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Anomalous Josephson effect controlled by an Abrikosov vortex

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The possibility of a fast and precise Abrikosov vortex manipulation by a focused laser beam opens the way to create laser-driven Josephson junctions. We theoretically demonstrate that a vortex pinned in the vicinity of the Josephson junction generates an arbitrary ground state phase which can be equal not only to 0 or $\pi$ but to any desired $\phi_0$ value in between. Such $\phi_0$ junctions have many peculiar properties and may be effectively controlled by the optically driven Abrikosov vortex. Also we theoretically show that the Josephson junction with the embedded vortex can serve as an ultrafast memory cell operating at sub THz frequencies.

I. INTRODUCTION

Josephson junctions (JJs) with nonzero spontaneous phase difference between the superconducting electrodes in the ground state (the so-called $\pi$, $\phi_0$, and $\varphi$ JJs) have become the subject of intensive theoretical and experimental study during the past decade [1,2]. The most striking feature of such junctions is their ability to generate a current in the superconducting circuit, thus acting as a phase battery [3–6]. In addition, relatively small variation of the system parameters may provoke dramatic changes in the ground state phase difference, which is believed to provide new effective tools for controlling the currents in microelectronic devices (see, e.g., Refs. [7,8]).

There are several generic mechanisms leading to the spontaneous appearance of nonzero Josephson phase. The basic one reveals when the superconducting (S) electrodes are separated by a ferromagnetic (F) layer. The exchange field inside the ferromagnet produces the spatial oscillations of the Cooper pair wave function, and depending on the ratio between the oscillation period and the F-layer thickness the ground state phase is equal to 0 or $\pi$ (these cases are referred as 0 or $\pi$ JJs, respectively) [9–11]. A peculiar situation is realized when the thickness $d$ of the ferromagnet varies along the junction in a way that in some parts of the ferromagnet the value of $d$ corresponds to the 0 state while in other parts to the $\pi$ state (see Ref. [12] and references therein). If there is only a slight difference between the areas of the “0” and “$\pi$” regions the phase frustration can result in the appearance of a state with the spontaneous phase difference $\phi = \pm \varphi \neq 0,\pi$ which is degenerate ($\varphi$ JJ) (see Ref. [13] and references therein). A similar effect takes place in Josephson junctions with the current injectors acting as an effective source of the phase jumps along the junction (see Refs. [14–16] and references therein). One can also obtain a $\phi_0$ JJ, where the ground state phase $\phi = \phi_0$ is not degenerate, e.g., using the JJs with broken inversion symmetry [17]. In this case the superconducting current $I$ through the junction should not be necessarily an odd function of the phase difference $\phi$ and can take the form $I = I_c \sin(\phi + \phi_0)$ [17]. Such situation is realized in the S|F|S junctions with strong spin-orbit coupling (see Refs. [17–20] and references therein) or complicated noncollinear distribution of the magnetization (see Refs. [21–23] and references therein).

However, despite the variety of opportunities for the engineering of the JJs with any desired ground state phase, it is extremely hard to tune this phase after the system is fabricated. There are just a few suggestions concerning the realization of such tuning. For example, in S|F|S junctions the transition between 0 and $\pi$ states can occur by changing the temperature [10,24–26]. In contrast, this transition requires the presence of magnetic impurities and very precise choice of the F-layer thickness. A more realistic situation is realized in the Josephson systems containing ballistic Bi nanowires or InSb quantum dots where the interplay between strong spin-orbit and Zeeman interactions enables the formation of $\pi$ and $\phi_0$ states, which can be tuned by an external magnetic field [18,20,27].

An alternative approach for the creation of the tunable $\pi$, $\pm \varphi$, and $\phi_0$ JJs is based on the embedding of some intrinsic source of the current into one of the superconducting electrodes. Such current induces the nonuniform Josephson phase difference along the junction and thus modifies its ground state. The straightforward way to realize this scenario is to implant a pair of tiny current injectors serving as a source and drain [14,15,28,29]. In this case the Josephson phase along the junction reveals a jump with the amplitude determined by the value of the injected current. Recently it was demonstrated that this jump causes the appearance of the spontaneous ground state Josephson phase like in $\phi_0$ or $\pm \varphi$ JJs depending on the parameters [16]. However, the resulting ground state appears to be very sensitive to the position of the current injectors, and thus the creation of the junction with the desired current-phase relation requires extremely precise positioning of the injectors during the fabrication process.

Interestingly, a very similar situation is realized in Josephson junctions where one of the superconducting electrodes contains Abrikosov vortices. The superconducting current circulating around the vortex core perturbs the profile of the Josephson phase and thus affects the junction ground state.
The specific details of this phenomenon strongly depend on the system geometry and the orientation of the vortex.

In the simplest case of the planar JJ with the vortex trapped in one of the superconducting electrodes near the Josephson barrier, the vortex currents cross this barrier and induce the nonuniform phase profiles along the junction [30–33]. This results in the transition from the 0 to π ground state with the variation of the distance between the vortex and the Josephson barrier. Also the JJs with the embedded Abrikosov vortex are promising for the design of the cryogenic random access memory cells [34].

In the Josephson junctions of the “overlap” geometry that are formed by two thin superconducting films with the quasi-two-dimensional insulating layer in between, the cores of Abrikosov vortices tend to become oriented perpendicular to the plane of the junction. In this case the vortex currents flowing parallel to the plane of the junction induce two-dimensional phase profiles which can modify the Fraunhofer oscillations of the critical current and current-voltage characteristics, create nonquantized Josephson vortices, or even induce the nonzero ground state Josephson phase [35–41]. A similar situations is realized when such JJ is affected by the magnetic field of a small magnetic particle [42].

