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Multi-phase structural optimization of multi-layered composites and viscoelastic materials

G. Delgado*, G. Allaire†, M. Hamdaoui‡

Abstract— This work is devoted to the application of the level-set method for topology optimization to multi-phase design of multi-layered composites and viscoelastic structures. In the case of composite laminates, we study their optimal design by allowing a variable stacking sequence and in-plane shape of each ply. In order to optimize both variables we rely on a decomposition technique which aggregates the constraints into one unique constraint margin function. Thanks to this approach, a rigorous equivalent bi-level optimization problem is established. On the other hand, we consider the optimal design of viscoelastic vibration damping treatments. We prove a general result describing the complex frequencies of the underlying non-linear eigenvalue problem. In both cases every layer of the structure is represented as a bi-material structure where a level set method is used to characterize the interfaces meanwhile the shape evolution is driven by a Hadamard method for boundary variations using the shape gradient. Two numerical test-cases are exhibited: In the case of multi-layered laminates, we minimize the weight of the structure subjected to a compliance and a first buckling load constraint, meanwhile for viscoelastic treatments, we maximize the the structure capacity to dissipate energy measured via the loss factor.

1 Introduction

Structural optimization usually looks for the lightest structure which sustains the forces and environmental conditions that for instance an aircraft or a car will typically find during operation. Classically, this optimization process has been done by the engineer expertise. However, the increment of the size of the design space does not allow to find the best design without automatizing the process. As a response to this challenge, several techniques for size, shape and topology optimization have successfully been developed and applied to structural design [3, 8, 13, 20, 24].

During the last years, a special type of material has become quite popular in automotive and aerospace industries: multi-layered composites. These materials benefit from very at-

tractive features such as low weight, high fatigue resistance and good endurance against corrosion and other harsh environmental conditions. The properties of multi-layered composite structures strongly depend on the shape, the orientation of the reinforcement and the stacking sequence of the laminate. Indeed the directional nature of the fibers in a fiber-reinforced laminate introduces directional dependence of the strength, thermal and electrical conductivity. Meanwhile the stacking sequence has a strong influence on the bending behavior of the laminate.

In view of the increasing use of composite materials within industry, their optimal design has drawn great attention of the scientific community. We refer e.g. to Gürdal, Haftka and co-workers [1, 12, 13] but also to [14, 18, 19]. Actually, composite materials possess a large number of design possibilities

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which makes their optimization a complex problem. A typical composite laminate may be characterized by design variables which are continuous (geometry, size of the structure, material distribution in each ply) or discrete (orientation of the fibers, lay-up or stacking sequence). Additionally, when designing composite structural components, one must take into account constraints on the structural performance (accelerations, buckling factors, displacement, material failure criteria, etc.) and equally constraints on the global and local manufacturing rules imposed during the composite manufacturing process. These constraints are specific to the type of technology used and industrial policies (symmetric and balanced laminates, ply drops and overlaps, etc.).

Structures incorporating viscoelastic materials for structural damping have also attracted a lot of interest of the engineering and scientific communities. Viscoelastic damping material behavior occurs in a wide variety of materials and can be characterized by liquid-like elastic behavior. Materials that experience viscoelastic behavior include acrylics, rubber, and adhesives. The characteristics of viscoelastic materials depend on temperature and excitation frequency. Structural damping reduces both impact-generated and steady-state noises at their source. It dissipates vibrational energy in the structure before it can build up and radiate as sound. A damping treatment consists of any material (or combination of materials) applied to a component to increase its ability to dissipate mechanical energy. Two categories of treatment for structural damping exist: Unconstrained layer damping (UCLD), where the material is simply attached with a strong bonding agent to the surface of a structure and energy is dissipated as a result of extension and compression of the damping material under flexural stress from the base structure; and Constrained-layer damping (CLD), where a “sandwich” is formed by laminating the base layer to the damping layer and adding a third constraining layer. In the latter case energy dissipation is achieved by shearing a viscoelastic polymer between a base structure and a constraining layer.

