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MIXED MODE THREE DIMENSIONAL CONTOUR INTEGRAL: FOR WOOD APPLICATION UNDER CREEP LOADING

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Abstract: The main goal of this work is to present a new integral for three dimensional crack problems under mixed mode loading. This work is based on a generalization of the M integral proposed initially by Chen and Shield [1] and developed by Moutou Pitti et al. [2] to viscoelastic crack growth process. The aim of this generalization is to propose a new parameter integral entitled $M_{3D}^\theta$ which computes the energy release rate combining real and auxiliary displacement fields in orthotropic material such as wood. The energy release rate distribution along the crack front line is calculated and the non-path dependence is proved with the use of numerical application.

1. Introduction
Two dimensional current approaches studying crack growth problems of a viscoelastic material such as wood, using independent path integrals, do not really include all stresses effects existing in timber structures. By considering a three dimensional orientation of external loading, the crack can be situated in a three dimensional state by considering torsion effects inducing the third crack mode, and which leads to a coupled pure opening mode. Also, we can consider thermal or hydrological effects in timber materials under mixed mode loading. All this cases require the generalization of the $J^{3D}$-integral formalism for a mixed mode loading problem and the adaptability of the M-theta method for a future finite element implementation.

2. Results
$M_{3D}^\theta$-integral is a combination between real and virtual strain fields which enables computing fracture parameter. Based on a conservative law, $M_{3D}^\theta$-integral enables to separate fracture mode under creep mixed load. Where $\sigma_{ij}^r$ and $\sigma_{ij}^v$ are real and virtual stresses associated with the real and virtual displacements fields $u$ and $v$. In order to make easy numerical integration we use the $\theta$ method, and by considering Ostrogradski transformation we obtain:

$$M_{\theta} = \frac{1}{2} \int_T p_{kj,i} \cdot \theta_{k,j} \cdot dV + \frac{1}{2} \int_{\Gamma_T} \left( \sigma_{ij}^r \cdot (\epsilon_{ij}^u)_{k} + \sigma_{ij}^v \cdot (\epsilon_{ij}^u)_{k} \right) \cdot \left( \sigma_{ij}^r \cdot (\epsilon_{ij}^v)_{k} + \sigma_{ij}^v \cdot (\epsilon_{ij}^v)_{k} \right) \cdot \theta_{k} \cdot dV - \frac{1}{2} \int_{\Gamma} \left( \sigma_{ij}^r \cdot u_{i,k} + \sigma_{ij}^v \cdot v_{i,k} \right) \cdot n_{j} \cdot \theta_{k} \cdot dS$$

(1)

The finite element implementation is based on a Double Cantilever Beam DCB specimen loaded in mixed mode configuration (Figure 1). In order to validate the non-dependence of

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the integration domain, the average energy release rate is computed. The second application considers the evaluation of the energy release rate along the crack front line (Figure 2).

The finite element implementation is proposed by computing a $M^{3D}$-integral based on a numerical definition of the integration domain allowing the computation of the average value of the energy release rate or its distribution along the crack front line.

### 3. Conclusions

In this work, a new M-integral is developed for three-dimensional problem under mixed mode loading. The evolution of the energy release rate versus crack front line has been determined. The non-dependence of path integral is proved. In future work, it will be necessary to extend the $M^{3D}$-integral in order to take into account a variable climate case for viscoelastic orthotropic materials. Also, experimental tests will be necessary to validate the numerical results.

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