Recently it was experimentally demonstrated that the position of a single Abrikosov vortex can be controlled by the tip of the magnetic force microscope [43,44], the electron beam [41], the probe of the scanning tunneling microscope [45], or the focused laser beam [46]. The latter technique allows the ultrafast optically controlled positioning of an individual Abrikosov vortex which opens a new avenue for the design of novel optoelectronic superconducting devices. In particular, it becomes possible to realize the optical tuning of the current-phase relation of the Josephson junction provided the vortex is trapped in one of its superconducting leads.

In this paper we propose the concept of the optically controlled Josephson device and show that the Abrikosov vortex pinned near the JJ is a promising tool for the ultrafast tuning of the ground state phase. We consider a planar Josephson junction we propose the concept of the memory cell based on the optically controlled positioning of the vortex near the JJ. In Sec. V we summarize our results.

II. TUNABLE PLANAR JOSEPHSON JUNCTION OF LARGE THICKNESS

A. Model

The system under consideration is shown in Fig. 2. The Josephson junction consists of two superconducting slabs (S1 and S2) of width 2w separated by a thin insulating (I) barrier of thickness d. We choose the origin of the Cartesian axes at the center of the insulator so that the superconductor/insulator interfaces are parallel to the xz plane and correspond to \( y = \pm d/2 \) while the outer boundaries of the superconductors are parallel to the yz plane and correspond to \( x = \pm w \). An Abrikosov vortex is pinned inside the S2 superconductor at the position \((x_0, y_0)\) and its core is parallel to the \( z \) axis. Also we introduce the external magnetic field \( H_0 \) parallel to the \( z \) axis. In contrast with Refs. [33,47] we assume that the thickness \( h \) of the slabs in the \( z \) direction well exceeds the London penetration depth \( \lambda \) so that the boundary effects at \( z = \pm h/2 \) can be neglected (for simplicity we will consider an infinite sample in the \( z \) direction). We also assume that the width \( 2w \) of the junction satisfies the condition \( \lambda \ll 2w \ll \lambda_J \), where \( \lambda_J \) is the Josephson length. Finally, we assume that \( d \ll \lambda \) and neglect the variation of the magnetic field across the Josephson barrier.
If the distance $y_0$ between the Abrikosov vortex and the insulating layer is on the order of $\lambda$, the vortex strongly modifies the properties of the Josephson junction. Indeed, in this case, the superconducting current flowing around the vortex core induces a nonuniform profile of the superconducting phase along the boundary of the $S_2$ electrode. In Sec. II C we show that this phase can result in the appearance of the spontaneous Josephson phase across the JJ. For simplicity we assume that the vortex is situated several $\lambda$ away from all outer boundaries of the superconductor. That is, its currents do not interact with them, i.e., $|x_0 \pm w| \gg \lambda$.

First we derive the analog of the Ferrell-Prange equation [48] which defines the spatial profiles of the Josephson phase $\phi(x)$ along the junction. In the $S_2$ superconductor the current density $j$ is the sum of the current density $J_{sc}$, which screens the external magnetic field penetrating into the superconducting electrodes, and the vortex-induced current $j_v$:

$$j = \frac{1}{\mu_0 \lambda^2} \left( \frac{\Phi_0}{2\pi} \nabla \phi - A \right) = j_{sc} + j_v,$$  

where $\phi(x,y)$ is the local phase of the superconducting gap function, $A$ is the vector potential, and $\Phi_0 \approx 2.07$ fwb is the magnetic flux quantum. In the vortex-free $S_1$ electrode, $j_v = 0$ and the current is determined only by the external magnetic field. Note that the expression (1) contains the density through the junction.

Now let us analyze the behavior of the Josephson phase across the full electrodynamic problem. Indeed, far from the insulating layer the current density can result in the appearance of the spontaneous Josephson phase $\phi(x)$ along the junction. For simplicity we assume that $j_v = j_s \sin \phi$, which can be neglected.

Here we assume that the external magnetic field $H_0$ is weak so that $\mu_0 H_0 \ll \Phi_0/(\lambda_j \lambda)$. In the opposite limit the variation of the magnetic field $H_0$ along the junction scales by the length $\Phi_0/(2\mu_0 H_0 \lambda)$ instead of $\lambda_j$.

The solution of the London equation inside the $S_1$ and $S_2$ electrodes allows us to calculate $\partial H_0/\partial y|_{y=\pm d/2} = -H_0 \delta \text{sgn}(y)/\lambda$ and then Eq. (3) transforms into

$$\frac{\Phi_0}{2\pi} \frac{\partial \phi}{\partial x} = -H_0 \delta(d + 2\lambda).$$  

Finally, taking the derivative of Eq. (4) over $x$ and taking into account that $\partial H_0/\partial x = -\mu_0 j_s$ with the Josephson current density through the junction $j_s = j_s \sin \phi$ (where $j_s$ is the critical current density) we obtain the Ferrell-Prange equation in the usual form:

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{1}{\lambda_j^2} \sin \phi,$$  

where $\lambda_j = \sqrt{\frac{\Phi_0}{2\pi \mu_0 j_s \lambda}}$ is the Josephson length and $\delta = (d + 2\lambda)$ is the thickness of the insulating layer renormalized by the screening currents in the superconductors.

