L^p-L^q Maximal Regularity for some Operators associated with Linearized Incompressible Fluid-Rigid Body Problems

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L^p-L^q MAXIMAL REGULARITY FOR SOME OPERATORS ASSOCIATED WITH LINEARIZED INCOMPRESSIBLE FLUID-RIGID BODY PROBLEMS

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ABSTRACT. We study an unbounded operator arising naturally after linearizing the system modelling the motion of a rigid body in a viscous incompressible fluid. We show that this operator is $\mathcal{R}$ sectorial in L^q for every $q \in (1, \infty)$, thus it has the maximal L^p-L^q regularity property. Moreover, we show that the generated semigroup is exponentially stable with respect to the L^q norm. Finally, we use the results to prove the global existence for small initial data, in an L^p-L^q setting, for the original nonlinear problem.

Key words. Fluid Structure interaction, Incompressible flow, Maximal L^p regularity **AMS subject classifications.** 76D03, 35Q30,76N10

1. INTRODUCTION AND MAIN RESULTS

The aim of this work is to show that the semigroup associated to the equations obtained by linearizing some systems modelling fluid-structure interactions has the maximal L^p-L^q regularity property. This result can be seen as an improvement of those in [22, 21], where the result was proved in a Hilbert space setting and in [24], where it has been shown that the corresponding semigroup is analytic in $L^{\frac{6}{5}}(\mathbb{R}^3) \cap L^q(\mathbb{R}^3)$, for $q \geqslant 2$. We then apply this result in proving a global existence and uniqueness result, for small data, for the original nonlinear problem. Such a result seems new in an L^p-L^q setting. The references [22, 21] and [11] contain closely related results and methods which are often used in the present paper.

Let us first remind the original free boundary system which motivates this work. We will come back to this system later on, in order to prove the global existence of solutions for small initial data. The smallness is measured in a Besov space and the solutions lie in function spaces which are L^p with respect to time and L^q with respect to the space variable. As far as we know, global existence results of this type were known only in an L^2 setting.

Consider a rigid body immersed in a viscous incompressible fluid and moving under the action of forces exerted by the fluid only. At time $t \geqslant 0$, this solid occupies a smooth bounded domain $\Omega_S(t)$. The fluid and rigid body are contained in a bounded domain $\Omega \subset \mathbb{R}^3$ with smooth boundary $\partial\Omega$. We assume that there exists a constant α with

$$\operatorname{dist}(\Omega_S(0), \partial\Omega) \geqslant \alpha > 0. \tag{1.1}$$

At any time $t \geqslant 0$, we denote by $\Omega_F(t) = \Omega \setminus \overline{\Omega_S(t)}$ the domain occupied by the fluid. We assume that the motion of the fluid is governed by the incompressible Navier-Stokes equations, whereas the motion of the structure is governed by the balance equation for linear and angular momentum. The full system of equations modelling the rigid body inside the fluid can be

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written as

$$
\begin{aligned}
&\partial_t u + (u \cdot \nabla) u - \operatorname{div} \sigma(u, \pi) = 0, && t \in (0, \infty),\ x \in \Omega_F(t),\\
&\operatorname{div} u = 0, && t \in (0, \infty),\ x \in \Omega_F(t),\\
&u = 0, && t \in (0, \infty),\ x \in \partial\Omega,\\
&u = a'(t) + \omega(t) \times (x - a(t)), && t \in (0, \infty),\ x \in \partial\Omega_S(t),\\
&m a''(t) = - \int_{\partial\Omega_S(t)} \sigma(u, \pi) n \, \mathrm{d}\gamma, && t \in (0, \infty),\\
&J\omega'(t) = (J\omega) \times \omega - \int_{\partial\Omega_S(t)} (x - a(t)) \times \sigma(u, \pi) n \, \mathrm{d}\gamma, && t \in (0, \infty),\\
&u(0, x) = u_0(x) && x \in \Omega_F(0),\\
&a(0) = 0, \quad a'(0) = \ell_0, \quad \omega(0) = \omega_0.
\end{aligned}
\tag{1.2}
$$

In the above equations, $u(t, x)$ denote the velocity of the fluid, $\pi(t, x)$ denote the pressure of the fluid, $a(t)$ denote the position of the centre of the mass and $\omega(t)$ denote the angular velocity of the rigid body. The domain $\Omega_S(t)$ is defined by

$$
\Omega_S(t) = a(t) + Q(t) y, \quad \forall y \in \Omega_S(0),\ \forall t > 0,
$$

where $Q(t) \in \mathbb{M}_{3\times 3}(\mathbb{R})$ is the orthogonal matrix giving the orientation of the solid. More precisely, $\omega(t)$ and $Q(t)$ are related to each other through the following relation

$$
\dot{Q}(t) Q(t)^{-1} y = A(\omega(t)) y = \omega(t) \times y, \quad \forall y \in \mathbb{R}^3, \quad Q(0) = I_3, \tag{1.3}
$$

where the skew-symmetric matrix $A(\omega)$ is given by

$$
A(\omega) = \begin{pmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{pmatrix}, \qquad \omega \in \mathbb{R}^3.
$$

The constant $m > 0$ denote the mass of the rigid structure and $J(t) \in \mathcal{M}_{3\times 3}(\mathbb{R})$ its tensor of inertia at time t. This tensor is given by

$$
J(t) a \cdot b = \int_{\Omega_S(0)} \rho_S(y) (a \times Q(t) y) \cdot (b \times Q(t) y) \, dy, \qquad \forall a, b \in \mathbb{R}^3, \tag{1.4}
$$

where $\rho_S > 0$ is the density of the structure. One can check that

$$
J(t) a \cdot a \geqslant C_J |a|^2 > 0, \tag{1.5}
$$

where C_J is independent of $t > 0$. In the above, we have denoted by $\partial\Omega_S(t)$ the boundary of the rigid structure at time t and by $n(t, x)$ the unit normal to $\partial\Omega_S(t)$ at the point x directed towards the interior of the rigid body. The Cauchy stress tensor $\sigma(u, \pi)$ is given by

$$
\sigma(u, \pi) = -\pi I_3 + 2\nu\varepsilon(u), \quad \varepsilon(u) = \frac{1}{2} \left(\nabla u + \nabla u^\top \right), \tag{1.6}
$$

where the positive constant ν is the viscosity of the fluid.

Linearizing the above equations around the zero solution we obtain a system coupling Stokes equations in a fixed domain and an ODE system. The corresponding equations read as

$$\begin{aligned}
&\partial_t u - \nu\Delta u + \nabla\pi = f, \quad \operatorname{div} u = 0, && t\in(0,\infty),\ y\in\Omega_F(0),\\
&u = 0, && t\in(0,\infty),\ y\in\partial\Omega,\\
&u = \ell + \omega\times y, && t\in(0,\infty),\ y\in\partial\Omega_S(0),\\
&m\ell' = -\int_{\partial\Omega_S(0)} \sigma(u,\pi)n\, d\gamma + g_1, && t\in(0,\infty),\\
&J(0)\omega' = -\int_{\partial\Omega_S(0)} y\times\sigma(u,\pi)n\, d\gamma + g_2, && t\in(0,\infty),\\
&u(0,y) = u_0(y), && y\in\Omega_F(0),\\
&\ell(0) = \ell_0, \quad \omega(0) = \omega_0,
\end{aligned} \tag{1.7}$$

where n is the unit normal to $\partial\Omega_S(0)$ directed towards the interior of the rigid body.

Let us now define the operator associated with the above linear fluid-structure interaction problem, which was first introduced in [22, 21]. The idea is to extend the fluid velocity u by $\ell(t) + \omega(t)\times y$ in $\Omega_S(0)$. More precisely, for any $1 < q < \infty$ we define

$$\mathbb{H}^q(\Omega) = \left\{\varphi \in L^q(\Omega)^3 \mid \operatorname{div}\varphi = 0 \text{ in } \Omega, \quad \varepsilon(\varphi) = 0 \text{ in } \Omega_S(0), \quad \varphi\cdot n = 0 \text{ on } \partial\Omega\right\} \tag{1.8}$$

We define

$$\mathcal{D}(\mathbb{A}) = \left\{\varphi \in W^{1,q}_0(\Omega)^3 \mid \varphi|_{\Omega_F(0)} \in W^{2,q}(\Omega_F(0))^3, \quad \operatorname{div}\varphi = 0 \text{ in } \Omega, \quad \varepsilon(\varphi) = 0 \text{ in } \Omega_S(0)\right\}. \tag{1.9}$$

For all $v \in \mathcal{D}(\mathbb{A})$ we set

$$\mathcal{A}v = \begin{cases} -\nu\Delta v & \text{in } \Omega_F(0),\\ 2\nu m^{-1}\displaystyle\int_{\partial\Omega_S(0)} \varepsilon(v)n\, d\gamma + \left(2\nu J(0)^{-1}\int_{\partial\Omega_S(0)} y\times\varepsilon(v)n\, d\gamma\right)\times y & \text{in } \Omega_S(0),\end{cases}$$

and

$$\mathbb{A}v = \mathbb{P}\mathcal{A}v, \tag{1.10}$$

where $\mathbb{P}$ is the projection from $L^q(\Omega)^3$ onto $\mathbb{H}^q(\Omega)$. The existence of such projector $\mathbb{P}$ can be found in [24, Theorem 2.2].

Takahashi and Tucsnak [22] proved that the operator $\mathbb{A}$ defined above generates an analytic semigroup on $\mathbb{H}^2(\Omega)$ when $\Omega = \mathbb{R}^2$. When Ω is a smooth bounded domain in $\mathbb{R}^2$ the same result was proved by Takahashi [21]. Later, Wang and Xin [24] proved that the operator $\mathbb{A}$ generates an analytic semigroup on $\mathbb{H}^{6/5}(\mathbb{R}^3)\cap\mathbb{H}^q(\mathbb{R}^3)$ if $q \geqslant 2$ and when the solid is a ball in $\mathbb{R}^3$ the operator $\mathbb{A}$ generates an analytic semigroup on $\mathbb{H}^2(\mathbb{R}^3)\cap\mathbb{H}^q(\mathbb{R}^3)$ if $q \geqslant 6$. In this article, as a corollary of our main result, we prove that the operator $\mathbb{A}$ generates an analytic semigroup on $\mathbb{H}^q(\Omega)$ for any $1 < q < \infty$.

Before we state our main result, we introduce the notion of maximal L^p-regularity. Let us consider the following Cauchy problem:

$$z'(t) = Az(t) + f(t), \quad z(0) = z_0, \tag{1.11}$$

where A is a closed, linear densely defined unbounded operator in a Banach space $\mathcal{X}$ with domain $\mathcal{D}(A)$, $f : \mathbb{R}^+ \mapsto \mathcal{X}$ is a locally integrable function and $z_0 \in \mathcal{X}$.

Definition 1.1. We say A has maximal L^p-regularity property for $1 < p < \infty$, on $[0,T)$, $0 < T \leqslant \infty$, if for $z_0 = 0$ and for every $f \in L^p(0,T;\mathcal{X})$ there exists a unique $z \in W^{1,p}_{\text{loc}}[0,\infty);\mathcal{X})$ satisfying (1.11) almost everywhere and such that z' and Az belong to $L^p(0,T;\mathcal{X})$. We denote the class of all such operators by $\mathcal{MR}_p([0,T);\mathcal{X})$.

Remark 1.2. In the above definition we do not assume that $z \in L^p(0,T;\mathcal{X})$. In fact, if $T < \infty$ or $0 \in \rho(A)$, where $\rho(A)$ is the resolvent set of A, $z' \in L^p(0,T;\mathcal{X})$ can be replaced by $z \in W^{1,p}(0,T;\mathcal{X})$ ([6, Theorem 2.4]).

We now state our first main result:

Theorem 1.3. *Let $1 < p, q < \infty$ and $T < \infty$. Then $\mathbb{A} \in \mathcal{MR}_p([0,T];\mathbb{H}^q(\Omega))$. In particular, the operator $\mathbb{A}$ generates an analytic semigroup on $\mathbb{H}^q(\Omega)$ for any $q \in (1,\infty)$.*

To prove the above result we use the characterization of maximal L^p regularity due to Weis ([25, Theorem 4.2]), which says that maximal L^p regularity property in a UMD Banach space, in particular for L^q spaces, is equivalent to the $\mathcal{R}$-sectoriality property of the operator (see Section 2 for definition and properties of $\mathcal{R}$-sectorial operators).

The maximal L^p-L^q regularity property in finite time interval of the system (1.7), when $\Omega = \mathbb{R}^3$, was already proved in [11, 13]. However, the approach of those papers is different from our approach. In fact, in those papers, fluid and structure equations are treated separately and maximal L^p-L^q regularity property of the linear system (1.7) is proved by a fixed point argument. In our approach, we solve the fluid and structure equations simultaneously. In the study of fluid-structure interactions, this method is known as a monolithic approach. We refer to Maity and Tucsnak [18] where a similar approach has been used to prove maximal L^p-L^q regularity for several other fluid structure models.

The main advantage of such approach is that, by studying resolvent of the linear operator $\mathbb{A}$, we can conclude that the operator $\mathbb{A}$ generates a C^0-semigroup of negative type. This allows us to obtain the maximal L^p-L^q regularity of the system (1.7) on $[0,\infty)$. As a consequence, we obtain global existence and uniqueness for the full non-linear system (1.2) under a smallness condition on the initial data.

