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Numerical Study of Novel Ratiometric Sensors Based on Plasmon–Exciton Coupling

Yuankai Tang¹, Xiantong Yu¹, Haifeng Pan¹, Jinquan Chen¹, Benjamin Audit², Françoise Argoul⁴, Sanjun Zhang¹,³, and Jianhua Xu¹

Abstract
We numerically studied the optical properties of spherical nanostructures made of an emitter core coated by a silver shell through the generalized Mie theory. When there is a strong coupling between the localized surface plasmon in the metallic shell and the emitter exciton in the core, the extinction spectra exhibit two peaks. Upon adsorption of analytes on these core-shell nanostructures, the intensities of the two peaks change with opposite trends. This property makes them potential sensitive ratiometric sensors. Molecule adsorption on these nanostructures can be quantified through a very simple optical configuration likely resulting in a much faster acquisition time compared with systems based on the traditional metal nanoparticle surface plasmon resonance (SPR) biosensors.

Keywords
Surface plasmon resonance, nanoparticles, biosensor, strong coupling

Introduction
Localized surface plasmon resonance (LSPR) is the resonance of oscillating excitation light with localized surface plasmon (LSP) in confined metal nanostructures. It has attracted the attention of many researchers due to its wide range of applications including surface-enhanced Raman scattering,¹ plasmon-enhanced fluorescence,² and modified spontaneous emission.³ Interactions between LSP and the surrounding environment can significantly change the resonance frequency. This makes, for example, LSPR sensitive to the change of local refractive index⁴ and to biomolecules adsorption on nanostructure surface.¹⁵ Various LSPR-based sensors have been designed based on gold and silver nanostructures of diverse shapes like star-shaped gold nanoparticles,⁶ silver nanocubes,⁷ gold nanorods,⁵ gold–silica–gold nanoshells,⁸ silver nanoparticles,⁴ gold–air–gold nanoparticles, and silver–air–silver nanoparticles.⁹ Most LSPR-based sensory systems operate via the measurement of a peak frequency shift in the extinction or scattering spectra.¹⁰

In this paper, we introduce a novel ratiometric core-shell sensor based on strong coupling between shell LSP and emitter exciton, allowing to quantify a local refractive change in the surrounding of the sensor through the intensity ratio of two extinction peaks. We used Mie’s theory which has been demonstrated to be an effective method to model the optical properties of spherical core-shell nanoparticles¹¹–¹³ and the strong coupling between plasmons and excitons.¹⁴–¹⁶ We numerically studied the scattering, absorption, and extinction spectra of an emitter core with a silver shell (emitter@silver core-shell) nanostructure. When there is a strong coupling between the shell LSP and the emitter exciton, the spectra exhibit two peaks resulting from the plasmon polariton splitting.

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The intensities of the two peaks change with opposite trends upon adsorption of analytes on the core-shell surface. This makes emitter@silver core-shell nanostructures potential sensitive ratiometric sensors. We discuss a very simple optical measurement configuration to quantify molecule adsorption on the sensor surface, likely presenting a much faster acquisition time compared with configurations based on the traditional metal nanoparticle sensors.

**Method of Numerical Simulation**

Mie’s theory has been widely used to describe light scattering from homogeneous spherical particles. It was generalized to spheres made of an arbitrary number of homogeneous layers: first in 1985 by Bhandari that developed a matrix formalism to compute the scattering coefficients for a multilayered sphere and more recently by Sinzig and Quinten that presented a recursive formalism to compute these scattering coefficients. We employed the latter formalism that is more practical for numerical calculations.

As in Sinzig et al, we considered core-shell structures with \( r \) homogeneous layers (Fig. 1) of refractive indices \( n_s \) (\( s = 1, \ldots, r \)), embedded in a non-absorbing medium with real refractive index \( n_{r+1} \). When they are illuminated by an unpolarized incident plane wave of vacuum wavelength \( \lambda \), so that in each medium the corresponding wavenumber is \( k_s = 2\pi n_s / \lambda \), the cross-sections of scattering \( C_{sca} \), extinction \( C_{ext} \) and absorption \( C_{abs} \) can be written as:

\[
C_{sca} = \frac{2\pi}{k_{r+1}^2} \sum_{n=1}^{\infty} (2n+1) \left( |a_n|^2 + |b_n|^2 \right)
\]

