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High-order hybrid discontinuous galerkin methods for diffusion-convection equations with evanescent diffusion
Loïc DJOUX, Vincent FONTAINE
University of La Réunion

Physical context
- Diffusion-advection equation
  \[ \nabla \cdot (-\sigma \nabla u + \beta u) = f \quad \text{in} \quad \Omega \]
  where \( \sigma \) : the conductivity tensor, \( u \) : the state variable, \( \beta \) : velocity advection field and \( f \) : source term.
- Application field:
  - Ground water flow
  - Heat and Mass transfer,...

A mixed formulation
We introduce an auxiliary variable \( Q_d \):
\[
\nabla \cdot \alpha \nabla u = f, \quad \text{in} \quad \Omega
\]
where \( \alpha = \sigma_d Q_d \) and \( Q_d \) is a field variable. We define two approximate variables \( \alpha u \) and \( Q_d \) of respectively \( u \) and \( Q_d \):
\[
Q_d = \sum_{\Omega_k} \alpha u_k \text{ and } Q_d = \sum_{\Omega_k} \alpha u_k \text{ on } \Omega_k
\]
where \( u_k \) and \( u_k \) are the degrees of freedom of \( Q_d \) and \( Q_d \) respectively, \( w \) and \( v \) are their corresponding interpolations functions.

Approximation spaces
We define finite element spaces:
\[
W_h^e = \{ w \in [L^2(\Omega)]^d : w |_{\Omega_k} \in W^p(\Omega_k), \quad \forall \Omega_k \in \Omega \},
\]
\[
V_h^e = \{ v \in [L^2(\Omega)] : v |_{\Omega_k} \in W^p(\Omega_k), \quad \forall \Omega_k \in \Omega \},
\]
and add a traced finite element space:
\[
M_h^e = \{ \mu \in L^2(\Omega) : \mu |_{\Omega_k} \in M^p(\Omega_k) \}, \quad \forall \Omega_k \in \Omega
\]

Discrete weak formulation
We impose the continuity of \( \alpha Q_d + \beta u \) on each edge \( e \in \partial \Omega_k \) through one more equation and finally write:
\[
\text{Find } (Q_d, u, \lambda) \in W_h^e \times V_h^e \times M_h^e \text{ such that:}
\]
\[
\begin{align*}
(Q_d, w_k)_{\Omega_k} - (u_k \nabla \cdot (\sigma_d w_k))_{\Omega_k} + (\lambda_k \alpha w_k)_{\Omega_k} - (\beta \nabla \cdot u_k) \cdot w_k &= (f, w_k)_{\Omega_k}, \quad \forall w_k \in \Omega
\end{align*}
\]

Numerical results : degenerate case
We propose to study a heterogeneous and anisotropic case with the exact solution defined in \( \Omega = [0,1] \times [0,1] \) as:
\[
\begin{align*}
\alpha(x, y) &= 1 \quad \text{for } Q_1 \in [0,1] \times [0,1] \\
\beta(x, y) &= \alpha^{\gamma} \quad \text{for } Q_2 \in [0,1] \times [0,1]
\end{align*}
\]

For \( Q_1, Q_2 \) we have the conductivity tensor \( \sigma = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \) and for \( Q_2 \) we have \( \sigma = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \). The velocity field is defined in \( \Omega \) and we have \( \beta = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \).

Bibliography

Contact
Loïc DJOUX : loic.dijoux2@univ-reunion.fr
Vincent FONTAINE : vincent.fontaine@univ-reunion.fr