Topology optimization of viscoelastic UCLD and CLD have been performed by many authors in the literature using different methods. Zheng *et al.* [30] used the Solid Isotropic Material with Penalization (SIMP) method with the Method of Moving Asymptote (MMA) to perform topology optimization of a CLD cantilever plate treated with DYAD606 where a sum of modal loss factors is maximized. Zheng *et al.* [31] used the same methodology to perform topology optimization of CLD with partial coverage, showing interesting performances of the optimized structure in terms of damping and mass savings. Kim *et al.* [17] used the rational approximation for material properties (RAMP) with the optimality criteria method (OC) to perform topology optimization of UCLD shell structures to maximize modal loss factors. El-Sabbagh *et al.* [11] used and the method presented in [7] along with the MMA method to perform optimization of periodic and non-periodic plates. Zhanpeng *et al.* [29] used evolutionary structural opti-

mization (ESO) to minimize viscoelastic CLD plate response. James *et al.* [15] used a time dependent adjoint method along with the MMA method to perform topology optimization of viscoelastically damped beams for minimum mass under time dependent loadings. Yun *et al.* [28] performed multimaterial topology optimization to maximize energy dissipation of viscoelastically damped structures subjected to unsteady loads using SIMP and MMA. Ansari *et al.* [6] used a level-set method to perform topology optimization of viscoelastic UCLD plate.

The present article addresses the structural optimization of composite laminate and viscoelastic treatments by means of the level-set method for topology optimization. For that purpose we rely on the level-set approach for multi-phase optimization detailed in [4]. First introduced in [21], the level-set method has the advantage of tracking the interfaces on a fixed mesh, easily managing topological changes without any need of remeshing. Allied to the Hadamard method of shape differentiation, the level-set approach is an efficient shape and topology optimization algorithm [2, 26], which gives a better description and control of the geometrical properties of the interface, avoiding typical drawbacks such as intermediate density penalization and possible spurious physical behavior during the optimization process.

In the case of composite laminate structures, we present the mathematical model and numerical analysis already exposed in [5]. Within this work, each ply was made up of two phases (one of them being void) and the design variables where the position of the interface as well as the fiber orientation and the lay-up sequence of the laminate.

2 Setting of the problem

2.1 Multi-layered composites

Physical modeling

Let \mathcal{L} be a symmetric laminated composite structure composed of the superposition of $2N$ anisotropic layers, each one of constant thickness $\varepsilon > 0$ and characterized by a shape $\Omega_i \subset \Omega$, where Ω is a regular sub-domain of \mathbb{R}^2 (typically a rectangle). We denote by \mathcal{O} the collection of shapes

$$(1) \quad \mathcal{O} = (\Omega_i)_{i=-N, \dots, -1, +1, \dots, +N}.$$

Since we suppose \mathcal{L} symmetric, i.e., $\Omega_{-i} = \Omega_i$, we consider only N layers, so from now on we rather write $\mathcal{O} = (\Omega_i)_{i=1, \dots, N}$. The index i grows from the inside to the outside of the laminated composite structure (see Figure 1).

Each layer is made of an orthotropic material, i.e., an anisotropic material where there are three mutually perpendicular planes of symmetry in material properties. In the case of an unidirectional reinforced composite, the material properties, which are that of an equivalent homogeneous orthotropic continuum, are thus parametrized by an angle of rotation, corre-

sponding to the orientation (at the microscopic level) of the fibers with respect to the canonical axis.

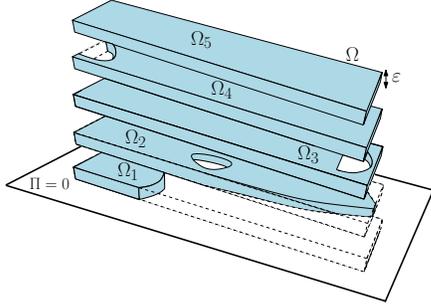


Figure 1: Half-part of a multi-shape composite design with plane of symmetry $\Pi = 0$. Each ply has its own shape $\Omega_i \subset \Omega$.