Equation (5) should be supplemented by the appropriate boundary conditions at $x = \pm w$. Using Eq. (4) we find

$$\frac{\partial \phi}{\partial x} \bigg|_{x=\pm w} = -\frac{2\pi \delta}{\Phi_0} H_0(\pm w).$$  

It is important to note that the magnetic field $H_0(\pm w)$ is the sum of the external field $H_0$ and the field $H_J$ induced by the Josephson current $j_s$ flowing through the junction. It is easy to demonstrate that this self-induced field $H_J$ has the same value and the opposite direction at the different sides of the junction. Indeed, far from the insulating layer the current flowing in the $y$ direction is localized in the thin layers of the thickness $\lambda$ near the planes $x = \pm w$ while near $y = 0$ it spreads over the whole weak link being transformed into the Josephson current. To calculate $H_J$ it is convenient to reproduce the current distribution in the sample as the sum of the uniform current layers near $x = \pm w$ and the closed current loops localized near the insulating layer. The latter current does not contribute to $H_J$. Thus, the field $H_J$ is effectively induced by two uniform current layers carrying the total current per unit length $I_s = \int_{-w}^{w} j_s(x)dx$: $H_J(\pm w) = \mp \mu_0 I_s/2$. Of course, the derivation of this condition is based on the assumption that the system is infinite in the $z$ direction. For the junction of a finite thickness $t$ the coefficient in the relation $H_J(\pm w) \propto I_s$ should be calculated from the solution of the full electrodynamic problem.

Now let us analyze the behavior of the Josephson phase in the region $x_0 - \delta < x < x_0 + \delta$ near the position $x_0$ of the vortex. Rewriting Eq. (1) for the two points $3$ and $4$ inside the $S_2$ superconductor, we obtain the analog of Eq. (4) but with the additional vortex contribution on the right-hand side:

$$\frac{\Phi_0}{2\pi} \frac{\partial \phi}{\partial x} = -H_0 \delta(d + \mu_0 \lambda^2 j_{sc,x}).$$  

Note that the expression (1) contains the density through the junction $\phi(x)$. For simplicity we assume that $j_v = j_s \sin \phi$, which can be neglected. Hereafter we assume that the external magnetic field $H_0$ is weak so that $\mu_0 H_0 \ll \Phi_0/(\lambda_j \lambda)$. In the opposite
Our goal is to match the solutions of Eq. (5) at different sides of this region with the boundary conditions for \( \phi \). First, since \( \delta \gg \lambda \), the vortex current \( j_{v,x} \) and the corresponding magnetic field at \( x = x_0 \pm \delta \) are negligibly small. Then at \( x = x_0 \pm \delta \) the magnetic field in Eq. (8) does not contain the vortex contribution and, thus, varies over distances much larger than \( \lambda \) so that \( H_{c2}(x_0 - \delta) \approx H_{c2}(x_0 + \delta) \). Then one gets the boundary condition matching the derivatives of the Josephson phase:

\[
\frac{\partial \phi}{\partial x} \bigg|_{x_0 + \delta} = \frac{\partial \phi}{\partial x} \bigg|_{x_0 - \delta}.
\]

(9)

Second, integrating Eq. (8) over the interval \( x_0 - \delta < x < x_0 + \delta \) and neglecting the term \( H_{c2}d\delta \) since \( \delta \ll \omega \) we find

\[
\phi(x_0 + \delta) - \phi(x_0 - \delta) = \kappa,
\]

(10)

where

\[
\kappa = \frac{2\pi \mu_0 \lambda^2}{\Phi_0} \int_{x_0 - \delta}^{x_0 + \delta} j_{c,x} dx.
\]

(11)

Note that in Eq. (11) we may put \( \delta \to \infty \) since \( \delta \) is much larger than the scale of the vortex current \( \lambda \). Then one sees that the phase jump \( \kappa \) does not depend on \( x_0 \) and is determined only by the distance \( y_0 \) between the vortex and the insulating-layer center. To calculate this phase jump let us in the first approximation neglect the small electron transparency of the layer center. Thus, the voltage drop caused by the Josephson current in the insulating layer can be calculated using the image technique. Note that the vortex does not create the stray magnetic field in the S1 electrode and thus does not induce the current there. To account for the S2 boundary, where the currents flow only in \( x \) direction, we consider a superconducting film without this boundary containing not only a vortex at \((x_0, y_0)\) but also an antivortex at \((x_0, -y_0)\). Then we find that the supercurrent flowing at \( y = +d/2 \) (along the edge of the S2 in original problem) is

\[
j_{c,x}(x) = \frac{\Phi_0}{2\pi \mu_0 \lambda^2} \frac{y_0}{\lambda} K_1\left(\frac{r}{\lambda}\right),
\]

(12)

where \( r = \sqrt{y_0^2 + (x - x_0)^2} \) is the distance between the point \((x, 0)\) at the S2-I interface and the vortex center. Substituting Eq. (12) into Eq. (11) we find

\[
\kappa(y_0) = \frac{2\pi y_0}{\lambda} \int_{-\infty}^{+\infty} \frac{K_1\left(\frac{y_0\sqrt{1 + t^2}}{\lambda}\right)}{\sqrt{1 + t^2}} dt.
\]

(13)

The resulting dependence \( \kappa(y_0) \) is shown in Fig. 3. If the vortex is far from the junction \( y_0 \gg 4\lambda \), then the phase jump \( \kappa \to 0 \). If the vortex is near the S2-I interface \( y_0 \approx 0 \), then \( \kappa \to 2\pi \). Note that in our case the phase jump of \( \pi \) occurs at \( y_0 \approx 0.8\lambda \), while in the case of the Pearls vortex in the ultrathin film the corresponding distance is \( \lambda \). Thus, the spatial profile of the Josephson phase \( \phi(x) \) is determined by Eq. (5) with the boundary conditions (7)–(10). Substituting this phase profile into the expression for the Josephson current and integrating over the junction length \( 2w \) one obtains the current-phase relation:

\[
I_y = \int_{-w}^{+w} j_{c,x} \sin(\phi(x)) dx.
\]

(14)

In the ground state the superconducting current \( I_y \) is zero. Among the profiles \( \phi(x) \) satisfying this condition the ground state corresponds to the one with the minimal free energy

\[
F = \varepsilon_j \int_{-w}^{+w} \left[ \frac{\lambda^2}{2} \left( \frac{\partial \phi}{\partial x} \right)^2 + (1 - \cos \phi) \right] dx,
\]

(15)

where \( \varepsilon_j = j_{c,0}\Phi_0/(2\pi) \) is the Josephson energy per unit of length.

