Robust observer-based H_∞ stabilization for a class of switched discrete-time linear systems with parameter uncertainties

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Robust Observer-Based $\mathcal{H}_\infty$ Stabilization of Switched Discrete-Time Linear Systems with Parameter Uncertainties

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***Abstract*— This paper presents a robust observer-based $\mathcal{H}_\infty$ controller design method via LMIs for a class of switched discrete-time linear systems with l_2-bounded disturbances and parameter uncertainties. The main contribution of this paper consists in a new and judicious use of the slack variables coming from Finsler's lemma. We show analytically how the proposed slack variables allow to eliminate some bilinear matrix coupling. The effectiveness of the proposed design methodology is shown through a numerical example.**

***Index Terms*— Observer-based control; Linear matrix Inequalities (LMIs); Switched Lyapunov Functions (SLF); Finsler's lemma.**

I. Introduction and Preliminaries

A. Introduction

Hybrid and Switched systems may be encountered in several engineering applications [1], [2]. Among them, we may cite the control of motor engine [3] and networked systems [4]. Stability issues of such switching processes have been the subject of growing interest in the last decades. An overview of some basic problems related to that are summarized in [1].

Usually, the considered switched systems, in the literature, consist of linear subsystems or first-order nonlinear subsystems. However, unfortunately up to now, no complex dynamics such as stochastic noises and unknown uncertainties have been taken into account. In addition to that, plenty of industrial systems cannot be described by simple switched system models. Hence, traditional control synthesis methods are no longer applicable for such systems. In this context, we target, in this paper, the study of a class of switching linear discrete-time systems affected by unknown disturbances. More precisely, we are interested in $\mathcal{H}_\infty$ observer-based controller problem in the synchronous switching case, using LMI approach. Control techniques by switching among different controllers have been applied extensively in recent years ([5], [6], [7], [2]). However, in this case, a fundamental pre-requisite for the design of feedback control systems is full knowledge of the state that may be impossible or costly. This obstacle motivated the researchers to investigate the problem of estimating the state of switching systems by different observer structures [8], [6], [9], [10]. Moreover, it is always desirable to design a control system which is not only stable, but also which guarantees an adequate level of performance. This is the reason why control systems design that can handle model uncertainties has been one of the most challenging problems, and has received considerable attention from control engineers and scientists [11], [12], [13]. Indeed, such a problem remains far from being solved especially when switched systems are concerned. Among the works dealing with the output feedback control for a class of switching discrete-time linear systems with parameters uncertainties, we cite [14], [15], [16] and [17], that constitute the main motivation of the present work.

The problem of observer-based stabilization of a class of switched linear systems has been first considered in [15] for systems without disturbances, using Finsler's lemma combined with a switching Lyapunov function [5]. Unfortunately, an error has been occurred when applying the Finsler lemma. Although a corrected version has been given in [17] where many LMI scenarios have been provided for several ways of use of the Finsler's Lemma, the inferred LMI synthesis conditions are conservative. Hence the observer-based stabilization problem for switching systems remains still open until now. Much remains to be done to improve the available LMI methods. The proposed work may be viewed as:

- (i) an extension of the technique in [17] to systems with disturbances in the dynamical equations and the output measurements;
- (ii) an improvement of the LMI techniques in [17] by introducing a more general structure of the slack variables coming from Finsler's lemma.

It is worth to notice that the obtained result can be applied to robust observer-based $\mathcal{H}_\infty$ control design problem for polytopic uncertain linear time varying systems. Indeed, asymptotic stability problem for switched linear systems with arbitrary switching is equivalent to the robust asymptotic stability problem for polytopic uncertain linear time-varying systems, for which conservative stability conditions are available in the literature [18].

B. Preliminary lemmas

In this subsection, we provide some useful lemmas, namely the Finsler's Lemma, the Young's relation, and the Schur Lemma. The main contribution of this paper is based on a convenient exploitation of Finsler's lemma.

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Lemma 1 (Finsler's Lemma [18]): Let $x \in \mathbb{R}^n, P \in \mathbb{S}^{n\times n}$, and $H \in \mathbb{R}^{m\times n}$ such that rank $(H) = r < n$. The following statements are equivalent:

1) $x^T P x < 0$, $\forall U x = 0, x \neq 0$,
2) $\exists X \in \mathbb{R}^{n\times m}$ such that $P + XU + U^T X^T < 0$.

Lemma 2 (Schur Lemma [19]): Let Q_1, Q_2, and Q_3 be three matrices of appropriate dimensions such that $Q_1 = Q_1^T$ and $Q_3 = Q_3^T$. Then, $Q_3 < 0$ and $Q_1 - Q_2 Q_3^{-1} Q_2^T < 0$ if and only if

$$\begin{bmatrix} Q_1 & Q_2 \\ Q_2^T & Q_3 \end{bmatrix} < 0\,.$$

Lemma 3 (Young's relation [19]): For given matrices X and Y of appropriate dimensions, we have for any matrix $S > 0$,

$$X^T Y + Y^T X \leq X^T S X + Y^T S^{-1} Y.$$

The rest of the paper is organized as follows. Section II is devoted to the problem formulation. The main contribution is presented and proved in Section III. A numerical example is presented in Section IV to show the superiority of the proposed methodology compared to the existing results in the literature. Finally, we end the paper by a conclusion.

II. Formulation of the Problem

A. System description and assumptions

Let us consider the class of switching discrete-time linear systems described by:

$$x_{t+1} = (A_{\sigma_t} + \Delta A_{\sigma_t})x_t + B_{\sigma_t} u_t + E_{\sigma_t}\omega_t \qquad (1a)$$
$$y_t = (C_{\sigma_t} + \Delta C_{\sigma_t})\, x_t + S_{\sigma_t}\omega_t \qquad (1b)$$
$$z_t = H_{\sigma_t} x_t + D_{\sigma_t} u_t + J_{\sigma_t}\omega_t \qquad (1c)$$

where $t \in \mathbb{N}$, $x_t \in \mathbb{R}^n$ is the state vector, $y_t \in \mathbb{R}^p$ is the output measurement, $u_t \in \mathbb{R}^m$ is the control input, $w_t \in \mathbb{R}^v$ is an unknown exogenous disturbance, $z_t \in \mathbb{R}^q$ is the controlled output, and $\sigma : \mathbb{N} \to \Lambda = \{1, 2, \ldots, N\}$, $t \mapsto \sigma_t$, is a switching rule.

