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To cite this version:
Loïc Dijoux, Vincent Fontaine. High-order hybrid discontinuous galerkin methods for diffusion-convection equations with evanescent diffusion. InterPore, May 2017, Rotterdam, Netherlands. <hal-01625960>

HAL Id: hal-01625960
https://hal.archives-ouvertes.fr/hal-01625960
Submitted on 30 Oct 2017

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High-order hybrid discontinuous galerkin methods for diffusion-convection equations with evanescent diffusion
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Physical context
- Diffusion-advection equation
  \[ \nabla \cdot (\alpha \nabla u + \beta u) = f \quad \text{in} \quad \Omega \]
  where \( \alpha \) : the conductivity tensor, \( u \) : the state variable, \( \beta \) : velocity advection field and \( f \) : source term.
- Application field:
  - Ground water flow
  - Heat and Mass transfer, ...

A mixed formulation
We introduce an auxiliary variable \( Q \) :
\[
\begin{align*}
\nabla \cdot Q &= \alpha \nabla u + \beta u = f \\
Q &= \frac{1}{\alpha} (\nabla \cdot Q - \nabla \cdot u) + \beta u
\end{align*}
\]

Approximation spaces
We define finite element spaces:
\[
\begin{align*}
W^0_K &= \{ w \in [L^2(\Omega)]^n : w |_K \in W^0(K), \quad \forall K \in T_h \}, \\
V^0_K &= \{ v \in [L^2(\Omega)]^n : v |_K \in W^0(K), \quad \forall K \in T_h \}, \\
and add a traced finite element space:
M^0_e &= \{ u \in L^2(\Omega) : u |_e \in M^0(e), \quad \forall e \in T_h \}
\end{align*}
\]

Discrete weak formulation
We impose the continuity of \( \nabla Q \beta + \beta u \) on each edge \( e \in \eta_h \) through one more equation and finally write:
\[
\begin{align*}
\text{Find } u_h, Q_h, \text{ and } Q_{\beta h} \in W^0_K \times V^0_K \times \mathcal{M}_e^0 \text{ such that:}
\end{align*}
\]

Numerical results : degenerate case
We propose to study a heterogeneous and anisotropic case with the exact solution defined in \( \Omega \in [0,1] \times [0,1] \) as:
\[
\begin{align*}
u(x, y) &= 1 \quad \text{for } \Omega_1 \in [0, \frac{1}{8}] \times [0, \frac{1}{2}] \\
u(x, y) &= 1 \quad \text{for } \Omega_2 \in [\frac{3}{8}, 1] \times [\frac{1}{2}, \frac{3}{4}] \\
u(x, y) &\sim \frac{1}{x+y} \quad \text{for } \Omega_3 \in [\frac{7}{8}, 1] \times [\frac{1}{4}, 1]
\end{align*}
\]

For \( \Omega_1 \), we have the conductivity tensor \( \alpha \in \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \) and for \( \Omega_2 \) we have \( \beta = \frac{1}{x+y} \).

Numerical results : degenerate case
Triangular mesh (GMsh : 7547 faces) & Hdg-mesh:

Bibliography