A simple spiking neuron model based on stochastic STDP

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1. Framework
STDP: Thought to be responsible for memory, synaptic plasticity is the change of strength of neuron’s links. Popular plasticity models are based on Spike Timing Dependent Plasticity (STDP):
Hobb’s law (1949): “When an axon of cell A, B, is excited it excites, or at any rate, potentiates, all the cells it reaches, it excites or potentiates so as to increase the probability of its discharging; if it is more or less completely excited, the probability of its discharging is increased” [1]

Problem: Current models use deterministic plasticity rules whereas the biological mechanisms involved are mainly stochastic ones.
Moreover, there exists few mathematical studies [2] taking into account the precise spikes timings. Finally, there is a need to understand how to bridge the time scale gap at the synaptic level and how weights dynamics interplay with the network one.

Novelty: Stochastic STDP rule with discrete synaptic weights which allows a mathematical analysis of their dynamics.

2. Model constraints
- Rich enough to reproduce biological phenomena
- Simple enough to be analyzed mathematically and simulated
- Observe global properties of the network due to neurons firing

5. Simulations
Biologically coherent parameters: Even if simple, our model depends on many parameters. First, let’s detail the probability to jump:

\[ p^+(s) = A \cdot e^{-s} \quad \text{and} \quad p^-(s) = A \cdot e^{s} \]

Such functions enable to be close to biological experiments.

Parameters for the figure are: \( A = 1, \ldots, 0.9, \tau = 2, w_{0} = 34 \) as in [6]. These parameters have to be added to the first ones: \( \tau_{A}, \alpha, \beta, \gamma \). Time of influence of a spike 1ms so \( \tau = 1 \). Firing rates of neurons are bounded by \( \alpha_{A} = 0.01 \) and \( \alpha_{B} = 0.1 \). STDP parameters are in the following range: \( \tau_{A} \in [1, 40] \), \( \alpha_{j} \in [0, 1] \).

Analytic versus Numeric: First, we wanted to visually show our limit model is licit. In simulations, an easy value to get is the sum of jump rates of weights:

\[ \sum_{i} \sum_{j} \alpha_{i} \]

We get similar results in the case of 2 neurons. In higher dimension it is hard to get equivalent analytic and numerical precision.

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6. Conclusions
We showed divergence of weights even when integral of the learning window is negative. Additive terms, depending on weights, seem necessary to avoid divergence in the context of biologically parameters. However, our first mathematical results are encouraging for deeper study and our model showed more interesting behavior than those already presented: bidirectional as unidirectional connections can be strong.

3. Neuronal Network Model

Individual neuron: Simple model for the membrane potential [3]

At time 0, the neuron is at \( V_{0} \) and \( a_{i} \) equal to 1.

Dynamic of \( V_{t} \):

\[ \frac{d}{dt} V_{t} = \frac{1}{\tau} (w_{12} \cdot r_{12} + w_{21} \cdot r_{21}) - \frac{1}{\tau} V_{t} \]

In that particular case, \( \lim_{r_{21} \to 0} \frac{d}{dt} V_{t} = \lim_{r_{21} \to 0} V_{t} \), exist and we can prove the process converges to its unique invariant measure if \( R_{E} - R_{I} < 0 \). We computed, thanks to (1), the difference \( \tau_{r} \cdot e^{-r} \) and \( \tau_{r} \cdot e^{r} \). It depends on \( w_{12} \) and \( w_{21} \). We found parameters:

\( A_{1} = 0.8, \quad A_{2} = 0.7, \quad w_{12} < 0, \quad \text{for which} \quad w_{12} \text{diverges when} \quad \text{the integral of the learning window is negative:} \]

\[ V_{t} \]

7. Perspectives

Maths:
- Weight dynamics
- Mean field approximations

Modelling:
- Simulations to test other plasticity rules
- Neuronal states from discrete to continuous

Markov process \( (V_{t}, S_{t}, W_{t})_{t \geq 0} \) between \( (v_{i}, s_{i}, w_{i}) \) and \( (v_{j}, s_{j}, w_{j}) \).

\[ W_{t} \in \mathbb{R}^{2}, \text{matrix of synaptic weights} \]

\[ S_{t+1} = S_{t} + \text{last spike of neuron} \text{ occurred at time} \quad t - S_{t} \]

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