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NSCD* method applied to mechanical simulation of masonry in historical buildings using MOMA**

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ABSTRACT

This paper introduces a new approach for the mechanical modelling of historical monuments built of blocks. The computational method, * **N**on **S**mooth **C**ontact **D**ynamics (NSCD), is used to simulate masonry as a large collection of bodies under unilateral constraints and frictional contact. The computational method, NSCD, within an architectural and archeological model, MOMA, ****M**odels and **O**pticals **M**easurements in **A**rchitecture are merged in a single tool providing a interdisciplinary approach of historical buildings. The morphology and initial deformations measures are obtained by a photogrammetrical survey monitored by an architectural and archeological knowledge. The implementation of the model and experimental techniques are presented introducing an example of case study on the Tour Saint-Laurent of the "Palais des papes" in Avignon (France).

1 INTRODUCTION

The modelling of the mechanical behaviour of a historical building may appear as a standard work on a basic mechanical system, i.e on a system where the history of strains and stresses of each physical datum is well known. Actually, an historical building, even from a mechanical point of view, appears as a complex system. As a complex system, the monument can not be easily split into uncoupled problems that concern essentially a single field at a time. A global view of the problem is essential to tackle the complexity of such a system. For instance, the history of the building can not be described by simple moduli in a analytical way; its modelling is all the more tricky. By history, different meanings are inferred: there exits a mechanical history closely related to an archeological and architectural history.

Let us provide a very concrete example of the connection between these histories. For a mechanical engineer, a historical building is a structure viewed as deformed and pre-stressed. This very mechanical state is not sufficient to fully take into account the crucial historical features of the monuments. Some modes of failure may have appeared. The actual mechanical state may be the result of a strongly non-linear evolution on several time-scales going from the minute up to the century. These phenomena concern first of all mechanical engineering. The deformations are then described by a standard variable: the strain measure. But the deformations which are recorded on the building, may result from other causes such as defects at time of construction. To put it more precisely, these deformations may be due to deviations from the original plan or to defects created when the stone was bonded.

This reality is at the heart of our approach. Indeed, the data acquisition is not only made by a photogrammetrical survey of the building, but also by monitoring it using an architectural and archeological model. This allows to take into account wrong interpretations of the deformations acquired.

The paper is organised as follows. Section 2 contains a short description of the architectural and archeological model and how it is informed by a photogrammetrical survey. The specificity of the photogrammetric method in the case of using it for an architectural and mechanical view, stone to stone simulation especially, is discussed. In Section 3 a global view of the NSCD method is given. Main features are presented. It is stressed on how a masonry structure is taken into account by a collection of bodies in interaction with contact friction. Then, in Section 4 one explains the way how the survey process has been implemented. This implementation takes into account the various human computer interfaces with respect to each discipline. Other processing phases like contact and collision detection and meshing application are also presented. Eventually, in Section 5, some results are given for the case study of the Tour Saint-Laurent.

2 MODELS AND OPTICALS MEASUREMENTS IN ARCHITECTURE

The MOMA research program (**M**odels and **O**pticals **M**easurements in **A**rchitecture), developed at the GAMSAU CNRS laboratory, formalises and implements representations of the architectural patrimony summarising three features of the domain: information, representation, conservation. Taking advantage of the consistency of object-oriented programming with complex knowledge, the project can be described through three phases:

1. Formalisation, measurement processing, and information of the architectural model,
2. Representation and implementation of archaeological hypothesis,
3. Platform-independent web interfacing of the research outputs.

MOMA therefore focuses on the field of architectural surveying (photogrammetry, knowledge representation) as well as on the field of the architectural heritage (identification of pathologies, simulation of hypothesis). We consequently develop a knowledge tool using the architectural model as the core of an interdisciplinary representation. Handling the exchange of experiences and skills covering the wide scope of the architectural conservation discipline requires the use of models able to represent diverse points of view and practices. Relevance of this interdisciplinary approach therefore relies on the ability of a computer-based system to convey within the simulation process the information detained [5].

