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1 **Spectral up- and downshifting of Akhmediev breathers under wind forcing**

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We experimentally and numerically investigate the effect of wind forcing on the spectral dynamics of Akhmediev breathers, a wave-type known to model the modulation instability. We develop the wind model to the same order in steepness as the higher order modification of the nonlinear Schrödinger equation, also referred to as the Dysthe equation. This results in an asymmetric wind term in the higher order, in addition to the leading order wind forcing term. The derived model is in good agreement with laboratory experiments within the range of the facility's length. We show that the leading order forcing term amplifies all frequencies equally and therefore induces only a broadening of the spectrum while the asymmetric higher order term in the model enhances higher frequencies more than lower ones. Thus, the latter term induces a permanent upshift of the spectral mean. On the other hand, in contrast to the direct effect of wind forcing, wind can indirectly lead to frequency downshifts, due to dissipative effects such as wave breaking, or through amplification of the intrinsic spectral asymmetry of the Dysthe equation. Furthermore, the definitions of the up- and downshift in terms of peak- and mean frequencies, that are critical to relate our work to previous results, are highlighted and discussed.

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11 I. INTRODUCTION

12 The interaction between wind and ocean waves lies at the heart of ocean dynamics, and
 13 since the 1950's^{1,2} significant progress has been made in understanding this interplay³⁻⁷.
 14 More recently, a notable number of studies has been devoted to understanding the occurrence
 15 of rogue waves^{8? -10}. Rogue waves are ‘monster waves’ that have a much higher amplitude
 16 than what can statistically be expected from the current sea-state. The role of wind in
 17 the formation and the evolution of such extreme waves has been investigated numerically
 18 and experimentally¹¹⁻¹³. However, the direct effect of the wind on rogue wave spectra still
 19 needs further investigation. An interesting physical phenomenon related to the wind-wave
 20 interaction is the frequency downshift observed in ocean waves^{14,15}. Apart from altering
 21 a fundamental property of a wave field, namely its frequency, downshift can affect the
 22 formation of rogue waves^{16,17}. Yet, a *unified* explanation for the physical origin of downshift
 23 seems to be missing, since proposed mechanisms such as the presence of wind¹⁸⁻²⁰, wave
 24 breaking^{21,22} and viscosity^{23,24} are opposite in nature as they respectively force and damp
 25 the waves. This study aims to clarify this downshift paradox.

26 Investigating up- and downshift needs a precise definition. To do so, two approaches can
 27 be outlined²⁴⁻²⁶. The first is to look at a shift in position of the mode with the highest
 28 amplitude in the spectrum, the *spectral peak* f_p . In doing so, any small asymmetries in the
 29 spectrum are disregarded. The second approach is to look at the position of the *spectral*
 30 *mean* f_m , defined as the ratio of the momentum M of the envelope to its norm N . In the
 31 time domain, these are defined in Carter and Govan²⁴. In the spectral domain, this can be
 32 written as

$$M = \sum_{n=-n_{\text{lim}}}^{n=+n_{\text{lim}}} f_n |\hat{a}_{f_n}|^2 \quad (1)$$

$$N = \sum_{n=-n_{\text{lim}}}^{n=+n_{\text{lim}}} |\hat{a}_{f_n}|^2 \quad (2)$$

$$f_m = \frac{M}{N} \quad (3)$$

33 where \hat{a} is the Fourier mode of the envelope, f the frequency, and n_{lim} the spectral mode
 34 that marks the limit of the main carrier wave mode, such that the interval of f excludes the
 35 bound waves. [The boundary between the main mode an the first harmonic is considered](#)

36 to be at $f_{\text{lim}} = f_0 + f_0/2$. From the spectral definition one can see that M/N is equal to
 37 the quadratic weighted average and detects any asymmetry in the spectrum.

38 In addition, up-/downshifts can be reported as temporary or permanent. In the former
 39 case, the mean or the peak shifts to a higher/lower frequency, but eventually shifts back
 40 to the original carrier wave frequency. This recovery does not occur in case of permanent
 41 up-/downshift.

42 In this paper, we aim to investigate the effect of the wind on spectra of **unidirectional**,
 43 modulationally unstable gravity wave trains, specifically on Akhmediev breathers. **Note**
 44 **that when comparing to the ocean, this excludes directionally spread waves.** Our main
 45 finding is to inject the wind terms found by Brunetti and Kasparian²⁷ into existing MNLS²⁸
 46 model with viscous dissipation²⁴. This forms a forced-damped MNLS model in which the
 47 wind, dissipation and nonlinearity are all of the same order. In section II, we present our
 48 model for this purely forcing effect of wind on water waves, that is, wind blowing in the
 49 direction of the wave propagation and consequently adding energy to the system. The wind
 50 contribution in the model consists of a leading order forcing term that amplifies all wave
 51 frequency components equally, as well as a higher order asymmetric term that amplifies
 52 higher frequencies more than lower ones. In sections III and IV, we corroborate the model
 53 with wave tank experiments. Subsequently, we perform long range simulations in section V
 54 to overcome the limited fetch in the experiment. We demonstrate that the wind forcing by
 55 itself can only cause an upshift in the spectral mean. Finally, in section VI, we clear up the
 56 aforementioned downshift paradox, discussing our model in light of previous literature, and
 57 taking into account the two downshift definitions.

58 II. WEAKLY NONLINEAR WIND-WAVE MODEL

59 In a simplified Euler approximation for the water-wave problem, the Coriolis term is
 60 neglected with respect to the advective term in the momentum equations. In addition,
 61 the water density is considered constant $\rho_w = \rho_{w,0}$. **Considering unidirectional waves, the**
 62 **transverse coordinate is neglected**, and the system of equations that has to be solved is^{29,30}:

$$\nabla^2 \phi \equiv \phi_{xx} + \phi_{zz} = 0 \quad -h \leq z \leq \eta(x, t) \quad (4)$$

$$\phi_z = 0 \quad z = -h \quad (5)$$

$$\eta_t + \phi_x \eta_x - \phi_z = 2\nu \eta_{xx} \quad z = \eta(x, t) \quad (6)$$

$$\phi_t + \frac{1}{2} \phi_x^2 + \frac{1}{2} \phi_z^2 + g\eta = -\frac{P}{\rho_w} - 2\nu \phi_{zz} \quad z = \eta(x, t) \quad (7)$$

63 where g is the gravitational acceleration, h is the water depth, x is the propagation direc-
 64 tion and z is the upward coordinate. The potential function $\phi = \phi(x, z, t)$ is defined as
 65 $u = \partial\phi/\partial x = \phi_x$, $w = \partial\phi/\partial z = \phi_z$, where u and w are the velocity components. The
 66 function $\eta(x, t)$ is the surface elevation with respect to the average level $z = 0$. The kine-
 67 matic viscosity of the fluid is denoted as ν [m²/s]. The wind is characterized by $P = P(x, t)$,
 68 the excess pressure at the water surface in the presence of wind, that in the framework of
 69 the Miles mechanism¹ is given by

$$\frac{P}{\rho_w} = \frac{\Gamma}{f_0} \frac{c_p^2}{2\pi} \eta_x = \frac{\omega_0}{k_0^2} \Gamma \eta_x \quad (8)$$

70 where c_p is the phase velocity of the carrier wave, k_0 its wave number, f_0 its frequency,
 71 and $\omega_0 = 2\pi f_0$ its angular frequency. Γ [1/s] is the growth rate of the energy E of the
 72 waves due to the wind blowing in the direction of the wave propagation, that is $\frac{\partial E}{\partial t} = \Gamma E$.
 73 In turn, an expression for Γ as a function of wind speed U and wave frequency f_0 can be
 74 modeled in various ways, as will be discussed in section III. In the approximation that the
 75 envelope varies slowly in comparison to the surface elevation, the Method of Multiple Scales
 76 (MMS) can be used to expand these boundary conditions and obtain a weakly nonlinear
 77 propagation equation for the envelope $a(x, t)$ at each order of interest. In the MMS, the
 78 small order parameter is the steepness of the wave $\epsilon = ak_0$. In addition, deep water waves,
 79 $k_0 h \rightarrow \infty$, are considered. The surface elevation to the first order in steepness is given by

$$\eta(x, t) = \text{Re}\{a(x, t) \exp[i(k_0 x - \omega_0 t)]\} \quad (9)$$

80 Note that the sign choice of the argument in the exponential is important for the spectral
 81 representation of envelope with respect to the carrier wave. For instance in Carter and
 82 Govan²⁴ the opposite sign choice has been made. To avoid confusion, the spectrum of the
 83 carrier wave is plotted throughout this paper. In the absence of both wind and viscosity

84 ($\nu = \Gamma = 0$), the MMS yields (i) the nonlinear Schrödinger (NLS) equation when developing
 85 the boundary conditions to $O(\epsilon^3)^{31-33}$, and (ii) the modified NLS (MNLS) or Dysthe equation
 86 when developing the boundary conditions to $O(\epsilon^4)^{28}$. Assuming the wind Γ/f_0 and the
 87 viscosity $\nu k_0^2/f_0$ contribution are both of order $O(\epsilon^2)$, and including these in the MMS
 88 development up to $O(\epsilon^4)$ using a similar method as Carter and Govan²⁴, yields the following
 89 damped-forced MNLS:

$$\begin{aligned} \frac{\partial a}{\partial t} + \frac{\omega_0}{2k_0} \frac{\partial a}{\partial x} = & \epsilon \left[-i \frac{\omega_0}{8k_0^2} \frac{\partial^2 a}{\partial x^2} - \frac{1}{2} i k_0^2 \omega_0 a |a|^2 - 2k_0^2 \nu a + \frac{1}{2} \Gamma a \right] \\ & + \epsilon^2 \left[4i k_0 \nu \frac{\partial a}{\partial x} - \frac{3i}{4k_0} \Gamma \frac{\partial a}{\partial x} \right. \\ & \left. - \frac{3}{2} k_0 \omega_0 |a|^2 \frac{\partial a}{\partial x} - \frac{1}{4} k_0 \omega_0 a^2 \frac{\partial a^*}{\partial x} + \frac{\omega_0}{16k_0^3} \frac{\partial^3 a}{\partial x^3} - i k_0 a \frac{\partial \bar{\phi}}{\partial x} \right] \end{aligned} \quad (10)$$