In order to state global existence and uniqueness result, we introduce some notation. Firstly $W^{s,q}(\Omega)$, with $s \geqslant 0$ and $q > 1$, denote the usual Sobolev spaces. We introduce the space

$$L^q_m(\Omega) = \left\{ f \in L^q(\Omega) \mid \int_\Omega f = 0 \right\}$$

and we set

$$W^{s,q}_m(\Omega) = W^{s,q}(\Omega) \cap L^q_m(\Omega).$$

Let $k \in \mathbb{N}$. For every $0 < s < k$, $1 \leqslant p < \infty$, $1 \leqslant q < \infty$, we define the Besov spaces by real interpolation of Sobolev spaces

$$B^s_{q,p}(\Omega) = (L^q(\Omega), W^{k,q}(\Omega))_{s/k,p}.$$

We refer to [23] for more details on Besov spaces. We also need a definition of Sobolev spaces in the time dependent domain $\Omega_F(t)$. Let $\Lambda(t,\cdot)$ be a C^1-diffeomorphism from $\Omega_F(0)$ onto $\Omega_F(t)$

such that all the derivatives up to second order in space variable and all the derivatives up to first order in time variable exist are continuous. For all functions $v(t,\cdot):\Omega_F(t)\mapsto\mathbb{R}$, we denote $\widehat{v}(t,y)=v(t,\Lambda(t,y))$. Then for any $1<p,q<\infty$ we define

$$\begin{aligned}
&L^p(0,T;L^q(\Omega_F(\cdot)))=\{v\mid \widehat{v}\in L^p(0,T;L^q(\Omega_F(0)))\},\\
&L^p(0,T;W^{2,q}(\Omega_F(\cdot)))=\left\{v\mid \widehat{v}\in L^p(0,T;W^{2,q}(\Omega_F(0)))\right\},\\
&W^{1,p}(0,T;L^q(\Omega_F(\cdot)))=\left\{v\mid \widehat{v}\in W^{1,p}(0,T;L^q(\Omega_F(0)))\right\},\\
&C([0,T];B^{2(1-1/p)}_{q,p}(\Omega_F(\cdot)))=\left\{v\mid \widehat{v}\in C([0,T];B^{2(1-1/p)}_{q,p}(\Omega_F(0))\right\}.
\end{aligned}$$

Theorem 1.4. *Let $1<p,q<\infty$ satisfying the conditions $\dfrac{1}{p}+\dfrac{1}{2q}\neq 1$ and $\dfrac{1}{p}+\dfrac{3}{2q}\leqslant\dfrac{3}{2}$. Let $\eta\in(0,\eta_0)$, where η_0 is the constant introduced in Theorem 4.1. Then there exist two constants $\delta_0>0$ and $C>0$, depending on p,q,η and $\Omega_F(0)$, such that, for all $\delta\in(0,\delta_0)$ and for all $(u_0,\ell_0,\omega_0)\in B^{2(1-1/p)}_{q,p}(\Omega_F(0))^3\times\mathbb{R}^3\times\mathbb{R}^3$ satisfying the compatibility conditions*

$$\begin{aligned}
&\operatorname{div} u_0=0 \text{ in } \Omega_F(0),\\
&u_0=\ell_0+\omega_0\times y \text{ on } \partial\Omega_S(0),\quad u_0=0 \text{ on } \partial\Omega \text{ if } \frac{1}{p}+\frac{1}{2q}<1\\
\text{and } &u_0\cdot n=(\ell_0+\omega_0\times y)\cdot n \text{ on } \partial\Omega_S(0),\qquad u_0\cdot n=0 \text{ on } \partial\Omega \text{ if } \frac{1}{p}+\frac{1}{2q}>1,
\end{aligned}$$

and

$$\|u_0\|_{B^{2(1-1/p)}_{q,p}(\Omega_F(0))^3}+\|\ell_0\|_{\mathbb{R}^3}+\|\omega_0\|_{\mathbb{R}^3}\leqslant\delta, \tag{1.12}$$

the system (1.2) admits a unique strong solution (u,π,ℓ,ω) in the class of functions satisfying

$$\begin{aligned}
&\|e^{\eta(\cdot)}u\|_{L^p(0,\infty;W^{2,q}(\Omega_F(\cdot)))^3}+\|e^{\eta(\cdot)}u\|_{W^{1,p}(0,\infty;L^q(\Omega_F(\cdot)))^3}+\|e^{\eta(\cdot)}u\|_{L^\infty(0,\infty;B^{2(1-1/p)^3}_{q,p}(\Omega_F(\cdot)))}\\
&\quad+\|e^{\eta(\cdot)}\pi\|_{L^p(0,\infty;W^{1,q}_m(\Omega_F(\cdot)))}+\|a\|_{L^\infty(0,\infty;\mathbb{R}^3)}++\|e^{\eta(\cdot)}a'\|_{L^p(0,\infty;\mathbb{R}^3)}\\
&\qquad+\|e^{\eta(\cdot)}a''\|_{L^p(0,\infty;\mathbb{R}^3)}+\|e^{\eta(\cdot)}\omega\|_{W^{1,p}(0,\infty;\mathbb{R}^3)}\leqslant C\delta.
\end{aligned}\tag{1.13}$$

Moreover, $\operatorname{dist}(\Omega_S(t),\partial\Omega)\geqslant\alpha/2$ *for all $t\in[0,\infty)$. In particular, we have*

$$\|u(t,\cdot)\|_{B^{2(1-1/p)}_{q,p}(\Omega_F(t))}+\|a'(t)\|_{\mathbb{R}^3}+\|\omega(t)\|_{\mathbb{R}^3}\leqslant C\delta e^{-\eta t}.$$

Remark 1.5. When $p=q=2$, the above result was proved in [21, Corollary 9.2].

Remark 1.6. Our proof of Theorem 1.4 also applies to the 2 dimensional case. In this case, we have to choose $1<p,q<\infty$ such that $\dfrac{1}{p}+\dfrac{1}{2q}\neq 1$ and $\dfrac{1}{p}+\dfrac{1}{2q}\leqslant\dfrac{3}{2}$.

The plan of this paper is as follows. In Section 2, we recall the definition and some basic properties of $\mathcal{R}$-sectorial operators. In Section 3, we prove Theorem 1.3. The stability of the operator $\mathbb{A}$ is proved in Section 4. Maximal L^p-L^q regularity of the linear fluid structure system on $(0,\infty)$ is studied in Section 5. Finally, in Section 6 we prove Theorem 1.4.

2. Some background on $\mathcal{R}$-sectorial operators

In this section we recall some definitions and basic results concerning maximal regularity and $\mathcal{R}$-boundedness in Banach spaces. For detailed information on these subjects we refer to [25, 16, 5] and references therein. For $\theta \in (0, \pi)$ we define the sector Σ_θ in the complex plane by

$$\Sigma_\theta = \{\lambda \in \mathbb{C} \setminus \{0\} \quad | \quad |\arg \lambda| < \theta\}. \tag{2.1}$$

In order to state Weis' theorem concerning maximal L^p-regularity of the Cauchy problem (1.11) we need to introduce so-called UMD spaces.

Definition 2.1. Let $\mathcal{X}$ be a Banach space. The Hilbert transform of a function $f \in \mathcal{S}(\mathbb{R}; \mathcal{X})$, the Schwartz space of $\mathcal{X}$-valued rapidly decreasing functions, is defined by

$$Hf(t) = \frac{1}{\pi} \lim_{\epsilon \mapsto 0} \int_{|s|>\epsilon} \frac{f(t-s)}{s} \, ds, \quad t \in \mathbb{R}.$$

A Banach space $\mathcal{X}$ is said to be of class $\mathcal{HT}$, if the Hilbert transform is bounded on $L^p(\mathbb{R}; \mathcal{X})$ for some (thus all) $1 < p < \infty$.

These spaces are also called UMD Banach spaces, where UMD stands for *unconditional martingale differences*. Hilbert spaces, all closed subspaces and quotient spaces of $L^q(\Omega)$ with $1 < q < \infty$ are examples of UMD spaces. We refer the reader to [1, pp. 141-147] for more information about UMD spaces.

We next introduce the notion of $\mathcal{R}$-bounded family of operators and $\mathcal{R}$-sectoriality of a densely defined linear operator.

Definition 2.2 ($\mathcal{R}$ - bounded family of operators)**.** Let $\mathcal{X}$ and $\mathcal{Y}$ be Banach spaces. A family of operators $\mathcal{T} \subset \mathcal{L}(\mathcal{X}, \mathcal{Y})$ is called $\mathcal{R}$- bounded on $\mathcal{L}(\mathcal{X}, \mathcal{Y})$, if there exist constants $C > 0$ and $p \in [1, \infty)$ such that for every $n \in \mathbb{N}$, $\{T_j\}_{j=1}^n \subset \mathcal{T}$, $\{x_j\}_{j=1}^n \subset \mathcal{X}$ and for all sequences $\{r_j(\cdot)\}_{j=1}^n$ of independent, symmetric, $\{-1, 1\}$ valued random variables on $[0, 1]$, we have

$$\left\| \sum_{j=1}^n r_j(\cdot) T_j x_j \right\|_{L^p([0,1];\mathcal{Y})} \leqslant C \left\| \sum_{j=1}^n r_j(\cdot) x_j \right\|_{L^p([0,1];\mathcal{X})}.$$

The smallest such C is called $\mathcal{R}$-bound of $\mathcal{T}$ on $\mathcal{L}(\mathcal{X}, \mathcal{Y})$ and denoted by $\mathcal{R}_{\mathcal{L}(\mathcal{X},\mathcal{Y})}(\mathcal{T})$.

Definition 2.3 ($\mathcal{R}$-sectorial operator)**.** Let A be a densely defined closed linear operator on a Banach space $\mathcal{X}$ with domain $\mathcal{D}(A)$. Then A is said sectorial of angle $\theta \in (0, \pi)$ if $\sigma(A) \subseteq \overline{\Sigma_\theta}$ and for any $\theta_1 > \theta$ the set $\left\{\lambda(\lambda I - A)^{-1} \quad | \quad \theta_1 \leqslant |\arg(\lambda)| \leqslant \pi\right\}$ is bounded. If this set is $\mathcal{R}$-bounded then A is $\mathcal{R}$-sectorial of angle θ.

We now state several useful properties concerning $\mathcal{R}$-boundedness, which will be used later on

Proposition 2.4.

(1) *If $\mathcal{T} \subset \mathcal{L}(\mathcal{X}, \mathcal{Y})$ is $\mathcal{R}$-bounded then it is uniformly bounded with*

$$\sup \{\|T\| \mid T \in \mathcal{T}\} \leqslant \mathcal{R}_{\mathcal{L}(\mathcal{X},\mathcal{Y})}(\mathcal{T}).$$

(2) *If $\mathcal{X}$ and $\mathcal{Y}$ are Hilbert spaces, $\mathcal{T} \subset \mathcal{L}(\mathcal{X}, \mathcal{Y})$ is $\mathcal{R}$-bounded if and only if $\mathcal{T}$ is uniformly bounded.*
(3) *Let $\mathcal{X}$ and $\mathcal{Y}$ be Banach spaces and let $\mathcal{T}$ and $\mathcal{S}$ be $\mathcal{R}$-bounded families on $\mathcal{L}(\mathcal{X}, \mathcal{Y})$. Then $\mathcal{T} + \mathcal{S}$ is also $\mathcal{R}$-bounded on $\mathcal{L}(\mathcal{X}, \mathcal{Y})$, and*

$$\mathcal{R}_{\mathcal{L}(\mathcal{X},\mathcal{Y})}(\mathcal{T} + \mathcal{S}) \leqslant \mathcal{R}_{\mathcal{L}(\mathcal{X},\mathcal{Y})}(\mathcal{T}) + \mathcal{R}_{\mathcal{L}(\mathcal{X},\mathcal{Y})}(\mathcal{S}).$$

(4) *Let $\mathcal{X}, \mathcal{Y}$ and $\mathcal{Z}$ be Banach spaces and let $\mathcal{T}$ and $\mathcal{S}$ be $\mathcal{R}$-bounded families on $\mathcal{L}(\mathcal{X}, \mathcal{Y})$ and $\mathcal{L}(\mathcal{Y}, \mathcal{Z})$ respectively. Then $\mathcal{S}\mathcal{T}$ is $\mathcal{R}$-bounded on $\mathcal{L}(\mathcal{X}, \mathcal{Z})$, and*

$$\mathcal{R}_{\mathcal{L}(\mathcal{X},\mathcal{Z})}(\mathcal{S}\mathcal{T}) \leqslant \mathcal{R}_{\mathcal{L}(\mathcal{X},\mathcal{Y})}(\mathcal{T})\mathcal{R}_{\mathcal{L}(\mathcal{Y},\mathcal{Z})}(\mathcal{S}).$$

The following characterization of maximal L^p regularity is due to Weis ([25, Theorem 4.2])

Theorem 2.5. *Let $\mathcal{X}$ be a Banach space of class $\mathcal{HT}$, $1 < p < \infty$ and let A be a closed, densely defined unbounded operator with domain $\mathcal{D}(A)$. Then A has maximal L^p-regularity on $\mathbb{R}^+$ if and only if*

$$\mathcal{R}_{\mathcal{L}(\mathcal{X})}\left\{\lambda(\lambda - A)^{-1} \mid \lambda \in \Sigma_\theta\right\} \leqslant C \text{ for some } \theta > \pi/2. \tag{2.2}$$

In other words, A has maximal L^p-regularity if and only if $-A$ is $\mathcal{R}$-sectorial of angle $\theta < \pi/2$.

Next we state a perturbation result due to Kunstmann and Weis [15], which states that $\mathcal{R}$-boundedness is preserved by A small perturbations.

Proposition 2.6. *Let $\mathcal{X}$ be a Banach space let A be a closed, densely defined unbounded operator with domain $\mathcal{D}(A)$. Let us assume that there exist $\gamma_0 \geqslant 0$ and $\theta \in (0, \pi)$ such that*

$$\mathcal{R}_{\mathcal{L}(\mathcal{X})}\left\{\lambda(\lambda - A)^{-1} \mid \lambda \in \gamma_0 + \Sigma_\theta\right\} \leqslant C.$$

Let B be a A-bounded operator with relative bound zero, i.e., for all $\delta > 0$ there exists $C(\delta) > 0$ such that

$$\|Bz\| \leqslant \delta\|Az\| + C(\delta)\|z\| \text{ for all } z \in \mathcal{D}(A). \tag{2.3}$$

Then there there exists $\mu_0 \geqslant \gamma_0$ such that

$$\mathcal{R}_{\mathcal{L}(\mathcal{X})}\left\{\lambda(\lambda - (A + B))^{-1} \mid \lambda \in \mu_0 + \Sigma_\theta\right\} \leqslant C.$$

We conclude this section by stating an existence and uniqueness result for the abstract Cauchy problem (1.11) on $\mathbb{R}^+$, which we will use to prove maximal L^p-L^q regularity of the system (1.7) (see [6, Theorem 2.4]).

