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Stability Analysis and Bandwidth Estimation of Free-Carrier Driven Kerr Frequency-Comb

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Abstract—Free-carrier dispersion-led nonlinear cavity detuning is observed through the reverse Kerr-tilt of the resonant-peaks of steady-state solutions of Lugiato-Lefever equation. Effects of nonlinear losses on the gain-spectrum and the gain-bandwidth of modulation-instability are also studied.

Keywords—frequency comb; microring resonators; bandwidth; modulation in-stability; nonlinear optics; Lugiato-Lefever Equation.

I. INTRODUCTION

Optical frequency-comb is a set of equidistant and coherent frequency lines in the infrared, visible or ultraviolet regions, which can be used in precision measurement, spectroscopy, sensing, WDM-communication or as an optical ruler [1, 2]. Silicon based platforms are preferred over other materials for the generation of on-chip optical frequency combs [2] due to several advantages of silicon such as tight optical confinement, high Kerr-coefficient, low-cost etc. Although, octave-spanning frequency combs have already been demonstrated both experimentally and numerically in Si₃N₄ and Si microring resonators (MRRs) [2-4], a more realistic theoretical study on comb dynamics for Si-MRRs becomes indispensable where all the nonlinear losses and higher order dispersion terms are being considered. Nonlinear losses include multi-photon absorptions (two photon absorption (TPA/2PA), 3PA, 4PA etc.), free-carrier absorption (FCA) and free-carrier dispersion (FCD). Generalized mean-field Lugiato-Lefever equation (LLE) can be used to model the Kerr frequency comb in a high- Q , high-finesse optical cavity where the dispersion, nonlinear phase accumulation over a round-trip and pump detuning are low [1]. It is known that temporal cavity soliton (CS) generates Cherenkov radiation/dispersive-wave in presence of higher-order dispersions which leads octave-spanning frequency comb. Effects of Raman scattering and self-steepening on frequency comb are also investigated [2, 3]. Recently, numerical studies have shown that FCA-induced FCD causes nonlinear cavity detuning which in turn helps to generate optical frequency-comb even in absence of linear-detuning of the CW-pump [2, 3], however, a detailed theoretical study on the stability analysis of free-carrier driven Kerr-comb is still scarce. In this work, for the first time to the best of our knowledge, we find the steady state homogeneous solutions of free-carrier driven generalized mean-field LLE and report negative Kerr-tilt as a consequence of FCA-FCD assisted nonlinear cavity detuning.

We generalize the existing formulations to study the modulation instability (MI) gain and bandwidth in presence of higher-order dispersions, TPA, 3PA, 4PA, FCA and FCD.

II. THEORETICAL STUDIES

A. Normalization of LLE

At first we normalize the LLE and the rate-equation including multi-photon absorptions, FCA and FCD terms [2] in convenient forms, as given by equations (1) and (2), respectively. The steady-state homogeneous solutions of such equations are discussed later.

$$\frac{\partial u}{\partial \xi} = -(1 + j\Delta)u - \frac{1}{2}(1 + jK)\phi_c u - j\frac{s}{2}\frac{\partial^2 u}{\partial \tau^2} + \sum_{k \geq 3} \frac{d_k}{k!} \frac{\partial^k u}{\partial \tau^k} \quad (1)$$

$$+ j|u|^2 u - \frac{Q_2}{2}|u|^2 u - \frac{Q_3}{3}|u|^4 u - \frac{Q_4}{4}|u|^6 u + S$$

$$\frac{\partial \phi_c}{\partial \tau} = \theta_{c_2}|u|^4 + \theta_{c_3}|u|^6 + \theta_{c_4}|u|^8 - \frac{\phi_c}{\tau_c} \quad (2)$$

$$X = (1 + \Delta^2)Y + (Q_2 - 2\Delta)Y^2 + \left\{1 + \frac{Q_2^2}{4} + C_2(1 + \Delta K)\right\}Y^3 \quad (3)$$