(1)

\[
C_{ext} = \frac{2\pi}{k_{r+1}^2} \sum_{n=1}^{\infty} (2n+1) \text{Re}(a_n + b_n)
\]

(2)

\[
C_{abs} = C_{ext} - C_{sca}
\]

(3)

![Figure 1. Scheme of the core shell structure with \( r \) homogenous layers. Innermost layer (\( s = 1 \)) is the spherical core, layers \( s = 2, \ldots, r \) are \( r-1 \) shells and layer \( s = r + 1 \) is the external medium. \( d_s \) is the external diameter of the \( s \)th layer (\( s = 1, \ldots, r \)).](image)

where \( a_n \) and \( b_n \) are related to the decomposition of the scattered electromagnetic field on the spherical vector harmonics of order \( n \). Writing \( y_s = k_s d_s \) where \( d_s \) is the external diameter of the \( s \)th layer and \( m_s = n_{s+1} / n_s \) for \( s = 1, \ldots, r \), the coefficients \( a_n \) and \( b_n \) can be computed as:

\[
a_n = -\frac{m_s \psi_n(m_s y_s) \left[ \psi_n'(y_s) + T_{n+1}^s \chi_n'(y_s) \right]}{m_s \xi_n(m_s y_s) \left[ \psi_n(y_s) + T_{n+1}^s \chi_n(y_s) \right]} - \frac{m_s \psi_n'(m_s y_s) \left[ \psi_n(y_s) + T_{n+1}^s \chi_n(y_s) \right]}{m_s \xi_n(m_s y_s) \left[ \psi_n'(y_s) + T_{n+1}^s \chi_n'(y_s) \right]}
\]

(4)

\[
b_n = -\frac{m_s \psi_n'(m_s y_s) \left[ \psi_n(y_s) + S_{n+1}^s \chi_n(y_s) \right]}{m_s \xi_n(m_s y_s) \left[ \psi_n(y_s) + S_{n+1}^s \chi_n(y_s) \right]} - \frac{m_s \psi_n(m_s y_s) \left[ \psi_n'(y_s) + S_{n+1}^s \chi_n'(y_s) \right]}{m_s \xi_n(m_s y_s) \left[ \psi_n'(y_s) + S_{n+1}^s \chi_n(y_s) \right]}
\]

(5)

where \( r - 1 \) is the number of shells, \( \psi_n, \chi_n \) and \( \xi_n \) are the Ricatti–Bessel functions: \( \psi_n(z) = j_n(z), \chi_n(z) = y_n(z) \), \( \xi_n = z b_n^{(1)}(z) \), where \( j_n \) and \( y_n \) are the first and second kind spherical Bessel functions, respectively, and \( h_n^{(1)} \) the first kind spherical Hankel function and the prime denotes derivation of the function. Finally, coefficients \( T_{n+1}^s \) and \( S_{n+1}^s \) can be computed recursively using the following expressions:

\[
T_{n+1}^s = -\frac{m_s \psi_n(m_s y_s) \psi_n'(y_s) - \psi_n'(m_s y_s) \psi_n(y_s)}{m_s \chi_n(m_s y_s) \psi_n(y_s) - \chi_n'(m_s y_s) \psi_n'(y_s)}
\]

(6a)

\[
T_{n+1}^s = -\frac{m_s \psi_n(m_s y_s) \psi_n'(y_s) - \psi_n'(m_s y_s) \psi_n(y_s)}{m_s \chi_n(m_s y_s) \psi_n(y_s) - \chi_n'(m_s y_s) \psi_n'(y_s)}
\]

(6b)

\[
S_{n+1}^s = -\frac{\psi_n(m_s y_s) \psi_n'(y_s) - m_s \psi_n'(m_s y_s) \psi_n(y_s)}{\chi_n(m_s y_s) \psi_n(y_s) - m_s \chi_n'(m_s y_s) \psi_n'(y_s)} - \frac{\psi_n'(m_s y_s) \psi_n(y_s) + S_{n+1}^s \chi_n'(y_s)}{\chi_n(m_s y_s) \psi_n(y_s) + S_{n+1}^s \chi_n(y_s)}
\]