Theorem 2.7. *Let $\mathcal{X}$ be a Banach space of class $\mathcal{HT}$, $1 < p < \infty$ and let A be a closed, densely defined unbounded operator with domain $\mathcal{D}(A)$. Let us assume that $A \in \mathcal{MR}_p([0, T]; \mathcal{X})$ and the semigroup generated by A has negative exponential type. Then for every $z_0 \in (\mathcal{X}, \mathcal{D}(A))_{1-1/p,p}$ and for every $f \in L^p(0, \infty; \mathcal{X})$, (1.11) admits a unique strong solution in $L^p(0, \infty; \mathcal{D}(A)) \cap W^{1,p}(0, \infty; \mathcal{X})$. Moreover, there exists a positive constant C such that*

$$\|z\|_{L^p(0,\infty;\mathcal{D}(A))} + \|z\|_{W^{1,p}(0,\infty;\mathcal{X})} \leqslant C \left(\|z_0\|_{(\mathcal{X},\mathcal{D}(A))_{1-1/p,p}} + \|f\|_{L^p(0,\infty;\mathcal{X})}\right). \tag{2.4}$$

3. $\mathcal{R}$-SECTORIALITY OF THE OPERATOR $\mathbb{A}$.

Let us recall the operator $\mathbb{A}$ introduced in (1.10). The aim of this section is to prove Theorem 1.3. Due to Theorem 2.5, it is enough to prove the following theorem:

Theorem 3.1. *Let $1 < q < \infty$. There exists $\mu_0 > 0$ and $\theta \in (\pi/2, \pi)$ such that $\mu_0 + \Sigma_\theta \subset \rho(\mathbb{A})$ and*

$$\mathcal{R}_{\mathcal{L}(\mathbb{H}^q(\Omega))} \left\{ \lambda(\lambda I - \mathbb{A})^{-1} \mid \lambda \in \mu_0 + \Sigma_\theta \right\} \leqslant C. \tag{3.1}$$

Let us remark that when $q = 2$ the above theorem is already proved in [22]. To prove the above theorem we will first obtain an equivalence formulation of the resolvent equation.

3.1. Reformulation of the resolvent equation.

Given $\lambda \in \mathbb{C}$, $f \in L^q(\Omega_F(0))^3$ and $(g_1, g_2) \in \mathbb{C}^3 \times \mathbb{C}^3$, we consider the system

$$\begin{aligned}
&\lambda u - \nu \Delta u + \nabla \pi = f, \quad \operatorname{div} u = 0 && \text{in } \Omega_F(0),\\
&u = 0 && \text{on } \partial\Omega,\\
&u = \ell + \omega \times y && y \in \partial\Omega_S(0),\\
&\lambda m \ell = -\int_{\partial\Omega_S(0)} \sigma(u, \pi) n \, d\gamma + g_1, &&\\
&\lambda J(0)\omega = -\int_{\partial\Omega_S(0)} y \times \sigma(u, \pi) n \, d\gamma + g_2, &&
\end{aligned} \tag{3.2}$$

of unknowns (u, π, ℓ, ω). Following [21, 22] we have the following equivalence

Proposition 3.2. *Let $1 < q < \infty$. Let us assume that $f \in L^q(\Omega_F(0))$ and $(g_1, g_2) \in \mathbb{C}^3 \times \mathbb{C}^3$. Then $(u, \pi, \ell, \omega) \in W^{2,q}(\Omega_F(0))^3 \times W^{1,q}_m(\Omega_F(0)) \times \mathbb{C}^3 \times \mathbb{C}^3$ is a solution to (3.2) if and only if*

$$(\lambda I - \mathbb{A})v = \mathbb{P}F \tag{3.3}$$

where

$$v = u \mathbb{1}_{\Omega_F(0)} + (\ell + \omega \times y)\mathbb{1}_{\Omega_S(0)}, \quad F = \mathbb{P}\left(f \mathbb{1}_{\Omega_F(0)} + \left(m^{-1} g_1 + J(0)^{-1} y \times g_2 \right) \mathbb{1}_{\Omega_S(0)} \right).$$

Next, we derive another equivalent formulation of the resolvent equation (3.2). In this case, we do not extend the fluid velocity by the structure velocity everywhere in the domain Ω, rather we work on the fluid domain $\Omega_F(0)$. The idea is to eliminate the pressure from both the fluid and the structure equations. To eliminate the pressure from the fluid equation we use Leray projector

$$\mathcal{P} : L^q(\Omega_F(0))^3 \mapsto \mathcal{V}^q_n(\Omega_F(0)) := \left\{ \varphi \in L^q(\Omega_F(0))^3 \mid \operatorname{div} \varphi = 0, \ \varphi \cdot n = 0 \text{ on } \partial\Omega_F(0) \right\}.$$

Note that the projector $\mathcal{P}$ is different from the projector $\mathbb{P}$ used in (1.10). Following [19], first, we decompose the fluid velocity into two parts $\mathcal{P}u$ and $(I - \mathcal{P})u$. Next, we obtain an expression of pressure, which can be broken down into two parts, one which depends on $\mathcal{P}u$ and another part which depends on (ℓ, ω). This will allow us to eliminate the pressure term from the structure equations and rewrite the system (3.2) as an operator equation of $(\mathcal{P}u, \ell, \omega)$.

The advantage of this formulation is that we can prove the $\mathcal{R}$-boundedness of the resolvent operator just by using the fact that Stokes operator with *homogeneous Dirichlet boundary* *conditions* is $\mathcal{R}$-sectorial and a perturbation argument. This idea has been used in several fluid-solid interaction problems in the Hilbert space setting and when the structure is deformable and located at the boundary (see, for instance, [20, 17] and references therein).

Let $1 < q < \infty$ and q' denote the conjugate of q, i.e., $\dfrac{1}{q} + \dfrac{1}{q'} = 1$. Let n denote the normal to $\partial\Omega_F(0)$ exterior to $\Omega_F(0)$. For $1 < q < \infty$, we first introduce the space

$$W_{q,div}(\Omega) = \left\{\varphi \in L^q(\Omega)^3 \mid \text{div } \varphi \in L^q(\Omega)\right\},$$

equipped with the norm

$$\|\varphi\|_{W_{q,div}(\Omega)} := \|\varphi\|_{L^q(\Omega)^3} + \|\text{div } \varphi\|_{L^q(\Omega)}.$$

It is easy to check that $W_{q,div}(\Omega)$ is a Banach space. We have the following classical lemma:

Lemma 3.3. *Let Ω be a bounded domain with smooth boundary. The linear mapping*

$$\varphi \mapsto \gamma_n \varphi := \varphi|_{\partial\Omega} \cdot n$$

defined on $C^\infty(\overline{\Omega})^3$ can be extended to a continuous and surjective map from $W_{q,div}(\Omega)$ onto $W^{-1/q,q}(\partial\Omega)$.

Proof. For proof see [9, Lemma 1]. □

Let us set

$$\mathcal{V}^q_n(\Omega_F(0)) = \overline{\{\varphi \in C^\infty_c(\Omega_F(0)) \mid \text{div } \varphi = 0\}}^{\|\cdot\|_{L^q}}.$$

As $\Omega_F(0)$ is bounded, we actually have

$$\mathcal{V}^q_n(\Omega_F(0)) = \left\{\varphi \in L^q(\Omega_F(0))^3 \mid \text{div } \varphi = 0,\ \varphi \cdot n = 0 \text{ on } \partial\Omega_F(0)\right\}.$$

We have the following Helmholtz-Weyl decomposition of $L^q(\Omega_F(0))^3$

Proposition 3.4. *The space $L^q(\Omega_F(0))^3$ admits the following decomposition in a direct sum:*

$$L^q(\Omega_F(0))^3 = \mathcal{V}^q_n(\Omega_F(0)) \oplus G^q(\Omega_F(0)),$$

where

$$G^q(\Omega_F(0)) = \left\{\nabla\varphi \mid \varphi \in W^{1,q}(\Omega_F(0))\right\}.$$

The projection operator from $L^q(\Omega_F(0))^3$ onto $\mathcal{V}^q_n(\Omega_F(0))$ is denoted by $\mathcal{P}$. The projector $\mathcal{P}$: $L^q(\Omega_F(0))^3 \mapsto \mathcal{V}^q_n(\Omega_F(0))$ is defined by

$$\mathcal{P}u = u - \nabla\varphi,$$

where $\varphi \in W^{1,q}(\Omega_F(0))$ solves the following Neumann problem

$$\Delta\varphi = \text{div } u \text{ in } \Omega_F(0), \quad \frac{\partial\varphi}{\partial n} = \varphi \cdot n \text{ on } \partial\Omega_F(0).$$

Proof. For the proof of the above result we refer to Section 3 and Theorem 2 of [9]. □

Let us denote by $A_0 = \nu\mathcal{P}\Delta$, the Stokes operator in $\mathcal{V}^q_n(\Omega_F(0))$ with domain

$$\mathcal{D}(A_0) = W^{2,q}(\Omega_F(0)) \cap W^{1,q}_0(\Omega_F(0))^3 \cap \mathcal{V}^q_n(\Omega_F(0)).$$

Proposition 3.5. *The Stokes operator $-A_0$ is $\mathcal{R}$-sectorial in $\mathcal{V}^q_n(\Omega_F(0))$ of angle 0. In particular, there exists $\theta \in (\pi/2, \pi)$ such that*

$$\mathcal{R}_{\mathcal{L}(\mathcal{V}^q_n(\Omega_F(0)))}\left\{\lambda(\lambda I - A_0)^{-1} \mid \lambda \in \Sigma_\theta\right\} \leqslant C. \tag{3.4}$$

Moreover, A_0 generates a C^0-semigroup of negative type.

Proof. For proof we refer to Theorem 1.4 and Corollary 1.4 of [12]. □

Now we are going to rewrite the first three equations of (3.2) in terms of $\mathcal{P}u$ and $(I-\mathcal{P})u$. Let us consider the following problem

$$\begin{cases} -\nu\Delta w + \nabla\psi = 0, \quad \text{div } w = 0, \quad y \in \Omega_F(0), \\ w = 0, \qquad\qquad\qquad y \in \partial\Omega, \\ w = \ell + \omega \times y, \qquad y \in \partial\Omega_S(0), \\ \displaystyle\int_{\Omega_F(0)} \psi \text{ dy} = 0. \end{cases} \tag{3.5}$$

Lemma 3.6. *Let $(\ell,\omega) \in \mathbb{C}^3 \times \mathbb{C}^3$ and let $\{e_i\}$ denote the canonical basis in $\mathbb{C}^3$. Then the solution (w,π) of (3.5) can be expressed as follows*

$$w = \sum_{i=1}^{3} \ell_i W_i + \sum_{i=4}^{6} \omega_{i-3} W_i, \quad \psi = \sum_{i=1}^{3} \ell_i \Psi_i + \sum_{i=4}^{6} \omega_{i-3}\Psi_i, \tag{3.6}$$

where (W_i, Ψ_i), $i = 1, 2, \cdots, 6$ solves the following system

$$\begin{cases} -\nu\Delta W_i + \nabla\Psi_i = 0, \quad \text{div} W_i = 0, \quad y \in \Omega_F(0), \\ W_i = 0, \qquad y \in \partial\Omega, \\ W_i = e_i, \ \textit{for } i = 1,2,3 \ \textit{and } W_i = e_{i-3} \times y, \ \textit{for } i = 4,5,6, \quad y \in \partial\Omega_F(0), \\ \displaystyle\int_{\Omega_F(0)} \Psi \text{ dy} = 0. \end{cases} \tag{3.7}$$

Moreover,

$$\begin{pmatrix} \displaystyle\int_{\partial\Omega_S(0)} \sigma(w,\psi)n \, d\gamma \\ \displaystyle\int_{\partial\Omega_S(0)} y \times \sigma(w,\psi)n \, d\gamma \end{pmatrix} = \mathbb{B}\begin{pmatrix} \ell \\ \omega \end{pmatrix},$$

where

$$\mathbb{B}_{i,j} = \int_{\Omega_F(0)} DW_i : DW_j. \tag{3.8}$$

and the matrix $\mathbb{B}$ is invertible.

Proof. The expressions of w and π are easy to see. The expression of the matrix $\mathbb{B}$ follows easily by putting the expressions of w and π and by integration by parts. The proof of invertibility of the matrix $\mathbb{B}$ can be found in [14, Chapter 5]. □

Let us introduce the following operators:

- The Dirichlet lifting operator $D \in \mathcal{L}(\mathbb{C}^6, W^{2,q}(\Omega_F(0)))$ and $D_{pr} \in \mathcal{L}(\mathbb{C}^6, W^{1,q}_m(\Omega_F(0)))$ defined by
$$D(\ell,\omega) = w, \quad D_{pr}(\ell,\omega) = \psi, \tag{3.9}$$
where (w,ψ) is the solution to the problem (3.5).
- The Neumann operator $N \in \mathcal{L}(W^{1-1/q,q}_m(\partial\Omega_F(0)), W^{2,q}_m(\Omega_F(0)))$ defined by $Nh = \varphi$, where φ is the solution to the Neumann problem
$$\Delta\varphi = 0 \text{ in } \Omega_F(0), \quad \frac{\partial\varphi}{\partial n} = h \text{ on } \partial\Omega_F(0). \tag{3.10}$$
We set
$$N_S h = N(\mathbb{1}_{\partial\Omega_S(0)} h) \text{ for } h \in W^{1-1/q,q}_m(\partial\Omega_S(0)). \tag{3.11}$$

We now rewrite the equations satisfied by u in system (3.2) as a new system of two equations, one satisfied by $\mathcal{P}u$ and another by $(I-\mathcal{P})u$. More precisely, we have the following proposition

Proposition 3.7. *Let $1 < q < \infty$. Let us assume that $(f,\ell,\omega) \in \mathcal{V}^q_n(\Omega_F(0)) \times \mathbb{C}^3 \times \mathbb{C}^3$. A pair $(u,\pi) \in W^{2,q}(\Omega_F(0)) \times W^{1,q}_m(\Omega_F(0))$ satisfies the system*
$$\begin{cases} \lambda u - \nu\Delta u + \nabla\pi = f, \quad \text{div } u = 0, \quad y \in \Omega_F(0), \\ u = 0 \quad y \in \partial\Omega, \\ u = \ell + \omega \times y \quad y \in \partial\Omega_S(0), \end{cases} \tag{3.12}$$
if and only if
$$\begin{cases} \lambda\mathcal{P}u - A_0\mathcal{P}u + A_0\mathcal{P}D(\ell,\omega) = \mathcal{P}f, \\ (I-\mathcal{P})u = (I-\mathcal{P})D(\ell,\omega) \\ \pi = N(\nu\Delta\mathcal{P}u \cdot n) - \lambda N_S((\ell+\omega\times y)\cdot n). \end{cases} \tag{3.13}$$