**B. Asymmetric magnetic oscillations of the critical current**

In this section we demonstrate that for the JJ with thick superconducting electrodes \( t \gg \lambda \) an Abrikosov vortex strongly affects the dependence of the Josephson critical current \( I_c \) on the external magnetic field \( H_0 \) producing the asymmetry of the \( I_c(H_0) \) pattern with respect to \( H_0 \to -H_0 \). Note that previously a similar phenomenon was found for the junctions with \( t \ll \lambda \) [47].

In the limit \( w \ll \lambda \), the expression for the superconducting phase profile \( \phi(x) \) along the junction can be expanded over the small parameter \( w/\lambda \). Our strategy is to find \( \phi(x) \) in the lowest approximation of the perturbation theory neglecting the corrections \( \sim O(w^2/\lambda^2) \) which come from the phase renormalization due to the Josephson current. The resulting profile \( \phi(x) \) then determines the total current \( I_y \) flowing through the junction:

\[
I_y = j_{c,0} \int_{-w}^{+w} \sin(\phi(x)) dx.
\]

The solution of the Ferrell-Prange equation (5) satisfying the boundary conditions with the accuracy \( \sim O(w/\lambda) \) reads

\[
\phi(x) = \begin{cases} 
\phi_0 - \kappa/2 - h(x - x_0)/w & \text{for } x < x_0, \\
\phi_0 + \kappa/2 - h(x - x_0)/w & \text{for } x > x_0,
\end{cases}
\]

(16)

where \( \phi_0 \) is the average phase in the vicinity of the point \( x = x_0 \) and \( h = 2\pi \mu_0 \Phi_0 / w \) is the dimensionless external magnetic field. Integrating the resulting Josephson current density over the width of the junction we obtain the averaged...
that the asymmetry of the dependencies
different spontaneous Josephson phase profiles. To find the
vortex can cause phase transitions between the states with
current reads
\( \lambda \). The corresponding dependencies for different
\( \kappa = 0, \pi, \pi/2 \), \( \kappa = 3\pi/2 \) are asymmetric with respect to
\( x \) at different distances
\( y_0 \). The Josephson phase jump
\( \kappa \) corresponding to \( y_0 \) is indicated in each panel.

\[ I_x = \frac{I_c}{h} \sin \phi_0 \left[ \sin \left( \frac{K}{2} \right) - \cos(hX) \sin \left( \frac{K}{2} - h \right) \right] \]
\[ - \frac{I_c}{h} \cos \phi_0 \sin(hX) \sin \left( \frac{K}{2} - h \right), \]

where \( X_0 = x_0/w \) and \( I_{c0} = 2j,w \). The corresponding critical

\[ I_x(h,\kappa,X_0) = \frac{I_{c0}}{h} \left[ 1 - \cos(hX_0) \cos(h) \right. \]
\[ \left. + \cos(\kappa - h) \left( \cos(hX_0) - \cos(h) \right) \right]^{1/2}. \]

The expression (18) clearly shows that the critical current satis-
ies the symmetry relations \( I_x(-h,\kappa,X_0) = I_x(h,-\kappa,X_0) \) and
\( I_x(h,\kappa,-X_0) = I_x(h,\kappa,X_0) \). Thus, for \( \kappa \neq 0,\pi \), the Fraun-
hofer dependencies \( I_x(h) \) are symmetric with respect to
\( h \rightarrow -h \). In particular, if the vortex is pinned at the central
line of the junction (\( X_0 = 0 \)) the critical current reads
\[ I_x(h,\kappa,0) = \frac{I_{c0}}{h} \frac{2}{(h)} \cos \left( \frac{h - K}{2} \right). \]

The corresponding dependencies for different \( \kappa \) are shown in
Fig. 4.

Note that at \( h = 0 \) the expression for the critical current reduces to \( I_x^{sym}(0,\kappa,0) = I_{c0} \cos(\kappa/2) \).

Comparing our results with the ones in Ref. [47] one sees that
the asymmetry of the dependencies \( I_x(H_0) \) is a generic
phenomenon revealing itself for arbitrary ratios between the
thickness of the superconducting electrodes \( t \) and the London
penetration depth \( \lambda \).

C. Phase transitions due to the shifting of the vortex

In this section we demonstrate that shifting of the Abrikosov
vortex can cause phase transitions between the states with
different spontaneous Josephson phase profiles. To find the
profile \( \phi(x) \) corresponding to the minimum of the free energy

\[ \phi(x) = \left\{ \begin{array}{ll}
\sum_{n=0}^{2} C_{n}^+ \left( \frac{x-x_0}{w} \right)^n & \text{for } x < x_0, \\
\sum_{n=0}^{2} C_{n}^- \left( \frac{x-x_0}{w} \right)^n & \text{for } x > x_0,
\end{array} \right. \]

where the coefficients \( C_{n}^+ = \phi_0 \pm \kappa/2 \) can be written right
away, while the other \( C_{n}^- \) should be determined from boundary
conditions. Substituting (20) into the Ferrell-Prange equation (5),
expanding the sine on the right-hand side up to \( O(w^2/\lambda J^2) \),
and collecting the terms in each order of the perturbation theory
separately, we find

\[ C_2^\pm = \frac{1}{2} \sin \left( \phi_0 \pm \frac{\kappa}{2} \right). \]