Without ambiguity and for shortness, we write σ instead of σ_t. The matrices $A_\sigma \in \mathbb{R}^{n\times n}$, $B_\sigma \in \mathbb{R}^{n\times m}$, $E_\sigma \in \mathbb{R}^{n\times v}$, $C_\sigma \in \mathbb{R}^{p\times n}$, $S_\sigma \in \mathbb{R}^{p\times v}$, $H_\sigma \in \mathbb{R}^{q\times n}$, $D_\sigma \in \mathbb{R}^{q\times m}$, and $J_\sigma \in \mathbb{R}^{q\times v}, \sigma \in \Lambda$, are constant with real coefficients.

The uncertainties ΔA_σ and ΔC_σ are structured and norm-bounded in the sense of conditions

$$[\Delta A_\sigma,\ \Delta C_\sigma] = [M_\sigma,\ N_\sigma]\,\Gamma_\sigma\,[\mathcal{E}_\sigma,\ \mathcal{S}_\sigma], \qquad (2)$$
$$\Gamma_\sigma^T \Gamma_\sigma \leq I, \qquad (3)$$

where the matrices $M_\sigma, N_\sigma, \mathcal{E}_\sigma, \mathcal{S}_\sigma$ are known constant which characterize the structure of the uncertainty, and the normalized matrix Γ_σ contains the uncertain parameters.

The pairs (A_σ, B_σ) and (A_σ, C_σ) are assumed to be stabilizable and detectable, respectively. Throughout the paper, the coming assumptions are to build (see e.g. [15], [17]). Assume without loss of generality that the switching rule σ_t satisfies the following two items:

- The switching rule σ is not known a priori, but its instantaneous value is available in real time.
- The switching of the observer for systems should coincide exactly with the switching of the system.

B. $\mathcal{H}_\infty$ Observer-based stabilization problem

The state observer-based controller we consider in this paper has the following standard structure:

$$\hat{x}_{t+1} = A_\sigma \hat{x}_t + B_\sigma u_t + L\Big(y_t - C_\sigma \hat{x}_t\Big) \qquad (4a)$$
$$u_t = K_\sigma \hat{x}_t \qquad (4b)$$

where $\hat{x}_t \in \mathbb{R}^n$ is the estimate of x_t, and for each $\sigma \in \Lambda$, $L_\sigma \in \mathbb{R}^{n\times p}, K_\sigma \in \mathbb{R}^{m\times n}$ is the observer-based controller gains to be determined such that the estimation error $e_t = x_t - \hat{x}_t$ and the state x_t satisfy a prescribed performance criterion, namely the $\mathcal{H}_\infty$ criterion considered in this paper.

From (1) and (4), the dynamics of the augmented vector $\bar{x}_t = [\hat{x}_t^T \quad e_t^T]^T$ is given by:

$$\begin{aligned} \overline{x}_{t+1} &= \underbrace{\begin{bmatrix} \Omega_{11}(\sigma) & \Omega_{12}(\sigma) \\ \Omega_{21}(\sigma) & \Omega_{22}(\sigma) \end{bmatrix}}_{\Omega_\sigma} \bar{x}_t + \underbrace{\begin{bmatrix} L_\sigma S_\sigma \\ L_\sigma S_\sigma - E_\sigma \end{bmatrix}}_{\Pi_\sigma} w_t \\ &\triangleq \Omega_\sigma \bar{x}_t + \Pi_\sigma w_t \end{aligned} \qquad (5)$$

where

$$\Omega_{11}(\sigma) = A_\sigma + B_\sigma K_\sigma + L_\sigma \Delta C_\sigma, \qquad (6a)$$
$$\Omega_{12}(\sigma) = -L_\sigma(C_\sigma + \Delta C_\sigma), \qquad (6b)$$
$$\Omega_{21}(\sigma) = -(\Delta A_\sigma - L_\sigma \Delta C_\sigma), \qquad (6c)$$
$$\Omega_{22}(\sigma) = A_\sigma + \Delta A_\sigma - L_\sigma(C_\sigma + \Delta C_\sigma). \qquad (6d)$$

Let us define the indicator function

$$\xi(t) = [\xi_1(t), \xi_2(t), \ldots, \xi_N(t)]^T$$

as follows:

$$\xi_i(t) = \begin{cases} 1, & \sigma_t = i; \\ 0, & \text{otherwise.} \end{cases}$$

Therefore, system (5) and z_t in (1c) can be rewritten in the unified form:

$$\begin{bmatrix} \bar{x}_{t+1} \\ z_t \end{bmatrix} = \sum_{i=1}^{N} \xi_i(t) \begin{bmatrix} \Omega_i & \Pi_i \\ H_i + D_i K_i & -H_i \end{bmatrix} \begin{bmatrix} \bar{x}_t \\ w_t \end{bmatrix}, \qquad (7)$$

where Ω_i are defined in (5)-(6), when $\sigma_t = i$.

In the aim to analyze stability of the closed-loop system (7), we use the switched Lyapunov function defined as:

$$\begin{aligned} V(\bar{x}_t, \xi(t)) &= \bar{x}_t^T \hat{P}(\xi(t)) \bar{x}_t \\ &= \sum_{i=1}^{N} \xi_i(t) \bar{x}_t^T \begin{bmatrix} \hat{P}_i^{11} & \hat{P}_i^{12} \\ (\star) & \hat{P}_i^{22} \end{bmatrix} \bar{x}_t. \end{aligned} \qquad (8)$$

