

# Sequential Quasi Monte Carlo for Dirichlet Process Mixture Models

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## Sampling: SMC, QMC, SQMC

• **Sequential Monte Carlo (SMC)**, or Particle filtering, is a principled technique which sequentially approximates the full posterior using particles (Doucet et al., 2001). It focuses on sequential state-space models: the density of the observations  $\mathbf{y}_t$  conditionally on Markov states  $\mathbf{x}_t$  in  $\mathcal{X} \subseteq \mathbb{R}^d$  is given by  $\mathbf{y}_t | \mathbf{x}_t \sim f^Y(\mathbf{y}_t | \mathbf{x}_t)$ , with kernel

$$\mathbf{x}_0 \sim f_0^X(\mathbf{x}_0), \quad \mathbf{x}_t | \mathbf{x}_{t-1} \sim f^X(\mathbf{x}_t | \mathbf{x}_{t-1}). \quad (1)$$

• The initial motivation of **quasi Monte Carlo (QMC)** is to use *low discrepancy* vectors instead of unconstrained random vectors in order to improve the calculation of integrals via Monte Carlo.

• Gerber and Chopin (2015) introduce a **sequential quasi Monte Carlo (SQMC)** methodology. This assumes the existence of transforms  $\Gamma_t$  mapping uniform random variables to the state variables. Requires that (1) can be rewritten as

$$\begin{aligned} \mathbf{x}_0^{(n)} &= \Gamma_0(\mathbf{u}_0^{(n)}) \leftrightarrow \mathbf{x}_0^{(n)} \sim f_0(d\mathbf{x}_0^{(n)}) \\ \mathbf{x}_{1:t}^{(n)} &= \Gamma_t(\mathbf{x}_{1:t-1}^{(n)}, \mathbf{u}_t^{(n)}) \leftrightarrow \mathbf{x}_{1:t}^{(n)} | \mathbf{x}_{1:t-1}^{(n)} \sim f_t(d\mathbf{x}_{1:t}^{(n)} | \mathbf{x}_{1:t-1}^{(n)}) \end{aligned}$$

where  $\mathbf{u}_t^{(n)} \sim \mathcal{U}([0, 1]^d)$  is to be a quasi random vector of uniforms.

## Dirichlet process & SQMC

• Nonparametric mixtures for density estimation: extension of finite mixture models when the number of clusters is unknown. Observations  $\mathbf{y}_{1:T}$  follow a DPM model with kernel  $\psi$  parameterized by  $\theta \in \Theta$ ,

$$\mathbf{y}_t | G \stackrel{\text{i.i.d.}}{\sim} \int \psi(\mathbf{y}; \theta) dG(\theta), \quad t \in (1 : T),$$

where  $G \sim \text{DP}(\alpha, G_0)$ .

• DPM cast as SMC samplers by Liu (1996); Fearnhead (2004); Griffin (2015) : observations are spread out into unobserved clusters whose labels, or allocation variables, are *latent* variables acting as observations *states* in the context of SMC. Transition is given by the (posterior) *generalized Pólya urn scheme*

$$p_{t,j} = P(\mathbf{x}_t = j | \mathbf{x}_{1:t-1}, \mathbf{y}_{1:t}).$$

• Complies with Gerber and Chopin (2015) need for a deterministic transform

$$\begin{aligned} \Gamma_t(\mathbf{x}_{1:t-1}^{(n)}, \mathbf{u}_t^{(n)}) &= \\ \min \left\{ j \in \{1, \dots, k_{t-1}^{(n)} + 1\} : \sum_{i=1}^j p_{t,i}^{(n)} > \mathbf{u}_t^{(n)} \right\} \end{aligned}$$

for any particle  $n$ , with  $\mathbf{u}_t^{(n)} \sim \mathcal{U}([0, 1])$ .