Proof. Let $(u,\pi) \in W^{2,q}(\Omega_F(0))^3 \times W^{1,q}_m(\Omega_F(0))$ satisfies the system (3.12). We set
$$\widetilde{u} = u - D(\ell,\omega), \quad \widetilde{\pi} = \pi - D_{pr}(\ell,\omega).$$
The pair $(\widetilde{u},\widetilde{p})$ satisfies the following system
$$\begin{aligned} &\lambda\widetilde{u} + \lambda D(\ell,\omega) - \nu\Delta\widetilde{u} + \nabla p = f, \quad \text{div } \widetilde{u} = 0, \text{ in } \Omega_F(0), \\ &\widetilde{u} = 0 \quad \text{on } \partial\Omega_F(0). \end{aligned}$$
Note that $\widetilde{u} \in \mathcal{D}(A_0)$ and $\mathcal{P}\widetilde{u} = \widetilde{u}$. Thus applying the projection $\mathcal{P}$ on the above system it is easy to see that $\mathcal{P}u$ satisfies the following
$$\lambda\mathcal{P}u - A_0\mathcal{P}u + A_0\mathcal{P}D(\ell,\omega) = \mathcal{P}f.$$
Since $(I-\mathcal{P})\widetilde{u} = 0$, we obtain
$$(I-\mathcal{P})u = (I-\mathcal{P})(\widetilde{u} + D(\ell,\omega)) = (I-\mathcal{P})D(\ell,\omega).$$
Note that, from the expression of $\mathcal{P}$ in Proposition 3.4, it follows that $\Delta(I-\mathcal{P})u = 0$ in $\Omega_F(0)$. Therefore the first equation of (3.12) can be written as
$$\lambda u - \Delta\mathcal{P}u + \nabla\pi = f \quad \text{in } \Omega_F(0).$$

By applying the divergence and normal trace operators to the above equation, we obtain that π is the solution of the problem

$$\begin{cases} \Delta\pi = 0 & \text{in } \Omega_F(0), \\ \dfrac{\partial\pi}{\partial n} = \nu\Delta\mathcal{P}u\cdot n - \lambda u\cdot n & \text{on } \partial\Omega_F(0). \end{cases} \tag{3.14}$$

Since $\operatorname{div}\Delta\mathcal{P}u = 0$ it follows that $\nu\Delta\mathcal{P}u\cdot n$ belongs to $W^{-1/q,q}(\partial\Omega)$ and satisfies the following condition

$$\langle \nu\Delta\mathcal{P}u\cdot n, 1\rangle_{W^{-1/q,q},W^{1-1/q',q'}} = 0$$

Since $\Omega_F(0)$ is a smooth domain, (3.14) admits a unique solution in $W^{1,q}_m(\Omega_F(0))$ ([8, Theorem 9.2]) and the expression of π in (3.13) follows from the definition of the operators N and N_S.

Conversely, let $(u,\pi) \in W^{2,q}(\Omega_F(0))^3 \times W^{1,q}_m(\Omega_F(0))$ satisfies the system (3.13). Since $(I-\mathcal{P})u = (I-\mathcal{P})D(\ell,\omega)$ we get $\widetilde{u} := u - D(\ell,\omega) \in \mathcal{D}(A_0)$. Thus $(3.13)_1$ can be written as

$$\mathcal{P}\left(\lambda\widetilde{u} - A_0\widetilde{u}\right) = \mathcal{P}(f - \lambda D(\ell,\omega)).$$

Therefore, there exists $\widetilde{\pi} \in W^{1,q}_m(\Omega_F(0))$ such that $(\widetilde{u},\widetilde{\pi})$ satisfies

$$\lambda\widetilde{u} - \nu\Delta\widetilde{u} + \nabla\widetilde{\pi} = f - \lambda D(\ell,\omega), \quad \operatorname{div}\widetilde{u} = 0 \text{ in } \Omega_F(0), \quad \widetilde{u} = 0 \text{ on } \partial\Omega_F(0).$$

Then (u,π), with $\pi = \widetilde{\pi} + D_{pr}(\ell,\omega)$, satisfies the system (3.12). $\square$

Using the expression of the pressure π obtained in (3.7), we rewrite the equations satisfied by ℓ and ω in (3.2) in the form

$$\begin{aligned} \lambda m\ell &= -2\nu\int_{\partial\Omega_S(0)} \varepsilon(u)n\,d\gamma + \int_{\partial\Omega_S(0)} \pi n\,d\gamma + g_1 \\ &= -2\nu\left[\int_{\partial\Omega_S(0)} \varepsilon(\mathcal{P}u)n\,d\gamma + \int_{\partial\Omega_S(0)} \varepsilon((I-\mathcal{P})D(\ell,\omega))n\,d\gamma\right] \\ &\quad + \int_{\partial\Omega_S(0)} N(\Delta\mathcal{P}u\cdot n)n\,d\gamma - \lambda\int_{\partial\Omega_S(0)} N_S((\ell+\omega\times y)\cdot n)n\,\mathrm{d}\gamma + g_1, \end{aligned}$$

and

$$\begin{aligned} J(0)\lambda\omega = -2\nu&\left[\int_{\partial\Omega_S(0)} y\times\varepsilon(\mathcal{P}u)n\,d\gamma + \int_{\partial\Omega_S(0)} y\times\varepsilon((I-\mathcal{P})D(\ell,\omega))n\,d\gamma\right] \\ &+ \int_{\partial\Omega_S(0)} y\times N(\Delta\mathcal{P}u\cdot n)n\,d\gamma - \lambda\int_{\partial\Omega_S(0)} y\times N_S((\ell+\omega\times y)\cdot n)n\,\mathrm{d}\gamma + g_2. \end{aligned}$$

The above two equations can be written as

$$\lambda\mathbb{K}\begin{pmatrix}\ell\\ \omega\end{pmatrix} = \mathcal{C}_1\mathcal{P}u + \mathcal{C}_2\begin{pmatrix}\ell\\ \omega\end{pmatrix} + \begin{pmatrix}g_1\\ g_2\end{pmatrix}, \tag{3.15}$$

where

$$\mathbb{K} = \begin{pmatrix} mI_3 & 0 \\ 0 & J(0) \end{pmatrix} + \mathbb{M}, \quad \mathbb{M}\begin{pmatrix} \ell \\ \omega \end{pmatrix} = \begin{pmatrix} \displaystyle\int_{\partial\Omega_S(0)} N_S((\ell+\omega\times y)\cdot n)n \,\mathrm{d}\gamma \\ \displaystyle\int_{\partial\Omega_S(0)} y\times N_S((\ell+\omega\times y)\cdot n)n \,\mathrm{d}\gamma \end{pmatrix}, \tag{3.16}$$

$$\mathcal{C}_1\mathcal{P}u = \begin{pmatrix} \displaystyle -2\nu\int_{\partial\Omega_S(0)} \varepsilon(\mathcal{P}u)n\, d\gamma + \int_{\partial\Omega_S(0)} N(\Delta\mathcal{P}u\cdot n)n\, d\gamma \\ \displaystyle -2\nu\int_{\partial\Omega_S(0)} y\times\varepsilon(\mathcal{P}u)\, n\, d\gamma + \int_{\partial\Omega_S(0)} y\times N(\Delta\mathcal{P}u\cdot n)n\, d\gamma \end{pmatrix}, \tag{3.17}$$

and

$$\mathcal{C}_2\begin{pmatrix} \ell \\ \omega \end{pmatrix} = \begin{pmatrix} \displaystyle -2\nu\int_{\partial\Omega_S(0)} \varepsilon((I-\mathcal{P})D(\ell,\omega))n\, d\gamma \\ \displaystyle -2\nu\int_{\partial\Omega_S(0)} y\times\varepsilon((I-\mathcal{P})D(\ell,\omega))n\, d\gamma \end{pmatrix}. \tag{3.18}$$

In the literature, the matrix $\mathbb{M}$ defined above is known as the added mass operator. We are now going to show that the matrix $\mathbb{K}$ is an invertible matrix.

Lemma 3.8. *The matrix $\mathbb{K}$ defined as in (3.16) is an invertible matrix.*

Proof. The proof may be adapted from that of [10, Lemma 4.6] (see also [11, Lemma 4.3]). Let us briefly explain the idea of the proof. We are going to show that the matrix $\mathbb{M}$ is symmetric and semipositive definite. For that, we first derive an representation formula of the matrix $\mathbb{M}$. Let us consider the following problem

$$\Delta\pi^i = 0 \text{ in } \Omega_F(0), \quad \frac{\partial\pi^i}{\partial n} = 0 \text{ on } \partial\Omega,$$
$$\frac{\partial\pi^i}{\partial n} = e_i\cdot n \text{ for } i=1,2,3 \text{ and } \frac{\partial\pi^i}{\partial n} = (e_{i-3}\times y)\cdot n \text{ for } i=4,5,6, \quad y\in\partial\Omega_S(0),$$

where $\{e_i\}$ denote the canonical basis in $\mathbb{C}^3$. Therefore, it is easy to see that

$$N_S((\ell+\omega\times y)\cdot n) = \sum_{i=1}^{3}\ell_i\pi^i + \sum_{i=4}^{6}\omega_{i-3}\pi^i.$$

We define

$$m_{ij} = \begin{cases} \displaystyle\int_{\partial\Omega_S(0)} \pi^i n^j & \text{for } 1\leqslant i\leqslant 6, 1\leqslant j\leqslant 3, \\ \displaystyle\int_{\partial\Omega_S(0)} \pi^i(e_{j-3}\times y)\cdot n & \text{for } 1\leqslant i\leqslant 6, 4\leqslant j\leqslant 6. \end{cases}$$

One can easily check that, $\mathbb{M} = (m_{ij})_{1\leqslant i,j\leqslant 6}$. With this representation and Gauss' theorem, we can verify that $\mathbb{M}$ is symmetric and semipositive definite. $\square$

Let us set

$$\mathcal{X} = \mathcal{V}^q_n(\Omega_F(0)) \times \mathbb{C}^3 \times \mathbb{C}^3. \tag{3.19}$$

and consider the operator $\mathcal{A}_{FS} : \mathcal{D}(\mathcal{A}_{FS}) \mapsto \mathcal{X}$ defined by

$$\mathcal{D}(\mathcal{A}_{FS}) = \{(\mathcal{P}u, \ell, \omega) \in \mathcal{X} \mid \mathcal{P}u - \mathcal{P}D(\ell, \omega) \in \mathcal{D}(A_0)\},$$

and

$$\mathcal{A}_{FS} = \begin{pmatrix} A_0 & -A_0\mathcal{P}D \\ \mathbb{K}^{-1}\mathcal{C}_1 & \mathbb{K}^{-1}\mathcal{C}_2 \end{pmatrix}. \tag{3.20}$$

Combining the above results, below we obtain an equivalence formulation of the system (3.2).

Proposition 3.9. *Let $1 < q < \infty$. Let us assume that $(f, g_1, g_2) \in \mathcal{V}^q_n(\Omega_F(0)) \times \mathbb{C}^3 \times \mathbb{C}^3$. Then $(u, p, \ell, \omega) \in W^{2,q}(\Omega_F(0)) \times W^{1,q}_m(\Omega_F(0)) \times \mathbb{C}^3 \times \mathbb{C}^3$ satisfy the system (3.2) if and only if*

$$(\lambda I - \mathcal{A}_{FS}) \begin{pmatrix} \mathcal{P}u \\ \ell \\ \omega \end{pmatrix} = \begin{pmatrix} \mathcal{P}f \\ \widetilde{g}_1 \\ \widetilde{g}_2 \end{pmatrix}, \tag{3.21}$$

$$(I - \mathcal{P})u = (I - \mathcal{P})D(\ell, \omega),$$

$$\pi = N(\nu\Delta\mathcal{P}u \cdot n) - \lambda N_S((\ell + \omega \times y) \cdot n),$$

where $(\widetilde{g}_1, \widetilde{g}_2)^\top = \mathbb{K}^{-1}(g_1, g_2)^\top$.

We end this subsection with the following lemma

Lemma 3.10. *The map*

$$(\mathcal{P}u, \ell, \omega) \mapsto \|\mathcal{P}u\|_{W^{2,q}(\Omega_F(0))} + \|\ell\|_{\mathbb{C}^3} + \|\omega\|_{\mathbb{C}^3},$$

is a norm on $\mathcal{D}(\mathcal{A}_{FS})$ equivalent to the graph norm.