Notice that the Lyapunov function (8) is well known in the literature, (see for instance [10] and [20]). For shortness we use $\sigma_t = i$ and $\sigma_{t+1} = j$. This means that $\xi_i(t) = 1$ and $\xi_j(t+1) = 1$. Then we have

$$\begin{aligned} \Delta V_{ij}(t) &\triangleq V(\overline{x}_{t+1}, \xi(t+1)) - V(\overline{x}_t, \xi(t)) \\ &= \overline{x}_{t+1}^T (\sum_{i=1}^{N} \xi_i(t+1) \hat{P}_i) \overline{x}_{t+1} - \overline{x}_t^T (\sum_{i=1}^{N} \xi_i(t) \hat{P}_i) \overline{x}_t \\ &= [\overline{x}_t^T \quad \overline{x}_{t+1}^T] \begin{bmatrix} -\hat{P}_i & 0 \\ 0 & \hat{P}_j \end{bmatrix} [\overline{x}_t^T \quad \overline{x}_{t+1}^T]^T \end{aligned} \qquad (9)$$

Hence the $\mathcal{H}_\infty$ performance criterion is fulfilled if the following inequality holds:

$$\vartheta_{ij}(t) := \Delta V_{ij}(t) + z_t^T z_t - \mu w_t^T w_t < 0. \tag{10}$$

It is worth to notice that the inequalities (10) are sufficient to ensure the $\mathcal{H}_\infty$ criterion

$$\|z\|_{\ell_2^n} \leq \sqrt{\mu\|w\|^2_{\ell_2^q} + \nu\|z_0\|^2} \tag{11}$$

where $\sqrt{\mu}$ is the disturbance attenuation level, representing the disturbance gain from w to z, and $\nu > 0$ is to be determined. To show how (10) implies the classical $\mathcal{H}_\infty$ criterion (11), we refer the reader to [21] and [22] for instance. This criterion is well known in the literature and the use of Lyapunov analysis like in (10) is standard.

C. Application of Finsler's Lemma

Now we will exploit the Finsler's lemma to get sufficient conditions ensuing $\vartheta_{ij}(t) < 0$ for all $t \geq 0$.

Let us introduce the following notations:

$$\zeta_t = \begin{bmatrix} \overline{x}_t \\ \overline{x}_{t+1} \\ w_t \end{bmatrix}, \; P_{ij} = \begin{bmatrix} -\hat{P}_i & 0 & 0 & \Upsilon_i \\ (\star) & \hat{P}_j & 0 & 0 \\ (\star) & (\star) & -\mu I & J_i^T \\ (\star) & (\star) & (\star) & -I \end{bmatrix},$$

$$U_i = \begin{bmatrix} \Omega_i & -I & \Pi_i \end{bmatrix},$$

$$\Upsilon_i = \begin{bmatrix} K_i^T D_i^T + H_i^T \\ -H_i^T \end{bmatrix}, \; \forall i, j \in \Lambda$$

We have $U_i\zeta_t = 0$ and $\vartheta(t) = \zeta_t^\top P_{ij}\zeta_t$. Then, from Lemma 1 (Finsler's Lemma), we deduce that

$$\vartheta(t) < 0, \; \forall U_i\zeta_t = 0, \zeta_t \neq 0$$

if there exists

$$X_{i,j} = \begin{bmatrix} F_{i,j} \\ G_{i,j} \\ T_{i,j} \end{bmatrix}$$

such that

$$P_{ij} + X_{i,j}U_i + U_i^T X_{i,j}^T < 0. \tag{12}$$

After developing the calculations, we get the following equivalent detailed form of (12):

$$\begin{bmatrix} \Im_{ij} & -F_{ij} + \Omega_i^T G_{ij}^T & F_{ij}\Pi_i + \Omega_i^T T_{ij}^T & \Upsilon_i \\ (\star) & \hat{P}_j - \mathrm{He}(G_{ij}) & G_{ij}\Pi_i - T_{ij}^T & 0 \\ (\star) & (\star) & -\mu I + \mathrm{He}(T_{ij}\Pi_i) & J_i^T \\ (\star) & (\star) & (\star) & -I \end{bmatrix} < 0, \tag{13}$$

for all $i, j \in \Lambda$, where $\Im_{ij} = \mathrm{He}(F_{ij}\Omega_i) - \hat{P}_i$, and $\mathrm{He}(Y) = Y + Y^T$, for any matrix Y.

Rewriting the matrices F_{ij}, G_{ij}, T_{ij} and $\hat{P}_i$ under the detailed forms:

$$F_{ij} = \begin{bmatrix} F_{ij}^{11} & F_{ij}^{12} \\ F_{ij}^{21} & F_{ij}^{22} \end{bmatrix}, \tag{14}$$

$$G_{ij} = \begin{bmatrix} G_{ij}^{11} & G_{ij}^{12} \\ G_{ij}^{21} & G_{ij}^{22} \end{bmatrix}, \tag{15}$$

$$\hat{P}_i = \begin{bmatrix} \hat{P}_i^{11} & \hat{P}_i^{12} \\ (\star) & \hat{P}_i^{22} \end{bmatrix}, \tag{16}$$

$$T_{ij} = \begin{bmatrix} T_{ij}^1 & T_{ij}^2 \end{bmatrix}, \tag{17}$$

we get the equivalent detailed form of (13):

$$\begin{bmatrix} \Psi_{ij} & \begin{bmatrix} \Upsilon_i^T & 0 & 0 & J_i \end{bmatrix}^T \\ (\star) & -I \end{bmatrix} < 0, \tag{18}$$

for all $i, j \in \Lambda$, where

$$\Psi_{ij} = \begin{bmatrix} \Omega_{11}^{ij} & \Omega_{12}^{ij} & \Omega_{13}^{ij} & \Omega_{14}^{ij} & \Omega_{15}^{ij} \\ (\star) & \Omega_{22}^{ij} & \Omega_{23}^{ij} & \Omega_{24}^{ij} & \Omega_{25}^{ij} \\ (\star) & (\star) & \Omega_{33}^{ij} & \hat{P}_j^{12} - G_{ij}^{12} - (G_{ij}^{21})^T & \Omega_{35}^{ij} \\ (\star) & (\star) & (\star) & \hat{P}_j^{22} - G_{ij}^{22} - (G_{ij}^{22})^T & \Omega_{45}^{ij} \\ (\star) & (\star) & (\star) & (\star) & \Omega_{55}^{ij} \end{bmatrix},$$

