STUDENTS' UNDERSTANDING OF EQUILIBRIUM AND STABILITY: THE CASE OF DYNAMIC SYSTEMS

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ABSTRACT. Engineering students in control courses have been observed to lack an understanding of equilibrium and stability, both of which are crucial concepts in this discipline. The introduction of these concepts is generally based on the study of classical examples from Newtonian mechanics supplemented with a control system. Equilibrium and stability are approached in different ways at the various stages of a typical engineering syllabus: at the beginning, they are mostly dealt with a static point of view, for example in mechanics, and are subsequently handled through dynamic analysis in control courses. In general, there is a little clarification of the differences between these concepts or the ways in which they are linked. We believe that this leads to much confusion and incomprehension among engineering students. Several studies have shown that students encounter difficulties when presented with simple familiar or academic static equilibrium cases in mechanics. Our study investigates students' conceptions and misconceptions about equilibrium and stability through a series of questions about several innovative non-static situations. It reveals that the understanding of these notions is shaken when the systems being studied are placed in inertial or non-inertial moving reference frames. The students in our study were particularly uncertain about the existence of unstable equilibrium positions and had difficulty in differentiating between the two concepts. The results suggest that students use a velocity-based approach to explain such situations. A poor grasp of the above fundamental concepts may result from previous learning experiences. More specifically, certain difficulties seem to be directly linked to a lack of understanding of these concepts, while others are related to misconceptions arising from everyday experiences and the inappropriate use of physical examples in primary school.

KEYWORDS: conceptions, equilibrium, stability, students' way of thinking

INTRODUCTION

Equilibrium and stability are fundamental scientific concepts that are widely covered in both French and foreign curricula, and more generally in all physics and chemistry programs. One can find them at the primary and secondary school level, and subsequently in higher education. For example, balances and levers are studied in primary school, while the pendulum is dealt with at the secondary and tertiary levels, both in France and abroad.

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In classical mechanics, equilibrium is a stationary state for variables describing an isolated system (Mathieu, Kastler & Fleury, 1991). When extended to the case of non-isolated systems, one may speak of dynamic equilibrium. Mathematically, both cases involve a fixed point for the differential equations describing the system.[1]

Stability is a characteristic of an equilibrium state in which the system, whether free or controlled, tends to remain. More precisely, the system returns to its state of equilibrium on its own after momentarily being displaced from this state, without any additional external action and within a finite period of time (Mathieu, et al., 1991). The two concepts are therefore closely related. Historically speaking, as soon as one sought to understand the equilibrium of the balance in the sixteenth century (within a static framework), one naturally sought to characterize the behavior of this state in response to external disturbances[2] (Renn & Damerow, 2012)—in other words, its stability.

Verification of the equilibrium state can be carried out in two ways : either by using a field-dependent criterion in classical mechanics, this amounts to checking that the vector sum, or *net* sum, of external efforts[3] is zero, or by using a general empirical criterion directly derived from the definition (e.g. checking that the state of the system remains constant over time in the absence of external disturbances, while noting that the input of a controlled system is not considered as a disturbance and must therefore remain constant). Often, it has been observed that this criterion is used without proper identification and naming at the lower academic levels, when students lack the necessary mathematical tools for applying a disciplinary criterion. Unfortunately, this general criterion is then reduced to the verification of a local sub-criterion such as the horizontality of the fulcrum of a balance scale, which leads to erroneous conceptions about the concept, by induction, for example.

As regards to the concept of stability, at higher academic levels (e.g. in French collèges/lycées or secondary schools in English-speaking countries), it is handled in an academic manner and mostly by means of an energy criterion involving the potential energy of a system. More specifically, the potential energy of a mechanical system is verified to be an extremum when the system is in an equilibrium state: this corresponds to a minimum in stable equilibrium and a maximum in unstable equilibrium. Alternatively, one can use an empirical criterion directly derived from the general definition of stability, which would simply be a case of observing the behavior of the system when the latter is displaced from the equilibrium position. A free system in a position (i.e. state) of stable equilibrium is observed to return to this initial position once the disturbance disappears at the end of a finite time period. By contrast, it moves away from the initial position indefinitely in the case of unstable equilibrium or remains in the state induced by the disturbance in the case of neutral equilibrium.

Most school assessments are aimed at checking students' knowledge of formulae (criteria) and calculation techniques pertaining to these concepts in various disciplines. However, most students are able to simply reproduce these formulae and techniques without necessarily mastering the concepts themselves. Thus, such assessments often fail to reveal the lack of understanding of equilibrium and stability, which are wrongly perceived to have been mastered for a good portion of a student's schooling, thereby delaying the onset of difficulties. It is only in higher education—when the systems being studied are no longer presented in a classic form—that these difficulties resurface. This may create real problems in understanding complex phenomena such as those occurring in many controlled or regulated engineering systems. Indeed, these concepts are difficult to grasp, and as Municio Pozo & Gómez Crespo (1998, p 116) has pointed out, understanding equilibrium may well be one of the most significant scientific achievements of humankind. It involves changing one of the most common and therefore most inflexible ways of thinking—one that is related to causal linear reasoning (Fauconnet, 1981) and consists in focusing on changes (actions) while forgetting the mutual effects (reactions) that ensure *conservation* (Inhelder & Piaget, 1955). This goes to show that the root of these difficulties is complex and deep-seated.