Each layer is a non-homogeneous two-phase material, where each ‘‘hole’’ is filled with another ‘‘weak’’ material with different physical properties (weight, electric or heat conductivity, etc.). We will denote this weak material as A_0 .

Let χ_i be the characteristic function of the i -layer. According to the classical laminate theory for plates, the composite structure \mathcal{L} is characterized by the superposition of the elastic properties of each layer, namely the extensional stiffness tensor A , which reads

$$(2) \quad 2\varepsilon \sum_{i=1}^N \left(\chi_i(x) A_i + (1 - \chi_i(x)) A_0 \right),$$

where A_i is the extensional stiffness of the i -layer (a symmetric fourth-order tensor), and the bending stiffness tensor D , which reads

$$(3) \quad \frac{2\varepsilon^3}{3} \sum_{i=1}^N \left\{ \left(i^3 - (i-1)^3 \right) \left(\chi_i A_i + (1 - \chi_i) A_0 \right) \right\},$$

The boundary of Ω is decomposed into two disjoint subsets $\partial\Omega = \Gamma_N \cup \Gamma_D$, $\Gamma_N \cap \Gamma_D = \emptyset$. On Γ_N a in-plane surface load is applied, $g \in L^2(\Gamma_N; \mathbb{R}^2)$, and on Γ_D the in-plane and vertical displacements are fixed to zero. Define the spaces

$$H_D^1(\Omega; \mathbb{R}^2) = \{v \in H^1(\Omega; \mathbb{R}^2) \text{ such that } v = 0 \text{ on } \Gamma_D\}$$

$$H_D^2(\Omega) = \{\eta \in H^2(\Omega) \text{ such that } \eta = \nabla\eta \cdot n = 0 \text{ on } \Gamma_D\}$$

Our mechanical model is the linearized buckling problem for the two-dimensional von Kármán plate model [9, 22]. The unknowns are the in-plane displacement $u \in H_D^1(\Omega; \mathbb{R}^2)$, the vertical displacement $w \in H_D^2(\Omega)$, $w \neq 0$, and the so-called ‘‘buckling load factor’’ or ‘‘buckling critical reserve fac-

tor’’ $\lambda \in \mathbb{R}$. They satisfy

$$(4) \quad \begin{cases} \nabla^2 : (D\nabla^2 w) = \lambda(Ae(u)) : \nabla^2 w & \text{in } \Omega, \\ w = 0, \nabla w \cdot n = 0 & \text{on } \Gamma_D, \\ (D\nabla^2 w)_{nn} = 0 & \text{on } \Gamma_N, \\ \nabla \cdot (D\nabla^2 w) \cdot n + \frac{\partial}{\partial \tau} (D\nabla^2 w)_{n\tau} = \lambda 2N\varepsilon g \cdot \nabla w & \text{on } \Gamma_N, \end{cases}$$

and

$$(5) \quad \begin{cases} -\operatorname{div}(Ae(u)) = 0 & \text{in } \Omega, \\ u = 0 & \text{on } \Gamma_D, \\ Ae(u) \cdot n = 2N\varepsilon g, & \text{on } \Gamma_N, \end{cases}$$

where $e(u) = \frac{\nabla u + (\nabla u)^T}{2}$ is the classical linearized strain tensor, (n, τ) is the orthonormal local basis of normal and tangent vectors on $\partial\Omega$.

We denote by λ_1 the smallest positive eigenvalue of (4) which can be expressed through the Rayleigh quotient

$$(6) \quad \frac{1}{\lambda_1} = \max_{w \in H_D^2(\Omega) \setminus \{0\}} \max \left(0, -\frac{\int_{\Omega} Ae(u) \cdot \nabla w \cdot \nabla w \, dx}{\int_{\Omega} D\nabla^2 w : \nabla^2 w \, dx} \right).$$

This is the only eigenvalue with a physical meaning since its inverse is the buckling load factor which is an indicator of the degree of safety against this particular mode of failure [13]. The computed vertical displacement eigenfunction w_1 is referred here as the ‘‘buckling mode’’.