2.1 The research field

2.1.1 Definition of the architectural model: Object technology

Elements of a building will be described as ”entities” (elementary elements) providing that they meet two requirements :

1. An entity is a unique ”object” identified by a single element of the architectural vocabulary,
2. An entity has an obvious and permanent role in the physical structure of the building.

A typical example of this approach can be found in the description of the roman column: its base, shaft and capital are entities (one word, one role, indivisible); the column itself isn’t (one word, several objects, divisible).

Entities are then physical individuals to which we attach various elements of knowledge such as non measurable data (historical informations and so forth); and geometrical informations (dimensions, orientation) which we use to organise topological relations between the entities.

2.1.2 Structure of the architectural model

The object-oriented programming approach lets us gather generic entities into hierarchies of elements sharing properties or common behaviours or attributes, each property added giving birth to a new, more specialized (lower in the hierarchy), generic element.

Architectural entities, structured by the inheritance of class mechanism, are analysed from the viewpoint of intension, thus isolating object categories for which similarities of structure or behaviour can be observed. Once the architectural entity’s proprieties are described (both nature and behaviour), relations between entities are formalised in order to organise the building.

2.2 Dimensioning the model: measurements and survey process

The survey process we present in this research makes use of photogrammetrical survey techniques to propose a method that integrates a pre-defined knowledge of the elements to be measured [6].

The process first gives dimensions to the independent elementary entities, using canonical definitions of the models involved, then makes use of one or more hypothesis on their combinations so as to link them to one another. The background of this research, and consequently the outputs to figure, is therefore a dimensioning process in which architectural models are informed by a photogrammetrical survey.

3 THE NON SMOOTH CONTACT DYNAMICS METHOD

3.1 Relevance of the NSCD method

When monuments are composed of masonry structures, their response to various loadings, such as gravity, earthquakes or ground level motions is characterized by large heterogeneities. Large local stresses and strains are generated causing failure by fracture, generally located in joints. These phenomena are not necessarily the premonitory signs of some dangerous situations but may just be signs of adaptation to external aggressions. As an example of this type of adaptation, failures usually appear in dry stones bond vault where contact between two voussoirs can not be realized on the whole surface of blocks.

Finite elements method applied to a building, may provide very interesting results concerning the location of the largest stresses and strains, even if a linear elastic stress-strain relationship is used. The monument is then considered as a single continuous body composed of a single heterogeneous material. Since masonry materials are not able to undergo large tensile stresses, a step forward in the analysis would be the use of a damage or plastic behavior coupled with tension failure criteria such as Mohr-Coulomb. Another method is the use of homogenization techniques, where the masonry structure is simulated with a homogenized equivalent material. All these methods belong to the mechanics of continuous media and give very good results when small perturbations are taken into account. However large local stresses and strains causing fracture are very difficult to compute with continuous media. Moreover, many other problems are connected with softening behavior such as spurious mesh sensitivity due to strain localization.

That is the reason why in our approach, which is proposed in the NSCD method, the choice of describing masonry structure as it is, a collection of isolated blocks with unilateral frictional contact, has been made. This technique may be found under the designation of "Distinct Elements Method", shortly DEM. It goes back to the pioneering work of P. Cundall. The NSCD method, which has been initiated and developed by M. Jean and J.J. Moreau [13, 11, 12, 10], is distinguishable from basic DEM by the use of these following items:

1. finite element discretization in order to fit the elastic behavior of stone,
2. implicit scheme for integrating the time discretized dynamical equation,
3. non-regularized interaction law (Signorini and Coulomb dry friction).

More details on NSCD may be found in [10] and [1]

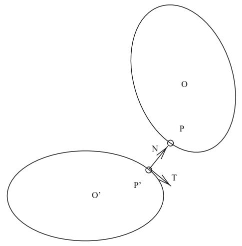
3.2 The frictional contact model

In order to describe the frictional contact relationship between two neighboring bodies, a local orthonormal frame is introduced.(See figure 1(a)). This local frame is defined by two unit vectors:

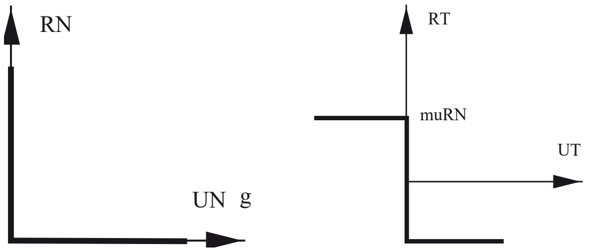
- A normal vector $\vec{N}$, which points from one body, considered as the antagonist to contact on the other body, considered as candidate to contact.
- A tangential vector $\vec{T}$, which completes the orthonormal frame.