Overview of Research

Many studies have focused on the understanding of these concepts beyond the mere knowledge of the aforesaid criteria, but most of them deal with equilibrium while almost none concern stability, which is probably considered to be a separate concept.[4] Much research has been devoted to investigating students' difficulties in understanding general physics concepts, including equilibrium, in order to identify alternative modes of reasoning.

Gunstone (1987), for example, observed that many students have trouble understanding various concepts in mechanics, especially equilibrium in static pulley/mass systems. He concluded that many students could actually be reasoning in ways that are quite different from those taught in classrooms. Albanese, Danhoni Neves & Vicentini (1998) examined this issue from a historical point of view by taking into account the influence of real-life (i.e. everyday), non-academic situations on the construction of students' reasoning, otherwise known as *spontaneous reasoning* or *common-sense reasoning* (Viennot, 1979). They brought to light certain energy and dissipative aspects that distinguish academic situations from those of everyday life and possibly influence or even account for some of the difficulties in understanding these phenomena.

Some studies have dealt with the equilibrium concept indirectly in their investigations on the mechanisms that could lead to alternative reasoning about general physics situations and therefore the above mentioned difficulties. One example is the study by Pozo, del Puy Pérez, Sanz & Limón (1992), who organized and classified students' alternative reasoning in the form of mini-theories or the ones by Sherin (2006), Steif & Dantzler (2005) or Palmer (2001). In this later study, the students were asked to reflect on simple static situations (e.g. a book on a table, on a ball, on a spring, etc.) and the authors were able to identify the strong role of context and the apparent inconsistency of the students' reasoning when faced with situations involving the very same physical principle.

Other investigations have been specifically centered on the understanding of equilibrium. Some were carried out at the child's psycho developmental level in line with the work of Piaget, such as Siegler & Chen (2002); Bonawitz, Lim & Schulz (2007); and others at the undergraduate level, for example the study by Newcomer & Steif (2008), in which students were asked to express their views on the equilibrium of static systems of rope-connected beams, that by Ortiz, Heron, & Shaffer (2005) involving prototypical static balance systems, or the work of Flores-García, Alfaro-Avena, Chávez-Pierce, Luna-González & González-Quezada (2010) concerning mixed static pulley/mass or pulley/spring systems.

Setting aside the study carried out on children—which nevertheless reveals some very interesting aspects pertaining to the basic reasoning about the moments of forces or distance/mass compensation—it is once again apparent that students' conception about systems or situations is highly dependent on the context and certain visual aspects: for example, it seems that most students believe that the horizontality of a fulcrum is the sole equilibrium criterion for systems that look like a balance scale (Ortiz et al., 2005) and hold the same belief about the identical height of two masses suspended on either side of a pulley (Gunstone, 1987).

As for the application of textbook criteria, rules, or methods such as the sum of forces and moments, these studies suggest that students have difficulty in applying all of them together (Ortiz et al., 2005; Newcomer & Steif, 2008) (i.e. they may check for equilibrium by using the sum of forces or the sum of moments, but not both) or fail to include all the forces in their reasoning, with some internal[5] forces being overlooked (Newcomer & Steif, 2008).

To date, only static equilibrium (within the field of mechanics) has been dealt with in the literature and its link with stability has yet to be considered. But while it is possible to scientifically separate these two concepts (i.e. to speak of equilibrium without speaking of stability), there is nothing to suggest that students knowingly and correctly adopt this strategy: on the contrary, classroom experience and interviews with students often reveal that they tend to mix up both terms (Pedreros Martinéz, 2013). Furthermore, many dynamic-equilibrium systems that students encounter in their everyday or academic life (e.g. the human body) are not thoroughly understood but are nonetheless readily considered as being as in equilibrium, stable, unbalanced, and so on. Such perceptions could give free rein to alternative interpretations.

In light of the above, we decided to specifically investigate students' perceptions of how equilibrium and stability are related. Our study examines the relationships between students and these concepts in an original context involving engineering students and a set of non-static situations. As the context dependence of student reasoning has already been demonstrated, our study has the potential to provide new information. The aim is to broaden the existing knowledge of the issue as well as to develop teaching strategies for overcoming the highlighted difficulties or for supplementing the current approach in various curricula. However, in this article, we only present the diagnosis part of the study and not the teaching design one. We seek to directly question the operational effectiveness of the physical definitions of these concepts and identify the factors that interfere with rational and relevant use of these definitions.

The following questions are addressed: How does a dynamic context influences the way in which student's reason about equilibrium and stability? Does such a context generate new ways of reasoning or alternative ideas other than those identified in the current literature?

Data Collection

Unlike the work by Newcomer & Steif (2008) and Ortiz et al. (2005), our study investigates students' opinions about the equilibrium and stability of a dynamic mechanical system.[6]

Theoretical Framework: The Facets of Thinking

This change from static to dynamic frame of reference must enable the identification of new *facets of knowledge* related to these concepts.