The boundary conditions (7)–(9) give the other two equations:

\[ C_1^+ = C_1^-, \]
\[ C_1^- - 2C_2^- \frac{w + x_0}{\lambda J} = -C_1^+ - 2C_2^+ \frac{w - x_0}{\lambda J}. \]

Writing Eq. (23) we take into account that in the absence of
the external magnetic field \( H_0(\pm w) = -H_0(\pm w) \). Obviously
in the ground state both \( H_0(\pm w) = 0 \) due to the absence of the
current through the junction. Thus,

\[ C_1^\pm = \frac{x_0}{\lambda j} \cos \left( \frac{K}{2} \right) - \frac{w}{\lambda j} \sin \left( \frac{K}{2} \right). \]

The expressions (21) and (24) define the Josephson phase
profile \( \phi(x) \) which now depends only on \( \phi_0 \). To find \( \phi_0 \) we
take into account that in the ground state the total Josephson
current \( I_y = 0 \). Substituting \( \phi(x) \) into Eq. (14) with \( I_y = 0 \) we
obtain the following equation:

\[ \cos \left( \frac{K}{2} \right) \sin \phi_0 - X_0 \sin \left( \frac{K}{2} \right) \cos \phi_0 \]
\[ + \frac{w^2}{12\lambda J^2} \left[ (1 - 3X_0^2 + 3X_0^2) \sin^2 \left( \frac{K}{2} \right) \right] \sin(2\phi_0) \]
\[ + X_0 \cos \kappa (3 + X_0^2) \cos \phi_0 (2X_0^2) \right) = 0, \]

where again we have introduced the dimensionless parameter
\( X_0 = x_0/w \).

The free energy is

\[ F(\phi_0) = F_0 + 2w_{eff} f(\phi_0), \]

where \( F_0 \) does not depend on the phase \( \phi_0 \), and the dimension-
less function \( f(\phi_0) \) is

\[ f(\phi_0) = -\cos \phi_0 \cos \left( \frac{K}{2} \right) - X \sin \phi_0 \sin \left( \frac{K}{2} \right) + \frac{w^2}{24\lambda J^2} \]
\[ \times \left( 2X^3 \sin(2\phi_0) \sin \kappa - 3(1 - X^2) \cos(2\phi_0) \right) \]
\[ - (1 - 3X^2) \cos(2\phi_0) \cos \kappa. \]
Before proceeding with the general analysis of Eqs. (25) and (26) let us consider two limiting cases (i) $X_0 = 0$ and (ii) $\kappa = \pi$ illustrating the general features of the Josephson phase behavior.

1. Phase jump equal to $\pi$

When the Abrikosov vortex produces the phase jump $\kappa = \pi$ (i.e., $\gamma_0 \approx 0.8\pi$) the situation formally coincides with the one for the S/F/S junction with the steplike thickness of the F layer [12]. In this case Eq. (25) for the phase $\phi_0$ transforms into

$$-X_0 \cos \phi_0 + \frac{w^2}{6\lambda J} \sin(2\phi_0) = 0. \quad (28)$$

For $|X_0| > w^2/3\lambda J^2$ this equation has the series of solutions $\phi_0 = \pm \pi/2 + 2\pi n$ ($n$ is an integer number) while in the narrow region $|X_0| < w^2/3\lambda J^2$ the additional solutions exist: $\phi_0 = \alpha + 2\pi n$ and $\phi_0 = \pi - \alpha + 2\pi n$ with $\alpha = \arcsin(3X_0\lambda J^2/w^2)$.

The function $f(\phi_0)$ which defines the free energy (26) takes the form

$$f(\phi_0) = -X_0 \sin \phi_0 - \frac{w^2}{12\lambda J^2} \cos(2\phi_0). \quad (29)$$

One can check that $f(\pm \pi/2) = \mp X_0 + w^2/12\lambda J^2$ and $f(\alpha) = f(\pi - \alpha) = -3X_0\lambda J^2/2w^2 - w^2/12\lambda J^2$. Thus, in the region $|X_0| > w^2/3\lambda J^2$ the equilibrium phase is $\phi_0 = (\pi/2)\text{sgn}(X_0) + 2\pi n$ while for $|X_0| < w^2/3\lambda J^2$ the minimum of the free energy corresponds to the degenerated states $\phi_0 = \alpha + 2\pi n$ and $\phi_0 = \pi - \alpha + 2\pi n$. The dependencies $\phi_0(X_0)$ near the transition point are shown in Fig. 5. For convenience for each phase we choose $n$ in a way that $0 < \phi_0 < 2\pi$.

Clearly, the vortex motion along the junction results in the series of two type-II phase transitions at $X_0 = \pm w^2/3\lambda J^2$ with continuously changing $\phi_0$ and the jump in the second derivative of the free energy over $X_0$. This situation is very similar to the formation of the $\phi$ JJ in the stacked S[F]S systems [13]. However, in contrast with the S[F]S JJs, the Abrikosov vortex enables the real-time tuning of the equilibrium phase.

2. Vortex at the central line of the junction

Now let us turn to the limiting case when the Abrikosov vortex is placed at the central line of the junction where $X_0 = 0$. In this case Eq. (25) takes the form

$$\cos \left(\frac{\kappa}{2}\right) \sin \phi_0 + \frac{w^2}{24\lambda J^2} (3 - \cos \kappa) \sin(2\phi_0) = 0. \quad (30)$$

The number of the solutions depends on the phase jump $\kappa$. For $|\kappa - \pi| > 2w^2/3\lambda J^2$ there are only two series of the solutions $\phi_0 = 0 + 2\pi n$ and $\phi_0 = \pi + 2\pi n$ while for $|\kappa - \pi| < 2w^2/3\lambda J^2$ two additional series $\phi_0 = \pm \beta + 2\pi n$ appear, where

$$\beta = \arccos \left[ \frac{-12\lambda J^2 \sin \left(\frac{\kappa}{2}\right)}{w^2 (3 - \cos \kappa)} \right]. \quad (31)$$

To simplify this expression it is convenient to introduce the deviation $\delta\kappa$ of the phase jump from $\pi$: $\kappa = \pi + \delta\kappa$. Since $|\delta\kappa| \ll 1$ one obtains $\beta \approx \arcsin(3X_0^2/2w^2\delta\kappa)$.