(7a)

\[
S_{n+1}^s = -\frac{\psi_n(m_s y_s) \psi_n'(y_s) - m_s \psi_n'(m_s y_s) \psi_n(y_s)}{\chi_n(m_s y_s) \psi_n(y_s) - m_s \chi_n'(m_s y_s) \psi_n'(y_s)} - \frac{\psi_n'(m_s y_s) \psi_n(y_s) + S_{n+1}^s \chi_n'(y_s)}{\chi_n(m_s y_s) \psi_n(y_s) + S_{n+1}^s \chi_n(y_s)}
\]

(7b)

We used Eqs. 1–7 to numerically estimate the cross-section versus wavelength spectra of emitter@silver core-shell nanostructures in order to study the behavior of the LSPR of the silver shell in the diverse geometries of interest. In Eqs. 1–3, the cross-sections are the sum of infinite number of terms. Hence, a suitable summation cutoff has to be
chosen that ensure calculation convergence. Terms of index $n$ correspond to the $n$th order eigenmode: dipole $n = 1$, quadrupole $n = 2$, octupole $n = 3$, and so on. The general trend is that higher order eigenmodes become more important with increasing nanoparticle size. For example, when the radius of the Ag sphere is less than 20 nm, the quadrupole has negligible contribution to the extinction cross section of the Ag spheres. However, when the radius is larger than 20 nm, the quadrupole turns on in the extinction spectrum, and when the radius is larger than 60 nm, the octopole has also to be considered. We performed these numerical calculations using Matlab and set $n = 10$ to avoid missing any high order eigenmodes.

Generalized Mie’s theory is thus an operational way to compute optical cross-sections of nanoparticle with core-shell structure, capturing special phenomena related to LSPR, such as strong coupling and Spasers. The cross-section calculations using our implementation of Sinzig and Quinten formalism agreed well with these reports. These observations prompted us to use Sinzig and Quinten formalism to investigate the strong coupling between LSPR and exciton in core-shell structure in this research.

Results and Discussion

Optical Properties of SiO$_2$@Ag Core-Shell Nanoparticles

We began by studying the LSPR properties of silver nanoparticles by computing their optical cross-section spectra as a function of wavelength using our implementation of the generalized Mie theory. The refractive index of silver was obtained from McPeak et al. Figure 2a shows the calculated optical cross-sections of a spherical Ag nanoparticle of diameter 40 nm. The optical spectra present a resonance wavelength at 395 nm, which is in good agreement with the extinction peak of Ag nanoparticles in water ($\lambda \approx 400$ nm). Note that the sensitivity of this resonance wavelength to the surrounding environment has been widely used to utilize Ag nanoparticles as label-free biosensors. With increasing size of the silver particle, the resonance peak in cross-section spectra is red shifted as demonstrated in Fig. 2b. When the diameter is larger than 80 nm, a quadrupole resonance peak becomes apparent at a shorter wavelength ($\lambda \approx 390$ nm), in accordance with experimental results.

Figure 2. (a) Scattering, extinction, and absorption spectra of a silver sphere in water; the diameter is $d_1 = 40$ nm. (b) Scattering cross section spectra of silver sphere of different size. (c) Scattering, extinction, and absorption spectra of a SiO$_2$@Ag core shell nanostucture in water; the diameters are $d_1 = 30$ nm and $d_2 = 40$ nm. (d) Scattering cross section spectra of SiO$_2$@Ag core shell nanostuctures of different sizes, the thickness of the silver shell was kept at 5 nm.
The refractive index of SiO$_2$ has been measured as $1.46$ in the wavelength range of 300–1000 nm, allowing us to numerically compute the cross-section spectra of SiO$_2$@Ag core-shell nanostructures (Fig. 2c, d). The SiO$_2$ core significantly changes the optical properties compared with pure Ag nanoparticles of same size (Fig. 2). The resonance peak is red-shifted and becomes narrower, and the quadrupole resonance is suppressed. The resonance wavelength of the 40 nm-diameter SiO$_2$@Ag nanostructure (Fig. 2c) is longer than that of the 100 nm-diameter Ag nanoparticles, but the width of the resonance peak is tenfold narrower. This signifies that SiO$_2$@Ag core-shell structures have a longer decay time of LSPR compared to pure silver particles with the same resonance wavelength.