$$\begin{aligned}
\Omega_{11}^{ij} =& -\hat{P}_i^{11} + \mathrm{He}\Big(F_{ij}^{11}A_i + F_{ij}^{11}B_iK_i + (F_{ij}^{11} + F_{ij}^{12})L_i\Delta C_i \\
& - F_{ij}^{12}\Delta A_i\Big), \\
\Omega_{12}^{ij} =& -\hat{P}_i^{12} + F_{ij}^{12}(A_i + \Delta A_i) - (F_{ij}^{11} + F_{ij}^{12})L_i(C_i + \Delta C_i) \\
& + K_i^T B_i^T (F_{ij}^{21})^T + A_i^T (F_{ij}^{21})^T + \Delta C_i^T L_i^T (F_{ij}^{21} + F_{ij}^{22})^T \\
& - \Delta A_i^T (F_{ij}^{22})^T, \\
\Omega_{13}^{ij} =& -F_{ij}^{11} + A_i^T (G_{ij}^{11})^T + K_i^T B_i^T (G_{ij}^{11})^T - \Delta A_i^T (G_{ij}^{12})^T \\
& + \Delta C_i^T L_i^T (G_{ij}^{11} + G_{ij}^{12})^T, \\
\Omega_{14}^{ij} =& -F_{ij}^{12} + A_i^T (G_{ij}^{21})^T + K_i^T B_i^T (G_{ij}^{21})^T - \Delta A_i^T (G_{ij}^{22})^T \\
& + \Delta C_i^T L_i^T (G_{ij}^{21} + G_{ij}^{22})^T, \\
\Omega_{15}^{ij} =& A_i^T (T_{ij}^1)^T + K_i^T B_i^T (T_{ij}^1)^T + \Delta C_i^T L_i^T (T_{ij}^1 + T_{ij}^2)^T \\
& - \Delta A_i^T (T_{ij}^2)^T + (F_{ij}^{11} + F_{ij}^{12})L_iS_i - F_{ij}^{12}E_i, \\
\Omega_{22}^{ij} =& -\hat{P}_i^{22} + \mathrm{He}\Big(F_{ij}^{22}A_i + F_{ij}^{22}\Delta A_i \\
& - (F_{ij}^{22} + F_{ij}^{21})L_i(C_i + \Delta C_i)\Big), \\
\Omega_{23}^{ij} =& -F_{ij}^{21} - (C_i + \Delta C_i)^T L_i^T (G_{ij}^{11} + G_{ij}^{12})^T + A_i^T (G_{ij}^{12})^T \\
& + \Delta A_i^T (G_{ij}^{12})^T, \\
\Omega_{24}^{ij} =& -F_{ij}^{22} + A_i^T (G_{ij}^{22})^T + \Delta A_i^T (G_{ij}^{22})^T \\
& - (C_i + \Delta C_i)^T L_i^T (G_{ij}^{22} + G_{ij}^{21})^T, \\
\Omega_{25}^{ij} =& -(C_i + \Delta C_i)^T L_i^T (T_{ij}^1 + T_{ij}^2)^T + A_i^T (T_{ij}^2)^T \\
& + \Delta A_i^T (T_{ij}^2)^T + (F_{ij}^{21} + F_{ij}^{22})L_iS_i - F_{ij}^{22}E_i, \\
\Omega_{33}^{ij} =& \hat{P}_j^{11} - (G_{ij}^{11})^T - G_{ij}^{11}, \\
\Omega_{35}^{ij} =& (G_{ij}^{11} + G_{ij}^{12})L_iS_i - G_{ij}^{12}E_i - (T_{ij}^1)^T, \\
\Omega_{45}^{ij} =& (G_{ij}^{21} + G_{ij}^{22})L_iS_i - G_{ij}^{22}E_i - (T_{ij}^2)^T, \\
\Omega_{55}^{ij} =& -\mu I + \mathrm{He}\Big((T_{ij}^1 + T_{ij}^2)L_iS_i - T_{ij}^2E_i\Big).
\end{aligned}$$

In the next section we will provide some techniques allowing to handle the BMI problem (18). By exploiting the Finsler's lemma, we will show that convenient choices of some matrices in X_{ij} lead to less conservative LMI conditions compared to the existing results in the literature.

III. New LMI Design Technique

This section is devoted tot the main contribution of this paper. We will propose new and less conservative LMI conditions to handle the observer-based stabilization problem for a class of linear switched systems in the presence of $\mathcal{L}_2$ bounded disturbances and norm-bounded parameter uncertainties.

A. Main Theorem

This subsection is devoted to the main result of the paper. We will provide less conservative LMI synthesis conditions ensuring the $\mathcal{H}_\infty$ criterion (11).

Theorem 1: If for $i, j \in \Lambda$ there exist positive definite matrices $\tilde{P}_i^{11}, \hat{P}_i^{22} \in \mathbb{R}^{n\times n}$, invertible matrices G_i^{22}, $\tilde{G}_{ij}^{11}$, $\tilde{F}_i^{11} \in \mathbb{R}^{n\times n}$, matrices $\tilde{K}_i \in \mathbb{R}^{m\times n}, \hat{L}_i \in \mathbb{R}^{n\times p}$, such that the following convex optimization problem holds for some positive constants ϵ_i and λ_i:

$$\min(\mu) \quad \text{subject to} \tag{19}$$

$$\begin{bmatrix} \Xi_{ij} & \mathbb{S}_i \\ (\star) & \mathbb{D}_i \end{bmatrix} < 0, \forall i, j \in \Lambda, \tag{20}$$

where

$$\Xi_{ij} = \begin{bmatrix} \Upsilon_{11}^i & -A_i & \Upsilon_{13}^{ij} & I & E_i & \Upsilon_{16}^i & 0 \\ (\star) & -\hat{P}_i^{22} & -A_i^T & \Upsilon_{24}^i & 0 & -H_i^T & 0 \\ (\star) & (\star) & \Upsilon_{33}^{ij} & I & E_i & 0 & \tilde{G}_{ij}^{11} \\ (\star) & (\star) & (\star) & \Upsilon_{44}^{ij} & \Upsilon_{45}^i & 0 & 0 \\ (\star) & (\star) & (\star) & (\star) & -\mu I & J_i^T & 0 \\ (\star) & (\star) & (\star) & (\star) & (\star) & -I & 0 \\ (\star) & (\star) & (\star) & (\star) & (\star) & (\star) & -\tilde{P}_j^{11} \end{bmatrix}, \tag{21}$$

