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Sequential Quasi Monte Carlo for Dirichlet Process Mixture Models

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Sampling: SMC, QMC, SQMC

- Sequential Monte Carlo (SMC), or Particle filtering, is a principled technique which sequentially approximates the full posterior using particles (Doucet et al., 2001). It focuses on sequential state-space models: the density of the observations \( y \) conditionally on Markov states \( x \) in \( X \subseteq \mathbb{R}^d \) is given by \( y|\tilde{x}_t \sim f^y(y|\tilde{x}_t) \), with kernel

\[
x_0 \sim f^0(x_0), \quad \tilde{x}_t|x_{t-1} \sim f^\tilde{x}(\tilde{x}_t|x_{t-1}).
\]

- The initial motivation of quasi Monte Carlo (QMC) is to use low discrepancy vectors instead of unconstrained random vectors in order to improve the calculation of integrals via Monte Carlo.

- Gerber and Chopin (2015) introduce a sequential quasi Monte Carlo (SQMC) methodology. This assumes the existence of transforms \( \Gamma \), mapping uniform random variables to the state variables. Requires that (1) can be rewritten as

\[
\tilde{x}^{(n)}_t = \Gamma_t(u^{(n)}_t) \leftrightarrow \tilde{x}^{(0)}_t \sim f^{\tilde{x}_t}(\tilde{x}^{(0)}_t),
\]

\[
x^{(n)}_{t,j} = \Gamma_t(x^{(n)}_{t-1,j}, u^{(n)}_{t,j}) \leftrightarrow x^{(0)}_{t,j} \sim f(x^{(0)}_{t,j}|x^{(0)}_{t-1,j}),
\]

where \( u^{(n)}_t \sim \mathcal{U}(0,1)^d \) is to be a quasi random vector of uniforms.

Dirichlet process & SQMC

- Nonparametric mixtures for density estimation: extension of finite mixture models when the number of clusters is unknown. Observations \( y_{1:T} \) follow a DPM model with kernel \( \psi \) parameterized by \( \beta \),

\[
y|\theta \sim \int \phi(y|\theta) \pi(G|\theta) d\theta,
\]

where \( G \sim \text{DP}(\alpha, G_0) \).

- DPM cast as SMC samplers by Liu (1996); Fearnhead (2004); Griffin (2015): observations are spread out into unobserved clusters whose labels, or allocation variables, are latent variables acting as observations states in the context of SMC. Transition is given by the (posterior) generalized Polya urn scheme

\[
p_{1:t} = P(x_1 = \tilde{x}_{1:1}, \ldots, x_T = \tilde{x}_{T:1}, y_{1:T}).
\]

- Complies with Gerber and Chopin (2015) need for a deterministic transform

\[
\Gamma_t(x^{(n)}_{t-1,j}, u^{(n)}_{t,j}) = \min \left\{ j \in \{1, \ldots, k^{(n)}_{t-1} + 1\} : \sum_{j'=1}^{j' - 1} p^{(n)}_{t,j'} > u^{(n)}_{t,j} \right\}
\]

for any particle \( n \), with \( u^{(n)}_t \sim \mathcal{U}(0,1) \).

Goal

- Peculiarity to the DPM setting:
  - state-space \( = (1 : T)^2 \) is discrete and varies
  - transition is not Markovian

- Goal: investigate how SQMC fares
  - compare allocation trajectories \( \tilde{x}^{(n)}_{1:T}, n = 1, \ldots, N \) in SMC & SQMC
  - measure their dispersion with a principal component analysis (PCA) \( \rightarrow \) proportion of variance explained by number of components in the PCA

References


Results II

Left: Density fit; Middle: Particles diversity (PCA); Right: Number of different particles for each data point.

Three samplers, non sequential Monte Carlo, MC, sequential Monte Carlo, SMC and sequential quasi Monte Carlo SQMC.

Sample size \( T = 200 \), number of particles \( N = 1000 \).
Top row: Heavy tailed distr. (student 2); Middle row: Skewed distr. (log-Gamma); Bottom row: Multimodal distr. (mixture of normals).