$$+ C_2\left(\frac{Q_2}{2} - K\right)Y^4 + \frac{C_2^2}{4}(1 + K^2)Y^5$$

$$X = (1 + \Delta^2)Y - 2\Delta Y^2 + \left(1 + \frac{2}{3}Q_3\right)Y^3 + C_3(1 + \Delta K)Y^4 \quad (4)$$

$$+ \left(\frac{Q_3^2}{4} - KC_3\right)Y^5 + \frac{C_3Q_3}{3}Y^6 + \frac{C_3^2}{4}(1 + K^2)Y^7$$

$$X = (1 + \Delta^2)Y - 2\Delta Y^2 + Y^3 + \frac{Q_4}{2}Y^4 + C_4(1 + \Delta K)Y^5 \quad (5)$$

$$- KC_4Y^6 + \frac{Q_4^2}{16}Y^7 + \frac{C_4Q_4}{4}Y^8 + \frac{C_4^2}{4}(1 + K^2)Y^9$$

$$X = \left[(1 + \Delta^2)Y - 2\Delta Y^2 + Y^3\right] \quad (6)$$

B. Steady-State Homogeneous Solutions

We find the steady-state ($\partial u / \partial \tau = 0$), stationary ($\partial u / \partial \xi = 0$) homogeneous solutions of (1) for Si-MRRs in presence of TPA (telecommunication and NIR $\sim 2.2\mu\text{m}$) which satisfies the quintic polynomial of output intensity, $Y (=|E|^2)$ [1] as given in (3). We show, septic and nonic polynomials of Y as given in equations (4) and (5) express the steady-state solutions of (1) in presence of only 3PA (short wavelength IR) and 4PA (mid-IR), respectively. Note that all these three equations are

reduced to the well-known cubic polynomial of Y (6) if the nonlinear losses are neglected [1]. C_j is defined as $(\theta_{c_j} \tau_c)$ while $j=2, 3, 4$ stands for TPA, 3PA and 4PA respectively.

Fig. 1. (a) Kerr-tilts for the steady-state solutions of Eq. (1) in the absence of nonlinear losses (blue curves) and in presence of TPA (red-curves). (b) Kerr-tilts in presence of TPA, 3PA, 4PA, FCA and FCD.

C. Observation of Negative Kerr-tilt

The dynamic behaviour of frequency comb can partially be understood through Kerr-tilt and bi-stability curve as the CS solutions of LLE results from the coexistence of patterned and CW solutions [1]. From Figs. 1 (a) and 1(b), it is evident that nonlinear losses introduce cavity detuning which causes reverse Kerr-tilts. We observe that the nonlinear cavity detuning introduced only by 3PA at a very low pump power does not play any vital role in comb dynamics. However, through reverse Kerr-tilt (shown by red-arrow in Fig. 1 (a)) we show that nonlinear losses especially TPA, FCA and FCD play crucial roles in determining the parameter spaces for MI, chaos formation and stable frequency-comb generations. Parameters required for the simulations are given in [2].

D. Modulation-Instability

Next we proceed for the stability analysis of the stationary CW solutions (u_0 and ϕ_0) of free-carrier driven LLE. The evolution of normalized carrier density and signal amplitude due to the spatio-temporal perturbations can be written as Eqs. (7) and (8), respectively while TPA, FCA-FCD are present.

$$\phi_c(\xi, \tau) = \phi_0(\xi) + \phi_+(\xi)e^{i\Omega\tau} + \phi_-(\xi)e^{-j\Omega\tau} \quad (7)$$

$$u(\xi, \tau) = u_0(\xi) + u_+(\xi)e^{i\Omega\tau} + u_-(\xi)e^{-j\Omega\tau} \quad (8)$$