90 where $\bar{\phi}$ is the potential mean flow. To obtain a propagation in space, in accordance with
 91 the motion in a 1D wave tank, the following coordinate transformation is applied:

$$\begin{cases} \tilde{t} = t - \frac{2k_0}{\omega_0} x \\ \tilde{x} = \epsilon x \end{cases}$$

92 The potential mean flow term in (10) is replaced by a Hilbert transform term, see
 93 Janssen³⁴. The Hilbert transform \mathcal{H} is defined as $\mathcal{F}[\mathcal{H}[u]] = -i \text{sign}(\omega) \mathcal{F}[u]$, where
 94 \mathcal{F} is the Fourier transform. The damped-forced MNLS becomes:

$$\begin{aligned} \underbrace{\frac{\partial a}{\partial \tilde{x}} + i \frac{k_0}{\omega_0^2} \frac{\partial^2 a}{\partial \tilde{t}^2} + i k_0^3 a |a|^2}_{\text{NLS}} = & \underbrace{\epsilon \frac{k_0^3}{\omega_0} \left(6|a|^2 \frac{\partial a}{\partial \tilde{t}} + 2a \frac{\partial |a|^2}{\partial \tilde{t}} + 2ia \mathcal{H} \left[\frac{\partial |a|^2}{\partial \tilde{t}} \right] \right)}_{\text{MNLS correction}} \\ & \underbrace{- 4 \frac{k_0^3}{\omega_0} \nu a - \epsilon 20i \frac{k_0^3}{\omega_0^2} \nu \frac{\partial a}{\partial \tilde{t}}}_{\text{Viscosity}} \\ & \underbrace{+ \frac{k_0}{\omega_0} \Gamma a + \epsilon 4i \frac{k_0}{\omega_0^2} \Gamma \frac{\partial a}{\partial \tilde{t}}}_{\text{Wind}} \end{aligned} \quad (11)$$

95 This full model consists of the corrected NLS, taking into account higher-order dispersion
 96 and mean flow (MNLS correction) as well as viscosity and wind effects, where the higher
 97 order terms are indicated by ϵ . In the following the tilde's are omitted. The dynamics of
 98 the MNLS equation are well known and have been studied numerically by Lo and Mei³⁵?

99 and applied to many experiments, see for instance Trulsen and Stansberg³⁶, Tulin and
 100 Waseda³⁷, Slunyaev *et al.*³⁸, Chabchoub *et al.*³⁹.

101 The viscosity consists of a linear damping term $-4\frac{k_0^3}{\omega_0}\nu a$ in the leading order, as proposed
 102 in Dias, Dyachenko, and Zakharov²⁹. In addition, following Carter and Govan²⁴, the higher
 103 order viscosity term is given by $-\epsilon 20i\frac{k_0^3}{\omega_0^2}\nu\frac{\partial a}{\partial t}$.

104 The wind action appears as a leading order linear forcing term $\frac{k_0}{\omega_0}\Gamma a$ which in past work
 105 has been included based on either the intuition of a simple forcing⁴⁰, or through more rigorous
 106 justification³⁰. There has been no experimental validation of a wind forcing term in an
 107 evolution equation for deep water waves. Our addition of a higher order wind term $\epsilon 4i\frac{k_0}{\omega_0^2}\Gamma\frac{\partial a}{\partial t}$
 108 completes the wind-wave model (11). This higher order wind term restores consistency in
 109 the sense that since the nonlinearity is developed up to $O(\epsilon^4)$ in the MNLS framework, the
 110 dissipation and forcing also have to be developed up to this order to have a coherent model.

111 The model reported here can be related to the wind-wave model presented in Brunetti
 112 and Kasparian²⁷, Brunetti *et al.*⁴¹ by noting that the higher order wind term can be ob-
 113 tained in two ways. To obtain (11), the wind forcing Γ/f_0 was assumed $O(\epsilon^2)$ in (7), and
 114 the kinematic and dynamic boundary conditions were developed up to $O(\epsilon^4)$ in the MMS.
 115 Alternatively, following Brunetti *et al.*⁴¹, one can assume that $\Gamma/f_0 = O(\epsilon)$ in (7), thus,
 116 strong in comparison to the steepness ϵ . Next, the kinematic and dynamic boundary condi-
 117 tions only have to be developed up to $O(\epsilon^3)$ in the MMS to obtain the same wind terms as
 118 in (11).

119 Surprisingly, the outcome of the MMS is that the viscosity and wind contributions in
 120 both the leading and higher order have the same form in (11), despite of their non-similar
 121 appearance in the kinematic and dynamic boundary conditions (6 and 7), respectively. As
 122 these terms have opposite signs, the viscosity can cancel out the wind forcing, and vice versa.
 123 However, note that their balance is different for different orders. This balance occurs as

$$\delta_0 \equiv \Gamma - 4k_0^2\nu = 0 \quad (12)$$

124 for the leading order, as studied by Kharif *et al.*³⁰. In our model (11), the higher order
 125 terms are balanced if

$$\delta_1 \equiv \Gamma - 5k_0^2\nu = 0 \quad (13)$$

126 The NLS is a spectrally symmetric evolution equation, that is, as the envelope propagates,
 127 the frequencies below f_0 evolve in the same way as their counterparts above f_0 . An odd

128 derivative in time is needed to have an asymmetric evolution of the spectrum³⁵. In the full
 129 model (11), the odd terms are the MNLS correction, the higher order wind and the higher
 130 order viscosity term. Indeed, the leading order term δ_0 has the effect of either damping or
 131 amplifying all frequencies in the spectrum, depending on its sign. For the higher order, if
 132 viscosity is dominant, $\delta_1 < 0$, the higher frequencies ($f > f_0$) are damped more than lower
 133 ones ($f < f_0$) causing a permanent downshift of the spectral mean, as evidenced by Carter
 134 and Govan²⁴. Our derivation of the higher order wind term shows that if the wind action
 135 is dominant, $\delta_1 > 0$, the higher order effect is opposite and the spectral mean is upshifted.

136 III. EXPERIMENTAL AND COMPUTATIONAL DETAILS

137 A. Initial condition

138 To validate our model, we are interested in cases in which the system experiences a
 139 spectral broadening. To model the modulation instability, the Akhmediev Breather (AB)⁴²
 140 has been used to generate our initial conditions for the experiment. Starting the dynamics
 141 from an exact AB expression is useful for experimental investigations, since it allows to
 142 trigger the modulation instability dynamics in relation to the length of the facility. The AB
 143 gives an approximate prediction of the growth and subsequent decay cycle that is to occur,
 144 rather than just a prediction of a linear growth rate as calculated from the Benjamin-Feir
 145 instability analysis⁴³. This theoretical expectation allows us to identify deviations from this
 146 growth-decay pattern. The AB is a solution to the *NLS* part of (11) and reads⁴⁴

$$a_{\text{AB}}(x^*, t) = a_0 \frac{\sqrt{2A} \cos(\Omega \frac{t}{T_0}) + (1 - 4A) \cosh(R \frac{x^*}{L_0}) + iR \sinh(R \frac{x^*}{L_0})}{\sqrt{2A} \cos(\Omega \frac{t}{T_0}) - \cosh(R \frac{x^*}{L_0})} \exp(i \frac{x^*}{L_0}), \quad (14)$$

147 where x^* is the distance to the focal point, $L_0 = (k_0^3 a_0^2)^{-1}$ and $T_0 = \sqrt{2}(k_0 \omega_0 a_0)^{-1}$ are
 148 the rescaling coordinates, $\Omega = 2\sqrt{1 - 2A}$ is the dimensionless modulation frequency,
 149 $R = \sqrt{8A(1 - 2A)}$ the growth rate, and $0 < A < 0.5$ is the Akhmediev parameter.
 150 The case $A = 0.25$ corresponds to a maximal growth rate in the Benjamin-Feir theory. Note
 151 that since the AB is a solution of the NLS only, the maximally unstable mode in the NLS
 152 framework does not exactly coincide with that of our full system. When $x \rightarrow \pm\infty$ the AB
 153 tends to a regular wave train, while at the focal point $x = 0$, the breather reaches maximal

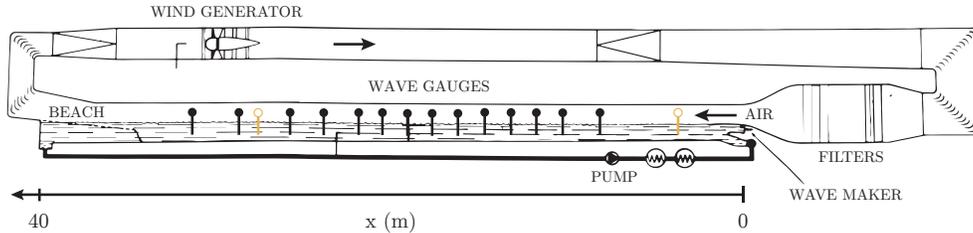


FIG. 1: IRPHE wind-wave tank facility: 40 m long, 3 m wide, water depth 0.80 m, air channel height 1.5 m. Wave gauges have been placed approximately evenly along the tank.