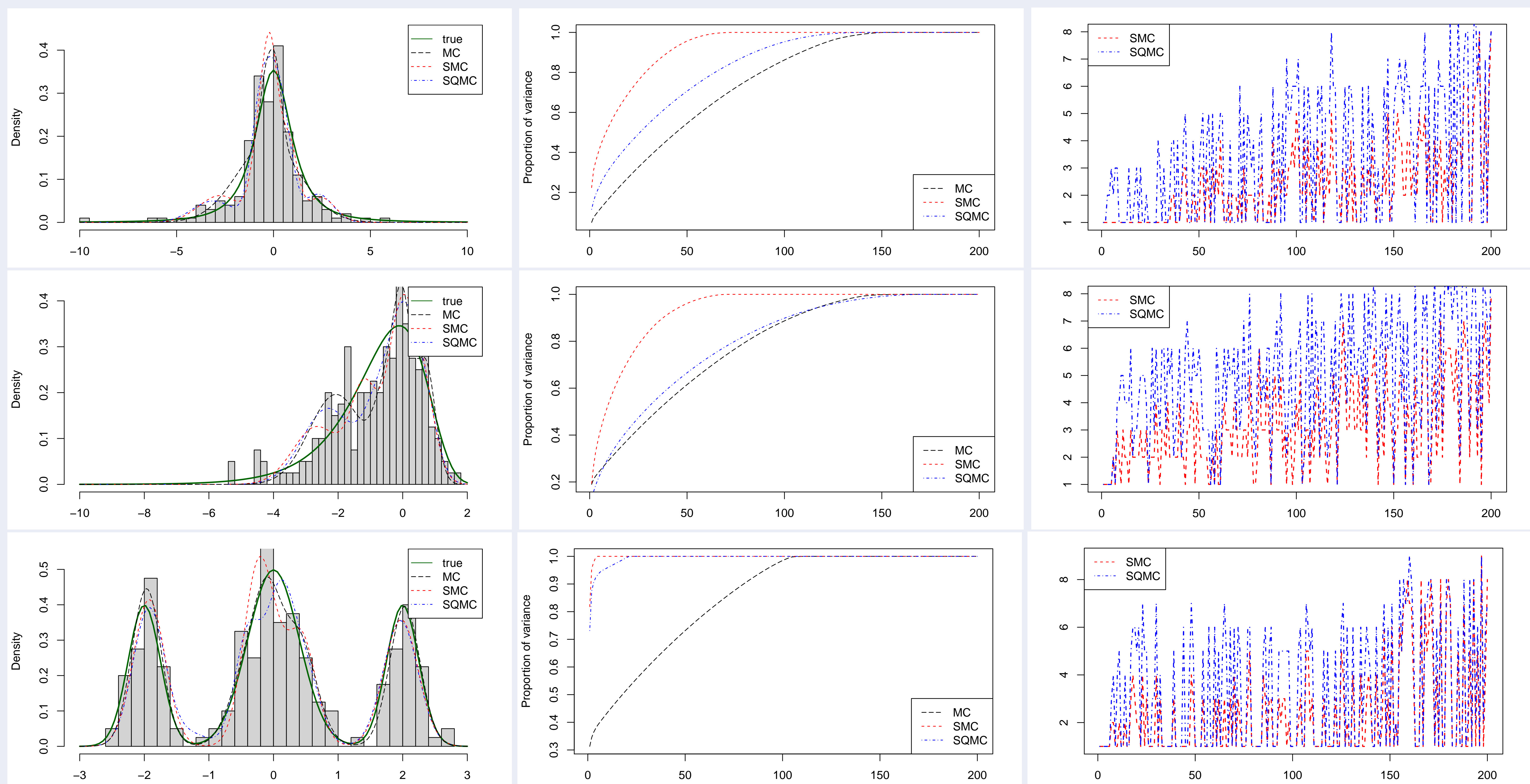
## Goal

- Peculiarity to the DPM setting:
  - state-space  $\approx (1 : T)^T$  is discrete and varies
  - transition is not Markovian
- **Goal**: investigate how SQMC fares
  - compare allocation trajectories  $\mathbf{x}_{1:T}^{(n)}$ ,  $n = 1, \dots, N$  in SMC & SQMC
  - measure their dispersion with a principal component analysis (PCA)  $\rightarrow$  proportion of variance explained by number of components in the PCA

## References

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## Results II



Left: **Density fit**; Middle: **Particles diversity (PCA)**; Right: **Number of different particles for each data point.**

Three samplers, non sequential Monte Carlo, **MC**, sequential Monte Carlo, **SMC** and sequential quasi Monte Carlo **SQMC**.

Sample size  $T = 200$ , number of particles  $N = 1000$ .

Top row: Heavy tailed distr. (student 2); Middle row: Skewed distr. (log-Gamma); Bottom row: Multimodal distr. (mixture of normals).