Online Min-Sum Flow Scheduling with Rejections
Giorgio Lucarelli, Kim Nguyen, Abhinav Srivastav, Denis Trystram

To cite this version:
Giorgio Lucarelli, Kim Nguyen, Abhinav Srivastav, Denis Trystram. Online Min-Sum Flow Scheduling with Rejections. The 13th Workshop on Models and Algorithms for Planning and Scheduling Problems (MAPSP 2017), Jul 2017, Seeon Abbey, Germany. hal-01672351

HAL Id: hal-01672351
https://hal.archives-ouvertes.fr/hal-01672351
Submitted on 24 Dec 2017

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
1 Introduction

A well-identified issue in online computation is the weakness of the worst case paradigm which underestimates the performance of an online algorithm. Many algorithms, which perform well in real-world, are known to admit a mediocre theoretical guarantee. Conversely, many theoretical sound algorithms behave poorly even in simple practical settings. The need of more accurate models is considered as a high importance in algorithmic community. Over the last two decades, several models have been proposed in this direction. We are interested in resource augmentation models in the context of scheduling. Kalyanasundaram and Pruhs [6] proposed the speed augmentation model, where an online algorithm is compared against an adversary with slower processing speed, while Phillips et al. [8] proposed the machine augmentation model in which the algorithm uses more machines than the adversary. Choudhury et al. [4] introduced the rejection model where an online algorithm is allowed to discard a small fraction of jobs.

In this paper, we study the problems of preemptive and non-preemptive online scheduling of jobs on unrelated machines in order to minimize the average time a job remains in the system. Both problems are known to be non-approximable by a constant factor [2, 5]. However, the preemptive variant has been extensively studied under the different resource augmentation models (see for example [1, 3] and the references therein). On the other hand, the non-preemptive variant is much less explored. An $O(\frac{1}{\epsilon})$-competitive algorithm has been presented in [7] for the non-preemptive average flow-time minimization problem on a set of unrelated machines if both an $\epsilon$-speed augmentation is used and an $\epsilon$-fraction of jobs is rejected. We are interested here in exploring the power of the rejection model and, mainly, in eliminating the need for speed augmentation in the latter result. On the road to this, we show how to replace speed augmentation with rejection in the preemptive variant. Our analysis is based on the dual-fitting paradigm.

Problem definition. We are given a set $\mathcal{M}$ of $m$ unrelated machines where jobs arrive online, that is we learn about the existence and the characteristics of a job only after its release. Let $\mathcal{J}$ denote the set of all jobs of our instance, which is not known a priori. Each job $j \in \mathcal{J}$ is characterized by its release time $r_j$, while if $j$ is executed on machine $i \in \mathcal{M}$ then it has a processing time $p_{ij}$. Moreover, each job has to be dispatched to one machine at its arrival and migration is not allowed. Given a schedule, let $C_j$ denote the completion time of the job $j$. The flow-time of $j$ is defined as $F_j = C_j - r_j$, that is the total time that $j$ remains in the system. Our objective is to create a schedule that minimizes the total flow-times of all jobs, i.e., $\sum_{j \in \mathcal{J}} F_j$. 

---

*giorgio.lucarelli,denis.trystram*@imag.fr. Univ. Grenoble Alpes, LIG, INRIA, France  
†thang@ibisc.fr. IBISC, University of Evry, France  
‡abhinav.srivastav@univ-grenoble-alpes.fr. Verimag, Univ. Grenoble Alpes, France
2 Linear Programming Formulation

For each machine $i \in \mathcal{M}$, job $j \in \mathcal{J}$ and time $t \geq r_j$, we introduce a binary variable $x_{ij}(t)$ which indicates if $j$ is processed on $i$ at time $t$. We consider the following linear programming relaxation and the corresponding dual program.

\[
\begin{align*}
\min & \sum_{i \in \mathcal{M}} \sum_{j \in \mathcal{J}} \int_{r_j}^{\infty} \frac{1}{p_{ij}} (t - r_j + p_{ij}) x_{ij}(t) dt \\
& \quad \sum_{i \in \mathcal{M}} \int_{r_j}^{\infty} \frac{x_{ij}(t)}{p_{ij}} dt \geq 1 \quad \forall j \in \mathcal{J} \\
& \quad \sum_{j \in \mathcal{J}} x_{ij}(t) \leq 1 \quad \forall i \in \mathcal{M}, t \\
& \quad x_{ij}(t) \in [0, 1]
\end{align*}
\]

\[
\begin{align*}
\max & \sum_{j \in \mathcal{J}} \lambda_j - \sum_{i \in \mathcal{M}} \int_{0}^{\infty} \gamma_i(t) dt \\
& \quad \lambda_j - \gamma_i(t) \leq (t - r_j + p_{ij}) \quad \forall i \in \mathcal{M}, j \in \mathcal{J}, t \geq r_j \\
& \quad \lambda_j, \gamma_i(t) \geq 0
\end{align*}
\]