$$\begin{aligned}
\Upsilon_{11}^i =& \tilde{P}_i^{11} + \mathrm{He}\Big(- \tilde{F}_i^{11} + A_i(\tilde{F}_i^{11})^T + B_i\tilde{K}_i \Big), \\
\Upsilon_{13}^{ij} =& \tilde{F}_i^{11}A_i^T + \tilde{K}_i^T B_i^T - (\tilde{G}_{ij}^{11})^T, \\
\Upsilon_{16}^i =& \tilde{K}_i^T D_i^T + \tilde{F}_i^{11} H_i^T, \\
\Upsilon_{24}^i =& A_i^T (G_i^{22})^T - C_i^T \hat{L}_i^T, \\
\Upsilon_{33}^{ij} =& - \mathrm{He}(\tilde{G}_{ij}^{11}), \\
\Upsilon_{45}^i =& \hat{L}_i S_i - G_i^{22} E_i, \\
\Upsilon_{44}^{ij} =& \hat{P}_j^{22} - G_i^{22} - (G_i^{22})^T,
\end{aligned} \tag{22}$$

$$\mathbb{S}_i = \begin{bmatrix} -\tilde{F}_i^{11}\mathcal{E}_i^T & -M_i & \tilde{F}_i^{11}\mathcal{S}_i^T & 0 \\ \mathcal{E}_i^T & 0 & -\mathcal{S}_i^T & 0 \\ 0 & -M_i & 0 & 0 \\ 0 & G_i^{22}M_i & 0 & \hat{L}_i N_i \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{23}$$

$$\mathbb{D}_i = \mathrm{diag}(-\epsilon_i^{-1} I, -\epsilon_i I, -\lambda_i^{-1} I, -\lambda_i I), \tag{24}$$

then the $\mathcal{H}_\infty$ criterion (11) is satisfied with the obtained minimum attenuation level μ and the observer-based controller gains:

$$K_i = \tilde{K}_i (\tilde{F}_i^{11})^{-T}, \; L_i = (G_i^{22})^{-1}\hat{L}_i. \tag{25}$$

B. Proof of Theorem 1

The proof is too long and uses different tools. To enhance the clarity of the contributions, we shared it into three steps. We will present the linearization of the BMI (18) to get the LMI (20) step by step until a full linearization.

1) First step: Linearization with respect to K_i:

From inequality (18), we can deduce that the matrices G_{ij}^{11}, and G_{ij}^{22} are invertible. Let us focus on the case where F_{ij}^{11} is invertible and independent of j. This is mainly due to the following principle of congruence. Indeed, by pre- and post-multiply the left hand side of (18) by

$$\mathrm{diag}\Big((F_i^{11})^{-1}, I, (G_{ij}^{11})^{-1}, I \Big)$$

and by using the change of variables

$$\tilde{G}_{ij}^{11} = (G_{ij}^{11})^{-1}, \tilde{F}_i^{11} = (F_i^{11})^{-1}, \tilde{K}_i = K_i(\tilde{F}_i^{11})^T$$

we get the equivalent inequality

$$\begin{bmatrix} \tilde{\Omega}_{11}^{ij} & \tilde{\Omega}_{12}^{ij} & \tilde{\Omega}_{13}^{ij} & \tilde{\Omega}_{14}^{ij} & \tilde{\Omega}_{15}^{ij} & \tilde{K}_i^T D_i^T + \tilde{F}_i^{11} H_i^T \\ (\star) & \Omega_{22}^{ij} & \tilde{\Omega}_{23}^{ij} & \Omega_{24}^{ij} & \Omega_{25}^{ij} & -H_i^T \\ (\star) & (\star) & \tilde{\Omega}_{33}^{ij} & \tilde{\Omega}_{34}^{ij} & \tilde{\Omega}_{35}^{ij} & 0 \\ (\star) & (\star) & (\star) & \Omega_{44}^{ij} & \Omega_{45}^{ij} & 0 \\ (\star) & (\star) & (\star) & (\star) & \Omega_{55}^{ij} & J_i^T \\ (\star) & (\star) & (\star) & (\star) & (\star) & -I \end{bmatrix} < 0, \tag{26}$$