Indeed, in the field of research on students' conceptions and reasoning, a theory developed in the 1990s by Minstrell suggests connecting these conceptions and reasoning with the problems encountered by the students. This has led to the classification of individual modes of reasoning or "abstractions of what students say or do when confronted with a situation in which they are asked to predict or explain a physical phenomenon" (Minstrell, 1992b) called *facets of knowledge* or *facets*, that are grouped around situations or ideas (called *clusters*). This point of view, which is also shared by DiSessa (1993) in his theory of *phenomenological primitives* (p-prims), comes from the following observation: Although inaccurate with regard to generally accepted theories within a given period, alternative conceptions or modes of reasoning do nonetheless have some sort of structure in a person's system of thought. These alternatives to academic knowledge usually enable individuals to solve certain problems in precise situations and are thus of significant operational value, but they also constitute significant hurdles when teaching scientific theories in school. They are not based on scientific theory, but because they depend on the salient (or surface) aspects or the context of situations, they may be understood as constructions of the mind that come from more basic pieces of knowledge or *p-prims* (diSessa, 1988, 1993), which have limited organization and are often abstracted from common experiences.

Facets differ from p-prims in that they are less basic and therefore of a higher level. They are units of thought that are large enough to characterize and analyse students' ways of thinking for teaching or assessment purposes, and represent cognitive units of reasoning or strategy applied by students when addressing particular situations (Galili & Hazan, 2000). Therefore, they can arise from the combination of a number of p-prims, which accounts for variations in type: Some are very general and are directly derived from a single p-prim, such as *more implies more*, while others may be a combination of p-prims and are more context dependent (Minstrell, 2000).

Facets may be grouped into clusters, which are sets of interconnected facets for a given physical situation, idea, or concept (Minstrell, 1992a). In every cluster, each facet is assigned a number and organized according to an approximate order of development corresponding to the distance between the facet and the reference knowledge (0 for a close relation and 9 for the most distant level; for more details, please refer to Jim Minstrell's Web page cited in the references). This order also indicates a certain level of difficulty, for the student who shows it, in understanding the concepts of nearby clusters (i.e. of similar theme). This classification aims to guide teachers in the early identification and/or handling of these facets in the classroom in order to change them.

There is no specific cluster associated with either equilibrium or stability in the literature,[7] but several mechanics clusters include facets relating to these concepts that may be encountered in a context of dynamic equilibrium. The two most relevant examples are clusters *Forces to Explain the At Rest Situation* (41-) and *Forces During Interactions* (47-):

- 410 Balanced forces on an at rest object (vector sum is zero).
- 411 At rest and constant velocity are relative.
- 412 Balanced forces cannot apply to both constant velocity and constant position conditions of motion.
- 470 All interactions involve equal magnitude and oppositely directed action and reaction forces that are on separate interacting bodies.
- 474 Effects (such as damage or resulting motion) dictate relative magnitudes of forces during interaction.
- 474–1 At rest, therefore interaction forces balance.
- 474–2 Moves, therefore interacting forces unbalanced.
- 475 Equal force pairs are identified as action and reaction but are on the same object.

Some facets of clusters relating to motion may also provide a basis for identifying the ways in which students' reason presented with a dynamic system. These include the facets of the *Perception of Motion in Different Frames* cluster like *Component motions are added, but without discriminating accelerated motion from constant velocity motion* (362), as well as facets of acceleration-related clusters such as *Acc. is not differentiated from displacement or velocity ideas* (259).

We have chosen to adopt a methodology that is a cross between that of Newcomer & Steif (2008) and Ortiz et al. (2005) by presenting students with the same system in different situations (i.e. configurations), the objective being to identify common units of reasoning among the students for each of these situations as well as the differences between these units according to the situation with which they are associated. The situations proposed call upon the students' cognitive components and involve both abstract aspects (static/dynamic, inertial reference frame or not) and aspects related to the system's appearance or spatial configuration (vertical/horizontal/oblique). The former are dealt with based on the paper by Newcomer & Steif (2008), which brings to light aspects that are unrelated to the system's configuration, such as the failure to take into account internal forces. As for the latter, Ortiz et al. (2005) have already identified the specific kinds of thinking relating to the equilibrium of balance-type systems that we aim to demonstrate here (e.g. the existence of natural positions for a given system). It is possible to group different elements of a given concept within a specific cluster. However, this has not been done in the present study.

Assessment Tool: Multiple Choice Questions

The system proposed to the students consists of a compound pendulum pulled along by a moving trolley. Although it is used here as a prototype (see Fig. 1), this kind of device can actually be found in many laboratories and therefore encountered in control courses. The students were presented with a system that is both simple, by virtue of the fact that it includes two well-known mechanical systems (the pendulum and a moving trolley), and original, in that this novel combination of systems offers the opportunity for developing unclassical modes of reasoning that may unveil new facets or combinations of known facets.

Because the situation consists of two known systems, the students may be expected to have used their existing knowledge about the individual behavior of these systems. Indeed, student was faced with the pendulum in mechanics courses at least twice before the test: when they studied harmonic oscillators (and the movement equations of the pendulum) and energetic stability criteria.[8] In the same way, the study of the rectilinear uniform and accelerated movement is part of the undergraduate curricula. Thus, they were not faced with a completely new situation but a familiar academic context, which prevented them from being too overwhelmed and encouraged them to apply previously acquired procedural[9] (e.g. determination of a net force) and declarative (e.g. Newton's second law) knowledge.