We will interpret the rejection model in the above primal and dual programs as follows. We assume that the algorithm is allowed to reject a set $\mathcal{R}$ of jobs. This corresponds to sum up in the primal objective only on the set of the non-rejected jobs. Hence, applying the concept of weak duality, the competitive ratio of an algorithm that rejects the jobs in $\mathcal{R}$ is at most:

\[
\frac{\sum_{i \in \mathcal{M}} \sum_{j \in \mathcal{J} \setminus \mathcal{R}} \int_{r_j}^{\infty} \frac{1}{p_{ij}} (t - r_j + p_{ij}) x_{ij}(t) dt}{\sum_{j \in \mathcal{J}} \lambda_j - \sum_{i \in \mathcal{M}} \int_{0}^{\infty} \gamma_i(t) dt}
\]

3 Preemptive Scheduling

**The algorithm.** Each job is immediately dispatched to a machine upon its arrival. We denote by $Q_i(t)$ the set of pending jobs at time $t$ dispatched to machine $i$, i.e., the set of jobs dispatched to $i$ that have been released but not yet completed or rejected at $t$. Let $q_{ij}(t)$ be the remaining processing time at time $t$ of a job $j$ that is dispatched to machine $i$. For each machine $i$, our scheduling policy is the following: at each time $t$ we execute on $i$ the job $j \in Q_i(t)$ with the smallest remaining processing time in $Q_i(t)$. In case of ties, we select the job that arrived the earliest. Moreover, we maintain a counter $c_i$ (initialized to 0) for each machine $i$. Every time a job is dispatched to $i$, $c_i$ is increased by 1. Then, the rejection policy is the following: given an arbitrary small constant $\epsilon \in (0, 1)$, whenever $c_i$ reaches $1/\epsilon + 1$, we reject the job with the largest remaining processing time from $Q_i(t)$ and reset $c_i$ to 0. Note that the rejected job does not immediately disappear from the system. We say that a job $\ell$ is definitely rejected at time $t + \sum_{j \in Q_i(t)} q_{ij}(t) + q_{i\ell}(t)$, that is at the time that it supposed to be completed. We denote by $R_i(t)$ the set of jobs dispatched to $i$ that are rejected but not yet definitely at time $t$. Let $\Delta_{ij}$ be the increase in the total flow-time occurred in the schedule of our algorithm following the above scheduling and rejection policies if we decide to dispatch a new job $j$ to machine $i$. Then, the dispatching policy is the following: we dispatch $j$ to the machine where $\Delta_{ij}$ is minimum.

**Dual variables.** Based on the dispatching policy, we set $\lambda_j = \min_i \Delta_{ij}$. Let $W_i(t)$ be the total number of jobs dispatched to machine $i$ that are either pending or not yet definitely rejected until $t$, i.e., $W_i(t) = |Q_i(t)| + |R_i(t)|$. We set $\gamma_i(t) = W_i(t)/(1 + \epsilon)$. Based on this definition, we can guarantee that, given any fixed time $t$, $\gamma_i(t)$ does not decrease due to rejections since the jobs remain in $R_i(t)$ for sufficient time after their rejection. Then, the following theorem holds.

**Theorem 1** Given any $\epsilon \in (0, 1)$, there is an $O(\frac{1}{\epsilon})$-competitive algorithm that rejects at most an $\epsilon$-fraction of jobs.
4 Non-preemptive Scheduling

The algorithm. We enhance the algorithm presented in the previous section in order to adapt it to a non-preemptive environment where a job is considered to be successfully executed only if its execution is performed without any interruption. The scheduling policy for each machine \(i\) is the following: at each time \(t\) when \(i\) is idle, we start executing on \(i\) the job that has the smallest processing time in \(Q_i(t)\); in case of ties, we select the job that arrived the earliest. We use two rejection rules. The first rejection policy is identical with the preemptive case. In order to define the second rejection rule, we maintain another counter \(v_j\) for each job \(j\) which is initialized to 0 when the execution of \(j\) begins. This counter is increased by one each time a new job is dispatched to machine \(i\) while \(j\) is executing on \(i\). Then, the second rejection policy is the following: we reject the job \(j\) when \(v_j = 1/\epsilon\). The dispatching policy is based again on the increase in the total flow-time \(\Delta_{ij}\) in the same vein as for the preemptive case.

Dual variables. We set \(\lambda_j = \epsilon \cdot \min_i \Delta_{ij}\). As before, we define the set \(R_i(t)\) of jobs that have been rejected due to the first rejection policy but not yet definitively. For each job \(j\), let \(D_j\) denote the set of jobs that are rejected due to the second rejection policy after \(r_j\) and before its completion or rejection. Let \(j_k\) denote the job released when the job \(k\) is rejected due to the second rejection policy. We say that a job \(j\) dispatched to machine \(i\) is definitively finished \(\sum_{k \in D_j} q_{ik}(r_{jk})\) time after its completion or rejection. Let \(U_i(t)\) be the set of jobs that are dispatched to machine \(i\) and are already completed or rejected due to the second rejection policy but not yet definitively finished at time \(t\). Let \(W_i(t)\) be the total number of jobs dispatched to machine \(i\) that are pending in \(Q_i(t)\), \(R_i(t)\) and \(U_i(t)\), i.e. \(W_i(t) = |Q_i(t)| + |R_i(t)| + |U_i(t)|\). We set \(\gamma_i(t) = \frac{\epsilon}{1+\epsilon} W_i(t)\). Based on these definitions, the following theorem holds.

Theorem 2 Given any \(\epsilon \in (0, 1)\), there is an \(O(\frac{1}{\epsilon^2})\)-competitive algorithm that rejects at most a \(2\epsilon\)-fraction of any job.

References