Local variables are introduced in order to write a frictional contact law:

- The relative velocity of P with respect to O' : $U = (U_T, U_N)$,
- The reaction force exerted by O' on O : $R = (R_T, R_N)$,
- The gap: $g = \overline{P'P}$.



(a) Local frame



(b) The Signorini graph

(c) Coulomb graph

Figure 1: Contact interactions

The first unilateral constraint is the Signorini condition which takes into account these three conditions:

- impenetrability: $g \geq 0$,
- no attraction acts between objects: $R_N \geq 0$,
- the reaction force vanishes when the objects do not contact: $g > 0 \Rightarrow R_N = 0$.

The graph of this relation is displayed in Figure 1(b). In the case of cohesive mortar, a shifting is applied to R_N which represents a cohesive force. This shifting is set to zero if the contact is broken. The second unilateral constraint is the *Coulomb dry friction* which takes into account these two following conditions:

- the friction force lies in Coulomb's cone: $\|R_T\| \leq \mu R_N$, μ friction coefficient,
- if the sliding velocity is different from zero, friction force is opposed to the sliding velocity with magnitude μR_N:

$$U_T^+ \neq 0 \Rightarrow R_T = \mu R_N \frac{U_T^+}{\|U_T^+\|}$$

More complicated friction laws may be introduced, for instance by making differences between static and dynamic friction coefficient. Coulomb law graph is displayed in Figure 1(c). Both Signorini and Coulomb graphs are monotonous multi-mapping graphs. The above relations are familiar in the context of Convex Analysis.

3.3 The dynamical equation

The configuration of the building is given by some generalized variables q. The time derivative is denoted by $\dot{q}$ (a function of time with bounded variations). For smooth motions the *dynamical equation* writes:

$$M(q)\ddot{q} = F(q, \dot{q}, t) + r \ , \tag{1}$$

where $M(q)$ is the mass matrix; $F(q, \dot{q}, t)$ represents internal forces, elastic forces for instance, external forces, and quadratic acceleration terms; r is the representative of local reaction forces. Since shocks are expected, the derivatives in the above equation are to be understood in the sense of distributions.

3.4 Discrete forms of the dynamical frictional contact problem

To avoid details necessary when embracing the general problem of collections of bodies, the presentation is restricted to the case of small perturbations of rigid or elastic bodies. In this case the mass matrix $M(q)$ is considered as constant and F takes the form:

$$F(q, \dot{q}, t) = -V\dot{q} - Kq + P(t) \ ,$$

where V is the damping matrix and K is the stiffness matrix. Setting $\dot{q}(i)$, $q(i)$, $\dot{q}(i+1)$, $q(i+1)$, respectively approximations of $\dot{q}(t_i)$, $q(t_i)$, $\dot{q}(t_{i+1})$, $q(t_{i+1})$, the Euler implicit scheme writes:

$$\left\{ \begin{array}{c} \dot{q}(i+1) - \dot{q}(i) = w(-hV\dot{q}(i) - hK(q(i) + h\dot{q}(i)) \\ +hP(i+1) + hr(i+1)) \ , \\ q(i+1) - q(i) = h\dot{q}(i+1) \ , \\ \text{where} \\ w = (M + hV + h^2K)^{-1} \quad , \quad P(i+1) = P(t_{i+1}) \ . \end{array} \right.$$

It is assumed that the matrix $M + hV + h^2K$ is invertible, assumption which is satisfied since M is positive definite and V and K are positive.

