A Block Parallel Majorize-Minimize Memory Gradient Algorithm

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***Abstract*—In the field of 3D image recovery, huge amounts of data need to be processed. Parallel optimization methods are then of main interest since they allow to overcome memory limitation issues, while benefiting from the intrinsic acceleration provided by recent multicore computing architectures. In this context, we propose a Block Parallel Majorize-Minimize Memory Gradient (BP3MG) algorithm for solving large scale optimization problems. This algorithm combines a block coordinate strategy with an efficient parallel update. The proposed method is applied to a 3D microscopy image restoration problem involving a depth-variant blur, where it is shown to lead to significant computational time savings with respect to a sequential approach.**

I. Introduction

In many inverse problems encountered in image processing, one has to generate an image estimate $\hat{\boldsymbol{x}} \in \mathbb{R}^N$ by minimizing an appropriate cost function F, which has the following composite form:

$$(\forall \boldsymbol{x} \in \mathbb{R}^N) \quad F(\boldsymbol{x}) = \sum_{s=1}^{S} f_s(\boldsymbol{L}_s \boldsymbol{x})$$

where, for every $s \in \{1, \dots, S\}$, $\boldsymbol{L}_s \in \mathbb{R}^{P_s \times N}$, $P_s \in \mathbb{N}^*$, and f_s is a function from $\mathbb{R}^{P_s}$ to $\mathbb{R}$. In the case of large scale image recovery problems, a major challenge is to design an optimization algorithm able to deliver reliable numerical solutions in a reasonable time.

When all the involved functions $(f_s)_{1 \leqslant s \leqslant S}$ are differentiable on $\mathbb{R}^N$ (but non necessarily convex), a very efficient strategy is the Majorize-Minimize Memory Gradient (3MG) algorithm [1]. It relies on a Majorize-Minimize (MM) approach, combined with a subspace acceleration technique. The 3MG algorithm enjoys nice convergence properties in both convex and non-convex cases and comparisons with state-of-the-art optimization methods on a number of image restoration problems have shown its good performance in terms of practical convergence speed [1], [2]. However, when the size of the problem becomes increasingly large, as it may happen in 3D image processing or video processing, running this kind of algorithm becomes difficult, due to memory limitation issues.

II. Proposed method

The MM approach relies on the existence of symmetric positive matrices

$$(\forall \boldsymbol{x} \in \mathbb{R}^N) \quad \boldsymbol{A}(\boldsymbol{x}) = \sum_{s=1}^{S} \boldsymbol{L}_s^\top \operatorname{Diag}\{\boldsymbol{\omega}_s(\boldsymbol{L}_s \boldsymbol{x})\} \boldsymbol{L}_s,$$

with for every $s \in \{1, \dots, S\}$, $\boldsymbol{\omega}_s : \mathbb{R}^{P_s} \to]0, +\infty[^{P_s}$, such that, for every $(\boldsymbol{x}, \boldsymbol{x}') \in (\mathbb{R}^N)^2$, the following majoration holds:

$$F(\boldsymbol{x}) \leqslant F(\boldsymbol{x}') + \nabla F(\boldsymbol{x}')^\top (\boldsymbol{x} - \boldsymbol{x}') + \frac{1}{2}(\boldsymbol{x} - \boldsymbol{x}')^\top \boldsymbol{A}(\boldsymbol{x}')(\boldsymbol{x} - \boldsymbol{x}').$$

In the 3MG algorithm, a new iterate results from the minimization of the latter quadratic majorant within a two-dimensional subspace spanned by the current gradient and the previous direction. In order to overcome difficulties related to memory requirements, we propose to combine 3MG with a parallel block alternating strategy. The target vector $\boldsymbol{x}$ is split into J non-overlapping block vectors $\boldsymbol{x}^{(j)}$ of reduced dimension $N_j \neq 0$. At each iteration, only a subset $\mathcal{S} \subset \{1, \dots, J\}$ of them is selected, and the associated entries $\boldsymbol{x}^{(\mathcal{S})} = (x_p)_{p \in \mathbb{S}}$ of $\boldsymbol{x}$ are updated. To this end, a clever use of Jensen's inequality allows us to show that, for every $\mathcal{S} \subset \{1, \dots, J\}$,

$$(\forall \boldsymbol{x} \in \mathbb{R}^N) \quad \boldsymbol{A}^{(\mathcal{S})}(\boldsymbol{x}) \preceq \boldsymbol{B}^{(\mathcal{S})}(\boldsymbol{x}) = \operatorname{BDiag}\left\{\left(\boldsymbol{B}^{(j)}(\boldsymbol{x})\right)_{j \in \mathcal{S}}\right\},$$

where, for every $j \in \mathcal{S}$,

$$(\forall \boldsymbol{x} \in \mathbb{R}^N) \quad \boldsymbol{B}^{(j)}(\boldsymbol{x}) = \sum_{s=1}^{S} \left((\boldsymbol{L}_s^{(j)})^\top \operatorname{Diag}\{\boldsymbol{b}_s(\boldsymbol{L}_s \boldsymbol{x})\} \boldsymbol{L}_s^{(j)}\right),$$

with, for every $s \in \{1, \dots, S\}$ and $p \in \{1, \dots, P_s\}$,

$$(\forall \boldsymbol{x} \in \mathbb{R}^N) \quad [\boldsymbol{b}_s(\boldsymbol{L}_s \boldsymbol{x})]_p = [\boldsymbol{\omega}_s(\boldsymbol{L}_s \boldsymbol{x})]_p [|\boldsymbol{L}_s^{(\mathcal{S})}| \mathbf{1}_{|\mathbb{S}|}]_p / [|\boldsymbol{L}_s^{(j)}| \mathbf{1}_{N_j}]_p.$$

Thanks to the block-diagonal structure of the majorant matrix, the selected blocks with indices $j \in \mathcal{S}$ can be updated in a parallel manner according to a 3MG scheme, leading to the so-called block-parallel 3MG algorithm. The monotonic convergence of the criterion sequence $(F(\boldsymbol{x}_k))_{k \in \mathbb{N}}$ to a (locally) optimal value is established, using the same theoretical tools as in [3].

III. Application to 3D microscopy

The proposed algorithm is applied for solving a 3D image restoration problem with depth-variant blur. Figure 1 illustrates its high efficiency in terms of acceleration for multi-core architectures.

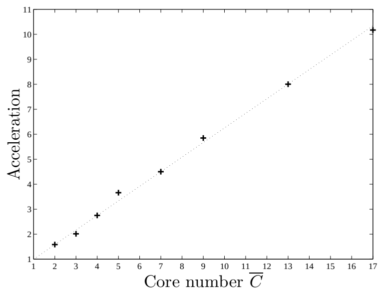

Fig. 1. Ratio between the computation time for one core and the computation time for $\overline{C}$ slave cores (crosses) with linear fitting (dotted line), for the restoration of a 3D microscopy image with size $N = 256 \times 256 \times 48$ pixels.

References

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