where

$$\begin{aligned}
\tilde{\Omega}_{11}^{ij} =& - \tilde{F}_i^{11}\hat{P}_i^{11}(\tilde{F}_i^{11})^T + \mathrm{He}\Big(A_i(\tilde{F}_i^{11})^T + B_i\tilde{K}_i \\
& + (I + \tilde{F}_i^{11}F_{ij}^{12})L_i\Delta C_i(\tilde{F}_i^{11})^T - \tilde{F}_i^{11}F_{ij}^{12}\Delta A_i(\tilde{F}_i^{11})^T \Big), \\
\tilde{\Omega}_{12}^{ij} =& \tilde{F}_i^{11}F_{ij}^{12}(A_i + \Delta A_i) - (I + \tilde{F}_i^{11}F_{ij}^{12})L_i(C_i + \Delta C_i) \\
& + \tilde{K}_i^T B_i^T (F_{ij}^{21})^T + \tilde{F}_i^{11} A_i^T (F_{ij}^{21})^T - \tilde{F}_i^{11}\hat{P}_i^{12} \\
& + \tilde{F}_i^{11}\Delta C_i^T L_i^T (F_{ij}^{21} + F_{ij}^{22})^T - \tilde{F}_i^{11}\Delta A_i^T (F_{ij}^{22})^T, \\
\tilde{\Omega}_{13}^{ij} =& - (\tilde{G}_{ij}^{11})^T + \tilde{F}_i^{11}{A_i}^T + \tilde{K}_i^T B_i^T - \tilde{F}_i^{11}\Delta {A_i}^T(\tilde{G}_{ij}^{11}G_{ij}^{12})^T \\
& + \tilde{F}_i^{11}\Delta C_i^T L_i^T (I + \tilde{G}_{ij}^{11}G_{ij}^{12})^T, \\
\tilde{\Omega}_{14}^{ij} =& - \tilde{F}_i^{11}F_{ij}^{12} + \tilde{F}_i^{11}A_i^T(G_{ij}^{21})^T + \tilde{K}_i^T B_i^T (G_{ij}^{21})^T \\
& - \tilde{F}_i^{11}\Delta {A_i}^T (G_{ij}^{22})^T + \tilde{F}_i^{11}\Delta C_i^T L_i^T (G_{ij}^{21} + G_{ij}^{22})^T, \\
\tilde{\Omega}_{15}^{ij} =& \tilde{F}_i^{11}A_i^T(T_{ij}^1)^T + \tilde{K}_i^T B_i^T (T_{ij}^1)^T + \tilde{F}_i^{11}\Delta C_i^T L_i^T (T_{ij}^1 + T_{ij}^2)^T \\
& - \tilde{F}_i^{11}\Delta A_i^T (T_{ij}^2)^T + (I + \tilde{F}_i^{11}F_{ij}^{12})L_i S_i - \tilde{F}_i^{11}F_{ij}^{12}E_i, \\
\tilde{\Omega}_{23}^{ij} =& - F_{ij}^{21}(\tilde{G}_{ij}^{11})^T - (C_i + \Delta C_i)^T L_i^T (I + \tilde{G}_{ij}^{11}G_{ij}^{12})^T \\
& + (A_i^T + \Delta A_i^T)(\tilde{G}_{ij}^{11}G_{ij}^{12})^T, \\
\tilde{\Omega}_{33}^{ij} =& \tilde{G}_{ij}^{11}\hat{P}_j^{11}(\tilde{G}_{ij}^{11})^T - \tilde{G}_{ij}^{11} - (\tilde{G}_{ij}^{11})^T, \\
\tilde{\Omega}_{34}^{ij} =& \tilde{G}_{ij}^{11}\hat{P}_j^{12} - \tilde{G}_{ij}^{11}G_{ij}^{12} - \tilde{G}_{ij}^{11}(G_{ij}^{21})^T, \\
\tilde{\Omega}_{35}^{ij} =& (I + \tilde{G}_{ij}^{11}G_{ij}^{12})L_i S_i - \tilde{G}_{ij}^{11}G_{ij}^{12}E_i - \tilde{G}_{ij}^{11}(T_{ij}^1)^T, \\
\Omega_{44}^{ij} =& \hat{P}_j^{22} - G_{ij}^{22} - (G_{ij}^{22})^T.
\end{aligned}$$

Note that F_{ij}^{11} must depend only on i, otherwise the previous congruence principle reveals the term $K_i(\tilde{F}_{ij}^{11})^T$, which prevents a change of variables with respect to the gain K_i.

There are still some bilinear terms related to $\tilde{K}_i$, which need to be avoided, namely the coupling of $\tilde{K}_i$ with $F_{ij}^{21}, G_{ij}^{21}, T_{ij}^1$ and T_{ij}^2. To do this, the best way we propose is the following convenient and judicious choice:

$$G_{ij}^{21} = F_{ij}^{21} = 0, T_{ij}^1 = T_{ij}^2 = 0.$$

This is mainly do to the presence of bilinear terms $\tilde{F}_i^{11}A_i^T(T_{ij}^1)^T, \tilde{F}_i^{11}\Delta A_i^T(T_{ij}^2)^T$, which may lead to very conservative conditions if they are not null. Consequently, by using the change of variable $\tilde{P}_i^{11} = (\hat{P}_i^{11})^{-1}$ and the following Young's inequality $-\tilde{F}_i^{11}\hat{P}_i^{11}(\tilde{F}_i^{11})^T \leq \tilde{P}_i^{11} - \tilde{F}_i^{11} - (\tilde{F}_i^{11})^T$, we deduce that (26) is fulfilled if the following inequality holds:

$$\begin{bmatrix} \hat{\Omega}_{11}^{ij} & \hat{\Omega}_{12}^{ij} & \hat{\Omega}_{13}^{ij} & \hat{\Omega}_{14}^{ij} & \hat{\Omega}_{15}^{ij} & \tilde{K}_i^T D_i^T + \tilde{F}_i^{11} H_i^T \\ (\star) & \hat{\Omega}_{22}^{ij} & \hat{\Omega}_{23}^{ij} & \hat{\Omega}_{24}^{ij} & \hat{\Omega}_{25}^{ij} & -H_i^T \\ (\star) & (\star) & \tilde{\Omega}_{33}^{ij} & \hat{\Omega}_{34}^{ij} & \hat{\Omega}_{35}^{ij} & 0 \\ (\star) & (\star) & (\star) & \Omega_{44}^{ij} & \hat{\Omega}_{45}^{ij} & 0 \\ (\star) & (\star) & (\star) & (\star) & -\mu I & J_i^T \\ (\star) & (\star) & (\star) & (\star) & (\star) & -I \end{bmatrix} < 0, \tag{27}$$

where

$$\begin{aligned}
\hat{\Omega}_{11}^{ij} =& \tilde{P}_i^{11} + \mathrm{He}\Big(- \tilde{F}_i^{11} + A_i(\tilde{F}_i^{11})^T + B_i\tilde{K}_i \\
& + (I + \tilde{F}_i^{11}F_{ij}^{12})L_i\Delta C_i(\tilde{F}_i^{11})^T - \tilde{F}_i^{11}F_{ij}^{12}\Delta A_i(\tilde{F}_i^{11})^T \Big),
\end{aligned}$$