Writing discrete forms of frictional contact relations needs much care since consistency is to be preserved. The proposed discrete forms of the Signorini condition are,

$$g^\alpha(i+1) \geq 0 \quad R_N^\alpha(i+1) \geq 0$$
$$g^\alpha(i+1) R_N^\alpha(i+1) \ = \ 0, \ .$$

Superscripts $^{\alpha}$ $^{\beta}$ are used to denote some candidates to contact $\alpha, \beta \in 1, ..., \chi$. The proposed discrete form for Coulomb law is:

$$R_T^\alpha(i+1) \in \mathrm{D}(\mu R_N^\alpha(i+1))$$

Any of these Signorini conditions, together with the Coulomb law may be shortly referred to as

$$\mathrm{SignCoul}\left(i, U^\alpha(i+1), R^\alpha(i+1)\right) .$$

The index i stands for data, $q(i), \dot{q}(i)$, and the gaps known from geometric computations when updating the local frames.

3.5 Solving the basic frictional contact problem

Using the kinematic relations, a linear equation relating relative velocities and mean-values of the impulses may be derived from the discrete form of the linearized equation 2, to be written together with frictional contact equations,

$$\begin{array}{c} U^\alpha(i+1) = U^\alpha_{\text{free}} + \sum_\beta W^{\alpha\beta} \ hR^\beta(i+1) \ , \\ \mathrm{SignCoul}\left(i, U^\alpha(i+1), R^\alpha(i+1)\right) , \\ \text{with,} \\ W^{\alpha\beta} = H^{*\alpha}(\bar{q}) \ w \ H^\beta(\bar{q}) \quad , \\ U^\alpha_{\text{free}} = H^{*\alpha}(\bar{q}) \ v_{\text{free}} \ , \\ v_{\text{free}} = \dot{q}(i) + w(-hV\dot{q}(i) - hK(q(i) + h\dot{q}(i)) \\ +hP(i+1)) \ . \end{array}$$

The mappings $H^\alpha(q)$ and $H^{*\alpha}(q)$, the transposed mapping of $H^\alpha(q)$, are linear and represent the *kinematic relations* between local and generalized variables of a contact α. The data are $q(i), \dot{q}(i)$, and the gaps known from geometric computations when updating the local frames. The unknowns are $U^\alpha(i+1), R^\alpha(i+1)$, which are computed through the following algorithm :

```
$N$: number of contact-antagonist
while $error > precision$ do
  for $\alpha = 1$ to $N$ do
    $U^\beta, R^\beta$, from the previous iteration
    for $\beta > \alpha$
    $U^\beta, R^\beta$, known from the current iteration for $\beta < \alpha$
    $U^\alpha, R^\alpha$, is found discussing the intersection of graphs of affine
  end for
end while
```

This algorithm is similar to a block non-linear Gauss-Seidel algorithm.

4 SURVEY PROCESS

Geometric information in our case study is obtained by a photogrammetric survey. An operator having a thorough knowledge about the building provides the connection between the architectural model and the survey process. This section presents the actual state of the acquisition phase that provides the link between the photogrammetric acquisition and the instantiated model.

4.1 Required geometrical and meshing data

The NSCD method needs the complete geometric data of every ashlar in masonry. This data is actually obtained by a photogrammetric survey of building which means that only data present in negatives can be acquired (for instance the four vertex of the ashlar face). For the acquisition to be complete, we need to infer some information about ashlars: their depth, their shape, their extrude vector, etc.. Even though almost all the blocks in masonry keep an hexahedral shape, there are some of them that are eroded or have a beveled edge. After survey, blocks can be reshaped to obtain hexahedral ones. This phase has been called blocky simplification and is executed after the photogrammetric survey. In addition to this, every block can be meshed. We can take into account their position in the wall when meshing; this means that every block could be meshed in a particular way as their position in masonry for example. The meshing technique used applies the Delaunays triangulation.

4.2 Survey environment

The exploitation of photogrammetric negatives is made using the Intergraph ImageStation. This digital photogrammetric workstation can deal with high-resolution images; it offers an accurate stereo viewing and high quality restitution. The 3D graphical survey environment works together with MicroStation(TM), this means that we work in the geometric space topographically defined during the photogrammetric and image processing. We have developed an acquisition interface using MDL, MicroStation Developing Language, that links the operator block by block survey with the theoretical architectural model and creates the instance in the database.