Capacitive wave gauges are indicated with open circles, others are resistive gauges

154 temporal compression and consequently maximal amplitude. In the spectral domain, the
 155 focusing of the breather corresponds to a broadening of the spectrum, see Wetzels *et al.*⁴⁵.
 156 Thus, the focal point is marked by maximal spectral width, and by minimal amplitude of
 157 the carrier wave Fourier component.

158 Choosing an initial condition at a given distance from the focal point allows us to control
 159 the number of developed sidebands. The signal given to the wave maker is the surface
 160 elevation $\eta(x^*, t)$ (9), based on the dimensional AB envelope $a_{AB}(x^*, t)$ (14).

161 B. Experimental Setup

162 Experiments have been performed in the closed wind-wave facility at IRPHE (Luminy)-
 163 Aix Marseille University. A schematic depiction of the facility is shown in figure 1. As
 164 detailed in Coantic *et al.*⁴⁶, the tank is 3 m wide, has a water depth of 80 cm, and a
 165 length of 40 m. At the end of the wave tank, an 8 m sloping beach was installed to prevent
 166 wave reflection. At the beginning of the tank, a 1.5 m long plastic sheet floating on the
 167 water surface allowed the incoming wind to be tangential to the water surface, and damped
 168 possible high-frequency mechanical wave modes. Mechanical waves have been generated
 169 by an underwater piston-like wavemaker controlled by a computer. The system was able
 170 to produce arbitrary surface gravity waves in the frequency range of 0.5-1.8 Hz. The air-
 171 channel above the tank is 1.5 m high. The wind was generated by a closed-loop air flow
 172 system, up to a maximum velocity of $U = 15$ m/s in the direction of the wave propagation.
 173 A total of 16 wave gauges have been placed at fixed positions along the tank to measure the
 174 surface elevation. The gauges were positioned approximately evenly between $x = 3$ m and

175 $x = 32$ m. The first and 14th gauge were capacitive-, the others were resistive-type wave
 176 probes. All gauges had a sampling rate of 400 Hz. The wind speed was measured by a pitot
 177 tube at different positions in the tank to verify a constant and homogeneous air flow.

178 The parameter space of the experiment is limited in several directions. Firstly, the
 179 steepness of the background plane wave of the AB (when $x^* \rightarrow -\infty$) is constrained to a
 180 range of $0.08 \leq \epsilon \leq 0.10$. The lower the steepness, the larger the propagation distance
 181 required for the modulation instability to develop in the small fetch, restricted by the size of
 182 the facility (figure 1). Conversely, if the carrier waves are initially too steep, wave breaking
 183 is inevitable as a consequence of focusing. Secondly, all measurements have been performed
 184 with $f_0 = 1.67$ Hz, thus $\omega_0 = 10.5$ rad Hz, and $k_0 = 11.2$ rad/m. This yields a Bond number
 185 of ~ 1000 , confirming these waves are in the gravity wave regime⁴⁷. Higher frequencies waves
 186 were not possible to generate with the installed wavemaker. Thirdly, wind speeds were
 187 limited, as for $U \gtrsim 4$ m/s breaking would occur for waves generated with initial steepness
 188 in the described range.

189 Besides the breather-type waves generated by the wavemaker, the wind naturally gener-
 190 ates additional waves. The frequency range of these wind-waves shifts down as a function
 191 of wind speed and fetch. At the last wave gauge, for $U = 4$ m/s, the wind-waves have been
 192 measured to be in the range of approximately 2.2 - 3.2 Hz. Therefore, these waves were
 193 considered not to overlap with the breather-type waves. The wind-generated waves showed
 194 micro-breaking at wind speeds of $U \sim 4$ m/s towards the end of the tank.

195 C. Simulation parameters

196 The envelope a , which is periodic in time, was integrated forward in space according
 197 to (11) by means of a split-step Fourier scheme⁴⁸. As described in Agrawal⁴⁹, the linear
 198 and nonlinear part of (11) can be solved in separate steps. The linear part is an ordinary
 199 differential equation (ODE) which was solved in Fourier space, and to integrate the nonlinear
 200 part, the fourth order Runge-Kutta method has been used.

201 This initial condition for the simulations is the envelope based on the surface elevation
 202 measured by the first wave gauge. To ensure periodic boundary conditions, we selected a
 203 time interval equal to a multiple of the envelope period (see figure 2). As such, the spectral
 204 resolution of the simulation is the same as that in the experiment. At an acquisition rate of

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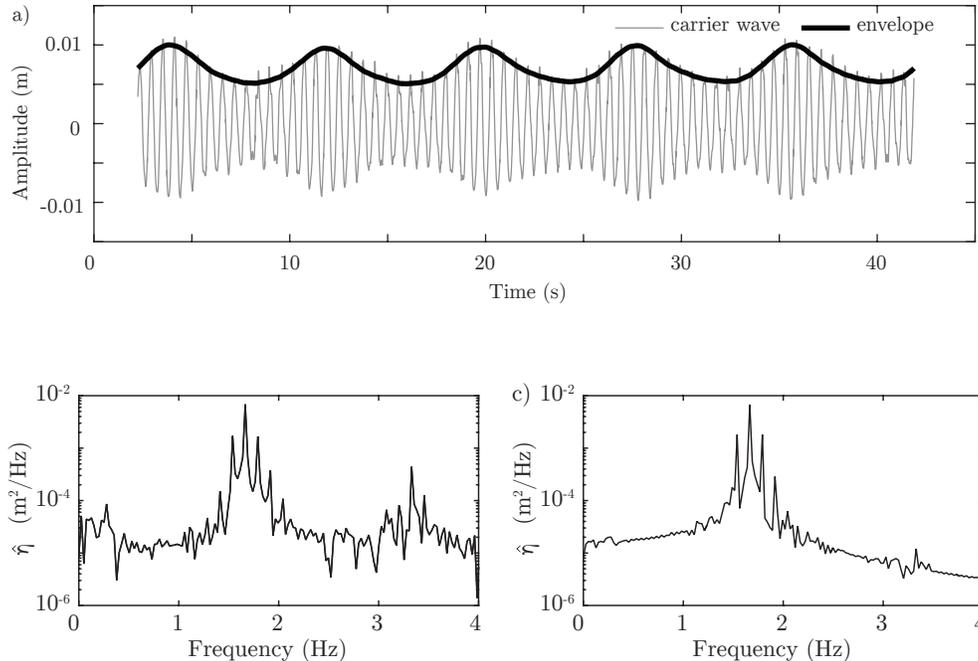


FIG. 2: a) Periodic initial condition used for the simulations, as measured by the first wave gauge at $x = 3.7$ m at an acquisition rate of 400 Hz. b) Log-linear plot of the spectrum of the surface elevation as measured by the first wave gauge. c) Log-linear plot of the spectrum of the carrier wave to first order as calculated from the envelope. Carrier wave parameters $\epsilon = 0.08$, $T = 0.6$ s ; AB parameters $x^* = -30$ m and $A = 0.25$. The envelope was calculated using the Hilbert transform.

205 400 Hz, depending on the exact conditions, the time series were ~ 40 s long, corresponding
 206 to ~ 60 periods of the carrier wave and spectral resolution of ~ 0.025 Hz. [The complex](#)
 207 [envelope was extracted from the real valued envelope \$\eta\$ as follows](#)

$$a(x_0, t) = [\eta(x_0, t) + i\tilde{\eta}(x_0, t)] e^{-i(k_0 x - \omega_0 t)} \quad (15)$$

208 [where \$\tilde{\eta}\$ is the Hilbert transform of the surface elevation, as described in Osborne⁵⁰ and \$x_0 =\$](#)
 209 [3.1 m is the position of the first wave gauge.](#) Starting the simulations from the experimental
 210 signal accounts for the possible imperfections of the wavemaker and reduces the number
 211 of free parameters in the model. The only free parameters to be determined are the wind
 212 growth rate Γ and the viscosity parameter ν , both of which are fitted from the experiment.
 213 Following the method of Segur *et al.*²³ and Carter and Govan²⁴, the viscosity parameter

TABLE I: Predicted values and values used in the simulations for the wind growth rate Γ

Wind speed	Γ_{Miles}	$\Gamma_{\text{Simulation}}$	leading order	higher order
4 m/s	$8.3 \times 10^{-3} \text{ s}^{-1}$	$8.5 \times 10^{-3} \text{ s}^{-1}$	$\delta_0 > 0$	$\delta_1 > 0$
2 m/s	$1.6 \times 10^{-3} \text{ s}^{-1}$	$2.0 \times 10^{-3} \text{ s}^{-1}$	$\delta_0 < 0$	$\delta_1 < 0$

²¹⁴ ν in (11) is in fact an effective term that includes not only viscosity, but all sources of
²¹⁵ dissipation in the experiment such as the side-wall effects and surface contamination. It
²¹⁶ has been determined experimentally by fitting the exponential decay of a the wave train
²¹⁷ propagating down the tank without wind forcing: $E = E_0 e^{-\frac{1}{2}\zeta x}$, where $\zeta = (4k_0^3/\omega_0)\nu$. All
²¹⁸ waves used in this work are of the same frequency and steepness, for which we found a
²¹⁹ measured value of the viscosity of $\nu = (1.18 \pm 0.35) \times 10^{-5} \text{ m}^2/\text{s}$. Consequently, we used
²²⁰ $\nu = 1 \times 10^{-5} \text{ m}^2/\text{s}$ in all simulations.