Proof. The proof is similar to that of [20, Proposition 3.3]. ☐

3.2. **$\mathcal{R}$-boundedness of the resolvent operator.** In this subsection we are going to prove Theorem 3.1. In view of Proposition 3.2 and Proposition 3.9, it is enough to prove the following theorem

Theorem 3.11. *Let $1 < q < \infty$. There exist $\mu_0 > 0$ and $\theta \in (\pi/2, \pi)$ such that $\mu_0 + \Sigma_\theta \subset \rho(\mathcal{A}_{FS})$ and*

$$\mathcal{R}_{\mathcal{L}(\mathcal{X})} \left\{ \lambda(\lambda I - \mathcal{A}_{FS})^{-1} \mid \lambda \in \mu_0 + \Sigma_\theta \right\} \leqslant C. \tag{3.22}$$

Proof. We write $\mathcal{A}_{FS}$ in the form $\mathcal{A}_{FS} = \widetilde{\mathcal{A}}_{FS} + B_{FS}$ where

$$\widetilde{\mathcal{A}}_{FS} = \begin{pmatrix} A_0 & -A_0\mathcal{P}D \\ 0 & 0 \end{pmatrix}, \quad B_{FS} = \begin{pmatrix} 0 & 0 \\ \mathbb{K}^{-1}\mathcal{C}_1 & \mathbb{K}^{-1}\mathcal{C}_2 \end{pmatrix}.$$

We first show that $\widetilde{\mathcal{A}}_{FS}$ with $\mathcal{D}(\widetilde{\mathcal{A}}_{FS}) = \mathcal{D}(\mathcal{A}_{FS})$ is a $\mathcal{R}$-sectorial operator on $\mathcal{X}$. Observe that

$$\lambda(\lambda I - \widetilde{\mathcal{A}}_{FS})^{-1} = \begin{pmatrix} \lambda(\lambda I - A_0)^{-1} & -(\lambda I - A_0)^{-1}A_0\mathcal{P}D \\ 0 & I \end{pmatrix}.$$

Since

$$-(\lambda I - A_0)^{-1} A_0 \mathcal{P} D = -\lambda(\lambda I - A_0)^{-1} \mathcal{P} D + \mathcal{P} D,$$

we get

$$\lambda(\lambda I - \widetilde{\mathcal{A}}_{FS})^{-1} = \begin{pmatrix} \lambda(\lambda I - A_0)^{-1} & -\lambda(\lambda I - A_0)^{-1}\mathcal{P}D + \mathcal{P}D \\ 0 & I \end{pmatrix}.$$

Therefore by Proposition 3.5 and Proposition 2.4, there exists $\theta \in (\pi/2, \pi)$ such that

$$\mathcal{R}_{\mathcal{L}(\mathcal{X})}\left\{\lambda(\lambda I - \widetilde{\mathcal{A}}_{FS})^{-1} \mid \lambda \in \Sigma_\theta\right\} \leqslant C. \tag{3.23}$$

Let us now show that, $\mathcal{C}_1 \in \mathcal{L}(\mathcal{D}(\mathcal{A}_{FS}), \mathbb{C}^3 \times \mathbb{C}^3)$. By Lemma 3.10, for any $(\mathcal{P}u, \ell, \omega) \in \mathcal{D}(\mathcal{A}_{FS})$ we have $(\mathcal{P}u, \ell, \omega) \in W^{2,q}(\Omega_F(0)) \times \mathbb{C}^3 \times \mathbb{C}^3$. Therefore, by trace theorem $\varepsilon(\mathcal{P}u)n \in W^{1-1/q,q}(\partial\Omega_S(0))$ and hence $\displaystyle\int_{\partial\Omega_S(0)} \varepsilon(\mathcal{P}u)n \, \mathrm{d}\gamma \in \mathbb{C}^3$. On the other hand, $\Delta\mathcal{P}u \in L^q(\Omega_F(0))$ and div $\Delta\mathcal{P}u = 0$. Therefore by Lemma 3.3, the term $\Delta\mathcal{P}u \cdot n$ belongs to $W^{-1/q,q}(\partial\Omega_F(0))$ and satisfies the following condition

$$\langle \Delta\mathcal{P}u \cdot n, 1\rangle_{W^{-1/q,q}, W^{1-1/q',q'}} = 0.$$

Thus by [8, Theorem 9.2], $N(\Delta\mathcal{P}u \cdot n) \in W^{1,q}(\Omega_F(0))$ and $\displaystyle\int_{\partial\Omega_S(0)} N(\Delta\mathcal{P}u \cdot n)n \, \mathrm{d}\gamma \in \mathbb{C}^3$. Other terms of the operator $\mathcal{C}_1$ can be checked in a similar manner. Thus $\mathcal{C}_1 \in \mathcal{L}(\mathcal{D}(\mathcal{A}_{FS}), \mathbb{C}^3 \times \mathbb{C}^3)$. Similarly, one can easily verify that $\mathcal{C}_2 \in \mathcal{L}(\mathbb{C}^3 \times \mathbb{C}^3, \mathbb{C}^3 \times \mathbb{C}^3)$. Therefore the operator B_{FS} with $\mathcal{D}(B_{FS}) = \mathcal{D}(\mathcal{A}_{FS})$ is a finite rank operator. By [7, Chapter III, Lemma 2.16], B_{FS} is a $\widetilde{\mathcal{A}}_{FS}$-bounded operator with relative bound zero. Finally using Proposition 2.6 we conclude the proof of the theorem. □

4. Exponential stability of linear fluid-structure interaction operator

The aim of this section is to show that the operator $\mathbb{A}$ or equivalently the operator $\mathcal{A}_{FS}$ generates an exponentially stable semigroup. More precisely, we prove:

Theorem 4.1. *Let $1 < q < \infty$. The operator $\mathcal{A}_{FS}$ generates an exponentially stable semigroup $\left(e^{t\mathcal{A}_{FS}}\right)_{t\geqslant 0}$ on $\mathcal{X}$. Equivalently, the operator $\mathbb{A}$ generates an exponentially stable semigroup $\left(e^{t\mathbb{A}}\right)_{t\geqslant 0}$ on $\mathbb{H}^q(\Omega)$. In other words, there exist constants $C > 0$ and $\eta_0 > 0$ such that*

$$\left\| e^{t\mathcal{A}_{FS}}(u_0, \ell_0, \omega_0)^\top \right\|_{\mathcal{X}} \leqslant C e^{-\eta_0 t} \left\| (u_0, \ell_0, \omega_0)^\top \right\|_{\mathcal{X}}. \tag{4.1}$$

To prove this theorem we first show that the set $\{\lambda \in \mathbb{C} \mid \mathrm{Re}\lambda \geqslant 0\}$, i.e, the entire right half plane is contained in the resolvent set of $\mathcal{A}_{\mathcal{F}\mathcal{S}}$.

Theorem 4.2. *Assume $1 < q < \infty$ and $\lambda \in \mathbb{C}$, with $\mathrm{Re}\lambda \geqslant 0$. Then for any $(f, g_1, g_2) \in \mathcal{X}$, the system (3.2) admits a unique solution satisfying the estimate*

$$\|u\|_{W^{2,q}(\Omega_F(0))^3} + \|p\|_{W^{1,q}_m(\Omega_F(0))} + \|\ell\|_{\mathbb{C}^3} + \|\omega\|_{\mathbb{C}^3} \leqslant C\|(f, g_1, g_2)\|_{\mathcal{X}}. \tag{4.2}$$

Proof. Let us recall, by Proposition 3.9, the system (3.2) is equivalent to

$$\begin{aligned}
&(\lambda I - \mathcal{A}_{FS}) \begin{pmatrix} \mathcal{P}u \\ \ell \\ \omega \end{pmatrix} = \begin{pmatrix} \mathcal{P}f \\ \widetilde{g}_1 \\ \widetilde{g}_2 \end{pmatrix}, \\
&(I-\mathcal{P})u = (I-\mathcal{P})D(\ell,\omega), \\
&\pi = N(\nu\Delta\mathcal{P}u\cdot n) - \lambda N_S((\ell+\omega\times y)\cdot n),
\end{aligned} \tag{4.3}$$

where $(\widetilde{g}_1,\widetilde{g}_2)^\top = \mathbb{K}^{-1}(g_1,g_2)^\top$. By Theorem 3.11, there exists $\widetilde{\lambda} > \mu_0$ such that $(\widetilde{\lambda}I - \mathcal{A}_{FS})$ is invertible. Consequently, (4.3) can be written as

$$\begin{aligned}
&\begin{pmatrix} \mathcal{P}u \\ \ell \\ \omega \end{pmatrix} = \left[I + (\lambda-\widetilde{\lambda})(\widetilde{\lambda}-\mathcal{A}_{FS})^{-1}\right]^{-1} \left(\widetilde{\lambda}-\mathcal{A}_{FS}\right)^{-1} \begin{pmatrix} \mathcal{P}f \\ \widetilde{g}_1 \\ \widetilde{g}_2 \end{pmatrix}, \\
&(I-\mathcal{P})u = (I-\mathcal{P})D(\ell,\omega), \\
&\pi = N(\nu\Delta\mathcal{P}u\cdot n) - \lambda N_S((\ell+\omega\times y)\cdot n).
\end{aligned} \tag{4.4}$$

Since $\left(\widetilde{\lambda}-\mathcal{A}_{FS}\right)^{-1}$ is a compact operator, in view of Fredholm alternative theorem, the existence and uniqueness of system (4.4) are equivalent. Therefore, in the sequel we show the uniqueness of the solutions (3.2). Once we prove the uniqueness, the estimate (4.2) follows easily from (4.4). Let $(u,\pi,\ell,\omega) \in W^{2,q}(\Omega_F(0))^3 \times W^{1,q}_m(\Omega_F(0)) \times \mathbb{C}^3 \times \mathbb{C}^3$ satisfies the homogeneous system

$$\begin{aligned}
&\lambda u - \nu\Delta u + \nabla\pi = 0, \quad \operatorname{div} u = 0, && \text{in } \Omega_F(0), \\
&u = 0 && \text{on } \partial\Omega, \\
&u = \ell + \omega\times y && \text{on } \partial\Omega_S(0), \\
&\lambda m\ell = -\int_{\partial\Omega_S(0)} \sigma(u,\pi)n\, d\gamma, \\
&\lambda J(0)\omega = -\int_{\partial\Omega_S(0)} y\times\sigma(u,\pi)n\, d\gamma.
\end{aligned} \tag{4.5}$$

We first show that $(u,\pi) \in W^{2,2}(\Omega_F(0))^3 \times W^{1,2}_m(\Omega_F(0))$. If $q \geqslant 2$, this follows from Hölder's estimate. Assume $1 < q < 2$. In that case, we can rewrite (4.5) as follows

$$\begin{aligned}
&(\widetilde{\lambda}I - \mathcal{A}_{FS}) \begin{pmatrix} \mathcal{P}u \\ \ell \\ \omega \end{pmatrix} = (\widetilde{\lambda}-\lambda) \begin{pmatrix} \mathcal{P}u \\ \ell \\ \omega \end{pmatrix}, \\
&(I-\mathcal{P})u = (I-\mathcal{P})D(\ell,\omega), \\
&\pi = N(\nu\Delta\mathcal{P}u\cdot n) - \lambda N_S((\ell+\omega\times y)\cdot n).
\end{aligned} \tag{4.6}$$

Since $W^{2,q}(\Omega_F(0)) \subset L^2(\Omega_F(0))$ and $(\widetilde{\lambda}I - \mathcal{A}_{FS})$ is invertible, we deduce that $(u,\pi) \in W^{2,2}(\Omega_F(0))^3 \times W^{1,2}_m(\Omega_F(0))$.

Multiplying $(4.5)_1$ by $\overline{u}$, $(4.5)_4$ by $\overline{\ell}$ and $(4.5)_5$ by $\overline{\omega}$, we obtain after integration by parts,

$$\lambda \int_{\Omega_F(0)} |u|^2 \,\mathrm{d}y + 2\nu \int_{\Omega_F(0)} \varepsilon(u) : \varepsilon(\overline{u}) \,\mathrm{d}y + \lambda m|\ell|^2 + \lambda J(0)\omega \cdot \overline{\omega} = 0.$$

Taking real part of the above equation we obtain

$$\mathrm{Re}\lambda \int_{\Omega_F(0)} |u|^2 \,\mathrm{d}y + 2\nu \int_{\Omega_F(0)} |\varepsilon(u)|^2 \,\mathrm{d}y + \mathrm{Re}\lambda m|\ell|^2 + \mathrm{Re}(\lambda J(0)\omega \cdot \overline{\omega}) = 0.$$

Since $\mathrm{Re}\lambda \geqslant 0$, we have

$$2\nu \int_{\Omega_F(0)} |\varepsilon(u)|^2 \,\mathrm{d}y = 0.$$

The above estimate and the fact that $u = 0$ on $\partial\Omega$ imply that $u = 0$. Next, using $u = \ell + \omega \times y$ for $y \in \partial\Omega_S(0)$, we get $\ell = \omega = 0$. Finally, as $\pi \in W^{1,q}_m(\Omega_F(0))$, we have $\pi = 0$. □

Proof of Theorem 4.1. From Theorem 4.2, we have

$$\{\lambda \in \mathbb{C} \mid \mathrm{Re}\ \lambda \geqslant 0\} \subset \rho(\mathcal{A}_{FS}).$$

Also, by Theorem 3.11 we have the existence of a constant $C > 0$ such that for any $\lambda \in \mu_0 + \Sigma_\theta$ with $\theta \in (\pi/2, \pi)$,

$$\left\|(\lambda - \mathcal{A}_{FS})^{-1}\right\|_{\mathcal{L}(\mathcal{X})} \leqslant C.$$

Since $\{\lambda \in \mathbb{C} \mid \mathrm{Re}\ \lambda \geqslant 0\} \setminus [\mu_0 + \Sigma_\theta]$ is a compact set, we deduce the existence of a constant $C > 0$ such that for any $\lambda \in \mathbb{C}$ with $\mathrm{Re}\ \lambda \geqslant 0$

$$\left\|(\lambda - \mathcal{A}_{FS})^{-1}\right\|_{\mathcal{L}(\mathcal{X})} \leqslant C.$$

This yields that

$$\{\lambda \in \mathbb{C} \mid \mathrm{Re}\ \lambda \geqslant -\eta\} \subset \rho(\mathcal{A}_{FS}),$$

for some $\eta > 0$. As $\mathcal{A}_{FS}$ generates an analytic semigroup, applying Proposition 2.9 of [3, Part II, Chapter 1, pp 120], we obtain exponential stability of $\mathcal{A}_{FS}$ in $\mathcal{X}$. □

5. Maximal L^p-L^q regularity of the system (1.7)

In this section we prove the maximal L^p-L^q regularity of a version of the system the (1.7) with non zero divergence. Treating a non zero divergence term will be useful in the next section in order to tackle some terms coming from a simple change of variables. More precisely, we consider the system

$$
\begin{aligned}
&\partial_t u - \nu\Delta u + \nabla\pi = f, \quad \text{div } u = \text{div } h && t\in(0,\infty),\ y\in\Omega_F(0),\\
&u = 0 && t\in(0,\infty),\ y\in\partial\Omega,\\
&u = \ell + \omega\times y && t\in(0,\infty),\ y\in\partial\Omega_S(0),\\
&m\frac{d}{dt}\ell = -\int_{\partial\Omega_S(0)} \sigma(u,\pi)n\,d\gamma + g_1 && t\in(0,\infty),\\
&J(0)\frac{d}{dt}\omega = -\int_{\partial\Omega_S(0)} y\times\sigma(u,\pi)n\,d\gamma + g_2 && t\in(0,\infty),\\
&u(0,y) = u_0(y) && y\in\Omega_F(0),\\
&\ell(0)=\ell_0, \quad \omega(0)=\omega_0.
\end{aligned}
\tag{5.1}
$$

