Taylor vortices versus Taylor columns

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Taylor-Couette flow is inevitably associated with the visually appealing toroidal vortices, waves, and spirals that are instigated by linear instability. The linearly stable regimes, however, pose a new challenge: do they undergo transition to turbulence and if so, what is its mechanism? Maretzke *et al.* (2014) begin to address this question by determining the transient growth over the entire parameter space. They find that in the quasi-Keplerian regime, the optimal perturbations take the form of Taylor columns and that the maximum energy achieved depends only on the shear.

Keywords. Transition to turbulence (Instability), Taylor-Couette flow (Convection)

1. Introduction

Taylor-Couette flow is an ideal test case for hydrodynamics – its Drosophila (van Gils *et al.* 2012) or its hydrogen atom (Tagg 1994). It has been extensively investigated and its parameters can be varied at will (at least numerically) to combine shear, rotation and curvature. Inner-cylinder-only rotation, the vertical axis of figure 1, is a textbook example of a now well-understood sequence of symmetry-breaking bifurcations. The validation of the Navier-Stokes equations is often thought to date from the observation in 1923 by Taylor of the formation of the now-famous toroidal vortices he had predicted for the linear instability. In later research, increasingly ornate and beautiful experimental patterns were discovered (e.g, Coles 1965; Andereck *et al.* 1986) and corresponding numerical, asymptotic, and theoretical calculations (e.g., Marcus 1984; Langford *et al.* 1988) reproduced and explained these patterns, again with remarkable accuracy.

In contrast, outer-cylinder-only rotation, the horizontal axis of figure 1, is an example of currently unexplained (sometimes called subcritical or bypass) transition to turbulence (Coles 1965; Borrero-Echeverry *et al.* 2010) despite linear stability. Transient growth was proposed in the 1980s and 1990s as a response to this puzzle in plane parallel shear flows, e.g. plane Couette and Poiseuille flow (Boberg & Brosa 1988; Trefethen *et al.* 1993). Although the eigenvalues governing the linear growth of perturbations all have negative real part, temporary linear growth in the energy norm may nevertheless takes place if flows are initialized with combinations of certain eigenvectors. Optimal perturbations are the initial conditions which achieve maximum growth. For plane-parallel shear flows, the famous theorem of Squire (1933) established that upon increasing the Reynolds number, the perturbations which first become linearly unstable are 2D, meaning that they vary only in the streamwise and cross-channel direction. The optimal perturbations are also 2D (or almost 2D), but in different directions, varying mainly in the spanwise and cross-channel directions. Indeed, spanwise-periodic structures (vortices and streaks) are a prominent feature of turbulent shear flows in experiments and numerical simulation. These are also present in other theories of transition, in particular the self-sustaining process of Waleffe (1997). A useful analogy can be drawn with the usual Taylor vortices,

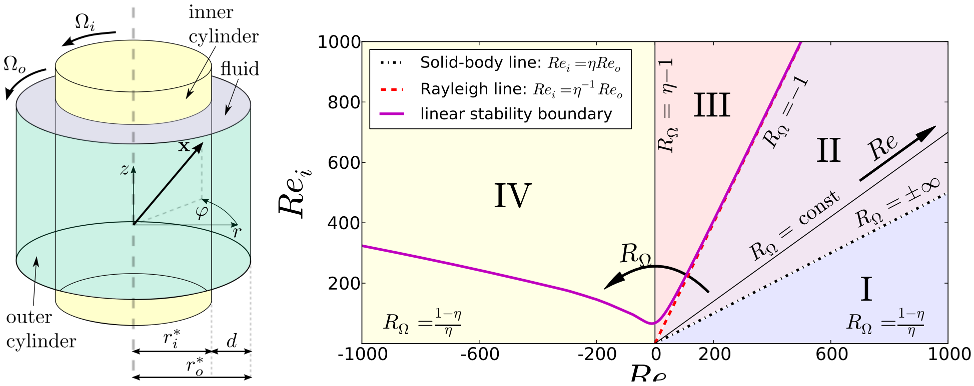

FIGURE 1. (a) Sketch of Taylor-Couette geometry and (b) of the (Re_i, Re_o) plane, from Maretzke *et al.* (2014). The parametrization of Dubrulle *et al.* (2005) divides the plane into four regimes, according to the value of the rotation number R_Ω. The linearly stable regime I encompasses the outer-cylinder-rotation axis and II is the quasi-Keplerian regime. Regime III contains the linearly unstable co-rotating region, while the counter-rotating regime IV contains both stable and unstable portions.

with the correspondence (streamwise ↔ azimuthal) and (spanwise ↔ axial); see, e.g., Nagata (1998) and Faisst & Eckhardt (2000).