The function $f(\phi_0)$ which defines the system free energy [see Eq. (27)] is

$$f(\phi_0) = \cos \phi_0 \sin \left(\frac{\delta\kappa}{2}\right) - \frac{w^2}{24\lambda J^2} (3 - \cos \delta\kappa) \cos(2\phi_0). \quad (32)$$

Expanding Eq. (32) over $\delta\kappa$ up to $O(\delta\kappa^2)$ and at the same time neglecting the terms $\sim O(w/\lambda J)^4$ we find

$$f(0) = \frac{\delta\kappa}{2} - \frac{w^2}{12\lambda J^2}, \quad f(\pi) = -\frac{\delta\kappa}{2} - \frac{w^2}{12\lambda J^2}, \quad f(\pm \beta) = \frac{3X^2}{8w^2} \delta\kappa^2 + \frac{w^2}{12\lambda J^2}. \quad (33)$$

Thus, the ground state phase is $\phi_0 = 0 + 2\pi n$ for $\kappa < \pi$ and $\phi_0 = \pi + 2\pi n$ for $\kappa > \pi$ while the states with $\phi_0 = \pm \beta + 2\pi n$ are not favorable at any phase jumps $\kappa$. As a result, the vortex motion along the central line of the junction (with $X \equiv 0$) causes the type-I phase transition accompanied by the discontinuous changing of the $\phi(x)$ profile and the jump in the first derivative of the free energy over $\delta\kappa$ [see Eq. (33)].

3. Arbitrary position of the vortex

In Secs. II C 2 and II C 3 it is shown that the type of the phase transition provoked by the vortex motion strongly depends on the direction of this motion. Here we analyze the general case of the arbitrary position of the vortex. To do this we numerically find all solutions of Eq. (25) and then choose the one which corresponds to the minimal energy (26). The results are presented in Figs. 6 and 7. Interestingly, the behavior of the value of $\phi_0$ in the ground state for different parameters $X_0$ and $\kappa$ is well reproduced by the simple estimate presented below.

One can check that since $|X_0| \leq 1$ the expression inside the braces in Eq. (25) clearly satisfies the inequality $|\ldots| \leq 12$. Thus, outside the narrow parameter region defined by the two simultaneous conditions $|X_0| < w^2/\lambda J^2$ and $|\kappa - \pi| < w^2/\lambda J^2$ the terms $\propto w^2/\lambda J^2$ in Eq. (25) are not sufficient and can be neglected. Then in the ground state the phase $\phi_0$ takes the value...
\[ \phi_0 = \tilde{\phi}_0, \ \text{where} \ \sin \tilde{\phi}_0 = \rho^{-1} X_0 \sin \left( \frac{\phi}{2} \right), \ \cos \tilde{\phi}_0 = \rho^{-1} \cos \left( \frac{\phi}{2} \right), \]

\[ \rho = \sqrt{\cos^2 \left( \frac{\phi}{2} \right) + X_0^2 \sin^2 \left( \frac{\phi}{2} \right)}. \]

In contrast, for \( |X_0| < w^2/\lambda_J^2 \) and \( |\kappa - \pi| < w^2/\lambda_J^2 \) the situation is more rich. In this parameter region we can expand Eq. (25) up to the terms \( -O(w^2/\lambda_J^2) \). Then one obtains the equation

\[ -\frac{\delta \kappa}{2} \sin \phi_0 - X_0 \cos \phi_0 + \frac{w^2}{6\lambda_J^2} \sin(2\phi_0) = 0, \quad (35) \]

where \( \delta \kappa = \kappa - \pi \).

First let us analyze the dependencies of the value of \( \phi_0 \) in the ground state as a function of \( X_0 \) for different \( \delta \kappa \neq 0 \). Depending on \( X_0 \), Eq. (35) has two or four solutions for \( \phi_0 \). To classify these solutions it is convenient to consider the function

\[ X_0(\phi_0) = \frac{w^2}{3\lambda_J^2} \sin \phi_0 - \frac{w^2}{2\lambda_J^2} \tan \phi_0 \]

which is uniquely defined.

The numerical analysis shows that the minimum of the free energy for the fixed \( X_0 \) corresponds to the monotonically increasing branch \( X_0(\phi_0) \) at the interval \( \pi/2 < \phi_0 < 3\pi/2 \) for \( \delta \kappa > 0 \) or at the union of the intervals \( 3\pi/2 < \phi_0 < 2\pi \) and \( 0 < \phi_0 < \pi/2 \). Also for each sign of \( \delta \kappa \) there is another branch of the function \( X_0(\phi_0) \) which determines one metastable \( \phi_0 \) for the fixed \( X_0 \) in the case \( |\delta \kappa| > w^2/3\lambda_J^2 \) and three metastable solutions in the case \( |\delta \kappa| < w^2/3\lambda_J^2 \) (see Fig. 6). Note that in the ground state the dependencies of \( \phi_0 \) and the free energy on \( X_0 \) are continuous which indicates the absence of the phase transitions for \( \delta \kappa \neq 0 \).