We set

$$
W^{2,1}_{q,p}(Q^F_\infty) = L^p(0,\infty;W^{2,q}(\Omega_F(0)))\cap W^{1,p}(0,\infty;L^q(\Omega_F(0))),
$$

with

$$
\|u\|_{W^{2,1}_{q,p}(Q^F_\infty)} := \|u\|_{L^p(0,\infty;W^{2,q}(\Omega_F(0)))} + \|u\|_{W^{1,p}(0,\infty;L^q(\Omega_F(0)))}.
$$

We prove the following theorem

Theorem 5.1. *Let $1<p,q<\infty$ such that $\dfrac{1}{p}+\dfrac{1}{2q}\neq 1$. Let $\eta\in[0,\eta_0)$, where η_0 is the constant introduced in Theorem 4.1. Let us also assume that $\ell_0\in\mathbb{R}^3$, $\omega_0\in\mathbb{R}^3$ and $u_0\in B^{2(1-1/p)}_{q,p}(\Omega_F(0))$ satisfying the compatibility conditions*

$$
\begin{aligned}
&\text{div } u_0 = 0 \ \text{in } \Omega_F(0),\\
&u_0 = \ell_0 + \omega_0\times y \ \text{on } \partial\Omega_S(0), \quad u_0 = 0 \ \text{on } \partial\Omega \ \text{if } \frac{1}{p}+\frac{1}{2q}<1
\end{aligned}
\tag{5.2}
$$
$$
\text{and } u_0\cdot n = (\ell_0+\omega_0\times y)\cdot n \ \text{on } \partial\Omega_S(0), \qquad u_0\cdot n = 0 \ \text{on } \partial\Omega \ \text{if } \frac{1}{p}+\frac{1}{2q}>1.
$$

Then for any $e^{\eta t}f\in L^p(0,\infty;L^q(\Omega_F(0)))^3$, $e^{\eta t}h\in W^{2,1}_{q,p}(Q^F_\infty)^3$, $e^{\eta t}g_1\in L^p(0,\infty;\mathbb{R}^3)$ and $e^{\eta t}g_2\in L^p(0,\infty;\mathbb{R}^3)$ satisfying

$$
h(0,y)=0 \ \text{for all } (t,y)\in(0,\infty)\times\Omega_F(0) \ \text{and } h|_{\partial\Omega_F(0)}=0,
$$

the system (1.7) admits a unique strong solution

$$
\begin{gathered}
e^{\eta t}u\in L^p(0,\infty;W^{2,q}(\Omega_F(0))^3)\cap W^{1,p}(0,\infty;L^q(\Omega_F(0))^3)\\
e^{\eta t}\pi\in L^p(0,\infty;W^{1,q}_m(\Omega_F(0)))\\
e^{\eta t}\ell\in W^{1,p}(0,\infty;\mathbb{R}^3), \quad e^{\eta t}\omega\in W^{1,p}(0,\infty;\mathbb{R}^3).
\end{gathered}
$$

Moreover, there exists a constant $C_L > 0$ depending only on Ω, p and q such that

$$\|e^{\eta(\cdot)}u\|_{W^{2,1}_{q,p}(Q^F_\infty)^3} + \|e^{\eta(\cdot)}\pi\|_{L^p(0,\infty;W^{1,q}(\Omega_F(0)))} + \|e^{\eta(\cdot)}\ell\|_{L^p(0,\infty;\mathbb{R}^3)}$$
$$+ \|e^{\eta(\cdot)}\omega\|_{L^p(0,\infty;\mathbb{R}^3)} \leqslant C_L\Big(\|u_0\|_{B^{2(1-1/p)}_{q,p}(\Omega_F(0))} + \|\ell_0\|_{\mathbb{R}^3} + \|\omega_0\|_{\mathbb{R}^3}$$
$$+ \|e^{\eta(\cdot)}f\|_{L^p(0,\infty;L^q(\Omega_F(0)))} + \|e^{\eta(\cdot)}h\|_{W^{2,1}_{q,p}(Q^F_\infty)^3} + \|e^{\eta(\cdot)}g_1\|_{L^p(0,\infty;\mathbb{R}^3)} + \|e^{\eta(\cdot)}g_2\|_{L^p(0,\infty;\mathbb{R}^3)}\Big). \quad (5.3)$$

Proof. We first consider the case $\eta = 0$. Let us set $v = u - h$. Then (v, π, ℓ, ω) satisfies the following system

$$\begin{aligned}
&\partial_t v - \nu\Delta v + \nabla\pi = F, \quad \operatorname{div} v = 0 && \text{in } (0,\infty)\times\Omega_F(0),\\
&v = 0 && \text{on } (0,\infty)\times\partial\Omega,\\
&v = \ell + \omega\times y && \text{on } (0,\infty)\times\partial\Omega_S(0),\\
&m\frac{d}{dt}\ell = -\int_{\partial\Omega_S(0)} \sigma(v,\pi)n\, d\gamma + G_1 && t\in(0,\infty),\\
&J(0)\frac{d}{dt}\omega = -\int_{\partial\Omega_S(0)} y\times\sigma(v,\pi)n\, d\gamma + G_2 && t\in(0,\infty),\\
&v(0,y) = u_0(y) && \text{in } \Omega_F(0),\\
&\ell(0) = \ell_0, \quad \omega(0) = \omega_0,
\end{aligned} \quad (5.4)$$

where

$$F = f - \partial_t h + \nu\Delta h, \quad G_1 = g_1 - \int_{\partial\Omega_S(0)} \varepsilon(h)n\, \mathrm{d}\gamma, \quad G_2 = g_2 - \int_{\partial\Omega_S(0)} y\times\varepsilon(h)n\, \mathrm{d}\gamma.$$

Proceeding as Proposition 3.9, it is easy to see that, the above system is equivalent to

$$\begin{cases}
\dfrac{d}{dt}\begin{pmatrix}\mathcal{P}v\\ \ell\\ \omega\end{pmatrix} = \mathcal{A}_{FS}\begin{pmatrix}\mathcal{P}v\\ \ell\\ \omega\end{pmatrix} + \begin{pmatrix}\mathcal{P}F\\ \widetilde{G}_1\\ \widetilde{G}_2\end{pmatrix}, \quad \begin{pmatrix}\mathcal{P}v(0)\\ \ell(0)\\ \omega(0)\end{pmatrix} = \begin{pmatrix}\mathcal{P}u_0\\ \ell_0\\ \omega_0\end{pmatrix},\\
(I-\mathcal{P})v = (I-\mathcal{P})D(\ell,\omega),
\end{cases} \quad (5.5)$$

where

$$\widetilde{G}_1 = \int_{\partial\Omega_S(0)} N((F-\nabla\varphi)\cdot n)n\, \mathrm{d}\gamma + G_1, \quad \widetilde{G}_2 = \int_{\partial\Omega_S(0)} y\times N((F-\nabla\varphi)\cdot n)n\, d\gamma + G_2,$$

and φ is the solution of the problem

$$-\Delta\varphi = \operatorname{div} F \text{ in } \Omega_F(0), \quad \varphi = 0 \text{ on } \partial\Omega_F(0).$$

(see also [20, Section 4.2] or [17, Proposition 3.7]). The operator $\mathcal{A}_{FS}$ is defined as in (3.20). Let us recall that the operator $\mathcal{A}_{FS}$ is an $\mathcal{R}$-sectorial operator in $\mathcal{X}$ (Theorem 3.11). One can easily verify that, under the hypothesis of the theorem, $(\mathcal{P}F, \widetilde{G}_1, \widetilde{G}_2) \in L^p(0,\infty;\mathcal{X})$ and

$$\|(\mathcal{P}F, \widetilde{G}_1, \widetilde{G}_2)\|_{L^p(0,\infty;\mathcal{X})} \leqslant C\Big(\|f\|_{L^p(0,\infty;L^q(\Omega_F(0)))} + \|h\|_{W^{2,1}_{q,p}(Q^F_\infty)^3}$$
$$+ \|g_1\|_{L^p(0,\infty;\mathbb{R}^3)} + \|g_2\|_{L^p(0,\infty;\mathbb{R}^3)}\Big).$$

From [2, Theorem 3.4], we obtain $(\mathcal{P}u_0, \ell_0, \omega_0) \in (\mathcal{X}, \mathcal{D}(\mathcal{A}_{FS}))_{1-1/p,p}$. Then by Theorem 2.7, the system (5.5) admits a unique solution $(\mathcal{P}v, \ell, \omega) \in L^p(0,\infty; \mathcal{D}(\mathcal{A}_{FS})) \cap W^{1,p}(0,\infty;\mathcal{X})$. From the expression of $(I-\mathcal{P})v$ together with Lemma 3.10, one can easily check that $v \in W^{2,1}_{q,p}(Q^F_\infty)^3$ and thus $u \in W^{2,1}_{q,p}(Q^F_\infty)^3$. The estimate (5.3) is easy to obtain.

The case $\eta > 0$ can be reduced to the previous case by multiplying all the function by $e^{\eta t}$ and using the fact that $\mathcal{A}_{FS} + \eta$ generates an C^0-semigroup of negative type for all $\eta \in (0, \eta_0)$. $\square$

6. Global in time existence and uniqueness

In this section we are going to prove Theorem 1.4. As the domain of the fluid equation for the full nonlinear problem is also a unknown of the problem, we first rewrite the system in a fixed spatial domain.

6.1. Change of variables. We describe a change of variable to rewrite the system (1.2) in a fixed spatial domain. We follow the approach of [4]. Let us assume that (1.1) is satisfied and we also assume

$$\|a\|_{L^\infty(0,\infty;\mathbb{R}^3)} + \|Q - I_3\|_{L^\infty(0,\infty;\mathbb{R}^{3\times 3})} \mathrm{diam}(\Omega_S(0)) \leqslant \frac{\alpha}{2}. \tag{6.1}$$

With the above choice we have $\mathrm{dist}\,(\Omega_S(t), \partial\Omega) \geqslant \alpha/2$ for all $t \in [0,\infty)$. We consider a cut-off function ψ which satisfies

$$\psi \in C^\infty(\overline{\Omega}), \quad \psi = 1 \text{ if } \mathrm{dist}(x,\partial\Omega) > \alpha/4, \quad \psi = 0 \text{ if } \mathrm{dist}(x,\partial\Omega) < \alpha/8. \tag{6.2}$$

We introduce a function ξ defined in $(0,\infty) \times \Omega$ by

$$\xi(t,x) = a'(t) + (x - a(t)) + \frac{|x-a(t)|^2}{2}\omega(t)$$

and Λ in $(0,\infty)\times\Omega$ by

$$\Lambda(t,x) = \psi(x)\ \left(a'(t) + \omega(t) \times (x - a(t))\right) + \begin{pmatrix} \dfrac{\partial \psi}{\partial x_2}(x)\xi_3(t,x) - \dfrac{\partial \psi}{\partial x_3}(x)\xi_2(t,x) \\ \dfrac{\partial \psi}{\partial x_3}(x)\xi_1(t,x) - \dfrac{\partial \psi}{\partial x_1}(x)\xi_3(t,x) \\ \dfrac{\partial \psi}{\partial x_1}(x)\xi_2(t,x) - \dfrac{\partial \psi}{\partial x_2}(x)\xi_1(t,x) \end{pmatrix}.$$

With the above definitions, it is easy to see that Λ satisfies the following lemma

Lemma 6.1. *Let us assume that $a \in W^{2,p}(0,\infty)$ and $\omega \in W^{1,p}(0,\infty)$. Let Λ be defined as above. Then we have*

- $\Lambda(t,x) = 0$ *for all $t \in [0,\infty)$ and for all x such that* $\mathrm{dist}(x,\partial\Omega) < \alpha/8$.
- $\mathrm{div}\Lambda(t,x) = 0$ *for all $t \in [0,\infty)$ and $x \in \Omega$.*
- $\Lambda(t,x) = a'(t) + \omega(t) \times (x - a(t))$ *for all $t \in [0,\infty)$ and $x \in \Omega_S(t)$.*
- $\Lambda \in C([0,\infty)\times\Omega;\mathbb{R}^3)$. *Moreover, for all $t \in [0,\infty)$, $\Lambda(t,\cdot)$ is a C^∞ function for all $x \in \Omega$, the function $\Lambda(\cdot,x) \in W^{1,p}(0,\infty;\mathbb{R}^3)$.*

Next we consider the characteristic X associated to the flow Λ, that is the solution of the Cauchy problem

$$\begin{aligned} \partial_t X(t,y) &= \Lambda(t, X(t,y)) \quad (t>0), \\ X(0,y) &= y \in \overline{\Omega}. \end{aligned} \tag{6.3}$$

We have the following lemma

Lemma 6.2. *For all $y \in \Omega$, the initial value problem (6.3) admits a unique solution $X(\cdot, y) : [0,\infty) \mapsto \mathbb{R}^3$, which is a C^1 function in $[0,\infty)$. Furthermore X satisfies the following properties*

- *For any $t \in [0,\infty)$, $X(t,\cdot)$ is a C^1- diffeomorphism from Ω onto Ω and $\Omega_F(0)$ onto $\Omega_F(t)$.*
- *For all $y \in \Omega$ and $t \in [0,\infty)$, we have*

$$\det\, \nabla X(t,\cdot) = 1.$$

- *For each $t \geqslant 0$, we denote by $Y(t,\cdot) = [X(t,\cdot)]^{-1}$ the inverse of $X(t,\cdot)$.*

Proof. See [4, Lemma 2.2]. □

We consider the following change of variables

$$\widetilde{u}(t,y) = Q^{-1}(t) u(t, X(t,y)), \qquad \widetilde{\pi}(t,y) = \pi(t, X(t,y)) \tag{6.4}$$

$$\widetilde{\ell}(t) = Q^{-1}(t)\dot{a}(t), \qquad \widetilde{\omega}(t) = Q^{-1}(t)\omega(t), \tag{6.5}$$

for $(t,y) \in (0,\infty) \times \Omega_F(0)$.