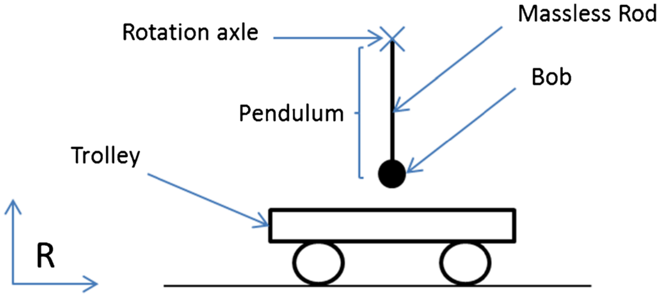


Fig. 1. The proposed system: a pendulum on a moving trolley

The situation was presented as a multiple choice questionnaire in paper form as a usual modality in such a research attends to map student reasoning in physics (and therefore involved static figures instead of a hands-on experience with the physical system). This may seem surprising given the dynamic nature of the systems, but it actually enabled simple application over a large scale while encouraging the conceptualization and application of a line of reasoning that is not directly derived from an observation of the system's true behavior. As pointed out by Albanese et al. (1998), simplified academic situations have been shown to reveal behavioral differences within the same system (or situation) of everyday life—differences that can sometimes be significant enough to lead to incomprehension among students. One example is the question of whether or not friction is present. However, many studies also show that the observation of a real phenomenon (whether natural or through experience), far from enlightening students, often lets them see only what their preconceptions lead them to observe (Champagne & Bunce, 1985; Gunstone & White, 1981). For example, in case of moving objects, as pointed out by Kariotoglou, Spyrtou & Tselfes (2009, p.857), a direct experience could drive students to automatically make a difference between motion and rest instead of considering one (rest) as a special case of the other (motion). Moreover, the presence of ambiguous sensory information,[10] such as the velocity or the acceleration of the trolley or pendulum, is an aggravating circumstance of this effect that can strongly skew the gathering of information about the system by influencing the orientation of students' erroneous conceptions (Brewer & Lambert, 1993).

Furthermore, we may observe that when students are presented with this kind of system, particularly because it contains an axis of rotation, they may overestimate the effect of the frictional forces and consider it to be the main cause of changes in the system in one direction or another; this then obscures the most important phenomenon here, which is related to the sum of the external efforts[11] and whether or not it is null. Additionally, it is interesting to note that it was partly this difficulty, encountered by Guidobaldo del Monte, Giovanni Battista Benedetti and Galilee during the sixteenth century (Duhem, 1905) when studying the equilibrium of the balance, that prevented a proper understanding of this phenomenon until Newton[12] finally grasped the concept. They wrongly attributed the characteristics of the equilibrium of the balance to uncertainties related to the quality of the pivot.

Mechanical Description of the System. From a mechanical point of view, the pendulum and trolley system are subjected to the force of gravity as well as the reaction of the pivot on the pendulum and the rails on which the trolley runs. As the trolley experiences translational motion with respect to the ground, it exerts a pseudo-force on the pendulum bob that is equal and opposite to its acceleration (to the nearest multiplicative factor). Applying the general theorems of classical mechanics (see for example Serway & Beichner, 2000) should lead students to include this pseudo-force when determining the net sum of forces and moments on the system, thereby enabling them to predict the behavior of the pendulum and the trolley for the given parameters of motion (velocity and acceleration of the trolley with respect to the ground and those of the pendulum with respect to the trolley). The equilibrium conditions of the trolley with respect to the ground and those of the pendulum with respect to the trolley and the ground are summarized in Table 1.

MCQ Structure

The questionnaire is composed of two parts.

Part 1. The first part of the questionnaire involves two equilibrium situations for the same system (see the figures in Table 2). The characteristics of the questions and their reference codes are given in the table for the corresponding figure. The students had to choose answers for the velocity and acceleration of the trolley with respect to the ground—constant, zero, or non-zero—corresponding to an equilibrium state of the pendulum for each of the system configurations (A and B). With regard to the assertions presented in the “necessary condition” column, they were asked if they (i) agreed, (ii) disagreed, or (iii) did not know. In actual fact, both situations involve the same equilibrium conditions, namely zero acceleration of the trolley with

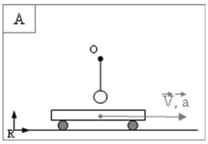
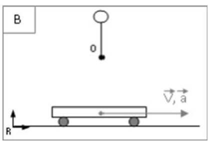
TABLE 1

Relationships between pendulum equilibrium and trolley motion characteristics

	Trolley motion in terrestrial frame:		
Pendulum state	*Stationary (i.e., at rest)*	*Constant velocity (≠ 0)*	*Constant acceleration (≠ 0)*
In equilibrium in terrestrial frame	Yes	Yes	No
Stationary in terrestrial frame	Yes	No	No
In equilibrium in trolley frame	Yes	Yes	Yes
Stationary in trolley frame	Yes	Yes	Yes

TABLE 2

Structure of Questionnaires and Associated Codes and Figures for Part 1

Situation reference	*Equilibrium*[a] *state*	*Frame of reference*	*Necessary condition*[b] *(question)*	*Code*
	Stable	Inertial	Zero velocity Constant velocity Zero acceleration Constant acceleration	AVN AVC AAN AAC
	Unstable	Inertial	Zero velocity Constant velocity Zero acceleration Constant acceleration	BVN BVC BAN BAC

[a]In the trolley reference frame
[b]For the trolley motion

respect to the ground, which implies constant (zero or non-zero) velocity for the system.