In contrast, the dependencies \( \phi_0(\delta \kappa) \) for the fixed \( X_0 \) demonstrate the type-I phase transition in the case when \( |X_0| < w^2/2\lambda_J^2 \). Indeed, Eq. (35) can be rewritten in the form

\[ \delta \kappa = \frac{w^2}{3\lambda_J^2} \cos \phi_0 - 2X_0 \csc \phi_0. \]

The values \( \phi_0 \) corresponding to the energy minimum belong to the interval \( 0 < \phi_0 < \pi \).
FIG. 8. The $X_{\kappa}-\kappa$ diagram showing the line of the type-I phase transition ending with two critical points (red straight curve). The blue dashed curves show the possible trajectories of the Abrikosov vortices corresponding to different type of the phase transitions (PT).

For $X_{0} > 0$ and $\pi < \phi_{0} < 2\pi$ for $X_{0} < 0$. For $|X_{0}| > w^{2}/2\lambda_{J}^{2}$ the stable branch of the function $\delta\kappa(\phi_{0})$ is monotonic, and thus the inverse dependence of the ground state phase $\phi_{0}$ vs $\delta\kappa$ is uniquely defined. However for $|X_{0}| < w^{2}/2\lambda_{J}^{2}$ the function $\delta\kappa(\phi_{0})$ becomes nonmonotonic and the ground state phase reveals a jump at $\delta\kappa = 0$ indicating the type-I phase transition (see Fig. 7).

Thus, on the parameter plane $X_{\kappa}-\kappa$ there is the segment $|X_{0}| < w^{2}/2\lambda_{J}^{2}$ at the line $\kappa = \pi$ separating the phases with different $\phi_{0}$ in the ground state corresponding to the points of the type-I phase transition (see Fig. 8). The ends of this segment are the critical points. Thus, the vortex motion along the plane of the Josephson junction can cause either the jump of the ground state phase $\phi_{0}$ or the continuous changing of $\phi_{0}$ depending on whether the vortex crosses the segment corresponding to the phase transition or not.

Note that the requirement $t \gg \lambda$ which was assumed throughout this section permits us to obtain the full analytical solution. However, qualitatively the formation of $\phi_{0}$ junctions should remain possible also in the intermediate regime $t \sim \lambda$ which is realized in typical experiments on the optical manipulation of vortices [46].

III. THIN PLANAR JOSEPHSON JUNCTION

The rich variety of possible phase transitions discussed in Sec. II opens a way for the optical sculpturing of the ground state of the JJ by performing controllable jumps of the vortex between several points. However, in such experiments the absorbed laser power needed for one jump can be substantially decreased by making the thickness of the superconducting electrodes on the order of $\lambda$ or even less. In this case the description of the nonlinear phenomena responsible for the emergence of the $\phi_{0}$ state requires sophisticated numerical calculations which are on the way and will be published elsewhere. In Sec. IV we will focus on the linear phenomena which allow the realization of promising devices for the superconducting electronics based on the JJ with $t \lesssim \lambda$. To simplify the calculations we will restrict ourselves for the limit $t \ll \lambda$ keeping in mind that the obtained results should qualitatively remain valid also in the intermediate regime $t \lesssim \lambda$. If the thickness $t \ll \lambda$, then the effective vortex size is $\lambda_{P} = \lambda^{2}/t$—the so-called Pearl length. The extreme case when $\lambda_{P} > 2w$ was studied in several works [33,47,50] and allows for the analytical solution. Therefore we do not rederive these case here, but rather review the key results in this section.

The phase distribution induced by a single vortex was derived in Refs. [33,47,50]. The phase along the JJ length is given by [50] (in our notations)

$$\phi(x) = -2 \arctan \left( -\sin \left( \frac{\pi x}{2w} \right) + \cos \left( \frac{\pi y_{0}}{2w} \right) \sin \left( \frac{\pi x_{0}}{2w} \right) \right).$$

(36)

It turns out that the vortex close to the JJ induces almost a steplike phase profile, with the step height (discontinuity of the Josephson phase) approaching $2\pi$ when $y_{0} \to 0$. As $y_{0}$ increases, the steplike profile of the induced phase smears. At the distances $y_{0} \geq 2w$ the vortex induces just a constant ($\phi$ independent) phase at the junction, which, however, depends on $x_{0}$, namely [50] $\phi = \pi x_{0}/w$. By changing the vortex coordinates $(x_{0}, y_{0})$, one can provoke the transitions between different junction ground states.

Further, the effect of the vortex (induced phase) on the critical current pattern $I_{c}(H)$ was calculated [47]. It was found that the vortex situated along the symmetry line $x_{0} = 0$ may result in vanishing $I_{c}(0)$ if $y_{0} \approx 0.350w$. This effect is similar to zero $I_{c}(0)$ in the 0-π JJ; however the phase profile is different from steplike. It is important to mention that $y_{0}$ corresponding to $I_{c}(0) = 0$ is on the order of $w$ in the case of a thin film, and $y_{0} \sim \lambda$ in the case of a thick film. Thus, it is much easier to control the JJ by moving the vortex in the thin film.

IV. OPTICALLY CONTROLLED 0-π TRANSITIONS IN THIN PLANAR JOSEPHSON JUNCTIONS

In this section we propose the concept of an optoelectronic superconducting device which exploits the possibility to realize the ultrafast switching of the critical current of the Josephson junction by the controllable displacement of the vortex. We consider the system shown in Fig. 9. The planar Josephson junction of the thickness $t \ll \lambda$ is embedded into a superconducting strip. The length $2w$ of the junction is assumed to be smaller or on the order of the effective Pearl length $\lambda_{P} = \lambda^{2}/t$ [52,53]. Previously, it was shown that in this case the vortex trapped inside the superconducting lead strongly influences the critical Josephson current $I_{c}$ [32,33]. Specifically, the shifting of the vortex along the central line of the strip (the $y$-axis) at the distances of the order of $w$ away from the junction results in the 0-π transition which is revealed through the vanishing of $I_{c}$.