²²¹ To determine Γ we rely on predictions by the Miles mechanism for wind growth⁵¹

$$\Gamma = \omega_0 \alpha \frac{\rho_{air} u_*^2}{\rho_w c_p^2} \quad (16)$$

²²² where u_* is the friction velocity, c_p is the phase velocity, ρ_{air} the density of air, ρ_w the density
²²³ of water, and α an empirical parameter of 32.5. Assuming a logarithmic profile of U as a
²²⁴ function of z , the measured wind speed U is related to the friction velocity u_* by¹

$$U(z) = \frac{u_*}{K} \log \left(\frac{z}{z_0} \right), \quad (17)$$

²²⁵ where z is the height where U is measured, $K = 0.41$ the Von Karman constant and z_0 is
²²⁶ the effective roughness length

$$z_0 = \kappa u_*^2 / g, \quad (18)$$

²²⁷ with $\kappa = 0.0144$ the Charnock constant.

²²⁸ Since estimations for Γ can have deviations up to a factor two with field and wave tank
²²⁹ measurements (figure 1 in Banner and Song⁵¹), we adjusted the latter value to match the
²³⁰ spectral widening and growth in the experiment for each wind speed, see table I. Values
²³¹ obtained for u_* through (17) are accurate within a range of 10 % compared to those measured
²³² in an elaborate study by Caulliez, Makin, and Kudryavtsev⁵² in the same facility.

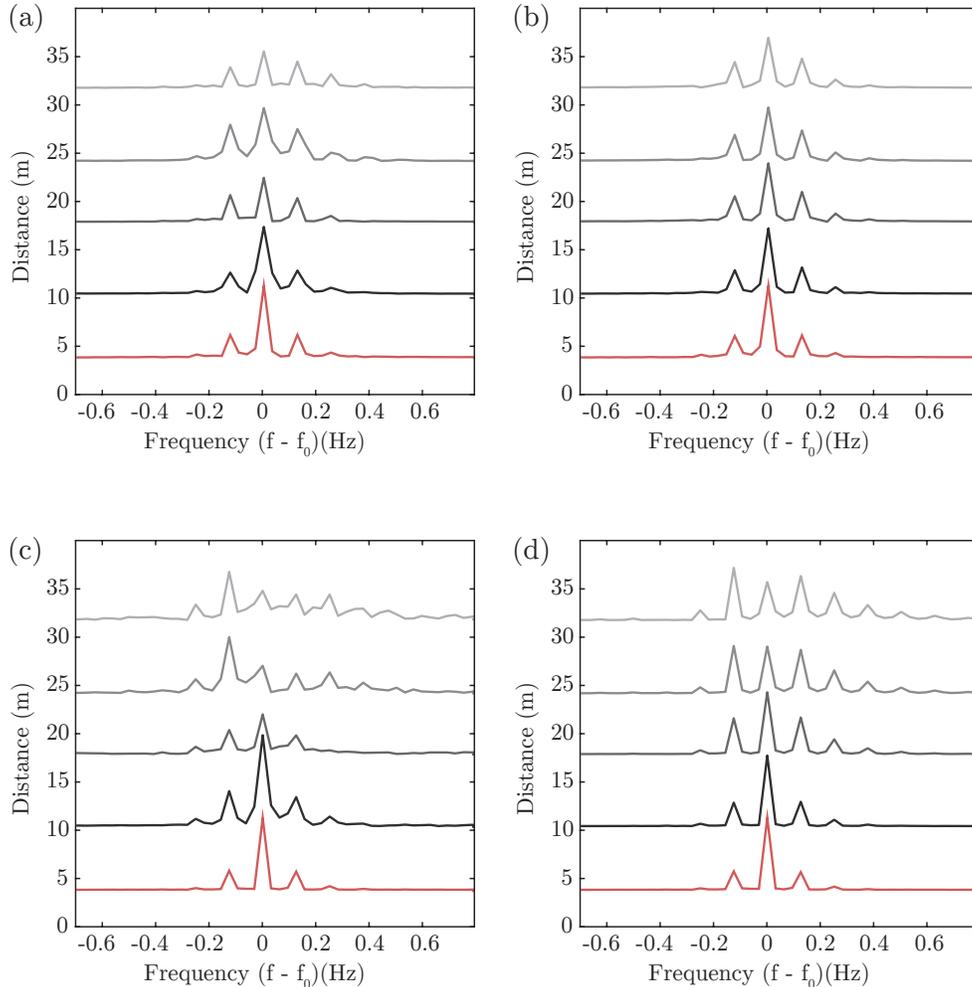


FIG. 3: Spectral amplitude $|\hat{\eta}|$ (normalized) offset by distance, for carrier wave parameters $\epsilon = 0.08$, $T_0 = 0.6$ s, AB parameters $x^* = -30$ m and $A = 0.25$, and simulation parameter $\nu = 1 \times 10^{-5}$ m²/s. (a,c) Experiments, (b,d) corresponding simulations. (a,b) $U = 0$ m/s, (c,d) $U = 4$ m/s. Initial condition (simulation) and measurement (experiment) at $x = 3.1$ m are indicated by the red line.

233 IV. EXPERIMENTAL RESULTS AND MODEL VALIDATION

234 Our numerical wind-wave model was compared to laboratory experiments on AB-type
 235 waves. All results presented here are based on carrier wave parameters $T = \frac{2\pi}{\omega_0} = 0.6$ s,
 236 $\epsilon = 0.08$ and AB parameter $A = 0.25$, the case for maximal modulation growth rate. For
 237 the simulations, we set $\nu = 1 \times 10^{-5}$ m²/s and Γ as presented in table I.

238 Based on the analytical solution to the NLS, it is expected for the AB to reach its
 239 maximal focusing after propagating 30 m from the wave maker ($x^* = -30$ m). Figure
 240 3 compares the evolution of the spectra retrieved from the experiment to the numerical
 241 simulations, without (a,b) and with the presence of wind $U = 4$ m/s (c,d). Note that the
 242 initial spectrum is still symmetric, both with and without wind, i.e. the lower sideband $\hat{\eta}_{-1}$
 243 and the upper sideband $\hat{\eta}_{+1}$ have a similar amplitude. In the experiment without wind, see
 244 figure 3a, due to the MNLS correction^{36,38}, and taking into account the effect of viscosity,
 245 the focal point is expected to be around $x = 36$ m. Note that the higher modes $\hat{\eta}_{+2}$ and $\hat{\eta}_{+3}$
 246 slightly grow towards the end of the tank. Indeed, our simulations in figure 3b reproduce
 247 these features. In the presence of wind, $U = 4$ m/s, both the experimental measurements
 248 in figure 3c and the simulations in figure 3d show an upstream shifting of the focal point,
 249 that is, $\hat{\eta}_0$ reaches a minimum around $x = 28$ m. In addition, under the wind action, the
 250 spectrum broadens, albeit in an asymmetric manner as a broad range of higher frequencies
 251 $\hat{\eta} > \hat{\eta}_0$ grow, while only a narrow range of the lower frequencies $\hat{\eta} < \hat{\eta}_0$ do.

252 In order to characterize the appearance of the spectral asymmetry, and see the effect of
 253 the wind on the shifting of the focal point, figure 4 displays the evolution of the carrier
 254 wave Fourier component $\hat{\eta}_0$ as well as the first upper and lower side bands as a function of
 255 propagation distance. With increasing wind speed the decay rate for $\hat{\eta}_0$ increases, equally,
 256 the growth rate of $\hat{\eta}_{-1}$ and $\hat{\eta}_{+1}$ increases. Consequently, the crossing point, at which the
 257 amplitude of the sidebands overtake $\hat{\eta}_0$, moves upstream with increasing wind speed. For
 258 $U = 4$ m/s, since $\hat{\eta}_{-1}$ has a higher growth rate than $\hat{\eta}_{+1}$, a downshift of the spectral peak
 259 originates, as measured at the last wave gauge. The numerical simulations of our developed
 260 model reproduce this shifting behavior. Similarly, the simulations matches the trend for the
 261 evolution of the total energy, in spite of the experimental fluctuations due to the inherent
 262 variability related to wind. Note that due to the width of the tank, a certain amount of this
 263 fluctuation in total energy measured by each wave gauge can also vary due to for instance
 264 reflected waves crossing the central line of the wave tank were the gauges were lined up,
 265 causing fluctuations superimposed on the trend set by the wind and viscosity input.