Stacking sequence

Even though the fiber orientation of each orthotropic laminate might take any possible rotation angle, in real applications due to manufacturing constraints, it only takes discrete values [12]. We will consider four values, namely: $0^\circ, 90^\circ, 45^\circ, -45^\circ$. We denote by $\mathcal{C}_{0^\circ}, \mathcal{C}_{90^\circ}, \mathcal{C}_{45^\circ}, \mathcal{C}_{-45^\circ}$ their respective in-plane reduced stiffness tensors. We assume that the fiber orientation is constant in each ply.

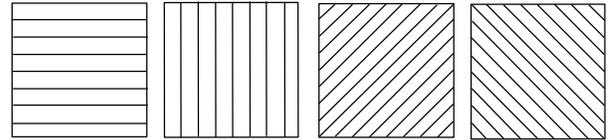


Figure 2: Fiber orientation of $0^\circ, 90^\circ, 45^\circ, -45^\circ$.

DEFINITION 1 We define the stacking sequence as the set of ply orientations and the way they are arranged in the normal direction of the composite laminate (see Figure 1). We represent it through a binary matrix $\xi = (\xi_{ij}) \in \{0, 1\}$, where $i = 1, \dots, N$, $j = 1, 2, 3, 4$, and

$$(7) \quad \xi_{ij} = \begin{cases} 1, & \text{if the layer in position } i \text{ has fiber orientation } j, \\ 0, & \text{if not.} \end{cases}$$

We identify fiber orientations 1, 2, 3, 4 to the angles $0^\circ, 90^\circ, 45^\circ, -45^\circ$, respectively.

From an engineering point of view, when a composite laminate is designed, some additional composite design rules must be respected. Following the typical industrial approach we consider the following rules:

- (R1) Continuity rule, no more than 4 successive plies with the same angle.
- (R2) Disorientation rule, maximum gap between two adjacent (superposed) plies is 45° .
- (R3) Balanced laminate with respect to the principal direction 0° , i.e. same number of plies at 45° and -45° .
- (R4) Minimum proportion of each fiber orientation (typically 8%). We note this proportion as p_j , $j = 1, 2, 3, 4$.
- (R5) Symmetric laminate. This ensures to avoid the coupling between in-plane traction and bending of the plate.

We will denote as \mathcal{Y}_{ad} the space of admissible values of ξ respecting the aforementioned constraints and for which one and only one orientation is possible in each ply.

Optimization problem

We look for a multi-layered composite plate with optimal stacking sequence and optimal ply shapes. Typically the optimization problem will be set as a mass minimization problem subject to a set of manufacturing constraints, local failure criteria, in-plane stiffness and avoidance of buckling.

From a mathematical point of view, our problem can be cast as a mixed optimization problem, namely

$$(8) \quad \begin{cases} \min_{\mathcal{O} \in \mathcal{U}_{ad}, \xi \in \mathcal{Y}_{ad}} J(\mathcal{O}) \\ \text{such that } G(\mathcal{O}, \xi) \leq 0, \end{cases}$$

where \mathcal{U}_{ad} denotes the space of admissible shapes. Problem (8) is called mixed because \mathcal{O} is a continuous variable while ξ is a discrete one. The objective function $J(\mathcal{O})$ is the mass of the structure and does not depend on the stacking sequence ξ . The function $G : (\mathcal{U}_{ad} \times \mathcal{Y}_{ad}) \rightarrow \mathbb{R}^m$ is a regular vector-valued constraint function with m components. Typically G is a mechanical constraint on the stiffness of the plate. Notably, we focus our attention on two kinds of stiffness measures, namely the compliance and the buckling avoidance through the load factor or first positive eigenvalue of (4) $\lambda_1^{-1} \leq 1$.