The strong sensitivity of $I_{c}$ to the vortex position allows us to use an optically controlled Josephson junction as an ultrafast optoelectronic superconducting memory cell where the information is stored by the vortex position. Specifically, the switching between the states “0” and “1” can be realized as the vortex displacement between two artificial pinning
The position and strength of the pinning centers and the described memory cell and highlight the main physical factors which have a strong influence on its working regimes. The position and strength of the pinning centers and the geometry of the superconducting leads should be chosen in a way which allows us to perform guaranteed laser-controlled optical switching of the critical current by the controllable vortex trapping the vortex. The laser beam focused at the nonoccupied pinning center creates an attractive thermal force acting on the vortex and provokes the jump of the vortex from one pinning center to another.

The most important factor affecting the properties of the suggested device is the pinning force $F_p$ which can efficiently trap the vortex. The laser beam focused at the nonoccupied pinning center creates an attractive thermal force acting on the vortex and provokes the jump of the vortex from one pinning center to another.

Let us calculate the force $F_{\text{att}}(y)$ attracting the vortex to the square boundaries as a function of the vortex displacement $y_v$ from the insulating layer assuming zero displacement from the junction central line in the direction perpendicular to the $y$ axis (in this case the force $F_{\text{att}}$ is also directed along the $y$ axis). To do this we need to solve the London equation for the magnetic field $H_z$ induced by the vortex currents with the boundary conditions requiring the absence of the normal component of the current at the $S_2$-electrode edges. Then the projection of the attraction force on the $y$ axis reads [53]

\[
F_{\text{att}} = -t \Phi_0 \frac{\partial H_z}{\partial y} \bigg|_{x=0,y=y_v}.
\] (37)

In the limit $t \ll \lambda$ the problem can be substantially simplified by neglecting the self-magnetic-fields generated by the vortex and the screening currents [54,55]. In this case the field $H_z$ satisfies the Poisson equation which can be solved with the help of the electrostatic analogy [56,57]. According to this analogy, the field $H_z$ plays the role of the electrostatic potential induced by the electrostatic charged wire with the linear charge density $Q = e_0 \Phi_0/(\mu_0 \lambda^2)$ and the boundary conditions at the sample edges can be fulfilled by introducing the set of the wire images which make the potential along the boundaries constant (in a full analogy with the well-known image method in electrostatics). Then the “potential” (magnetic field) at the point corresponding to the vortex center is the sum of the potentials created by all the “wire” images (see Fig. 10). At the same time, only the images whose distance from the real vortex changes with the variation of $y_v$ make nonzero contribution into the attraction force (they are filled in Fig. 10), while the relative position of all other images does not depend on $y_v$ and, thus, they can be excluded from further calculations. The coordinates of the contributing images have the form $(x_n, y_m)$ where $y_m = 2(2m+1)w - (y_v - w)$ and $x_n = 2(2n+1)w$ for the images with the charge $Q$ and $x_n = 4nw$ for the ones with the opposite charge $-Q$; $n$ and $m$ are arbitrary integers ($-\infty < n, m < +\infty$).
the system temperature \( T \) at the point of the vortex, and the depinning occurs when

\[
\Phi_0 \frac{\partial H_c}{\partial T}(\nabla T)_v \geq f_p.
\]

For \( f_p \approx 10 \text{ pN/\mu m} \) one finds that the condition (42) is satisfied for \((\nabla T)_v \approx 2 \text{ K/\mu m}\). Modeling the temperature profile in the superconductor under the influence of the laser beam in the spirit of Ref. [46] we obtain that to provoke the vortex motion at a distance \( \Delta y \approx 600 \text{ nm} \) from the initial position one needs a beam of the power \( W \approx 500 \mu \text{W} \) (we took the absorption coefficient equal to 50%).

The optically controlled vortex motion between the points \( y = (1 \pm 0.2)w \) should produce the variation of the Josephson critical current which exceeds 15%, namely, from 0.9\( I_c \) to 0.73\( I_c \) (see Fig. 12). This change may be easily detected on transport experiments allowing us to realize the effective readout procedure in the described memory cell. The numerical analysis of the heat transport equation with the parameters relevant to Nb and the substrates used in Ref. [46] shows that the characteristic switching time between the different vortex positions in the memory cell is on the order of \( 10 \text{ ps} \) which gives the energy per operation \( E \approx 3 \times 10^{-15} \text{ J} \).
Thus, the Josephson junction with the embedded vortex is a promising system which can serve as a basic optically controlled memory cell working at the subterahertz frequencies.

V. CONCLUSION

To sum up, we demonstrate that an Abrikosov vortex pinned near the Josephson junction produces the anomalous Josephson effect revealing itself through the emergence of the arbitrary Josephson phase in the ground state. Changing the vortex position along the specific trajectories, e.g., with the help of the focused laser beam, one can produce controllable phase transitions between the states with different Josephson phase values. In particular, for planar junctions of the thickness of the focused laser beam, one can produce controllable phase transitions between the states or the vortex-induced 0–π transitions seem to be a promising tool for the design of the ultrafast optically controlled memory cells for the cryogenic electronics where the information is encoded in the position of the vortex, the writing is performed by the laser, and the readout is realized on the basis of the Josephson current measurement. Note that in spite of the assumptions about the ratio $t/\lambda$ which allow us to simplify the analytical analysis in different limiting cases all discovered phenomena should be generic and remain qualitatively the same for arbitrary relations between $t$ and $\lambda$. Thus, we hope that our results can be directly verified in existing experimental setups and will stimulate future progress in the domain of optofluxonics.

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