266 Figure 5 shows the evolution of spectral peak f_p (dots) and the spectral mean f_m (solid
 267 line) as a function of distance. The spectral peak remains equal to f_0 for $U = 0$ m/s, while
 268 for $U = 4$ m/s a downshift of the peak occurs. Due to the length limitations of the tank
 269 and the short considered fetch, we are not able to experimentally assess and state whether

Spectral up- and downshifting of Akhmediev breathers under wind forcing

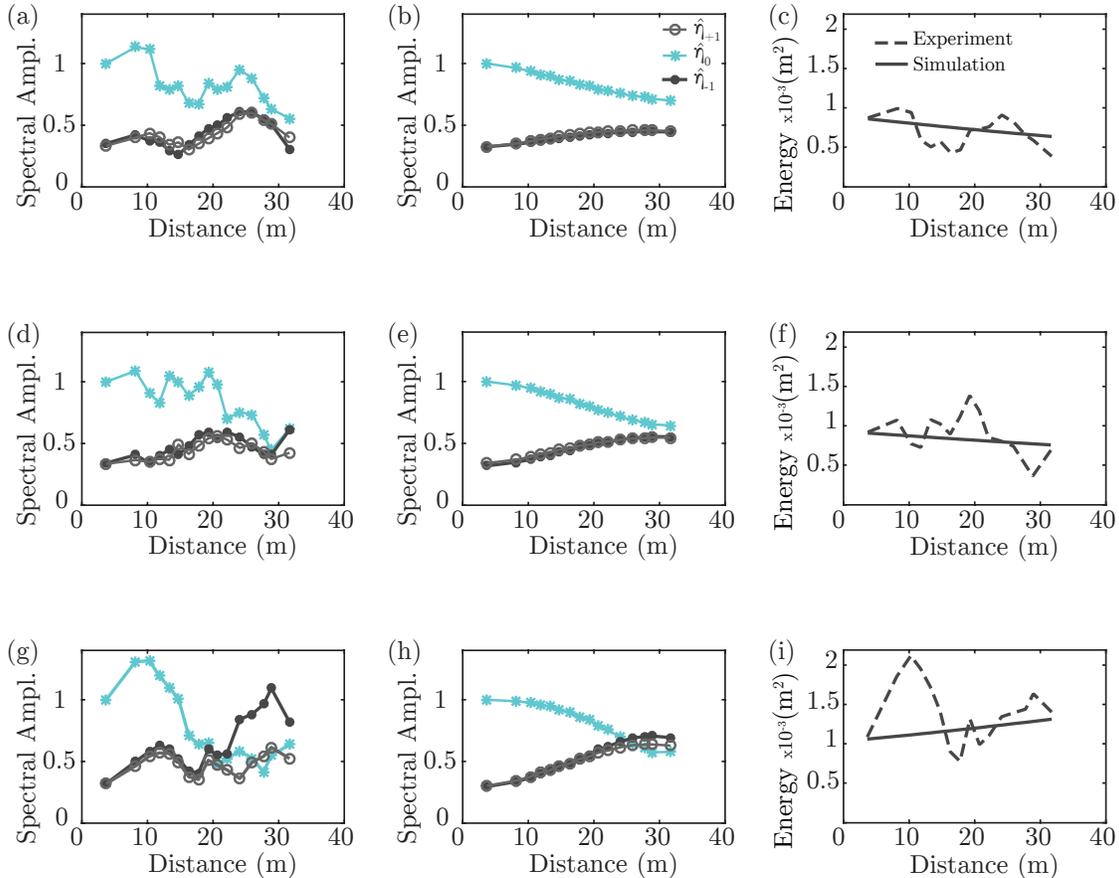


FIG. 4: Evolution of the three central Fourier components $\hat{\eta}_0$ (\star), $\hat{\eta}_{+1}$ (\circ) and $\hat{\eta}_{-1}$ (\bullet), for the same parameters as in figure 3. (a,d,g) Experiments, (b,e,h) simulations. Values are normalized to the amplitude of $\hat{\eta}_0$ at the first wave gauge. (c,f,i) Energy evolution. (a,b,c) $U = 0$ m/s, (d,e,f) $U = 2$ m/s, (g,h,i) $U = 4$ m/s.

270 this shift is permanent or temporary. In contrast to the spectral peak, the spectral mean
 271 in figure 5 demonstrates a clear upshift. This dissimilar behavior illustrates the need for a
 272 careful definition of up- and downshift in order to allow for an accurate description of the
 273 physics and dynamics at play.

274 Performing another experimental investigation for $U = 2$ m/s and for a shorter expected
 275 focal distance of about 20 m from the wave maker ($x^* = -20$ m), as shown in figure
 276 6, allows to quantify the spectral dynamics after the focal point. The spectral evolution
 277 indeed shows the same downshift behavior in the spectral peak, and a broad growth of the
 278 higher frequencies ($f > f_0$). However, for both of these features a reversion towards the

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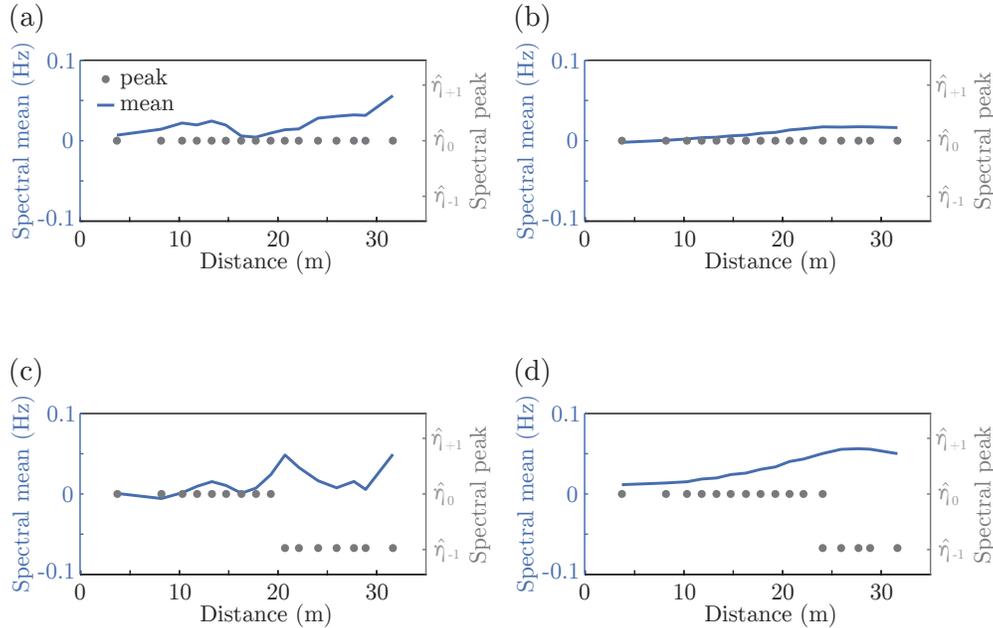


FIG. 5: Evolution of the spectral mean f_m (blue solid line, left vertical axis) and spectral peak f_p (gray dots, right vertical axis), for same parameters as in figure 3. (a,c) Experiments, (b,d) simulations. (a,b) $U = 0$ m/s, (c,d) $U = 4$ m/s. Two dots at the same distance indicates the spectral heights are within 1 percent range.

279 initial condition occurs at the end of the tank. The same is observed in figure 9 in Tulin
 280 and Waseda³⁷ or figure 4 in Chabchoub *et al.*³⁹. Indeed, for the homogeneous NLS, the
 281 Fermi-Pasta-Ulam recurrence⁵³ predicts a cyclic pattern after which the initial condition
 282 is completely recovered and then repeats several times. However, the MNLS framework
 283 predicts only a near-recovery^{35,37}, that is, a quasi-recurrence. With respect to the spectral
 284 mean, this quasi-recurrence causes an increase of the value of f_m near the focal point and
 285 a decrease to the original value when the cycle is finished. As a consequence, the spectral
 286 mean ‘oscillates’ if multiple quasi-recurrence cycles occur. The same oscillatory behavior is
 287 observed for the spectral peak as it shifts down and recovers. Together with the long range
 288 simulations in section V, this reveals that the spectral peak downshift downshift due to wind
 289 in the previous case, when the focal point was expected to be 30 m from the wave generator,
 290 is temporary too.

291 The upper limit of the model, with respect to the steepness, is reached when wave breaking
 292 occurs. Figure 7 shows the spectral evolution for the expected focal point at 30 m from

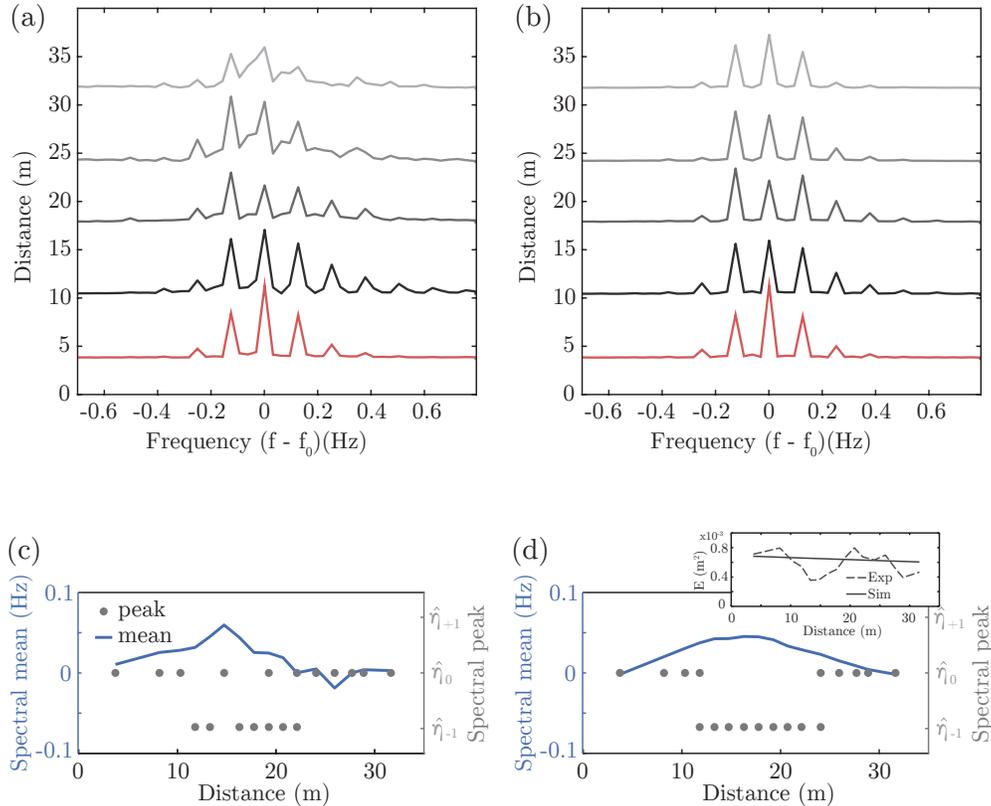


FIG. 6: (a) Spectral amplitude $|\hat{\eta}|$ (normalized) offset by distance at $U = 2$ m/s for (a) experiment and (b) simulations. Evolution of f_m (blue solid line) and f_p (gray dots) for experiment (c) and simulation (d). For the same parameters as in figure 3, except shorter focal distance $x^* = -20$ m. The inset shows the Energy evolution for the simulations and experiment

293 the wave maker ($x^* = -30$ m) and a stronger wind of $U = 6$ m/s. Clearly, a downshift
 294 is observed in the experiments, from which $\hat{\eta}_0$ does not recover. In the simulations, this
 295 permanent downshift cannot be reproduced, instead, a quasi-recurrence occurred. This
 296 disagreement can be attributed to the wave breaking that has in fact been observed to occur
 297 around $x \approx 15$ m, and is not considered in the model.