Then $(\widetilde{u}, \widetilde{p}, \widetilde{\ell}, \widetilde{\omega})$ satisfies the following system

$$\begin{aligned}
&\partial_t \widetilde{u} - \nu\Delta\widetilde{u} + \nabla\widetilde{p} = \mathcal{F}, \quad \text{div } u = \text{div } \mathcal{H}, && t\in(0,\infty),\ y\in\Omega_F(0),\\
&\widetilde{u} = 0, && t\in(0,\infty), y\in\partial\Omega,\\
&\widetilde{u} = \widetilde{\ell} + \widetilde{\omega}\times y, && t\in(0,\infty),\ y\in\partial\Omega_S(0),\\
&m\widetilde{\ell}' = -\int_{\partial\Omega_S(0)} \sigma(\widetilde{u},\widetilde{p})n\, d\gamma + \mathcal{G}_1, && t\in(0,\infty),\\
&J(0)\widetilde{\omega}' = -\int_{\partial\Omega_S(0)} y\times\sigma(\widetilde{u},\widetilde{p})n\, d\gamma + \mathcal{G}_2, && t\in(0,\infty),\\
&u(0,y) = u_0(y) && y\in\Omega_F(0),\\
&\ell(0) = \ell_0, \quad \omega(0) = \omega_0,
\end{aligned} \tag{6.6}$$

where

$$\dot{Q} = QA(\tilde{\omega}), \quad Q(0) = I_3, \tag{6.7}$$

is the rotation matrix of the solid at instant t,

$$X(t,y) = y + \int_0^t \Lambda(s, X(s,y))\, \mathrm{d}s, \quad \text{and} \quad \nabla Y(t, X(t,y)) = [\nabla X]^{-1}(t,y), \tag{6.8}$$

for every $y \in \Omega_F(0)$ and $t \geqslant 0$. Using the notation

$$Z(t,y) = (Z_{i,j})_{1\leqslant i,j\leqslant 3} = [\nabla X]^{-1}(t,y) \qquad (t\geqslant 0,\ y\in\Omega_F(0)), \tag{6.9}$$

the remaining terms in (6.6) are defined by

$$\mathcal{F}_i(\widetilde{u},\widetilde{\pi},\widetilde{\ell},\widetilde{\omega}) = -[(Q-I_3)\partial_t\widetilde{u}]_i - (\omega\times Q\widetilde{u})_i + \partial_t X\cdot Z^T\nabla(Q\widetilde{u})_i - \widetilde{u}\cdot Z^T\nabla(Q\widetilde{u})_i \\ + \nu\sum_{l,j,k}\frac{\partial^2(Q\tilde{u})_i}{\partial y_l\partial y_k}\left(Z_{k,j}-\delta_{k,j}\right)Z_{l,j} + \nu\sum_{l,k}\frac{\partial^2(Q\tilde{u})_i}{\partial y_l\partial y_k}\left(Z_{l,k}-\delta_{l,k}\right) \\ + \nu\left[(Q-I)\Delta\tilde{u}\right]_i + \nu\sum_{l,j,k}Z_{l,j}\frac{\partial(Q\tilde{u})_i}{\partial y_k}\frac{\partial Z_{k,j}}{\partial y_l} - \left((Z^\top - I_3)\nabla\widetilde{p}\right)_i, \tag{6.10}$$

$$\mathcal{H}(\widetilde{u},\widetilde{\pi},\widetilde{\ell},\widetilde{\omega}) = (I_3-[ZQ]^T)\widetilde{u}, \tag{6.11}$$

$$\mathcal{G}_1(\widetilde{\ell},\widetilde{\omega}) = -m(\widetilde{\omega}\times\widetilde{\ell}), \qquad \mathcal{G}_2(\widetilde{\ell},\widetilde{\omega}) = J(0)\widetilde{\omega}\times\widetilde{\omega}. \tag{6.12}$$

6.2. **Estimate of nonlinear terms.** In this section, we are going to estimate the nonlinear terms $\mathcal{F},\mathcal{H},\mathcal{G}_1$ and $\mathcal{G}_2$ defined as in (6.10) - (6.12).

Throughout this section we assume $1<p,q<\infty$ satisfying the conditions $\dfrac{1}{p}+\dfrac{1}{2q}\neq 1$ and $\dfrac{1}{p}+\dfrac{3}{2q}\leqslant\dfrac{3}{2}$. Let p' denote the conjugate of p, i.e., $\dfrac{1}{p}+\dfrac{1}{p'}=1$. Let us fix $\eta\in(0,\eta_0)$, where η_0 is the constant introduced in Theorem 4.1 and we introduce the following ball

$$\mathcal{S}_\gamma = \left\{(\widetilde{u},\widetilde{\pi},\widetilde{\ell},\widetilde{\omega}) \mid \widetilde{\rho}(t,y) = \left\|(\widetilde{u},\widetilde{\pi},\widetilde{\ell},\widetilde{\omega})\right\|_{\mathcal{S}} \leqslant \gamma\right\},$$

where

$$\left\|(\widetilde{u},\widetilde{p},\widetilde{\ell},\widetilde{\omega})\right\|_{\mathcal{S}} := \|e^{\eta(\cdot)}\widetilde{u}\|_{L^p(0,\infty;W^{2,q}(\Omega_F(0)))^3} + \|e^{\eta(\cdot)}\widetilde{u}\|_{W^{1,p}(0,\infty;L^q(\Omega_F(0)))^3} \\ + \|e^{\eta(\cdot)}\widetilde{p}\|_{L^p(0,\infty;W^{1,q}(\Omega_F(0)))} + \|e^{\eta(\cdot)}\widetilde{\ell}\|_{W^{1,p}(0,\infty;\mathbb{R}^3)} + \|e^{\eta(\cdot)}\widetilde{\omega}\|_{W^{1,p}(0,\infty;\mathbb{R}^3)}. \tag{6.13}$$

Our aim is to estimate the nonlinear terms in (6.10) - (6.12).

Proposition 6.3. *Let us assume $1<p,q<\infty$ satisfying the condition $\dfrac{1}{p}+\dfrac{3}{2q}\leqslant\dfrac{3}{2}$. There exist constants $\gamma_0\in(0,1)$ and $C_N>0$ both depending only on p,q,η and $\Omega_F(0)$ such that for every $\gamma\in(0,\gamma_0)$ and for every $(\widetilde{u},\widetilde{\pi},\widetilde{\ell},\widetilde{\omega})\in\mathcal{S}_\gamma$, we have*

$$\|e^{\eta(\cdot)}\mathcal{F}\|_{L^p(0,\infty;L^q(\Omega_F(0)))} + \|e^{\eta(\cdot)}\mathcal{H}\|_{W^{2,1}_{q,p}(Q_F^\infty)} \\ + \|e^{\eta(\cdot)}\mathcal{G}_1\|_{L^p(0,\infty;\mathbb{R}^3)} + \|e^{\eta(\cdot)}\mathcal{G}_2\|_{L^p(0,\infty;\mathbb{R}^3)} \leqslant C_N\gamma^2. \tag{6.14}$$

Proof. The constants appearing in this proof will be denoted by C and depends only on p,q,η and $\Omega_F(0)$. Let us first show that, there exists $\gamma_0\in(0,1)$, such that, for every $\gamma\in(0,\gamma_0)$ and for every $(\widetilde{u},\widetilde{\pi},\widetilde{\ell},\widetilde{\omega})\in\mathcal{S}_\gamma$ the condition (6.1) is verified.

The solution of (6.7) satisfies $Q\in SO(3)$ and thus $|Q(t)|=1$ for all $t\geqslant 0$. We can rewrite Q as follows

$$Q(t) = I + \int_0^t e^{-\eta s}e^{\eta s}\widetilde{\omega}(s)\times Q(s)\,ds.$$

Therefore

$$\|Q - I_3\|_{L^\infty(0,\infty;\mathbb{R}^{3\times 3})} \leqslant \int_0^\infty e^{-\eta s} e^{\eta s} |\widetilde{\omega}(s)| \, ds$$
$$\leqslant \left(\int_0^\infty e^{-p'\eta t} \, dt \right)^{1/p'} \|e^{\eta(\cdot)} \widetilde{\omega}\|_{L^p(0,\infty;\mathbb{R}^3)} \leqslant \left(\frac{1}{p'\eta} \right)^{1/p'} \gamma. \tag{6.15}$$

Similarly,

$$\|a\|_{L^\infty(0,\infty;\mathbb{R}^3)} \leqslant \int_0^\infty e^{-\eta t} e^{\eta t} |Q(s)| |\widetilde{\ell}(t)| \, dt \leqslant \left(\frac{1}{p'\eta} \right)^{1/p'} \gamma. \tag{6.16}$$

Combining (6.15) and (6.16), we get

$$\|a\|_{L^\infty(0,\infty;\mathbb{R}^3)} + \|Q - I_3\|_{L^\infty(0,\infty;\mathbb{R}^{3\times 3})} \mathrm{diam}(\Omega_S(0)) \leqslant \gamma \left(\frac{1}{p'\eta} \right)^{1/p'} (1 + \mathrm{diam}(\Omega_S(0))).$$

Let us set

$$\gamma_0 = \min \left\{ 1, \frac{\alpha}{2C_{p,\eta}(1 + \mathrm{diam}(\Omega_F(0)))} \right\}, \quad \text{with } C_{p,\eta} = \left(\frac{1}{p'\eta} \right)^{1/p'}. \tag{6.17}$$

With the above choice of γ_0, we can easily verify the condition (6.1).

Let X be defined as in (6.8). Differentiating (6.8) with respect to y we obtain

$$\nabla X(t, y) = I_3 + \int_0^t \nabla \Lambda(s, X(s, y)) \nabla X(s, y) \, ds$$

From the definition of Λ and X we obtain

$$\begin{aligned}
&\|\nabla X(t, \cdot)\|_{C^2(\Omega)} \\
&\leqslant 1 + C \int_0^t e^{-\eta s} e^{\eta s} \left(|\widetilde{\omega}(s) + |\widetilde{\ell}(s)| \right) \|\nabla X(s, \cdot)\|_{C^\infty(\Omega)} \, ds \\
&\leqslant 1 + C \left(\|e^{\eta(\cdot)} \widetilde{\ell}\|_{L^\infty(0,\infty;\mathbb{R}^3)} + \|e^{\eta(\cdot)} \widetilde{\omega}\|_{L^\infty(0,\infty;\mathbb{R}^3)} \right) \int_0^t e^{-\eta s} \|\nabla X(s, \cdot)\|_{C^2(\Omega)} \, ds \\
&\leqslant 1 + C \int_0^t e^{-\eta s} \|\nabla X(s, \cdot)\|_{C^2(\Omega)} \, ds,
\end{aligned}$$

By Gronwall's inequality

$$\|\nabla X(t, \cdot)\|_{C^2(\Omega)} \leqslant \exp \left(C \int_0^t e^{-\eta s} \, ds \right) \leqslant e^{C/\eta} \text{ for all } t \in (0, \infty).$$

With the above estimate we obtain

$$\|\nabla X(t, \cdot) - I_3\|_{L^\infty(0,\infty;C^2(\Omega))} \leqslant C \int_0^\infty e^{-\eta s} e^{\eta s} \left(\|\widetilde{\omega}(s)\|_{\mathbb{R}^3} + \|\widetilde{\ell}(s)\|_{\mathbb{R}^3} \right) \, ds \leqslant C\gamma. \tag{6.18}$$

It is also easy to see that

$$\|\mathrm{Cof} \nabla X\|_{L^\infty(0,\infty;C^2(\Omega))} \leqslant C.$$

From Lemma 6.2, we have $\det\nabla X(t,y) = 1$ for all $t \geqslant 0$ and $y \in \overline{\Omega}$. Thus from the relation

$$Z = [\nabla X]^{-1} = \frac{1}{\det\nabla X}\mathrm{Cof}\nabla X,$$

we obtain

$$\|Z\|_{L^\infty(0,\infty;C^2(\Omega))} \leqslant C. \tag{6.19}$$

Using the above estimate and (6.18), we get

$$\|Z - I_3\|_{L^\infty(0,\infty;C^2(\Omega))} \leqslant \|Z\|_{L^\infty(0,\infty;C^2(\Omega))}\|\nabla X - I_3\|_{L^\infty(0,\infty;C^2(\Omega))} \leqslant C\gamma. \tag{6.20}$$

In a similar manner we can obtain the following estimates

$$\begin{aligned} &\|\partial_t X\|_{L^\infty((0,\infty)\times\Omega_F(0))} \leqslant C\gamma, \quad \|\partial_t Z\|_{L^\infty((0,\infty)\times\Omega_F(0))} \leqslant C\gamma,\\ &\|ZQ - I_3\|_{L^\infty(0,\infty;C^2(\Omega))} \leqslant C\gamma. \end{aligned} \tag{6.21}$$

We are now in a position to estimate the nonlinear terms.
Estimate of $\mathcal{F}$.

$$\|e^{\eta(\cdot)}\mathcal{F}\|_{L^p(0,\infty;L^q(\Omega_F(0)))} \leqslant C\gamma^2. \tag{6.22}$$

•Estimate of first, second and third term of $\mathcal{F}$: Using (6.15), (6.19) and (6.21) we have

$$\begin{aligned} &\left\|e^{\eta(\cdot)}\Big(- [(Q - I_3)\partial_t\widetilde{u}]_i - (\omega \times Q\widetilde{u})_i + \partial_t X \cdot Z^T\nabla(Q\widetilde{u})_i\Big)\right\|_{L^p(0,\infty;L^q(\Omega_F(0)))}\\ &\leqslant C\left(\|Q - I_3\|_{L^\infty(0,\infty;\mathbb{R}^{3\times 3})} + \|\widetilde{\omega}\|_{L^\infty(0,\infty;\mathbb{R}^3)} + \|\partial_t X\|_{L^\infty((0,\infty)\times\Omega_F(0))}\right)\|e^{\eta(\cdot)}\widetilde{u}\|_{W^{2,1}_{q,p}(Q_F^\infty)}\\ &\leqslant C\gamma^2. \end{aligned}$$

