A simple spiking neuron model based on stochastic STDP
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1. Framework

STDP: Thought to be responsible for memory, synaptic plasticity is the change of strength of neuron’s links. Popular plasticity models are based on Spike Timing Dependent Plasticity (STDP):

Hebb’s law (1949): “When an axon of cell A [spikes] repeatedly and persistently takes part in firing cell B [spikes]—i.e., if A’s firing is a constant concomitant of B’s firing—then A and B may come to be functionally connected.”

Problem: Current models use deterministic plasticity rules whereas the biological mechanisms involved are mainly stochastic ones. Moreover, there exists few mathematical studies taking into account the precise spikes timings. Finally, there is a need to understand how to bridge the time scale gap at the synaptic scale and how weights dynamics interplay with the network one.

Novelty: Stochastic STDP rule with discrete synaptic weights which allows a mathematical analysis of their dynamics.

2. Model constraints

- Rich enough to reproduce biological phenomena
- Simple enough to be analyzed mathematically and simulated
- Observe global properties of the network due to neurons firing

3. Neuronal Network Model

Individual neuron: Simple model for the membrane potential [3]

\[ V(t) = \frac{1}{C} \int_{0}^{t} I(t) \, dt + \frac{1}{C} \int_{0}^{t} \frac{dV}{dt} \, dt \]

Dynamic of \( V \):

- \( \beta \): constant
- \( \alpha (v, w) = f \left( \sum_{j=1}^{N} W_{ij} v_{j} \right) + \omega_{v0} \) with \( f(x) = \max(0, x) \)

Remark: \( \alpha \) depends on current neurons states and weights

Dynamic of synaptic weights \( W \):

Weights have probability to change only when a neuron jumps from 0 to 1:

\[ \frac{dW_{ij}}{dt} = \alpha (v_{i}, w_{ij}) - \beta W_{ij} \]

5. Simulations

Biologically coherent parameters:

If even simple, our model depends on many parameters. First, let’s detail the probability to jump:

\[ p_{w}(s) = \frac{1}{2} \quad \text{and} \quad p_{v}(s) = \frac{1}{2} \]

Such functions enable to be close to biological experiments.

Parameters for the figure are: \( A_{1} = 1, A_{2} = 0.5, r_{w} = 2 \), \( r_{v} = 4 \) as in [6]. These parameters have been fixed to the first ones. Time of influence of a spike 1ms so \( \beta = 1 \). Firing rates of neurons are bounded by \( \alpha_{w} = 0.01 \) and \( \alpha_{v} = 0.5 \). STDP parameters are in the following range: \( \tau_{w,j} \in [0.01, 0.1] \), \( \tau_{v,j} \in [0, 1] \). Finally, \( \epsilon [0, 0.01, 0.001] \).

Analytic versus Numeric:

First, we wanted to visually show our limit model is licit. In simulations, an easy value to get is the sum of jump rates of weights:

\[ \sum_{i,j} \tau_{w,j} + \sum_{i,j} \tau_{v,j} = \sum_{i,j} (\tau_{w,j} + \tau_{v,j}) \]

We get similar results in the case of 2 neurons. In higher dimension it is hard to get equivalent analytic and numerical precision.

Weight divergence:

A big problem in plasticity models is the divergence of weights. We could have put hard bounds or soft bounds but we wanted to see in which limits weights diverged without them. We tested some criteria of non divergence of weights in our model. In particular in the article [7], such a criteria is to have the integral of the learning window is negative:

\[ \sum_{i,j} \tau_{w,j} + \sum_{i,j} \tau_{v,j} = \sum_{i,j} (\tau_{w,j} + \tau_{v,j}) \]

In that particular case, \( \lim_{t \to \infty} \phi_{w,\tau_{v,j}}(w)=\phi_{w,\tau_{v,j}}(w) \). We can prove the process converges to its unique invariant measure if \( \sum_{i,j} \tau_{w,j} + \sum_{i,j} \tau_{v,j} < 0 \). We computed, thanks to (1), the difference \( r_{w}(w) - r_{v}(w) \). It depends on \( \tau_{w,j} \) and \( \tau_{v,j} \). We found parameters: \( A_{1} = 0.8, A_{2} = 0.7, \tau_{w,j} < 0.7 \), for which \( w_{ij} \) diverges when the integral of the learning window is negative:

\[ \int_{0}^{\infty} \left( \sum_{j=1}^{N} \tau_{w,j} + \sum_{j=1}^{N} \tau_{v,j} \right) \, dt = \text{constant} \]

6. Conclusion

We showed divergence of weights even when integral of the learning window is negative. Additive terms, depending on weights, seem necessary to avoid divergence in the context of bi-ological parameters. However, our first mathematical results are encouraging for deeper study and our model showed more interesting behavior than those already presented: bidirectional as unidirectional connections can be strong.

7. Perspectives

Maths:

- Weight dynamics
- Mean field approximations

Modelling:

- Simulations to test other plasticity rules
- Neurps states from discrete to continuous

References