**Strong Coupling Between TDBC Core and Silver Shell**

The strong interaction between LSPR and emitter leads to plasmon–exciton states behaving like vacuum Rabi splitting, i.e., splitting of energy levels and change of peak positions. There are three models to describe the strong coupling between plasmon and exciton, namely, classical, semi-classical, and quantum description. Mie’s theory describes light as a classical field and uses the refractive index to account for the material properties based on Maxwells’s theory. Here, we used TDBC as the active material to form exciton. A Lorentz model is used to describe the permittivity of TDBC:

$$\varepsilon_{TDBC}(e) = \varepsilon_m(e) + \frac{f g^2 \hbar^2}{m_e q L_z} \frac{1}{e^2 - e^2 - i \frac{\gamma_0}{4} e}$$

where $\hbar$ is the reduced Planck constant, $\varepsilon_m$ is the dielectric constant of the matrix in which the TDBC is doped, $e$ is the energy, $e_0 = 2100$ meV is the absorption energy of TDBC, $f = 1.8 \times 10^{18}$ m$^{-2}$ is the oscillator strength of the J-aggregate, $\gamma_0 = 49$ meV is the the exciton linewidth, $q$ the electron charge magnitude, $m$ the electron mass, $e_0$ the permittivity of free space, and $L_z$ the thickness of the TDBC. The imaginary part of the permittivity shows that absorption wavelength of TDBC is 590 nm.

We numerically computed the absorption, extinction, and scattering cross-section spectra of TDBC@Ag core-shell nanostructures in water while keeping the thickness of the silver shell at 5 nm (Fig. 3). For the particle with $d_2 = 50$ nm, a clear Rabi splitting of 110 meV (~30 nm) is observed, characteristic of strong coupling between plasmon and exciton in a TDBC@Ag core-shell nanostructure in water (Fig. 3a). Changing the diameter of the TDBC core while keeping the thickness of the Ag shell constant allow us to tune the frequency of LSPR to different wavelengths. Increasing the particle size, the two split peaks redshift and their intensities change with opposite trends due to the interaction between LSPR and emitter exciton. The central wavelengths $\omega_{\pm}$ of the two split peaks depend on the resonance wavelength $\omega_0$ of LSPR and the absorption wavelength $\omega_0$ of TDBC as:

$$\omega_{\pm} = \frac{1}{2} (\omega_0 + \omega_0) \pm \sqrt{g^2 + \delta^2} / 4$$

where $g$ is the coupling rate and $\delta$ is the detuning, $\delta = \omega_\text{pl} - \omega_0$. In fact, Eq. 9 is a description of Rabi splitting. The anti-crossing of the $\omega_{\pm}$ versus diameter curves observed in the inserts of Figs. 3b–d can be seen in strong coupling or weak coupling regime (e.g., induced transparency). To discriminate which regime the plasmon–exciton interaction locates, we extract the coupling rate $g$ by equation:

$$\left(\omega_{\pm} - \omega_0\right)^2 = \left(\omega_\text{pl} - \omega_0\right)^2 \left(\Gamma_\text{pl} - \gamma_0\right)^2$$

where $\Gamma_\text{pl}$ is the full width half–maximum (FWHM) linewidths of the plasmon. The coupling rate $g$ in Fig. 3 changes from 63 meV to 49 meV oppositely with the size of nanoparticle (Fig. S1), which reaches strong coupling regime according to $g^2 > (\Gamma_\text{pl}^2 + \gamma_0^2)/16$ (Table S1). It indicates that peak splitting is caused by plasmon–exciton strong coupling. However, increase of exciton material $\gamma_0$ and decrease of oscillator strength $f$ decrease coupling rate $g$. In such circumstance, the peak splitting becomes unobvious, and consequently not suitable for ratiometric measurement (Fig. S2). This numerical analysis shows that, by adapting the diameter of the exciton core and the thickness of the metal shell, the resonance frequency of LSPR can easily...
be tuned to match the absorption frequency of the exciton. In our case of TDBC@Ag core-shell nanostructures, the shape of the two peaks is similar and symmetric around the central wavelength \( \lambda = 590 \text{ nm} \) when the diameter of TDBC core and the thickness of Ag shell are 41 nm and 5 nm, respectively. In practice, this kind of core-shell nanostructures can easily be prepared by reducing silver nitrate on the emitter doped SiO\(_2\) nanoparticles.43–46

