Equilibrated Warping: Finite Element Image Correlation with Mechanical Regularization

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Context. Image correlation/registration is playing an increasing role in many domains such as biomedical engineering [1]. Despite significant progress in the past decades, robustness, efficiency and precision of existing methods and tools must still be improved to translate them into medical and engineering applications. This abstract describes a finite element-based image correlation method with, as regularization, a novel continuum large deformation formulation of the equilibrium gap principle (introduced in [2] at the discrete level for linearized elasticity).

Methods. Let us denote I_0 & I_t as the intensity fields of two images representing an object occupying the domains Ω_0 & Ω_t in the reference and deformed states, respectively. The problem is to find the mapping $\underline{\varphi}$ between Ω_0 & Ω_t, or equivalently the displacement field $\underline{U}$ ($\underline{\varphi}\left(\underline{X}\right) = \underline{X} + \underline{U}\left(\underline{X}\right)$):

$$\text{find } \underline{U} = \text{argmin}_{\{\underline{U}^*\}} \left\{ J^2\left(\underline{U}^*\right) = \frac{1-\beta}{2} \int_{\Omega_0} \left(I_t \circ \underline{\varphi}^* - I_0\right)^2 d\Omega_0 + \beta \psi^{\text{reg}}\left(\underline{U}^*\right) \right\}, \tag{1}$$

where ψ^{reg} is required to regularize the otherwise ill-posed problem, and β is the regularization strength. Many regularizers have been proposed. One common approach, called hyperelastic warping [3], is to use the strain energy potential directly, i.e., $\psi^{\text{reg,hyper}} = \rho_0 \psi$, thus penalizing strain. Here we propose an alternate regularizer, which essentially penalizes any deviation from the solution of an hyperelastic body in equilibrium with arbitrary external loads: $\psi^{\text{reg,equil}} = \frac{1}{2} \left\|\underline{\text{div}}\left(\underline{\underline{P}}\right)\right\|^2_{L^2(\Omega_0)}$, where $\underline{\underline{P}} = \frac{\partial \rho_0 \psi}{\partial \underline{\underline{F}}}$ is the first Piola-Kirchhoff stress tensor. However, we discretize Problem (1) using standard Lagrange elements, so that $\underline{\underline{P}}$ belongs to $L^2\left(\Omega_0\right)$ but not $H\left(\text{div}; \Omega_0\right)$. Thus, the following equivalent norm is used instead:

$$\psi^{\text{reg,equil}} = \sum_K \frac{1}{2} \left\|\underline{\text{div}}\left(\underline{\underline{P}}\right)\right\|^2_{L^2(K)} + \sum_F \frac{1}{2h} \left[\!\left[\underline{\underline{P}} \cdot \underline{N}\right]\!\right]^2_{L^2(K)}, \tag{2}$$

where K denotes the set of finite elements, F the interior faces, h a characteristic length of the mesh.

Results on synthetic data. Here we consider the simple problem of a uniformly compressed square domain, and study the influence of the regularization strength β on the computed strain. Figure 1 shows the initial and final images superimposed with the undeformed and deformed mesh obtained.

In case of hyperelastic warping, if the regularization strength is close to 1, the mesh does not deform. And when regularization strength decreases, measured strain converges toward the exact value. For noise-free images, it does converge exactly. For noisy images, there is an optimum where the mean strain is close to the exact solution and standard deviation is still limited.

Conversely, with equilibrated warping, the registration is almost perfect over a wide range of regularization strengths, even on noisy images.

Results on *in vivo* images. Here we consider 3D CSPAMM cardiac magnetic resonance images of a healthy human subject. Figure 2 shows the resulting strains computed by both methods. Main features of left ventricular deformation are well captured. Equlibrated warping produces larger absolute strain values than hyperelastic warping, closer to expected values [1].

Conclusion. Equilibrated warping is a powerful method for non-rigid registration of images involving large deformation. Penalizing the equilibrium gap regularizes the image correlation problem, even in presence of noise, and without affecting strain measurement. The method has been implemented based on FEniCS and VTK, providing an efficient tool for 2D & 3D images registration.