Thus, the stationary solutions (ϕ_0 , u_0) and ϕ_+ can be given as:

$$\phi_0 = \theta_{c_2} \tau_c |u_0|^4 = C_2 |u_0|^4 \quad (9)$$

$$\frac{\partial u_0}{\partial \xi} = -(1+j\Delta)u_0 - \frac{1}{2}(1+jK)\theta_{c_2} \tau_c |u_0|^4 u_0 + \left(j - \frac{Q_2}{2}\right) |u_0|^2 u_0 + S \quad (10)$$

$$\phi_+ = \frac{2\theta_{c_2} \tau_c |u_0|^2 [u_0^* u_+ + u_0 u_+^*]}{(1+j\Omega\tau_c)} = \phi_-^* \quad (11)$$

We assume, $u_+ = C \exp(\lambda \xi)$. Finally, the MI-gain, λ can be found by solving the quadratic equation of λ (12),

$$\left(\frac{R^2 + I^2}{\alpha^2 + \beta^2} - Y^2 \right) = 0 \quad (12)$$

Here, LLE is truncated up to the 3rd order dispersions and,

$$R(\lambda) = - \left[(1+\lambda) + \frac{C_2(1+K\Omega\tau_c)}{1+\Omega^2\tau_c^2} Y^2 + \frac{C_2}{2} Y^2 + YQ_2 \right] \quad (13)$$

$$I(\lambda) = - \left[\Delta + \frac{C_2(K-\Omega\tau_c)}{(1+\Omega^2\tau_c^2)} Y^2 + \frac{KC_2}{2} Y^2 - \frac{s}{2} \Omega^2 + \frac{\Omega^3}{6} d_3 - 2Y \right] \quad (14)$$

$$\alpha = \left[C_2 \frac{(1+K\Omega\tau_c)}{(1+\Omega^2\tau_c^2)} Y + \frac{Q_2}{2} \right] \quad (15)$$

$$\beta = \left[C_2 \frac{(K-\Omega\tau_c)}{(1+\Omega^2\tau_c^2)} Y - 1 \right] \quad (16)$$

Note that, in absence of nonlinear losses, λ reduces to the well-known expression (17) for MI-gain:

$$\lambda_{\pm} = -1 \pm \sqrt{4Y \left(\Delta - \frac{s\Omega^2}{2} \right) - \left(\Delta - \frac{s\Omega^2}{2} \right)^2 - 3Y^2} \quad (17)$$

Figs. 2(a) and (b) show that the presence of third-order dispersion, $d_3(=0.1)$ can enhance the gain-bandwidth whereas, the MI-gain for Si-waveguide, in presence of TPA, FCA and FCD is insufficient to create any sidebands.

Fig. 2 (a) Effect of 3rd order dispersion and (b) TPA, FCA-FCD on λ . $s=1$, $Y=2.5$, $\Delta=7.5$, $Q_2=1$, $K=0.05$, $C_2=0.1$, $\theta_{c_2}=0.05$ taken. For Si at wavelength $< 2.2\mu\text{m}$ TPA is even higher to yield $\lambda \sim 100$ for the entire Ω range [0-6] shown here.

III. NUMERICAL RESULTS

Our theoretical predictions are confirmed numerically by solving the LLE (1) and the rate-equation (2) through split-step Fourier method (Fig. 3). It is observed that TPA (with FCA-FCD) inhibits the comb formation whereas 3PA can reduce the comb-bandwidth significantly.

Fig. 3 Numerical simulations confirm that a different dynamics is being played for different parameter spaces of Δ and X in presence of nonlinear losses. Inset shows the final pulses in time domain. $\theta_{c_2}=0.0005$, $Q_2=0.93$, $C_2=29.81$, $K_2=7.5$, $d_3=0.1$ (at $\lambda_0 \sim 1.56\mu\text{m}$). $\theta_{c_3}=6.31$, $Q_3=0.087$, $C_3=0.049$, $K_3=4.9$ (at $\lambda_0 \sim 2.4\mu\text{m}$).

IV CONCLUSION

To conclude, nonlinear phase detuning of cavity has been observed through negative Kerr-tilt. For the first time to the best of our knowledge, we also find out the MI-gain and bandwidth for Kerr-comb in presence of nonlinear losses.

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