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Cut algorithm for the Multi-Trip Vehicle Routing Problem with Time Windows

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1 Introduction

The Vehicle Routing Problem with Time Windows (VRPTW) deals with the determination of the trips of a homogeneous fleet of capacitated vehicles based at a central depot to deliver a set of customers, while complying delivery time intervals imposed by these latter. The Multi-Trip VRP with Time Windows (MTVRPTW) goes one step further by allowing vehicles to perform not one but a sequence of trips, called a journey, under a maximum overall shift length constraint. Moreover, the time to load a vehicle is service-dependent, i.e. proportional to the total service time of the subsequent trip. Hence, the MTVRPTW consists in finding a set of back-to-depot delivery trips and assigning them to vehicles so that the total travelled distance is minimized and :

- ▶ each customer is visited exactly once and service starts within the associated time window ;
- ▶ vehicle capacity is respected ;
- ▶ trips assigned to one vehicle do not overlap in time and do not exceed a maximum duration.

2 Short literature review

Recently, MTVRPTW has got the attention of scholars mostly due to its application to city logistics (see [4]). However, as far as we are aware of, the literature of exact methods is still scarce. In [6], an algorithm based on the Branch&Price paradigm is proposed which can solve most of the adapted Solomon instances (derived from the benchmark VRPTW instances first proposed in [8]) with 25 customers and 2 vehicles. In [5], the same authors study a variant where the length of a trip is limited. Another Branch&Price algorithm is proposed in [1], where another variant of the problem is tackled where the visit of customers is not mandatory but rewarded with a profit. Finally, [7] proposes an exact algorithm for a variant with both profits and limited trip duration.

3 Solution method

We propose a two-index MILP formulation for the MTVRPTW that makes use of base and replenishment arcs. The former model the direct connection between two nodes, whereas the latter imply a reload operation between two client nodes. Replenishment arcs, which have been used e.g. in [3], allow to represent a journey as an elementary path with both endpoints in the

instance	$\%_r$	T_r	T_t
C201_2_25	1.41	1.8	7.5
C202_2_25	2.30	6.5	616.8
C203_2_25	0.91	7.3	930.1
C204_2_25	0.22	9.3	100.6
C205_2_25	4.35	2.7	71.3
C206_2_25	2.22	4.1	221.3
C207_2_25	0.08	4.5	3.2
C208_2_25	0.73	3.9	379.9
average	1.53	5.0	291.3

TAB. 1 – Results on clustered instances with 25 customers and 2 vehicles.

depot. This ensures the connectivity of journeys by separating Subtour Elimination Constraints on a transformation of the graph. Further sets of two-indexed variables impose time windows, shift length, and service-dependent loading time constraints. The use of classical capacity constraints to enforce the load limit on vehicles leads to a Branch & Cut algorithm. In order to tighten the lower bound obtained from the linear relaxation of the proposed MILP model, we consider connectivity and path valid inequalities. The latter are an adaptation to our case of those found in [2] : an exact procedure based on LP is proposed to efficiently separate them.

4 Computational results

Tests have been conducted on a set of 54 adapted Solomon instances with 25 to 50 customers and a fleet of 2 to 4 vehicles. The algorithm is coded in C++ and makes use of CPLEX 12.6.1. Experiments are run on a Intel Core i7-5500U 2.4 Ghz machine with 15.56Gb RAM. We allocate 30s to solve the root node and extra 1200s after branching on it.

Table 1 shows the results on the *class C* (*clustered*) instances of of [4] with 25 customers. For these instances, the optimal value (which is known from [6]) is always found. T_r and $\%_r$ report the root computation time and the gap of the root lower bound with respect to the optimal value, while T_t is the additional running time to explore the Branch & Bound tree.

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