298 This observation underlines the importance of dissipation associated with wave breaking
 299 to induce a permanent downshift for both the spectral peak and mean. To further exemplify
 300 this fact, figure 8a shows the experimentally measured spectral peak shifts down several
 301 modes, up to $\hat{\eta}_{-3}$, for increasing wind speeds. A similar downward trend is observed for the

Spectral up- and downshifting of Akhmediev breathers under wind forcing

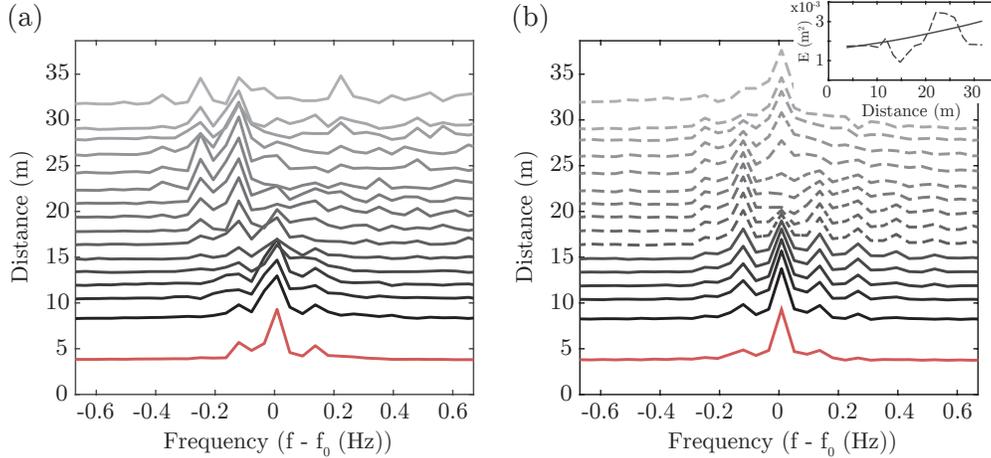


FIG. 7: Spectral amplitude $|\hat{\eta}|$ (normalized) offset by distance at $U = 6$ m/s for (a) experiment and (b) simulation, for the same parameters as in figure 3. The simulation is no longer accurate after the wave breaking event at $x \approx 15$ m, indicated by the dashed lines. The inset shows the Energy evolution for the simulations and experiment.

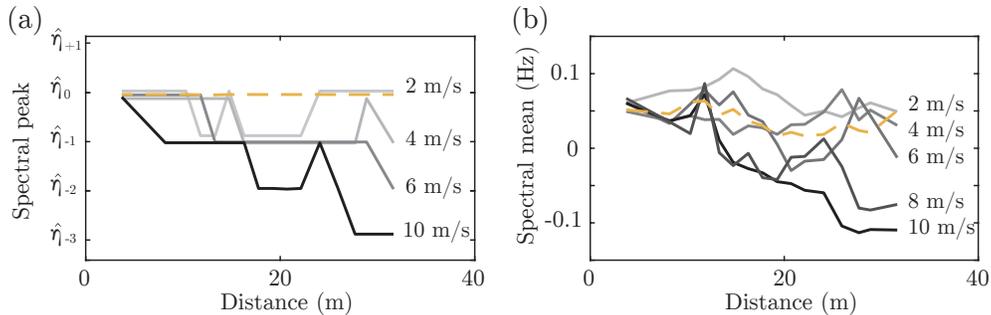


FIG. 8: Experimental evolution of (a) the spectral peak f_p and (b) the spectral mean f_m as a function of distance for increasing wind speeds, for same parameters as in figure 3, except shorter focal distance $x^* = -20$ m.. A systematic downshift occurs for strong winds of (a) the spectral peak up to the position of $\hat{\eta}_{-3}$ and (b) the spectral mean. Dashed line indicates $U = 0$ m/s. Wavebreaking occurs for $U \geq 4$ m/s.

302 spectral mean in figure 8b. For our carrier wave parameters, breaking occurs for $U > 4$ m/s,
 303 and therefore cannot be compared to simulations.

304 While the focus of this work lies on the forcing aspect of the wind, that is, up to the

305 point of wave breaking, it is worth mentioning efforts to model the wave evolution after
 306 the point of wave breaking. Kato and Oikawa²² propose an ad-hoc higher order term that
 307 activates at high steepness and leads to a permanent downshift of the peak. In a more
 308 theoretically structured approach, Trulsen and Dysthe^{21,40} add a symmetric source term
 309 to the MNLS equation and observe a permanent downshift to the most unstable mode.
 310 However, applying this model to our data did not yield the permanent downshift observed
 311 in for instance figure 7a. A rigorously derived model for the symmetric NLS is proposed
 312 and validated by experimental data in Tulin and Li⁵⁴, Hwung *et al.*⁵⁵, where a downshift
 313 to the most unstable mode is also observed. Here, the asymmetry is caused by an integral
 314 over the envelope. This model seems a promising candidate to simulate wave breaking in
 315 the framework of an MNLS equation.

316 Since the NLS inherently only applies to narrow banded spectra, it seems an open question
 317 whether a wave breaking term in an NLS-like evolution equation can account for a downshift
 318 of multiple modes, as observed in figure 8a, since this means the spectrum is broadened
 319 significantly. For fully nonlinear simulations, efforts to simulate wave breaking have been
 320 successfully conducted by Tian, Perlin, and Choi^{56,57}.

321 In summary, we observe a temporary downshift of the spectral peak towards the lower
 322 satellite, and a temporary upshift of the mean. Our model (11) reproduces the qualitative
 323 features of the experimental results well. These include the correct length scale and magni-
 324 tude of the spectral mean and peak shift, the crossing of the sidebands, and the broadening of
 325 the spectrum. Considering the variability inherent to wind experiments, a qualitative agree-
 326 ment on the spectral dynamics is the best one can expect. To overcome the length limit of
 327 our wave tank and investigate multiple semi-recurrence cycles, long range simulations have
 328 been performed as described in the next section

329 V. LONG RANGE SIMULATIONS

330 Numerical simulations have been performed over a length of 100 m on AB with parameters
 331 $x^* = -20$ m, $A = 0.25$ and carrier wave parameters $T_0 = 0.6$ s, $\epsilon = 0.1$, $\nu = 1 \times 10^{-5}$ m²/s,
 332 $\Gamma = 8.0 \times 10^{-3}$ s⁻¹ under wind forcing, in which case $\Gamma > 5\nu k_0^2$. To demonstrate the effect
 of our higher order wind term, three simulation cases are compared:

333

- 334 1. *Absence of wind*: Equation (11) including the MNLS correction and both leading
 335 and higher order viscosity terms, however without wind. In this case, $\delta_0 < 0$ and
 336 $\delta_1 < 0$.
- 337 2. *Leading order wind*: Equation (11) including the MNLS correction and both order
 338 viscosity terms, with only the leading order wind term, as posed in Kharif *et al.*³⁰,
 339 Trulsen and Dysthe⁴⁰, Onorato and Proment⁵⁸. In this case, $\delta_0 > 0$ and $\delta_1 < 0$.
- 340 3. *Full model*: Equation (11) including the MNLS correction and both order viscosity
 341 terms, with both the leading order and the higher order wind term. In this case, $\delta_0 > 0$
 342 and $\delta_1 > 0$.

343 Figure 9 compares the envelope amplitudes for these three cases. In absence of wind,
 344 simulation (i), the viscosity attenuates the amplitude. In addition, as described by Kim-
 345 moun *et al.*⁵⁹, it induces a shift in the quasi-recurrence pattern of the envelope, as indicated
 346 in figure 9a. Here, the dashed line is perpendicular on the gradient lines of the envelope
 347 amplitude. In simulation (ii), the wind amplifies the amplitude of the envelope with in-
 348 creasing fetch, as shown in figure 9b. In addition, the forcing cancels the shift caused by the
 349 viscosity term as now only maxima occur on the dashed line in figure 9b. In line with results
 350 reported in Kharif *et al.*³⁰, due to wind forcing and the increasing steepness, the position
 351 of the focal point is moved upstream to $x = 25$ m, and every subsequent quasi-recurrence
 352 cycle is more compressed in space compared to the previous one. The result of simulation
 353 (iii) is [similar to](#) that of simulation (ii). The consideration of the additional higher order
 354 wind term causes a slight further increase of the envelope's amplitude, and shortens the
 355 length of the quasi-recurrence cycle: figure 9c now fits 3 maxima of the envelope. Figure 9d
 356 displays the corresponding evolution of the normalized amplitude spectrum. Several MNLS
 357 quasi-recurrence cycles can be observed in which the spectrum broadens asymmetrically and
 358 narrows again. This quasi-recurrence pattern is superimposed on the general broadening of
 359 the spectrum due to wind action.

