Poster summarizing "The abc conjecture and some of its consequences"
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The \( abc \) conjecture and some of its consequences

Remarks

- For \( a, b, c > 0 \), natural numbers, if \( a + b = c \) then the conjecture states that \( \text{rad}(abc) \leq c - 1 \), where \( \text{rad}(n) \) denotes the product of the distinct prime factors of \( n \).

- Over the years, there have been several failed attempts to prove this conjecture.

- The authors have considered the cases where \( a \) or \( b \) are 0, which reduces the problem to a lower-dimensional space.

Best unconditional result

- Stewart and Yu (1994) showed that \( \text{rad}(abc) > c^{0.37} \) for all sufficiently large \( c \) if \( \text{rad}(abc) \leq c - 1 \) for a certain proportion of \( a, b, c \).

- This result is based on the inequality \( \text{rad}(abc) > c^{0.37} \) for a certain proportion of \( a, b, c \).

Pillai's conjecture (1948)

- Let \( \text{rad}(x) \) be the product of the distinct prime factors of \( x \).

- Then, for any \( x, y, p, q \) such that \( x, y, p, q \) are sufficiently large, \( \text{rad}(x) \neq \text{rad}(y) \) unless \( x = y \).

The case \( a = 1 \)

- In this special case, various cases were considered:
  - The Lang-Vojta conjecture (1978)
  - The abc conjecture implies Lang-Vojta and therefore Pillai's conjecture (1971)

The abc conjecture implies that \( \text{rad}(abc) > c^{0.37} \) for a certain proportion of \( a, b, c \).


- The Format-Wiles theorem states that if \( a + b = c \), then the number of solutions to this equation is finite.

The abc conjecture implies asymptotic Form-Wiles

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Waring's theorem (1919)

- Let \( a, b, c \) be positive integers such that \( a + b = c \) and \( a, b, c \) are the smallest positive integers satisfying this equation.

The abc conjecture implies asymptotic Waring's

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The Erdős-Woods conjecture (1981)

- Let \( n \) be a number, and let \( f(n) \) be the number of distinct prime factors of \( n \).

The abc conjecture implies Erdős-Woods

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The abc conjecture implies Langley

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Dirichlet's approximation theorem (1830)

- For any \( \epsilon > 0 \), there exists a number \( x \) such that \( \left| x - \frac{a}{b} \right| < \frac{\epsilon}{b^2} \) for some integers \( a, b \).

The abc conjecture implies Dirichlet's

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The abc conjecture implies the Thue-Siegel-Roth theorem (1909, 1925, 1955)

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The abc conjecture implies Siegel's theorem (1983)

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The abc conjecture implies Sierpinski-Walfisz (1990)

- The abc conjecture implies Sierpinski-Walfisz.

The abc conjecture implies the abc theorem (1994)

- The abc conjecture implies the abc theorem.

A conjecture on \( \tau \) (2003)

- For any \( \epsilon > 0 \), there exists a number \( x \) such that \( \left| x - \frac{a}{b} \right| < \frac{\epsilon}{b^2} \) for some integers \( a, b \).

The abc conjecture implies the abc theorem

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Linnik's theorem (1997)

- For any \( \epsilon > 0 \), there exists a number \( x \) such that \( \left| x - \frac{a}{b} \right| < \frac{\epsilon}{b^2} \) for some integers \( a, b \).

The abc conjecture implies Linnik's theorem

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Effective bound assuming abc (2015)

- For any \( \epsilon > 0 \), there exists a number \( x \) such that \( \left| x - \frac{a}{b} \right| < \frac{\epsilon}{b^2} \) for some integers \( a, b \).

The abc conjecture implies the abc theorem

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Heuristic:

- Baker's effective bound is independent of the abc conjecture.

Roth's theorem (1955)

- For any \( \epsilon > 0 \), there exists a number \( x \) such that \( \left| x - \frac{a}{b} \right| < \frac{\epsilon}{b^2} \) for some integers \( a, b \).

The abc conjecture implies Roth's theorem

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The ABC conjecture for polynomials

- For any \( \epsilon > 0 \), there exists a number \( x \) such that \( \left| x - \frac{a}{b} \right| < \frac{\epsilon}{b^2} \) for some integers \( a, b \).

The abc conjecture implies the abc theorem

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In the quest for examples

- For any \( \epsilon > 0 \), there exists a number \( x \) such that \( \left| x - \frac{a}{b} \right| < \frac{\epsilon}{b^2} \) for some integers \( a, b \).

The abc conjecture implies the abc theorem

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References


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