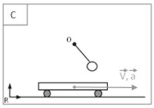
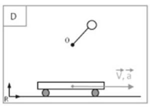
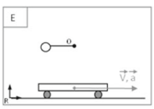
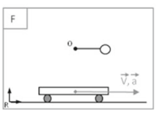
To reach this conclusion, the students had to correctly apply appropriate criteria, such as the criterion of nullity for the sum of the external forces and moments on the system. Any other answer provides information about the (erroneous) reasoning used, which must be directly related to the position of the pendulum (low or high) because it is the only system variable that differs between the two situations.

Part 2. The second part of the questionnaire comprises of four situations (see the figures in Table 3) involving the same system. In two of these situations (C and D), the velocity of the trolley with respect to the ground cannot be zero if the pendulum is to be in equilibrium. The other two situations (E and F) are physically impossible. We included a statement specifying that the velocity and the acceleration of the trolley were not zero in any of these situations. We then asked the students if they thought that the proposed positions were feasible situations of equilibrium (yes/no/I don't know) and whether they were stable (yes/no/I don't know).

The students were also asked to justify their answer. The correct answers to all these situations are obtained by simply applying the criterion of nullity to the net sum of forces and moments. However, the pendulum-on-trolley configuration may once again be expected to have

TABLE 3

Structure of questionnaires and associated codes and figures for Part 2

Situation reference	Equilibrium state [a]	Frame of reference	Necessary condition (question) [c]	Code
	No	Non-inertial	Equilibrium	CE
			Stability	CS
			Justification [b]	CJ
	Unstable	Non-inertial	Equilibrium	DE
			Stability	DS
			Justification [b]	DJ
	No	Non-inertial	Equilibrium	EE
			Stability	ES
			Justification [b]	DJ
	No	Non-inertial	Equilibrium	FE
			Stability	FS
			Justification [b]	FJ

[a]In the trolley reference frame
[b]In the case of non-equilibrium
[c]For the trolley motion

influenced the students' reasoning in cases where this criterion was not fully operational

These situations should allow us to detect the influence of certain aspects, for example if the forward position of the pendulum was systematically rejected, which could indicate a direct influence of the velocity of the trolley on the calculation of the net force, when in fact all that matters here is the acceleration (velocity/acceleration identification, Facet 259).

Data Analysis

Part 1. In this part, a correct answer is the {AVC, AAN, BVC, BAN} quadruplet (see the coding for every answer in the table) comprising of answers to case A {AVC, AAN} and those to case B {BVC, BAN}. In order to observe changes in each student's reasoning according to the situation (A or B), we defined typical profiles or "patterns" of answers and counted the number of students in every profile for each situation. Below is the list of profiles that were used:

- RRA: Right Responses for situation A corresponding to AVC, AAN
- RRB: Right Responses for situation B corresponding to BVC, BAN
- RRP1: Right Responses for the whole of Part 1 corresponding to AVC, AAN, BVC, BAN
- ImA: Immobility for situation A
- ImB: Immobility for situation B
- CA: Consistent Answers

Certain predictable profiles were defined even before the data were processed; these include profiles corresponding to the right answers for situation A, B, or both A and B (namely RRA, RRB, and RRP1, respectively). They correspond to facets relating to the correct understanding of mechanical equilibrium as being the zero sum of all the external forces and moments on the system (which implies a correct understanding of the acceleration of the trolley).

A facet corresponding to the understanding of equilibrium only in cases where the system is immobile (zero velocity of the trolley with respect to the ground) is also predictable and is associated with the ImA and ImB profiles. It is related to difficulties that students have understanding on mechanical equilibrium in cases of rectilinear and uniform motion, which were previously reported in the literature.

In order to detect inconsistencies in the students' answers with regard to the choice of reasoning strategy, the CA profile was created. Indeed, we can expect the students to have answered in a certain way in situation A by using a particular scheme of thought based on a certain combination of constituent p-prims of a given facet, and we may also expect them to have used the same reasoning or combination in situation B, thereby leading to a predetermined answer. Thanks to this profile, it was possible to detect the use of the same scheme in both situations or deduce the appearance of new facets otherwise. Thus, we expected to observe the following facets in this first part:

- Facet 0: Equilibrium is explained by the sum of forces and moments being zero
- Facet 1: Equilibrium is explained by the sum of forces being zero
- Facet 8: Equilibrium exists only if v=0
- Facet 9: Equilibrium exists only if the system is in a natural position

One can notice that ones of those facets can be linked to another existing cluster related to "At Rest" or "Motion" but the ultimate one, for example, is more specific. Thus, it could be interesting to create a new cluster for those facets of thinking.

Part 2. The joint action of situations involving a non-inertial reference frame (non-zero acceleration) should reveal the influence of motion-related facets on equilibrium-related ones. Similarly, the question about stability should make it possible to detect whether, in the students' minds, the concepts of equilibrium and stability have a mutual influence on each other. The MCQ coding developed for Part 1 was employed in conjunction with a specific method for processing the students' justifications, similar to that used by Newcomer & Steif, (2008). As there are currently no clusters on stability in the literature, we analysed the students' arguments to extract lexical information that might point to previously identified or new facets. The first facet of this new cluster is that which best corresponds to the piece of scientific knowledge: "Stability is a characteristic of equilibrium".