$$
\begin{aligned}
\hat{\Omega}_{12}^{ij} =& \tilde{F}_i^{11}F_{ij}^{12}(A_i+\Delta A_i) - (I+\tilde{F}_i^{11}F_{ij}^{12})L_i(C_i+\Delta C_i)\\
&+\tilde{F}_i^{11}\Delta C_i^T L_i^T(F_{ij}^{22})^T - \tilde{F}_i^{11}\Delta A_i^T(F_{ij}^{22})^T - \tilde{F}_i^{11}\hat{P}_i^{12},\\
\hat{\Omega}_{13}^{ij} =& -(\tilde{G}_{ij}^{11})^T + \tilde{F}_i^{11}A_i{}^T + \tilde{K}_i^T B_i^T - \tilde{F}_i^{11}\Delta A_i{}^T(\tilde{G}_{ij}^{11}G_{ij}^{12})^T\\
&+\tilde{F}_i^{11}\Delta C_i^T L_i^T(I+\tilde{G}_{ij}^{11}G_{ij}^{12})^T,\\
\hat{\Omega}_{14}^{ij} =& -\tilde{F}_i^{11}F_{ij}^{12} - \tilde{F}_i^{11}\Delta A_i{}^T(G_{ij}^{22})^T + \tilde{F}_i^{11}\Delta C_i^T L_i^T(G_{ij}^{22})^T,\\
\hat{\Omega}_{15}^{ij} =& (I+\tilde{F}_i^{11}F_{ij}^{12})L_iS_i - \tilde{F}_i^{11}F_{ij}^{12}E_i,\\
\hat{\Omega}_{22}^{ij} =& -\hat{P}_i^{22} + \mathrm{He}\Big(F_{ij}^{22}A_i + F_{ij}^{22}\Delta A_i - F_{ij}^{22}L_i(C_i+\Delta C_i)\Big),\\
\hat{\Omega}_{23}^{ij} =& -(C_i+\Delta C_i)^T L_i^T(I+\tilde{G}_{ij}^{11}G_{ij}^{12})^T\\
&+(A_i^T+\Delta A_i^T)(\tilde{G}_{ij}^{11}G_{ij}^{12})^T,\\
\hat{\Omega}_{24}^{ij} =& -F_{ij}^{22} + A_i^T(G_{ij}^{22})^T + \Delta A_i^T(G_{ij}^{22})^T\\
&-(C_i+\Delta C_i)^T L_i^T(G_{ij}^{22})^T,\\
\hat{\Omega}_{25}^{ij} =& F_{ij}^{22}L_iS_i - F_{ij}^{22}E_i,\\
\hat{\Omega}_{33}^{ij} =& \tilde{G}_{ij}^{11}\hat{P}_j^{11}(\tilde{G}_{ij}^{11})^T - \tilde{G}_{ij}^{11} - (\tilde{G}_{ij}^{11})^T,\\
\hat{\Omega}_{34}^{ij} =& \tilde{G}_{ij}^{11}\hat{P}_j^{12} - \tilde{G}_{ij}^{11}G_{ij}^{12},\\
\hat{\Omega}_{35}^{ij} =& (I+\tilde{G}_{ij}^{11}G_{ij}^{12})L_iS_i - \tilde{G}_{ij}^{11}G_{ij}^{12}E_i,\\
\hat{\Omega}_{45}^{ij} =& G_{ij}^{22}L_iS_i - G_{ij}^{22}E_i.
\end{aligned}
$$

Now that the BMI (18) is linearized with respect to the matrices $\tilde{K}_i$, we will proceed to the linearization with respect to the observer gains L_i. The next second step is dedicated to this issue.

2) Second step: Semi-linearization with respect to L_i:

The gains L_i are coupled with the matrices $(I+\tilde{F}_i^{11}F_{ij}^{12})L_iC_i$, $(I+\tilde{G}_{ij}^{11}G_{ij}^{12})L_iC_i$, $F_{ij}^{22}L_iC_i$, and $G_{ij}^{22}L_iC_i$. Since G_{ij}^{22} is necessarily invertible and L_i depends only on i, then to avoid complicated bilinearities, we take $G_{ij}^{22}=G_i^{22}$ independent of j. With the following convenient matrices:

$$F_i = \begin{bmatrix} F_i^{11} & -F_i^{11}\\ 0 & 0\end{bmatrix}, \tag{28a}$$

$$G_{ij} = \begin{bmatrix} G_{ij}^{11} & -G_{ij}^{11}\\ 0 & G_i^{22}\end{bmatrix}, \tag{28b}$$

$$\hat{P}_i^{12} = 0, \tag{28c}$$

and the change of variable $\hat{L}_i = G_i^{22}L_i$, we get the following semi-linearized version of (27) with respect to L_i:

$$\begin{bmatrix}
\Theta_{11}^i & \Theta_{12}^i & \Theta_{13}^{ij} & \Theta_{14}^i & E_i & \Theta_{16}^i & 0\\
(\star) & -\hat{P}_i^{22} & \Theta_{23}^i & \Theta_{24}^i & 0 & -H_i^T & 0\\
(\star) & (\star) & \Theta_{33}^{ij} & I & E_i & 0 & \tilde{G}_{ij}^{11}\\
(\star) & (\star) & (\star) & \Theta_{44}^{ij} & \Theta_{45}^i & 0 & 0\\
(\star) & (\star) & (\star) & (\star) & -\mu I & J_i^T & 0\\
(\star) & (\star) & (\star) & (\star) & (\star) & -I & 0\\
(\star) & (\star) & (\star) & (\star) & (\star) & (\star) & -\tilde{P}_j^{11}
\end{bmatrix} < 0, \tag{29}$$

where

$$
\begin{aligned}
\Theta_{11}^i =& \tilde{P}_i^{11} + \mathrm{He}\Big(-\tilde{F}_i^{11} + A_i(\tilde{F}_i^{11})^T + B_i\tilde{K}_i + \Delta A_i(\tilde{F}_i^{11})^T\Big),\\
\Theta_{12}^i =& -(A_i+\Delta A_i),\\
\Theta_{13}^{ij} =& -(\tilde{G}_{ij}^{11})^T + \tilde{F}_i^{11}A_i{}^T + \tilde{K}_i^T B_i^T + \tilde{F}_i^{11}\Delta A_i{}^T,\\
\Theta_{14}^i =& I - \tilde{F}_i^{11}\Delta A_i{}^T(G_i^{22})^T + \tilde{F}_i^{11}\Delta C_i^T\hat{L}_i^T,\\
\Theta_{16}^i =& \tilde{K}_i^T D_i^T + \tilde{F}_i^{11}H_i^T,\\
\Theta_{23}^i =& -(A_i^T+\Delta A_i^T), \qquad \Theta_{33}^{ij} = -\tilde{G}_{ij}^{11} - (\tilde{G}_{ij}^{11})^T,\\
\Theta_{24}^i =& A_i^T(G_i^{22})^T + \Delta A_i^T(G_i^{22})^T - (C_i+\Delta C_i)^T\hat{L}_i^T,\\
\Theta_{44}^{ij} =& \hat{P}_j^{22} - G_i^{22} - (G_i^{22})^T, \qquad \Theta_{45}^i = \hat{L}_iS_i - G_i^{22}E_i.
\end{aligned}
$$