Using TDBC@Ag Core-Shell Nanostructures to Detect the Change of Local Refractive Index

The above discussion shows that the strong coupling in core-shell nanostructures is sensitive to LSPR. This property can be used in sensing application because the LSPR is influenced by the particle environment. When molecules adsorb on the core-shell nanostructure, the local refractive index near the nanoparticle is changed. This can be taken into account in our numerical formalism by considering an extra shell of refractive index \( n_s \), whose thickness was fixed to 2 nm (Fig. 4a). When the refractive index of the surrounding shell changes, the extinction spectrum of the strong coupling core-shell nanostructure changes as well. Figs. 4b and c shows the variation of the intensities and the wavelengths of the two split peaks as a function of the surrounding shell refractive index \( n_s \). With the increase of \( n_s \), the intensities of the two peaks change with opposite trends. Meanwhile, the wavelength of both peaks redshifts without crossing the 590-nm line, which is the absorption wavelength of TDBC. This feature can be used to perform ratiometric measurement, and the optical configuration is schematically illustrated in Fig. S3. Figure 4d shows the computed ratio of intensities of the two peaks versus the refractive index of surrounding shell \((n_s)\). When analytes adsorb on the surface, the local index changes, so does the ratio. The ratio changes with respect to the thickness of analyte shell (Figs. S4a and S4b). The thicker the analyte shell is, the larger the ratio changes. Furthermore, when the analyte shell is fixed to 2 nm, changing the thickness of the silver shell or the diameter of the TDBC core can tune the spectra (Figs. S4c and S4e), the intensity ratio still...
varies as a function of \( n_s \) (Figs. S4d and S4f). Since the detection employs the ratio of transmitted and reflected intensities, the intensity fluctuation of incident white light is eliminated. This is a big advantage over traditional LSPR sensors. For comparison, we simulated the response of traditional LSPR Ag sensor of 51 nm in the presence of a shell of adsorbed molecules (Fig. S5). When the refractive index of the surrounding shell \( (n_s) \) increases, the spectrum intensity changes and the peak shifts to a longer wavelength. However, the intensity variation cannot be used in practice to quantify the change in the surrounding shell because it is influenced by many factors, such as, the concentration of nanoparticles and the fluctuation of light source. Hence, the variation in the surrounding shell is usually quantified by the peak wavelength of the extinction spectrum. This implies that light has to be dispersed in a spectrometer in order to acquire the full spectrum, which is time-consuming. In contrast, the measurement framework (Fig. S3) that can be deployed when using core-shell nanostructures is simpler, and the acquisition time is much shorter because the response of many commercial photodetectors can exceed GHz, which is orders of magnitude higher than those used in full spectrum, such as charge-coupled device (CCD).

Hence, in principle, core-shell nanostructures presenting strong coupling between the shell LSP and the core emitter plasmon have unparalleled advantages over traditional LSPR sensors in the characterization of fast dynamic processes.

**Conclusion**

We numerically studied the strong coupling between LSPR and exciton in emitter@Ag core-shell nanostructures through generalized Mie theory. In this geometry, the resonance frequency of LSP in the Ag shell can easily be tuned to match the absorption frequency of the core exciton by changing the diameter of the emitter core and the thickness of the Ag shell. When tuning is achieved, strong coupling happens resulting in the optical cross-section spectra to exhibit a pair of split peaks. Upon adsorption of analytes on the surface of the core-shell nanostructure, the intensities of the two-peak change in opposite trends, which in principle allows core-shell nanostructures to be sensitive and quantitative ratiometric sensors. We proposed an optical experimental configuration to perform this measurement that is very likely to present a fast response time, much faster than what can be achieved with the traditional
biosensors based on surface plasmon resonance of pure metal nanoparticles.

Conflict of Interest
The authors declare that they have no conflict of interest.

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Supplemental Material
All supplemental material mentioned in the text, including five figures, is available in the online version of the journal.

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