Furthermore, we may expect that when justifying their answer, students used either an energy-based criterion involving the derivative of potential energy or a criterion related to the sum of the external forces and/or moments. The use of the first criterion was more foreseeable on the account of the students' educational background and the fact that the second criterion is more complex to handle (indeed, the latter criterion provides information on equilibrium first, and in order to determine the stability of this state of equilibrium, one must consider how the sum of forces and/or moments would change during a movement about the equilibrium position). A second facet could thus be one relating to an energy criterion: There is stability when potential energy is minimum. We gathered this information under two categories: argument criteria such as potential energy or net force and elements of incomplete answers such as speed, acceleration, or weight. For every element (speed, acceleration, weight, net force or net moment, and potential energy), we identified and counted the cases where they were given directly, cases where they were not (given directly), and cases where their use could be deduced; this was carried out for every student. For example, for a student who mentioned acceleration, we considered this argument to be an element of a partial direct answer, acceleration; additionally, if we deduced the use of a net force in the justification, we included it in the net argument criterion. Other arguments were grouped accordingly and, when present in sufficient numbers, enabled the creation of new facets.

Findings

Part 1 was presented to 51 French students at the university level (L3) while Part 2 was presented to 80 students at the same level. Such students are generally considered to have mastered these concepts, having previously learnt them and being required to apply them in such programs as control courses. Their level of competence in physics is acknowledged as being good on account of the mathematics courses they would have taken as part of the "classes préparatoires aux grandes écoles" curriculum prior to enrolment in a Grande Ecole (French Engineering School).

Part 1

Descriptive Statistics. We began by counting the students' answers to each of the two questions in Part 1 of the questionnaire (see Table 4 for situations A and B).

We may observe that the percentage of correct answers is different for the two situations and also depends on the physical variable being considered (speed or acceleration). A χ^2-test shows that the difference between the overall results for the two situations is statistically significant (p value=0.0002, modified Pearson test). As this is not related to a random effect, we can infer that the students did not use the same reasoning to handle both situations. Furthermore, given that the only variable that changes between situations A and B is the pendulum configuration, we can hypothesize that it is this difference in configuration that led to a change of strategy among the students. A subsequent analysis of the profiles enabled us to identify the factors or groups of factors that changed from one case to another for each student.

TABLE 4

Results for Part 1, cases A and B. Correct answers are highlighted (*V* velocity, *A* acceleration, *N* null, *C* constant), n=51

Answer	*AVN*	*AVC*	*AAN*	*AAC*
Yes	12 (23.5 %)	44 (86.3 %)	44 (86.3 %)	7 (13.7 %)
No	39 (76.5 %)	7 (13.7 %)	7 (13.7 %)	38 (74.5 %)
IdK	0	0	0	6 (11.8 %)
Ans.	BVN	BVC	BAN	BAC
Yes	23 (45 %)	25 (49 %)	(62.7 %)	11 (21.5 %)
No	27 (53 %)	25 (49 %)	19 (37.3 %)	38 (74.5 %)
IdK	1 (2 %)	1 (2 %)	0	2 (4 %)

According to Fig. 2, only 17.65 % of the students correctly answered the whole of Part 1 (RRP1) and situation B (RRB) was the most problematic. This finding is consistent with the overall statistical analysis. Yet, although situation A appears to have been less confusing to the students, only 55 % of them answered correctly (see Table 2), and out of the 45 % who gave a wrong answer, 34 % assumed that the trolley was immobile (AVN). The additional information provided by this profile analysis is that 24 % of the students changed their minds about the immobility of the trolley between situations A and B (see percentage of ImB/A) and only 33 % answered in a consistent manner, that is by considering that the same conditions of velocity and acceleration applied to both cases in order to justify equilibrium. Thus, 66 % or 34 out of 51 students thought that equilibrium in both cases could not be understood by the same conditions of velocity and/or acceleration, which confirms that the students did not use the same strategies in their line of thought for situations A and B.

Out of these 34 students, 53 % considered that the trolley needed to have zero velocity in situation B in order for the pendulum to be in equilibrium, which corresponds to 35 % of all the students with whom we may associate facet 8 (zero velocity for equilibrium). Among the other respondents, no other profile could be identified except for facet 1, which demonstrates the proper use of a sum of forces leading to a correct answer. We can deduce the influence of the stability of the pendulum on the students' reasoning because we know that the respondents had, at the

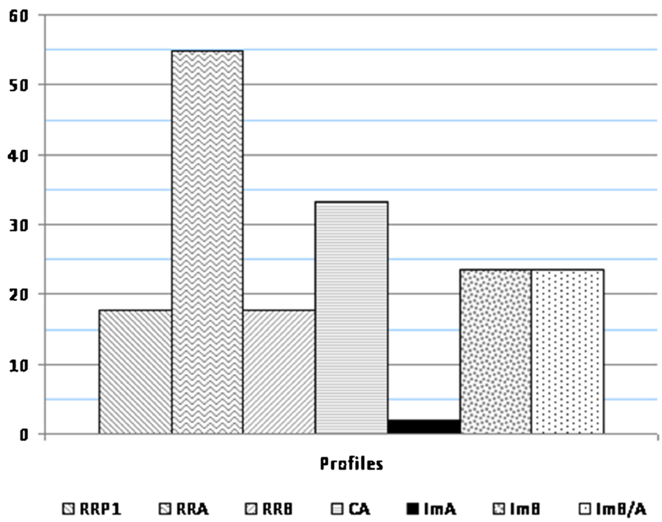


Fig. 2. Frequencies in each profile for Part 1 (expressed as a percentage of all answers)

very least, declarative knowledge of the stability conditions of the pendulum. More specifically, the lower equilibrium position is stable while the higher equilibrium position is unstable.