The pictures were taken in December 1997, in Avignon using camera, theodolite and a truck with a 20 m high cradle. The photogrammetric approach is very traditional : network of control points is used to orient a set of independent stereo pairs. The topometric network is made of around 200 control points distributed in 22 stations and allows the orientation of external and internal stereo pairs. Seventy photographs were taken from the ground, from the nearest building's roof and from the truck's cradle.

Material

- The topometric network was carried out with a co-axial distance-meter theodolite TC2002 (WILD).
- All the topometric stations were adapted in the same referential using the Metheor software package, developed by the photogrametric company SETP.
- Picture was made using half-metric camera and meric camera (Rollei 6006 an UMK 100mm).

Pictures configuration

The survey is as exhaustive as possible so as to anticipate on problems or needs that will probably come to the light when, collaborating with architects, archaeologists and mechanical engineers who take part in the project. Survey and use of the architectural model The relevancy of our approach relies on the link between a survey tool, in this case photogrammetry, and a theoretical architectural model finalised with the collaboration of architects and archaeologists. Choosing photogrammetry introduces in the measurement phase a human operator. This approach of course excludes an automation of the system but ensures a human controlled monitoring of the data capture. More precisely, the operator can understand the monitoring interface developed by our team and adapt it to the context being studied.

4.3 Survey-mechanical simulation communication

Data gathered and some other included in the architectural model is needed to get the distinct element model. The communication between the acquired 3D-model in MicroStation and LMGC has been experienced using an ASCII file. The objective of this test was to show feasibility of communication and to prove that information gathered and attached were enough. During the stereo plotting phase an exhaustive determination of an architectural object's geometry is most ofen impossible. The example of an ashlar wall's measurement speaks for itself: the photogrammetrical process permits a measurement of only the points that are visible on the picture, yet the mechanical software needs a thorough geometrical description for each ashlar block studied. In this experimentation we operated as follows :

- Computation of the least square plane fitting the blocks' faces (the block face is the only visible surface of a block)
- Visible blocks vertex measurement
- Extrusion of the measured face. The extrusion vector is given by the computed plane, the depth is given by the archaeologist expertise (the dimension was determined during a previous excavation phase).

The interface between survey process and architectural model is here developed on the digital stereo system Image Station made by Intergraph. We often need stereo display and high resolutions (up to 100 mb for one picture) in the understanding and analysis of the building. Actually It can be difficult to see for instance a block's edge because of the block's erosion or sometimes of vegetation. The high diversity of hardware and operational systems used in this project (Image Station Z, Windows NT, Unix, Microstation 95, Java, fortran 77) made the automation of the link a bit difficult, we have in this experimentation used ASCII file communication.

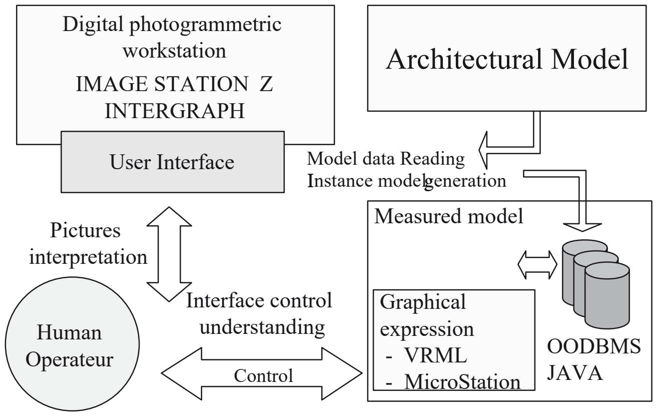


Figure 2: Survey process synoptic scheme

5 CASE STUDY OF THE TOUR SAINT-LAURENT

The raising of the 47 meter high Tour Saint-Laurent in the Palais des Papes of Avignon, under the reign of Innocent VI, lasted two years concerning its main structure, and five for its completion, from 1353 to 1358. It is located close to the Palais-Neuf south aisle, only three walls have been raised for five of the six vaulted rooms it encloses. This building was one of the last to be erected in the Palais des Papes.[7]

Applying the MOMA project approach to this medieval monument required, before the actual survey process, a complete description of the architectural entities concerned, most of them being encountered here for the first time as for example tracery windows, buttresses, machicolation, crenelation, rib vaults or barrel vaults. This full description and its computational formalisation implied a constant dialogue between archaeologists, architects, mechanical engineers and computer scientists. Focus is put in this research on the simulation of mechanical behaviour. The Tour Saint-Laurent walls being constructed of two ashlar facings with mortar infill in-between, another task was to add to the architectural entity's description new components that would enable the computation of heterogeneous walls.