•Estimate of fourth term of $\mathcal{F}$: By Hölder's inequality and using (6.19), we obtain

$$\begin{aligned} &\|e^{\eta(\cdot)}\widetilde{u} \cdot Z^T\nabla(Q\widetilde{u})_i\|_{L^p(0,\infty;L^q(\Omega_F(0)))}\\ &\leqslant C\|e^{\eta(\cdot)}\widetilde{u} \cdot \nabla\widetilde{u}_i\|_{L^p(0,\infty;L^q(\Omega_F(0)))}\\ &\leqslant C\|e^{\eta(\cdot)}\widetilde{u}\|_{L^{3p}(0,\infty;L^{3q}(\Omega_F(0)))}\|\nabla\widetilde{u}_i\|_{L^{3p/2}(0,\infty;L^{3q/2}(\Omega_F(0)))}. \end{aligned}$$

Since $\dfrac{1}{p} + \dfrac{3}{2q} \leqslant \dfrac{3}{2}$, one has the following embeddings (see for example [11, Proposition 4.3])

$$W^{2,1}_{q,p}(Q_F^\infty) \hookrightarrow L^{3p}(0,\infty;L^{3q}(\Omega_F(0))) \text{ and } W^{2,1}_{q,p}(Q_F^\infty) \hookrightarrow L^{3p/2}(0,\infty;W^{1+3q/2}(\Omega_F(0))).$$

Therefore, using the above embeddings we obtain

$$\|e^{\eta(\cdot)}\widetilde{u} \cdot Z^T\nabla(Q\widetilde{u})_i\|_{L^p(0,\infty;L^q(\Omega_F(0)))} \leqslant C\gamma^2$$

•Estimate of fifth term of $\mathcal{F}$(estimates of remaining terms of $\mathcal{F}$ are similar) : Using (6.19) and (6.20) we have

$$\begin{aligned} &\left\|\nu e^{\eta(\cdot)}\sum_{l,j,k}\frac{\partial^2(Q\tilde{u})_i}{\partial y_l\partial y_k}\left(Z_{k,j} - \delta_{k,j}\right)Z_{l,j}\right\|_{L^p(0,\infty;L^q(\Omega_F(0)))}\\ &\leqslant C\|Z - I_3\|_{L^\infty(0,\infty;C^2(\Omega))}\|e^{\eta(\cdot)}\widetilde{u}\|_{W^{2,1}_{q,p}(Q_F^\infty)} \leqslant C\gamma^2. \end{aligned}$$

Estimate of $\mathcal{H}$.

$$\|e^{\eta(\cdot)}\mathcal{H}\|_{W^{2,1}_{q,p}(Q^\infty_F)} \leqslant C\gamma^2. \tag{6.23}$$

Using (6.20) and (6.21), we obtain

$$\begin{aligned}
&\|e^{\eta(\cdot)}(I_3 - [ZQ]^T)\widetilde{u}\|_{W^{2,1}_{q,p}(Q^\infty_F)} \\
&\leqslant C\Big(\|ZQ - I_3\|_{L^\infty(0,\infty;C^2(\Omega))} + \|\partial_t Z\|_{L^\infty((0,\infty)\times\Omega_F(0))} \\
&\qquad\qquad + \|\partial_t Q\|_{L^\infty(0,\infty;\mathbb{R}^3\times\mathbb{R}^3)}\Big)\|e^{\eta(\cdot)}\widetilde{u}\|_{W^{2,1}_{q,p}(Q^\infty_F)} \\
&\leqslant C\gamma^2.
\end{aligned}$$

Estimate of $\mathcal{G}_1$ and $\mathcal{G}_2$: From the expressions of $\mathcal{G}_1$ and $\mathcal{G}_2$ it is easy to see that

$$\|e^{\eta(\cdot)}\mathcal{G}_1\|_{L^p(0,\infty;\mathbb{R}^3)} + \|e^{\eta(\cdot)}\mathcal{G}_2\|_{L^p(0,\infty;\mathbb{R}^3)} \leqslant C\gamma^2. \tag{6.24}$$

Combining (6.22) - (6.24), we obtain (6.14). ☐

Proposition 6.4. *Let us assume $1 < p, q < \infty$ satisfying the condition $\dfrac{1}{p} + \dfrac{3}{2q} \leqslant \dfrac{3}{2}$. Let γ_0 is defined as in (6.17). There exist constant $C_{lip} > 0$ depending only on p, q, η and $\Omega_F(0)$ such that for every $\gamma \in (0, \gamma_0)$ and for every $(\widetilde{u}^j, \widetilde{\pi}^j, \widetilde{\ell}^j, \widetilde{\omega}^j) \in \mathcal{S}_\gamma$, $j = 1, 2$ we have*

$$\begin{aligned}
&\|e^{\eta(\cdot)}\mathcal{F}(\widetilde{u}^1, \widetilde{\pi}^1, \widetilde{\ell}^1, \widetilde{\omega}^1) - e^{\eta(\cdot)}\mathcal{F}(\widetilde{u}^2, \widetilde{\pi}^2, \widetilde{\ell}^2, \widetilde{\omega}^2)\|_{L^p(0,\infty;L^q(\Omega_F(0)))} \\
&\qquad + \|e^{\eta(\cdot)}\mathcal{H}(\widetilde{u}^1, \widetilde{\pi}^1, \widetilde{\ell}^1, \widetilde{\omega}^1) - e^{\eta(\cdot)}\mathcal{H}(\widetilde{u}^2, \widetilde{\pi}^2, \widetilde{\ell}^2, \widetilde{\omega}^2)\|_{W^{2,1}_{q,p}(Q^\infty_F)} \\
&+ \|e^{\eta(\cdot)}\mathcal{G}_1(\widetilde{\ell}^1, \widetilde{\omega}^1) - e^{\eta(\cdot)}\mathcal{G}_1(\widetilde{\ell}^2, \widetilde{\omega}^2)\|_{L^p(0,\infty;\mathbb{R}^3)} + \|e^{\eta(\cdot)}\mathcal{G}_2(\widetilde{\ell}^1, \widetilde{\omega}^1) - e^{\eta(\cdot)}\mathcal{G}_2(\widetilde{\ell}^2, \widetilde{\omega}^2)\|_{L^p(0,\infty;\mathbb{R}^3)} \\
&\qquad\qquad \leqslant C_{lip}\gamma\left\|(\widetilde{u}^1, \widetilde{\pi}^1, \widetilde{\ell}^1, \widetilde{\omega}^1) - (\widetilde{u}^2, \widetilde{\pi}^2, \widetilde{\ell}^2, \widetilde{\omega}^2)\right\|_{\mathcal{S}}
\end{aligned} \tag{6.25}$$

Proof. The proof is similar to the proof of Proposition 6.3. ☐

6.3. **Proof of Theorem 1.4.** At first we prove global existence and uniqueness theorem for the transformed system (6.6) -(6.12) under the smallness assumption on the initial data. More precisely we prove the following theorem

Theorem 6.5. *Let $1 < p, q < \infty$ satisfying the conditions $\dfrac{1}{p} + \dfrac{1}{2q} \neq 1$ and $\dfrac{1}{p} + \dfrac{3}{2q} \leqslant \dfrac{3}{2}$. Let $\eta \in (0, \eta_0)$, where η_0 is the constant introduced in Theorem 4.1. There exist a constant $\widetilde{\gamma} > 0$ depending only on p, q, η and $\Omega_F(0)$ such that, for all $\gamma \in (0, \widetilde{\gamma})$ and for all $(u_0, \ell_0, \omega_0) \in B^{2(1-1/p)}_{q,p}(\Omega_F(0)) \times \mathbb{R}^3 \times \mathbb{R}^3$ satisfying the compatibility conditions*

$$\begin{aligned}
&\operatorname{div} u_0 = 0 \text{ in } \Omega_F(0), \\
&u_0 = \ell_0 + \omega_0 \times y \text{ on } \partial\Omega_S(0), \quad u_0 = 0 \text{ on } \partial\Omega \text{ if } \frac{1}{p} + \frac{1}{2q} < 1 \\
\text{and } &u_0 \cdot n = (\ell_0 + \omega_0 \times y) \cdot n \text{ on } \partial\Omega_S(0), \qquad u_0 \cdot n = 0 \text{ on } \partial\Omega \text{ if } \frac{1}{p} + \frac{1}{2q} > 1,
\end{aligned}$$

and

$$\|u_0\|_{B^{2(1-1/p)}_{q,p}(\Omega_F(0))} + \|\ell_0\|_{\mathbb{R}^3} + \|\omega_0\|_{\mathbb{R}^3} \leqslant \frac{\gamma}{2C_L}, \tag{6.26}$$

where C_L is the continuity constant appear in (5.3), the system (6.6) -(6.12) admits a unique strong solution $(\widetilde{u}, \widetilde{\pi}, \widetilde{\ell}, \widetilde{\omega})$ such that

$$\left\|(\widetilde{u}, \widetilde{\pi}, \widetilde{\ell}, \widetilde{\omega})\right\|_{\mathcal{S}} \leqslant \gamma. \tag{6.27}$$

Proof. Let us set

$$\widetilde{\gamma} = \min\left\{\gamma_0, \frac{1}{2C_L C_N}, \frac{1}{2C_L C_{lip}}\right\} \tag{6.28}$$

where γ_0 is defined as in (6.17) and C_L, C_N and C_{lip} are the constants appearing Theorem 5.1, Proposition 6.3 and Proposition 6.4 respectively. Let us choose $\gamma \in (0, \widetilde{\gamma})$ and $(v, \varphi, \kappa, \tau) \in \mathcal{S}_\gamma$. We consider the following problem

$$\begin{aligned}
&\partial_t \widetilde{u} - \nu\Delta\widetilde{u} + \nabla\widetilde{\pi} = \mathcal{F}(v, \varphi, \kappa, \tau), \quad \operatorname{div} u = \operatorname{div} \mathcal{H}(v, \varphi, \kappa, \tau), && t \in (0,\infty),\ y \in \Omega_F(0),\\
&\widetilde{u} = 0, && t \in (0,\infty),\ y \in \partial\Omega,\\
&\widetilde{u} = \widetilde{\ell} + \widetilde{\omega} \times y, && t \in (0,\infty),\ y \in \partial\Omega_S(0),\\
&m\widetilde{\ell}' = -\int_{\partial\Omega_S(0)} \sigma(\widetilde{u}, \widetilde{\pi}) n \, d\gamma + \mathcal{G}_1(\kappa, \tau), && t \in (0,\infty),\\
&J(0)\widetilde{\omega}' = -\int_{\partial\Omega_S(0)} y \times \sigma(\widetilde{u}, \widetilde{\pi}) n \, d\gamma + \mathcal{G}_2(\kappa, \tau), && t \in (0,\infty),\\
&u(0, y) = u_0(y), && y \in \Omega_F(0),\\
&\ell(0) = \ell_0, \quad \omega(0) = \omega_0.
\end{aligned} \tag{6.29}$$

We are going to show the mapping

$$\mathcal{N} : (v, \varphi, \kappa, \tau) \mapsto (\widetilde{u}, \widetilde{\pi}, \widetilde{\ell}, \widetilde{\omega})$$

where $(\widetilde{u}, \widetilde{\pi}, \widetilde{\ell}, \widetilde{\omega})$ is the solution to the system (6.29), is a contraction in $\mathcal{S}_\gamma$. As $(v, \varphi, \kappa, \tau) \in \mathcal{S}_\gamma$, we can apply Theorem 5.1 and Proposition 6.3 to the system (6.29) and using (6.26) and definition of $\widetilde{\gamma}$ we obtain

$$\begin{aligned}
&\|\mathcal{N}(v, \varphi, \kappa, \tau)\|_{\mathcal{S}}\\
&\leqslant C_L\Big(\|u_0\|_{B^{2(1-1/p)}_{q,p}(\Omega_F(0))} + \|\ell_0\|_{\mathbb{R}^3} + \|\omega_0\|_{\mathbb{R}^3}\Big) + C_L C_N \gamma^2\\
&\leqslant \gamma.
\end{aligned}$$

Thus $\mathcal{N}$ is a mapping from $\mathcal{S}_\gamma$ to itself for all $\gamma \in (0, \widetilde{\gamma})$. Next, using Theorem 5.1 and Proposition 6.4, we obtain

$$\begin{aligned}
&\left\|\mathcal{N}(v^1, \varphi^1, \kappa^1, \tau^1) - \mathcal{N}(v^2, \varphi^2, \kappa^2, \tau^2)\right\|_{\mathcal{S}}\\
&\leqslant C_L C_N \gamma \left\|(v^1, \varphi^1, \kappa^1, \tau^1) - (v^2, \varphi^2, \kappa^2, \tau^2)\right\|_{\mathcal{S}},
\end{aligned}$$

for all $(v^j, \varphi^j, \kappa^j, \tau^j)$, $j = 1, 2$. Again using the definition of $\widetilde{\gamma}$ one can easily verify that $\mathcal{N}$ is a strict contraction of $\mathcal{S}_\gamma$ for any $\gamma \in (0, \widetilde{\gamma})$, which implies our existence and uniqueness result. $\square$

Proof of Theorem 1.4: Let $(\widetilde{u}, \widetilde{\pi}, \widetilde{\ell}, \widetilde{\omega})$ be the solution of the system (6.6) -(6.12), constructed in Theorem 6.5. Since $\gamma < \widetilde{\gamma}$, (6.1) is verified and $X(t, \cdot)$ is a well defined mapping and it is a C^1-diffeomorphism from $\Omega_F(0)$ into $\Omega_F(t)$. Therefore, there is a unique $Y(t, \cdot)$ from $\Omega_F(t)$ into $\Omega_F(0)$ such that $Y(t, \cdot) = X(t, \cdot)^{-1}$. We set, for all $t \geqslant 0$ and $x \in \Omega_F(t)$

$$u(t, x) = \widetilde{u}(t, Y(t, x)), \quad \pi(t, x) = \widetilde{\pi}(t, Y(t, x)),$$
$$a'(t) = Q(t)\widetilde{\ell}(t) \text{ and } \omega(t) = Q(t)\widetilde{\omega}(t).$$

We can easily check that (u, π, a, ω) satisfies the original system (1.2) satisfying (1.13).

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