute an efficient tool to separate and calculate the karstic properties and could be a promising tool worthwhile to apply on other sites.

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Author Contributions: Philippe Machetel and David Yuen are both involved in numerical solving of non-linear differential equations, thermodynamics and physics of fluids.

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

A.1. Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
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<tbody>
<tr>
<td>CS</td>
<td>Conduit System of fluviokarstic system</td>
</tr>
<tr>
<td>CV</td>
<td>Control volume of the OTS</td>
</tr>
<tr>
<td>OTS</td>
<td>Open Thermodynamic System</td>
</tr>
<tr>
<td>PFM</td>
<td>Porous Fractured Matrix</td>
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A.2. Cent-Fonts Fluviokarst Flows, White’s and Pumping Test Notations

A.2.1. CS Inflows (m³/s)

| Q_{ULAI} | Upper allogenic stream intrusion (Buèges intrusion at swallow zone—M) |
| Q_{NAI}  | Neighbor allogenic stream intrusion (Hérault intrusion at base level—C) |
| Q_{PFM}  | Porous Fractured Matrix to CS flow (C) |
| Q_{PFMB} | Porous Fractured Matrix to CS base flow (C) |
| Q_{PFMD} | Porous Fractured Matrix to CS drawdown induced flow (C) |
| Q_{D}    | Drawdown induced Porous Fractured Matrix to CS flow (C) |
A.2.2. CS outflows (m³/s)

- \( Q_S \): Spring discharge (Resurgence discharge in Hérault M or C)
- \( Q_P \): Pump discharge (Pump discharge in Hérault during pumping tests—M)
- \( Q_{CS} \): CS Dewatering corresponding to the lowering of water table (C)

A.2.3. Temperatures (°C)

- \( T_{UA} \): Upper allogenic stream temperature (M)
- \( T_{NA} \): Neighbor allogenic stream temperature (M)
- \( T_P \): Pump output temperature (M)
- \( T_{CS} \): CS water temperature (C or M)
- \( T_\infty \): Far field temperature-P7 borehole temperature (M)

A.3. Equations Notations (units)

- \( \mathbf{v} \): Fluid velocity vector (m/s)
- \( q_i \): CS algebraic value (positive for incoming flow) (m³/s)
- \( q_o \): CS algebraic value (negative for outgoing flow) (m³/s)
- \( \delta E \): CS Internal Energy (J)
- \( j \): CS incoming or outgoing flows (-)
- \( h_j \): Specific enthalpy of CS flow j (J/kg)
- \( e_{pot,j} \): Potential energy of CS flow j (J/kg)
- \( e_{cin,j} \): Kinetic energy of CS flow j (J/kg)
- \( \delta \Phi_j \): CS-PFM Thermal exchanges (J)
- \( \delta W_j \): CS-PFM Work exchanges (J)
- \( \rho_j \): Water density in flow j (kg/m³)
- \( C_{pj} \): Specific thermal capacity in flow (J/kg K)
- \( T_i \): Temperature of CS incoming flow (°C or °K)
- \( T_o \): Temperature of CS incoming flow (°C or °K)

A.4. Error Assessment Notations, (Units), Values

- \( D_m \): PFM thermal diffusivity (m²/s) \( 1.42 \times 10^{-6} \)
- \( D_w \): Water thermal diffusivity (m²/s) \( 1.43 \times 10^{-7} \)
- \( v \): Water kinematic viscosity (m²/s) \( 10^{-6} \)
- \( L \): CS Length (m) \( 5 \times 10^3 \)
- \( r_h \): Half hydraulic radius (m) \( 5 \)
- \( V \): Velocity scale (m/s)
- \( D \): Thermal diffusivity ratio (\( D_m/D_w \)) (-)
- \( Pe \): Conduit Peclet number (\( LV/Dw \)) (-)
- \( Pr \): Prandtl number (\( \rho \cdot C_p \cdot V \)) (-)
- \( Red \): CS Reynolds number (\( 2 \cdot V \cdot r_h / \nu \)) (-)
- \( \delta T \): Conservative hypothesis error (°C or °K)

References


42. Patterson, C. Lead in sea water. Sciences 1974, 183, 553–558. [CrossRef] [PubMed]


44. Tunusoglu, O.; Shahwan, T.; Eroglu, A.E. Retention of aqueous Ba(2+) ions by calcite and aragonite over a wide range of concentrations: Characterization of the uptake capacity, and kinetics of sorption and precipitate formation. Geochem. J. 2007, 41, 379–389. [CrossRef]