Archaeologists from L.A.M.M. (Laboratoire d'Archéologie Médiévale Méditerranéenne, C.N.R.S. 6572) team have worked on scaffoldings raised for a restoration campaign of the tower's external facings in July-October 1997 [9]. Observing the differences of depth in the facings (connected with the types of hewn stones) and the nature of in-fill mortar, they could provide the project with what is known of medieval masonry techniques, with a specific vocabulary, and with a traditional stone by stone survey of the tower. Corresponding to their shape or position, stones in the facings now have in our model different names : headers, stretchers, voussoirs and their specificity is now integrated to the architectural entity renewed definition. This restoration work also gave a possibility to the archaeologists to proceed to a manual survey of all the ashlars of the external facings, thereby recording data on blocks, differences of size, mason marks, building site organisation. The archaeological analysis of this survey raised questions about the period during which course deformations appeared. It is hoped that mechanical simulation will provide useful clues.

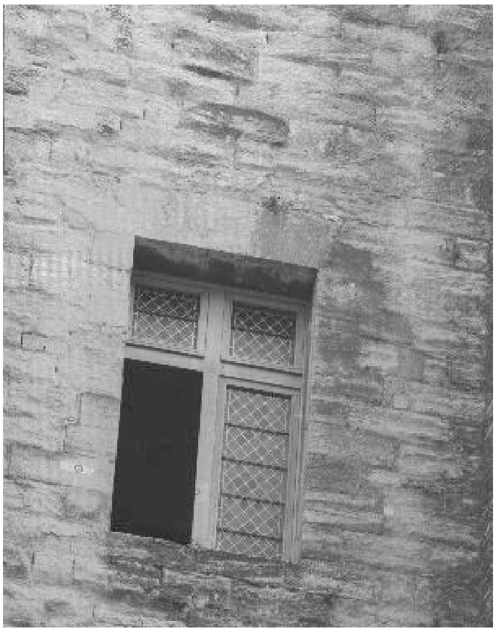
Soon before this restoration work began, thirty or so negatives have been shot using an UMK camera in order to record the stones condition. In a photogrammetrical survey, only what is visible on photographs can be observed. However, mechanical simulation requires an entire description of the walls, including what cannot be seen. Full description of in-fill and blocks shape therefore had to be provided through another source. An "acquisition interface", developed for that purpose, enables the calculation of hidden points of a stone using both the observation of the facing plane and a depth chosen according to the stone type. Because several parts of the tower could not be caught by the stereo-photogrammetrical survey, hybrid techniques can be used as 2D photogrammetry or measurements on rectified photographs, but always in strong link with our theoretical architectural model.

A sub-part of the tower, the fourth floor, is chosen for our experiment. It features forty courses (layers) of hewn stones on three sides of the tower. A stone by stone survey is carried out using our acquisition interface, including external and internal facings as well as the third floor's rib vaults and the second floor's barrel vaults.