Another subset of the stable regime, termed quasi-Keplerian, region II of figure 1, has attracted attention as a model for accretion disks (Pringle 1981), currently one of the most controversial topics in theoretical astrophysics. Ensembles of stellar matter rotating under gravitational attraction must lose angular momentum at a rate sufficient to collapse inwards. One line of research views this stellar matter as an incompressible fluid rotating with a Keplerian velosity distribution; the issue then becomes that of whether its low "molecular" viscosity can be replaced by a much higher "turbulent" viscosity. Turbulent viscosity requires turbulence, of course, raising the question of whether a Keplerian velocity profile is hydrodynamically stable (Yecko 2004; Ji *et al.* 2006; Paoletti & Lathrop 2011; Balbus 2011; Avila 2012; Busse 2007). When maintained by rotating cylinders in Taylor-Couette flow, the Keplerian profile is linearly stable. Can it nevertheless undergo transition to turbulence as do planar shear flows, or is another mechanism involving other physical phenomena, such as the magneto-rotational instability (Balbus & Hawley 1991) the strato-rotational instability (Le Bars & Le Gal 2007), or radial throughflow (Dubrulle *et al.* 2005; Gallet *et al.* 2010) required? It is in this context that the stability of the quasi-Keplerian regime of Taylor-Couette flow has taken on significance.

2. Summary of Paper

The first calculations of transient growth for Taylor-Couette flow were carried out for counter-rotating cylinders, region IV of figure 1, by Hristova *et al.* (2002), who considered axisymmetric perturbations in the plane Couette limit of exact counter-rotation and nearly equal radii, and by Meseguer (2002), who investigated the linearly stable region in which transition to turbulence had been observed by Coles (1965).

Maretzke, Hof & Avila (2014) have accomplished a tour de force by surveying the transient growth for the entire stable three-parameter space of Taylor-Couette flow. In this task, they have been guided by the reparametrization proposed by Dubrulle *et al.* (2005), replacing the usual inner and outer Reynolds numbers Re_i and Re_o by a shear Reynolds number Re and a rotation number R_Ω, based on the difference and ratio between the

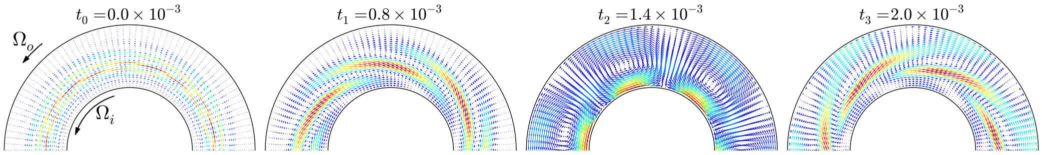

FIGURE 2. Axial cut of an optimal perturbation in the quasi-Keplerian regime and the evolution from it in time. The field is axially symmetric and, in the plane perpendicular to the axis, has a spiral shape which changes orientation over the course of time via the Orr mechanism. From Maretzke *et al.* (2014).

angular velocities, respectively, of the two cylinders (see figure 1). By astute variation of Re, R_Ω and the radius ratio η, Maretzke *et al.* (2014) have been able to catalog the optimal growth and wavenumbers for all three stable regimes. In the course of their survey, they discovered that in most of the quasi-Keplerian regime, transient growth is optimized by perturbations that vary with the azimuthal angle but are independent (or nearly so) of the axial coordinate; see figure 2. The Taylor-Proudman theorem predicts that rapidly rotating flows are axially invariant and, indeed, Maretzke *et al.* (2014) find that this effect is strongest for larger R_Ω, near the solid-body-rotation line. Thus, the optimal perturbations are approximately perpendicular to the axisymmetric stacked tori of Taylor vortices and the eigenvectors which lead to them.

Motivated by this discovery, Maretzke *et al.* (2014) studied the case of axially-independent perturbations asymptotically using WKB theory. They arrive at the startling conclusion that the associated linear problem depends only on Re and not on R_Ω. The fate of axially independent perturbations necessarily provides a lower bound of the energy that can be attained by optimal growth. This bound is independent of Re_Ω and scales like $Re^{2/3}$ (see Yecko (2004)) multiplied by a universal function of η. New exact results for basic flows are few and far between. Here, Maretzke *et al.* (2014) have accomplished two extremely powerful reductions, from three non-dimensional parameters to two and from three spatial directions to two.

3. The Future

The transient growth calculation of the Taylor-Couette problem by Maretzke *et al.* (2014) is exhaustive, powerful and general. The question is that of its applicability. It remains to be established whether and how quasi-Keplerian Taylor-Couette flow undergoes transition to turbulence. Transient growth alone cannot lead to sustained transition (Waleffe 1995). What is its role in predicting transition to turbulence? If traces of columnar vortices are seen in experiment or simulations in the quasi-Keplerian regime, in the same way that streamwise vortices and streaks are seen in transitional regimes in planar shear flows, this would provide evidence for the relevance of transient growth.

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