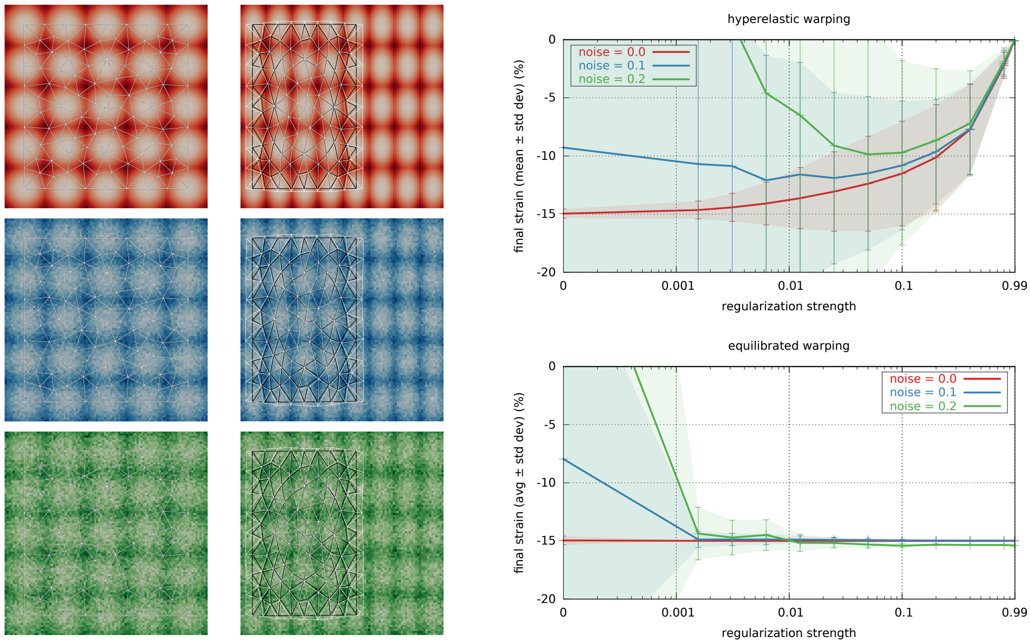

Figure 1: Results on synthetic data. Left: hyperelastic (white mesh) *vs.* equilibrated (black mesh) warping, for a regularization strength of 0.1. Right: influence of regularization strength on hyperelastic (top) and equilibrated (bottom) warping strains. Ground truth is -15% homogeneous strain.

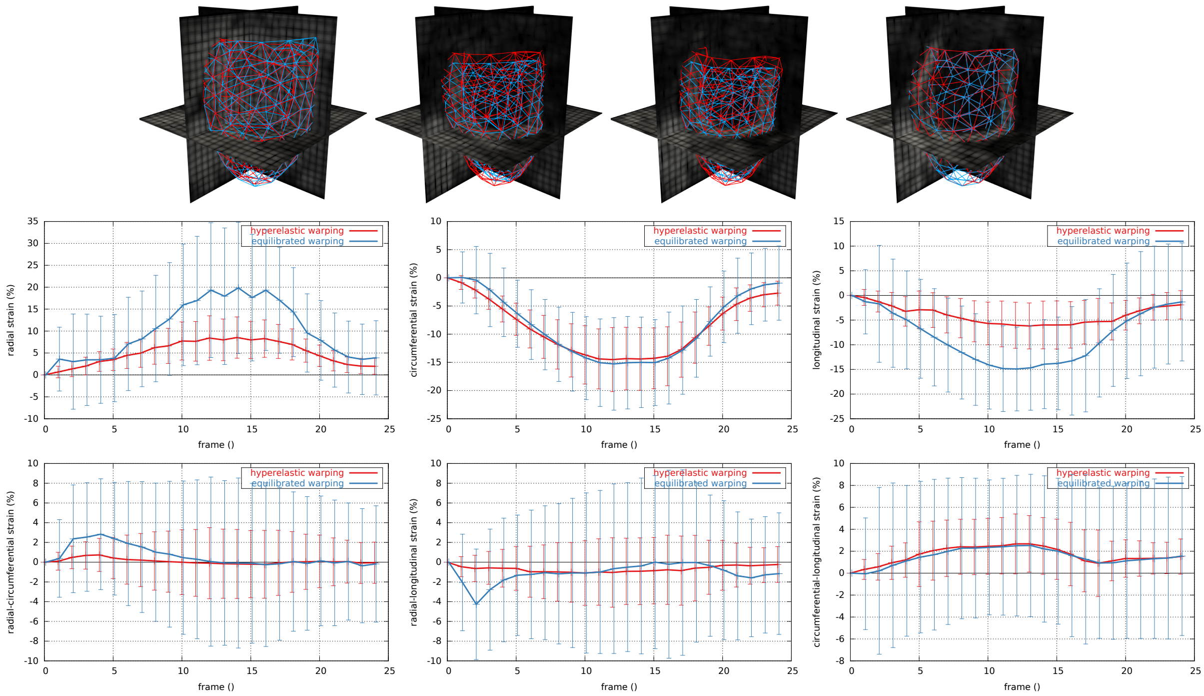

Figure 2: Results on *in vivo* data, hyperelastic (red) *vs.* equilibrated (blue) warping. Top: Sequence of 3D CSPAMM images with superimposed mesh. Bottom: Sequence of local strain components.

References

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