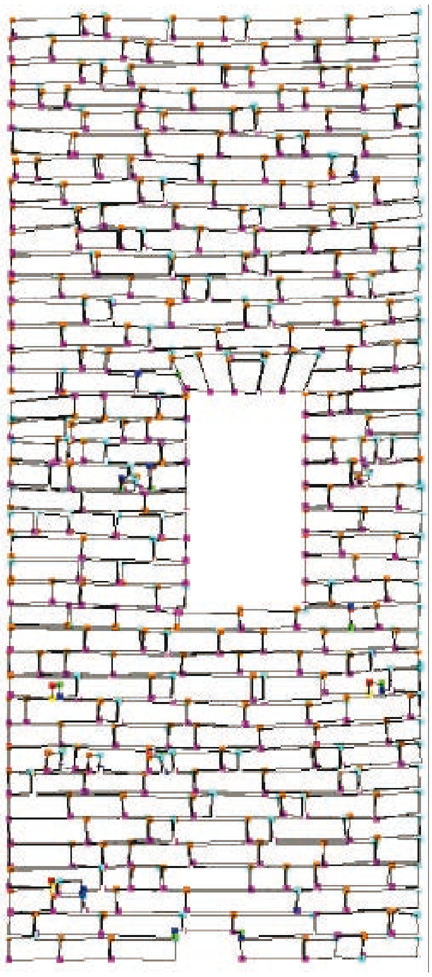
6 CONCLUSIONS AND PERSPECTIVES

Simulating a building's mechanical behaviour needs an assessment of their structure and physical properties. To get to have a model that could be studied by simulation we need to gather and model as much information as possible. We propose to study a building beyond its geometry as a collection of well-defined architectural entities whose morphology, some dimensions, mechanical properties and their relations with the other entities are known.

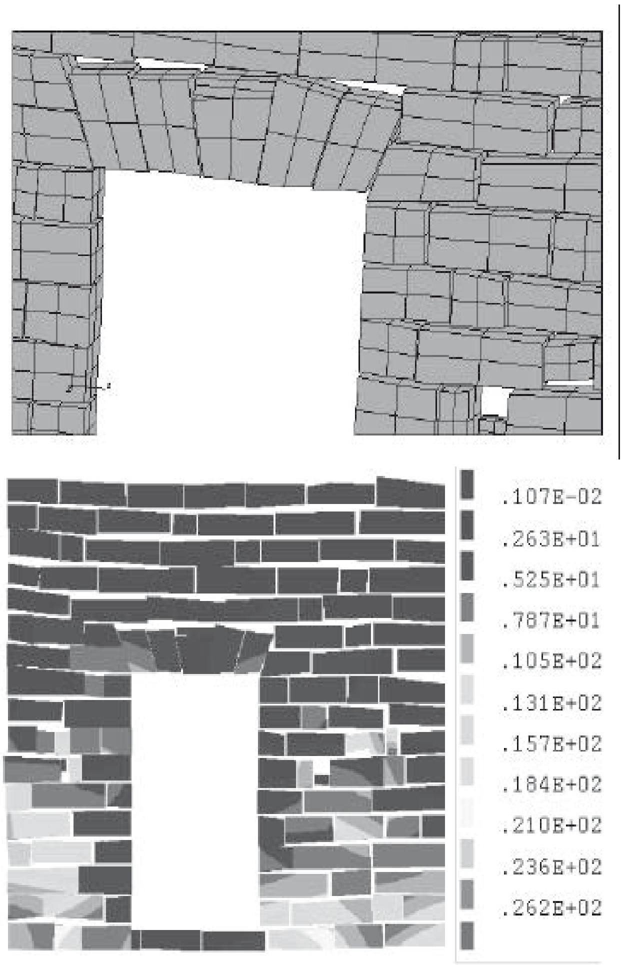
Tour Saint-Laurent south-east overview



West side, third floor, window detail.



West side detail, photogrammetric survey.



West side, third floor, window detail. Von Mises stress.

Figure 3: From the survey to the simulation

In using architectural models developed at gamsau-map and a photogrammetric monitored survey, we want to get a relevant building model. A java connection will enable a link between the survey process and the OODBMS architectural model [2]. The result of this communication will be the instantiated model that contains the identified and measured entities in the hierarchical tree. Communication test described in the survey process section let us also define the way entities will be seized.

At the present time, the whole process, from survey to simulation, is tested on one west side external facing part (see ill.). Because the survey is not yet achieved, it does not provide a relevant system from a mechanical point of view. Therefore no results is detailed here. A future photogrammetric campaign is planed in order to work on internal facings and to obtain the complete morphology. Then, a relevant simulation will throw a light on the Tour Saint-Laurent mechanical role in relation to the Grande Chapelle aisle.

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