In fact, equations (28a)-(28b) imply

$$I+\tilde{F}_i^{11}F_{ij}^{12} = 0, \tag{30a}$$

$$I+\tilde{G}_{ij}^{11}G_{ij}^{12} = 0, \tag{30b}$$

which mean that the bilinear terms $(I+\tilde{F}_i^{11}F_{ij}^{12})L_iC_i$, $(I+\tilde{G}_{ij}^{11}G_{ij}^{12})L_iC_i$ related to L_i vanish. Finally, with (28c) and from Schur Lemma applied on the term $\tilde{G}_{ij}^{11}\hat{P}_j^{11}(\tilde{G}_{ij}^{11})^T$, we get easily (29).

Hence all the bilinear terms except those related to ΔA_i and ΔC_i, are avoided. The linearization of the terms related to the uncertainties is the aim of the next and last step of the proof.

3) Third step: Full linearization :

In this subsection, we use the Young relation for linearize the uncertainties. By developing ΔA_i and ΔC_i , we can rewrite the previous inequality in the following more suitable form:

Inequality (29) may be rewritten under the form:

$$\Xi_{ij} + \mathrm{He}\Big(Z_{i1}^T\Gamma_i^T Z_{i2} + Z_{i3}^T\Gamma_i^T Z_{i4}\Big) < 0, \tag{31}$$

where

$$
\begin{aligned}
Z_{i1} &= \begin{bmatrix} -\mathcal{E}_i(\tilde{F}_i^{11})^T & \mathcal{E}_i & 0 & 0 & 0 & 0 & 0\end{bmatrix},\\
Z_{i3} &= \begin{bmatrix} \mathcal{S}_i(\tilde{F}_i^{11})^T & -\mathcal{S}_i & 0 & 0 & 0 & 0 & 0\end{bmatrix},\\
Z_{i2} &= \begin{bmatrix} -M_i^T & 0 & -M_i^T & M_i^T(G_i^{22})^T & 0 & 0 & 0\end{bmatrix},\\
Z_{i4} &= \begin{bmatrix} 0 & 0 & 0 & N_i^T\hat{L}_i^T & 0 & 0 & 0\end{bmatrix},
\end{aligned}
$$

and Ξ_{ij} is defined in (21). Consequently, applying the well known Young's inequality, we deduce that (31) holds if the following one is satisfied:

$$\Xi_{ij} + \epsilon_i Z_{i1}^T Z_{i1} + \epsilon_i^{-1}Z_{i2}^T Z_{i2} + \lambda_i Z_{i3}^T Z_{i3} + \lambda_i^{-1}Z_{i4}^T Z_{i4} < 0 \tag{32}$$

where ϵ_i, λ_i are some positive scalars coming from Young's relation. finally, by using again Schur Lemma (2), inequality (32) is fulfilled if the LMI (20) is feasible. This ends the proof of Theorem 1.

IV. Numerical Example

In this section, we present a numerical example to show the validity and effectiveness of the proposed design methodology. Through this example, we will show that the proposed LMIs (20) are less conservative than those provided in [12]. Then, we reconsider the same example given in [12], which is a linear system without parameter uncertainties. The system is described by the following matrices:

$$A = \begin{bmatrix} \delta & 0.8 & -0.4\\ -0.5 & 0.4 & 0.5\\ 1.2 & 1.1 & 0.8\end{bmatrix},\ B = \begin{bmatrix} 0 & 1\\ 2 & -1\\ 0 & 1.3\end{bmatrix},$$

$$E^T = \begin{bmatrix} 0.1 & 0.4 & 0.1\end{bmatrix},\ C = \begin{bmatrix} -1 & 1.2 & 1\\ 0 & -3 & 1\end{bmatrix}, D = 0,$$

$$S^T = \begin{bmatrix} 0.1 & 0.4 \end{bmatrix},\ H = \begin{bmatrix} -1 & 0 & 2 \end{bmatrix},\ \ J = 0.3$$

Obviously, this example can be viewed as a switching system under the form (1) with only one mode (there are no switching) and with $\Delta A_i = \Delta C_i = 0$. We test the feasibility for different values of δ and we look for the minimum value of $\mu_{\min}$ provided by each method. Table I summarizes the results.

δ	Theorem 1 in [12]		LMI (20)
	(α, β)	$\mu_{\min}$	$\mu_{\min}$
0.5	$(-0.03, -2.85)$	0.5523	0.3703
1.2	$(0.11,\ 3.66)$	0.6307	0.3703
1.7	$(1.03,\ 4.08)$	2.9248	0.3703
1.9	$(-1.20,\ -1.69)$	16.1504	0.3703

TABLE I

EXAMPLE 2: VALUE OF $\mu_{\min}$ FOR TWO LMI DESIGN METHODS

V. CONCLUSION

In this paper, new LMI conditions have been developed for the problem of the stabilization of a class of switching discrete-time linear systems with parameter uncertainties and l_2-bounded disturbances. We have shown that a judicious choice of slack variables coming from Finsler's lemma leads to less conservative LMIs. Analytical developments have been provided to clarify how the proposed choice allows to eliminate some bilinear matrix coupling without using any conservative inequality. The validity of the proposed design method is shown through a numerical example.

In future work, we hope to extend our technique to more general classes of switching systems, namely nonlinear systems, systems with uncertainties in all the matrices of the model, and linear parameter varying systems with inexact parameters.

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