360 Comparing the spectral evolution of the simulation cases, however, the influence of the
 361 higher order wind term can be clearly observed. Figure 10a displays the spectral mean of the
 362 three simulations. Simulation (i) confirms the result of Carter and Govan²⁴, namely that
 363 the higher order viscosity term causes a downshift in the spectral mean. Since $\Gamma = 0$, $\delta_1 < 0$,

Spectral up- and downshifting of Akhmediev breathers under wind forcing

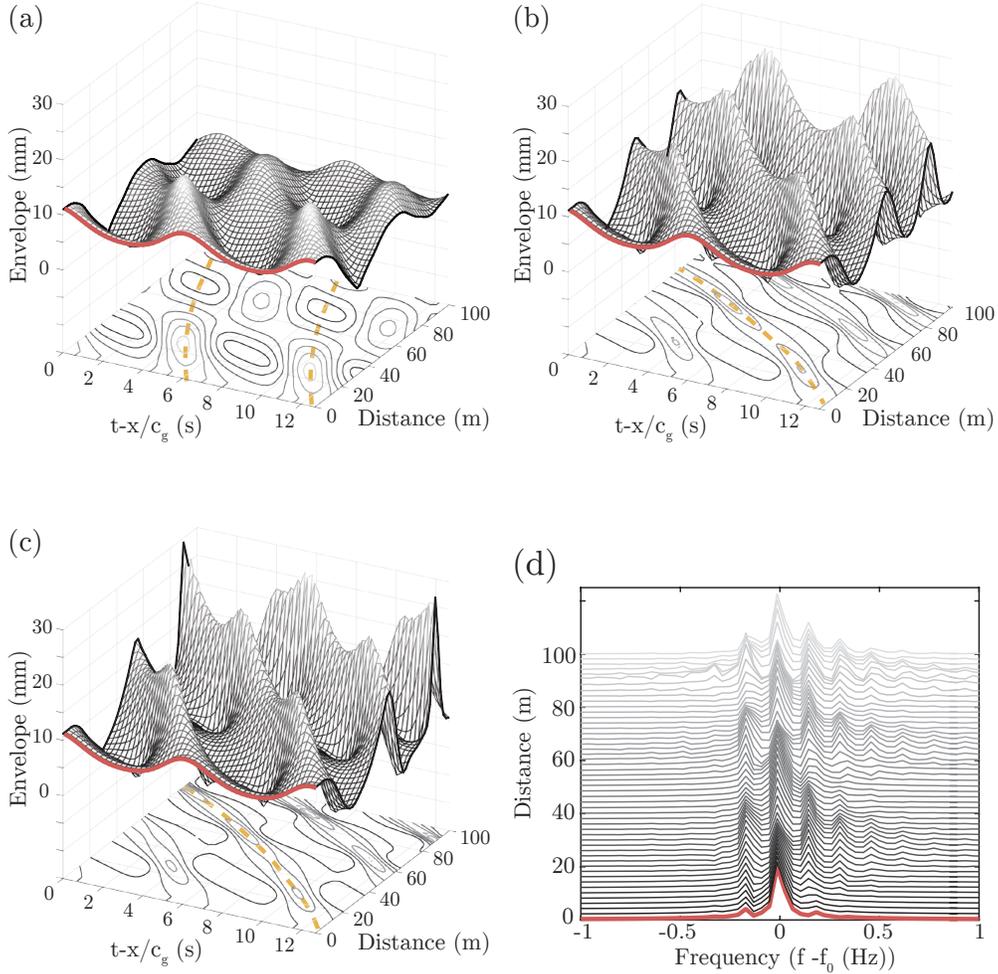


FIG. 9: Evolution of envelope in space and time for the (a) *no wind*, (b) *leading order wind*, (c) *full model* simulations. The orange dashed line shows the space-time evolution of the wave packets. (d) Simulated spectral amplitude $|\hat{\eta}|$ (normalized), for the *full model* simulation. Each spectrum is offset according to the distance Red line indicates the initial condition based on the theoretical Akhmediev solution at $x^* = -20$ m and $A = 0.25$ that is subsequently propagated in space by numerical integration. For carrier wave parameters $\epsilon = 0.1$, $T_0 = 0.6$ s, and simulation parameters $\nu = 1 \times 10^{-5} \text{ m}^2/\text{s}$ and $\Gamma = 8.0 \times 10^{-3} \text{ s}^{-1}$ when wind is active.

364 and the light dashed line, indicating the trend of the spectral mean, has a negative slope.
 365 In simulation (ii), the addition of the leading order wind term accelerates the oscillation of
 366 the spectral mean, without affecting the trend: the oscillations follow the same downward

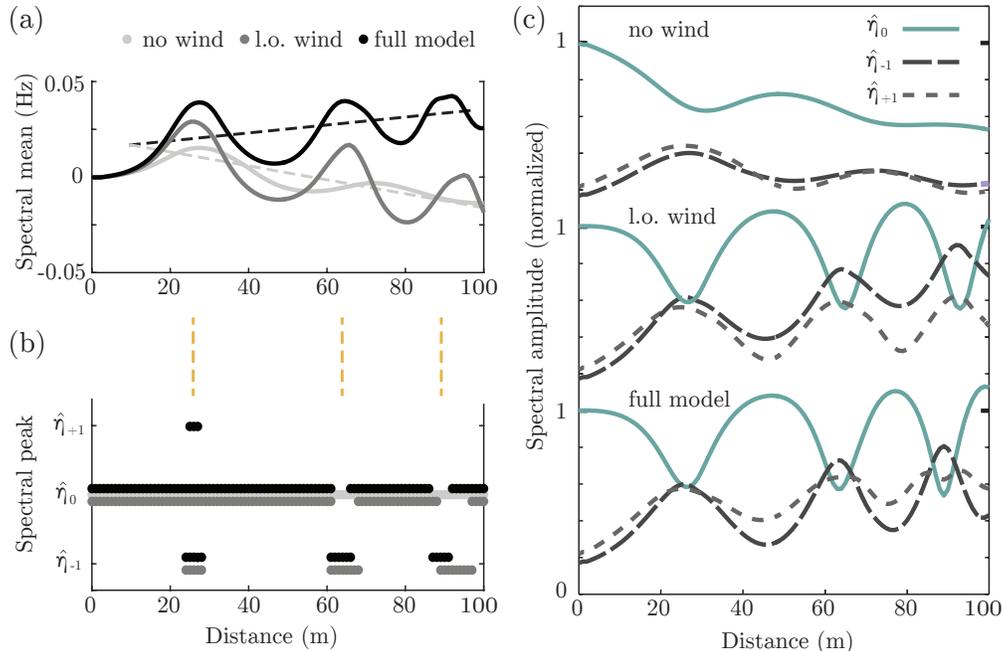


FIG. 10: (a) Simulated evolution of the spectral mean f_m . Dark dashed line is the trend set by the higher order wind term, light dashed line the trend set by the higher order viscosity term. (b) Evolution of the spectral peak f_p . Dots are offset for clarity. The dashed lines between (a) and (b) indicate the focal points of the quasi-recurrence cycles. (c) Simulated evolution of the Fourier amplitudes for of the three central modes $\hat{\eta}_0$ (green solid line), $\hat{\eta}_{-1}$ (dashed dark gray line) and $\hat{\eta}_{+1}$ (dashed light gray line). Comparing the *no wind*, *leading order (l.o.) wind* and *full model* simulations. Simulations based on the same parameters as figure 9.

367 slope as set by the higher order viscosity term ($\delta_1 < 0$). In contrast, in the full model
 368 simulation (iii), there is indeed a clear tendency towards a permanent upshift in the spectral
 369 mean, indicated by the dark dashed line, as now there is a forcing effect in the higher order
 370 ($\Gamma > 5\nu k_0^2$, $\delta_1 > 0$). In brief, due to the MNLS modification terms the mean oscillates around
 371 the slope induced by a balance between the higher order viscosity term and the higher order
 372 wind term.