Part 2

As the question in Part 2 refers directly to the concept of stability, we were able to obtain more precise information about the relationships between stability and equilibrium in the minds of the students. Table 5 shows that the percentage of correct answers to item S (stability) is higher than the percentage of Yes answers for item E (equilibrium) of the corresponding situation. This amounts to 43 students in situation C, 44 in D, 13 in E[13], and 35 in F, out of all 80 students. These students believed that the pendulum could be stable without being in equilibrium. These data indicate that almost half of the respondents did not perceive stability to be a characteristic of equilibrium and believed that it could very well be independent of the latter—a rather unexpected finding that gave rise to a new facet for the cluster stability: stability is independent of equilibrium.

As regards equilibrium itself, we observe that the percentage of No answers pertaining to equilibrium is similar in both situations C and D; in fact, the latter corresponds to a realistic equilibrium position while the former does not (because of the direction of the acceleration of the trolley). We can thus conclude that these students used a line of reasoning other than that involving the sum of the external forces on the system. Furthermore, in situation D, equilibrium in the upper position was mainly considered to be impossible (for 72 % of the students), which is consistent with the finding obtained in Part 1: The upper equilibrium position does not seem to resist any trolley motion, be it uniform or uniformly accelerated. This information led to the identification of an additional facet for unstable equilibrium in cases of non-zero velocity: unstable

TABLE 5

Unprocessed Results for Part 2, cases C, D, E, and F (*E* realistic equilibrium, *S* stability). Correct answers are highlighted. n=51

	CE	*CS*	*DE*	*DS*	*EE*	*ES*	*FE*	*FS*
Yes	18 (22.58)%	11 (17.46 %)	21 (26.25 %)	5 (7.25 %)	61 (76.25 %)	45 (62.50 %)	26 (32.50 %)	12 (18.46 %)
No	62 (77.5 %)	49 (77.78 %)	58 (72.50 %)	61 (88.41 %)	19 (23.75 %)	25 (34.72 %)	51 (63.75 %)	50 (76.92 %)
IdK	0	3 (4.76 %)	1 (1.25 %)	3 (4.35 %)	0	2 (2.78 %)	3 (3.75 %)	3 (4.62 %)
Total	80	63	80	69	80	72	80	65

equilibrium only exists if v=0. Finally, we observe that the proportions of Yes and No answers pertaining to equilibrium in situations E and F are almost opposite, as are the positions of the pendulum. Once again, this suggests that the direction of the system's motion, and more specifically that of its velocity rather than its acceleration, played an important role in the reasoning used by the students and led to a "non-symmetric" handling of the situations.

Table 6 summarizes the number of arguments used by the students to justify their answers. Energy-based arguments are few in number; those related to the use of a net force are more common, but in fact only three students applied it properly and were able to obtain the correct answers in all the situations.

By allocating weights to the various arguments according to how close they are to the correct reasoning (or facet) that is efficient for the situation (−2 for a velocity-type argument, −1 for an implicit velocity argument, 0 for the absence of an argument, 1 for an implicit net-force argument, and 2 for an explicit net-force argument or a potential-energy argument), we were able to search for correlations between the situations and the type of argument used. This coding is in agreement with our purpose, which is about the presence or not of arguments in relation to some conceptions and how are those arguments close to the "right" conception. On the whole, a significant link between the situations and the arguments was identified (χ^2-test, p value= 0.003). In fact, a Wilcoxon (paired) signed rank test revealed a significant difference between situations C and D (p value=0.002). We then grouped the situations according to the type of argument used, which gave rise to two possible partitions (highlighted in Table 6):

1. {C,E} and {D,F} if we consider arguments relating to the movement of the trolley (mainly a velocity argument)
2. {C,D} and {E,F} if we consider the use of an argument criterion

TABLE 6

Arguments used for Part 2: Number of arguments and percentage of total arguments (Tot. Arg.) used, n=51

Case	*Velocity*	*Acceleration*	*Weight*	*Net force*	*PE*[a]	*Tot. Arg.*
C	22 (27.5)%	12 (15 %)	4 (5 %)	4 (5 %)	0	42
D	9 (11 %)	6 (7.5 %)	10 (12.5 %)	5 (6.2 %)	2 (2.5 %)	32
E	20 (25 %)	17 (21 %)	10 (12.5 %)	14 (17.5 %)	0	61
F	13 (16 %)	9 (11 %)	12 (15 %)	12 (15 %)	0	46

[a]Potential energy

If we compare these groups to the salient aspects of the situations, in the second grouping, we can hypothesize that the horizontal position of the pendulum would more frequently involve the use of a net force—which is an effective strategy in every case—without necessarily enabling students to arrive at the correct answer, probably because of a failure to correctly interpret the results of their calculation. Therein lies a manifestation of the "equilibrium = horizontal position" facet, which in this case would lead students to use the most commonly observed criterion for considering the equilibrium of a balance. Finally, if we match the arguments to each case, we may form a group with situations C, D, and F. Indeed, in these situations, there is a statistically significant correlation between the direction of the pendulum and the use of the velocity criterion to refute the hypothesis of equilibrium (χ^2-test, p value= 0.0009). If we compare this to the abovementioned second grouping, we may observe a manifestation of the equilibrium = natural position facet (alongside its negation/generalization: "not in equilibrium because not in a natural position"). Indeed, the arguments of some of the students refer to this rather directly: logic according to the direction of motion or wrong side.