373 Figure 10b shows the position of spectral peak f_p as a function of distance. As observed
 374 in the experimental data in Section IV, without wind forcing f_p remains equal to f_0 . In
 375 simulation (ii) a temporary downshift is observed. The origin of this downshift can be

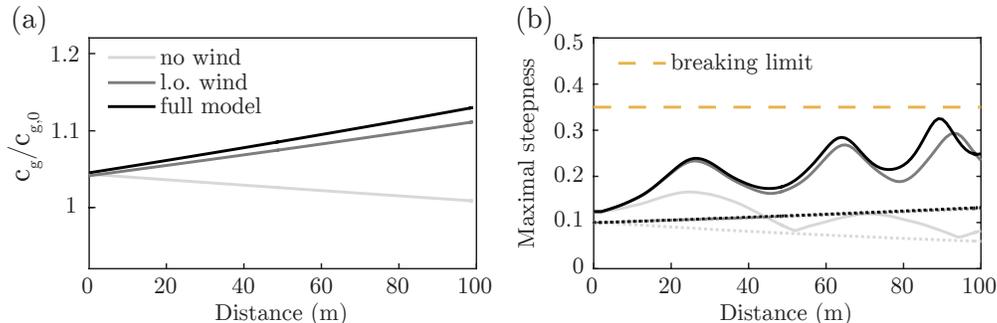


FIG. 11: Simulated evolution of (a) the group velocity and (b) the maximal steepness. Comparing the *no wind*, *leading order (l.o.) wind* and *full model* simulations. The dashed line indicates the critical steepness for wave breaking Simulations based on the same parameters as figure 9. The dotted lines indicate the characteristic steepness

$$\epsilon_{\text{char}} = \text{RMS}(\eta^2)k_0$$

376 revealed by analyzing figure 10c. Without wind action, the amplitudes of the modes oscillate,
 377 but the sidebands do not overtake the main mode. Considering the leading order wind effect,
 378 the wave steepness is amplified and consequently the growth and decay rates of all modes,
 379 increasing the frequency and amplitude of their oscillation. In figure 10c the sidebands do
 380 overtake the main peak within a quasi-recurrence cycle. Due to the initially slightly different
 381 growth rates, $\hat{\eta}_{-1}$ reaches a slightly higher amplitude than $\hat{\eta}_{+1}$ and a temporary downshift
 382 in the spectral peak sense occurs. The observations of a similar spectral peak downshift
 383 pattern by Tulin and Waseda³⁷ without wind forcing but at higher steepness, confirm that
 384 the spectral peak downshift is not a direct effect of wind forcing. Rather, it is an consequence
 385 due to the wind's influence on the steepness. That is, the spectral asymmetry inherent to
 386 the MNLS, where the lower sideband has a slightly higher growth rate than the upper
 387 sideband, is amplified by the wind as wave steepness is naturally increased. In simulation
 388 (iii) this downshift pattern is not significantly altered, although the quasi-recurrence cycles
 389 are slightly shorter. We can notice in figure 10c that instead of $\hat{\eta}_{-1}$ continuously being the
 390 dominant sideband, $\hat{\eta}_{-1}$ and $\hat{\eta}_{+1}$ alternate.

391 The addition of the forcing terms affects the group velocity. The homogeneous NLS
 392 implies a linear group velocity $c_{g,0}$. Taking into account the MNLS correction terms increases
 393 the wave packets' speed⁶⁰. The higher the steepness, the higher the importance of the MNLS

394 correction and the higher the increase in c_g . For the *no wind* simulation the dashed line
 395 in figure 9a shows a curvature of the direction of the wave packet propagation in the $x - t$
 396 plane that is attenuated towards the end of the tank due to the decrease in steepness caused
 397 by viscosity and dissipation. In contrast, for the wind simulations, the dashed lines indicate
 398 that this induced variation of group velocity increases with the increase of the steepness.
 399 Figure 11a quantifies the deviation from the linear group velocity for the three simulation
 400 cases (i), (ii) and (iii). In the experiment, a similar increase in the group velocity is indeed
 401 observed.

402 It should be noted that the regime in which the higher order wind term becomes relevant
 403 is hard to reach experimentally due to its proximity to the wave breaking threshold. An in-
 404 creased steepness makes the higher order wind term of greater influence and hereby increases
 405 the deviation from simulation (ii). However, at the same time, high steepness brings waves
 406 closer to the breaking threshold, beyond which our model (11) is incomplete. Furthermore,
 407 the significance of the higher order wind term increases with the wind strength and fetch,
 408 see figure 10a. Indeed, the steepness is also increased as a consequence of wind strength and
 409 fetch, as displayed in figure 11b. Thus it is important to monitor the steepness in the simula-
 410 tions to signal possible wave breaking. In our long range simulations, the maximal steepness
 411 of the wave $\epsilon_{\max} = k_0 a_{\max}$, remains below the breaking limit of $\epsilon = 0.35$ as considered by
 412 Trulsen and Dysthe⁴⁰. While this value for the threshold number was calculated for Stokes
 413 waves in the absence of wind, Saket *et al.*⁶¹ show the breaking threshold is very similar for
 414 wind driven waves. Note that other studies suggest an even higher critical steepness⁶²⁻⁶⁴.

415 VI. DISCUSSION

416 For the spectral peak, data and simulations alike show that a forcing wind can induce
 417 a downshift. However, the underlying cause for the faster growth of the lower sideband is
 418 the asymmetry introduced by the MNLS correction terms, which is amplified by the wind.
 419 Moreover, this downshift is only temporary. Considering the spectral mean, our long range
 420 simulations show that the higher order wind term creates a permanent upward trend, while
 421 the higher order viscosity term causes a permanent downward trend. As both terms have
 422 the same form in (11), the balance between these two, the sign of δ_1 , determines whether an
 423 upshift or a downshift in the spectral mean will be observed. Finally, when the wind action

TABLE II: Our observations of the influence on the spectrum of the MNLS correction, and the leading order (l.o.) and higher order (h.o.) wind and viscosity terms in (11).

US = upshift, DS =downshift.

Terms	Effect	
	Mean	Peak
MNLS correction	temporary DS	temporary US
l.o. viscosity \rightarrow lower ϵ	slower MNLS dynamics	slower, damped oscillation
h.o. viscosity	-	permanent DS
l.o. wind \rightarrow higher ϵ	faster MNLS dynamics	faster, amplified oscillation
h.o. wind	-	permanent US

424 is sufficiently strong, wave breaking is a natural result. We experimentally confirm the well
 425 known notion that wave breaking induces a permanent downshift in both the spectral peak
 426 and spectral mean. Our observations on the effect of the wind, viscosity, and the MNLS
 427 modification are summarized in table II.

428 These findings might seem contradictory with respect to existing literature in which wind
 429 is often associated with spectral downshift of gravity waves^{18–20,22,40}, as discussed. However,
 430 by taking into account the different downshift interpretations and making the distinction
 431 between wind forcing, in which energy is added to the system and forms of energy dissipation
 432 that can be triggered by wind, our results extend the existing framework, as upon closer
 433 inspection instances of permanent downshift are associated with dissipation. This distinction
 434 between the direct and potential indirects effects of wind solves the downshift paradox. [This](#)
 435 [idea is confirmed for unidirectional waves in the review of the frequency downshift by Dias](#)
 436 [and Kharif⁴⁷.](#)

437 The most obvious dissipative phenomenon that can occur as a result of wind forcing is
 438 that waves reach a critical steepness and break. Wave breaking shifts the spectral peak
 439 to a lower frequency. This has indeed been already experimentally observed by Tulin and
 440 Waseda³⁷, Melville⁶², Lake *et al.*⁶⁵ and is explained along the general line of energy being
 441 dissipated from the higher modes into the lower modes. Efforts have been made to model
 442 wave breaking theoretically^{21,22,40,54,55,57}, as discussed. Another instance in which wind can

443 have a dissipative effect is when it blows in opposite direction of the wave propagation and
 444 as such damps the waves. This configuration has been modeled by Schober and Strawn²⁰,
 445 a study in which the dissipation term as defined by Kato and Oikawa²² has been taken into
 446 account and a permanent downshift is modeled as well. Finally, even when the direct forcing
 447 effect of wind is included in a study, the dominant regime for spectral movement can still
 448 be dissipative when the viscosity is strong. For example, Touboul and Kharif¹⁹ observed
 449 a permanent downshift in the spectral peak due to the effect of wind. However, they are
 450 exactly on the balance of forcing and dissipation in the leading order, $\Gamma \approx 4k_0^2\nu$, and thus,
 451 in the higher order regime the dissipation is slightly dominating, $\delta_1 < 0$. A similar argument
 452 holds for Hara and Mei¹⁸. While our study applies to surface gravity waves, it is interesting
 453 to note that for capillary-gravity waves Hara and Mei⁶⁶ numerically found a frequency
 454 upshift of the spectral peak due to wind, and Skandrani, Kharif, and Poitevin⁶⁷ have shown
 455 numerically that the shift to lower frequencies is promoted by a damping mechanism.

456 VII. CONCLUSION

457 We derive a higher order $O(\epsilon^4)$ wind forcing contribution to the MNLS framework, re-
 458 sulting in a forced-damped MNLS equation (11). The direct effect of this term, when it
 459 exceeds the viscosity at the same order, is an upshift of the spectral mean. This trend is
 460 superimposed on the oscillation caused by the MNLS correction. For significant wind ac-
 461 tion, the higher order wind term cancels the downshifting effect of the higher order viscosity
 462 term and moves towards an upward trend of the spectral mean. The leading order wind
 463 term is symmetric but can amplify the asymmetric growth initiated by the MNLS correction
 464 terms, resulting in a temporary downshift of the spectral peak. Finally, we confirm that the
 465 permanent downshift of the spectral mean and of the spectral peak often observed in wind
 466 experiments is an indirect effect associated with dissipation, including wave breaking.

467 This is the first time a propagation equation for deep water waves including wind forcing
 468 is validated by laboratory experiments. The evolution of an Akhmediev breather in the
 469 presence of wind shows good agreement with the model in the limited fetch, dictated by the
 470 length of the facility. A natural continuation of this work is an experimental exploration of
 471 the full damping-forcing range that we have now modeled in the leading and higher order in
 472 (2.8), in particular the upshift in the spectral mean predicted by the higher order wind term.

473 In addition, increasing the fetch in the laboratory environment would improve the validation
474 analysis of the numerical results. However, it is a delicate balance between observing the
475 effect of the higher order term, and driving the model out of its validity range due to wave
476 breaking, since the elements that increase the effect of the higher order term, *i.e.* fetch,
477 wind speed and steepness, at the same time drive waves toward breaking.

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483 infrastructure.

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