The differences between the answers to these six situations reveal specific difficulties that the students experienced with the concepts of equilibrium and stability. To begin with, it seems that the link between these two concepts is not standardized: although stability is, in physics, a characteristic of equilibrium, half of the students in the study perceived it as being unconnected with the latter. Furthermore, the students had difficulty in perceiving the unstable equilibrium position as being created by the same conditions as those for the stable position. They found it hard to combine the notions of equilibrium with the motion of the system being studied. They did, however, make a distinction between the conditions that give rise to a stable equilibrium position and those leading to an unstable position. The students seemed to use explanatory strategies that depended on the configuration of the system. At times, these strategies were appropriate (e.g. the determination of a net force), while at others, they were derived from empirical rules (e.g. the influence of velocity). This led to a certain asymmetry in the understanding of the concept of equilibrium and to the disadvantage of the unstable equilibrium position. We observe that in the case of traction, the entire notion of equilibrium disappeared, almost as if stability was perceived as a quality enabling equilibrium to remain in equilibrium in response to external disturbances (motion), rather than as an additional characteristic of this state of the system.

Discussion

The chosen population sample was comprised of students who had attended a comprehensive control course—a field of engineering that involves the analysis and control of dynamic systems. Equilibrium and stability are fundamental concepts in this discipline. In fact, one of the very skills required of a control engineer is the ability to identify the equilibrium points of any system and, wherever possible, to stabilize a naturally unstable system at these points. Indeed, the pendulum-on-trolley setup is a widely used system in this area because it calls upon several important aspects of systems control. It is a second-order system with two equilibrium points: one stable (lower vertical position and zero angular velocity) and the other unstable (upper vertical position and zero angular velocity). With the help of an appropriate regulation system, one generally attempts to stabilize the pendulum in the upper vertical position by controlling the trolley movement, which provides the opportunity to use all the available mathematical tools in a control engineer's toolkit: system modeling, search for equilibrium states, linearization, control by state feedback, and so on.

Because the operation of the pendulum or other simple (from a mechanical point of view) systems is considered to be thoroughly understood at university level, the focus is generally on the establishment of (differential) equations for the system and the mathematical study of its properties. Thus, equilibrium and stability are often approached from an essentially mathematical point of view, through the application of algebraic (calculation of roots of polynomials, eigenvalues of matrices, etc.) or graphical (root locus, Nyquist criterion, etc.) criteria. Yet, these concepts appeared to be foreign to the students of our sample, despite the latter having encountered them at least twice in recent years (in high school and the first year of engineering school). What have been demonstrated through this research is that students failed to make the connection between their experience of these concepts in Newtonian mechanics and the mathematical basics presented in control courses. We believe that the reason for this is their limited operational knowledge of these concepts from the mechanical point of view, which has been observed to stray very far from true scientific knowledge. This necessarily has consequences, and difficulties have indeed been observed when students are asked to study real and complex electromechanical systems (e.g. a battery-driven vehicle (Rajamani, 2006)), as a thorough understanding of their operation, i.e. behavior, is required in order to establish the right equations.

The concepts of equilibrium and stability in non-mechanical systems are encountered in one's early school years, for example in chemistry or physical education classes, both in France and in English-speaking countries. Thus, it is perfectly possible for these different viewpoints and approaches to influence students' understanding of these concepts or they way in which they develop ideas about the latter (e.g. conceptions). This is especially true if no academic connections are made between these various elements. Thus, we believe that a coherent and unified teaching approach that draws examples from a variety of fields can be a solution for improving students' understanding of this subject.

NOTES

[1] For a broader concept of a system and equilibrium in various fields, see Von Bertalanffy (1969)

[2] Terminology from the Control Systems Theory, which in the case of the balance scale, refers to a manually induced change in position of the fulcrum.

[3] Forces and moments of these forces.

[4] Stability does tend to be introduced in the later stages of schooling.

[5] When they are actually external forces with respect to the system, for example the reaction force exerted by a pivot.

[6] One that is in accelerated motion with respect to a given frame of reference.

[7] Minstrell's Web page contains a compilation of all the referenced facets and clusters

[8] See « Programmes des classes préparatoires aux Grandes Ecoles » for example at http://www.enseignementsup-recherche.gouv.fr

[9] (Solaz-Portolés & López, 2008).

[10] Information that is subject to various interpretations because direct measurement is impossible.

[11] Moments, forces, and pseudo-forces.

[12] 1642–1727

[13] this lower number can be explained by the fact that this situation mainly appeared to be in equilibrium, which translated into fewer No answers compared to the other situations

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