2.2 Viscoelastic structures

Physical modeling

Viscoelastic damping materials follow a liquid-like elastic behavior whose characteristics depend on the excitation frequency. For these materials a linear elastic constitutive relationship using Hooke's law is not an accurate representation. Instead, the complex modulus is extensively used to

describe the dynamic characteristics of viscoelastic materials. The stress-strain relationship of a viscoelastic damping material subjected to steady-state oscillatory conditions can be represented by the structural damping model considering the complex (or dynamic) modulus A as follows:

$$(9) \quad \hat{\sigma}(\omega) = A(\omega)\hat{e}(\omega),$$

where ω a complex pulsation and $\hat{\sigma}$ and \hat{e} are the Laplace transforms of stress and linearized strain, respectively. The real part of A represents the purely elastic behavior of the material meanwhile the imaginary part of A represents the purely viscous behavior.

Now let $\Omega \subset \mathbb{R}^d$ ($d = 2$ or 3) be a bounded open set occupied by a viscoelastic material with complex Hooke's law A and density $\rho > 0$. The boundary of Ω is made of two disjoint parts

$$\partial\Omega = \Gamma_N \cup \Gamma_D,$$

with Dirichlet boundary conditions on Γ_D , and Neumann boundary conditions on Γ_N . The two boundary parts Γ_N and Γ_D are allowed to vary in the optimization process, although is possible to fix some portion of it. We denote by $\omega \in \mathbb{C}$ the complex pulsation and by $u \in H_D^1(\Omega; \mathbb{C}^d)$ the associated mode (for simplicity we drop the Laplace transformation notation \hat{u}), i.e. the corresponding displacement field in Ω with

$$H_D^1(\Omega; \mathbb{C}^d) = \{v \in H^1(\Omega; \mathbb{C}^d) \text{ such that } v = 0 \text{ on } \Gamma_D\}$$

The pair (ω, u) is solution of the non-linear eigenvalue problem of the linearized elasticity problem

$$(10) \quad \begin{cases} -\operatorname{div}(A(\omega)e(u)) = \omega^2 \rho u & \text{in } \Omega, \\ u = 0 & \text{on } \Gamma_D, \\ A(\omega)e(u) \cdot n = 0 & \text{on } \Gamma_N. \end{cases}$$

We remark that the above eigenvalue problem is non-linear since A depends on ω .

REMARK 2 From now on we will denote as \bar{v} the conjugate transpose of the vector v .

Non-linear eigenvalue problem

Problem (10) can be cast as a generalized eigenvalue problem

$$(11) \quad \mathcal{T}(\omega)u = 0$$

where $\mathcal{T}(\omega)$ is a linear operator depending (non-linearly) on a parameter ω . A solution $u \neq 0$ will exist only for some particular values of ω (also called eigenvalues).

Herein we cite two results describing the existence and characterization of the solutions of (10). The first one stands that when the Hooke law $A(\omega)$ has a particular structure, the solutions of (10) coincide with the eigenvalues of a compact non-selfadjoint operator [10]. The second one corresponds to a

standard result on analytical perturbations of compact operators for local solutions of (11) in a more general framework (consult for instance [16], chapter VII, Th. 1.9).

PROPOSITION 3 *Suppose A with a polynomial or rational structure w.r.t. ω so that for some $N \in \mathbb{N}$ with $N > 2$, the variational formulation of (10) reads:*

$$(12) \quad \sum_{k=1}^{N-1} \omega^k \int_{\Omega} C_k e(u) : e(\bar{v}) \, dx = \omega^N \int_{\Omega} u \cdot \bar{v} \, dx$$

$\forall v \in H_D^1(\Omega; \mathbb{C}^2)$, where for each k

$$(13) \quad C_k = z_k \mathcal{C}_k,$$

$z_k \in \mathbb{C}$, $z_0 \neq 0$ and \mathcal{C}_k coercive (i.e. $\exists \alpha_k > 0 : \mathcal{C}_k \xi \bar{\xi} \geq \alpha_k |\xi|^2 \, \forall \xi \in \mathbb{C}^d$). Let the operators $T, S : (a_1, \dots, a_N) \in H_D^1(\Omega; \mathbb{C}^2)^N \rightarrow H_D^1(\Omega; \mathbb{C}^2)^N$ be the solutions of the variational systems described in [10]. Then:

1. The operator $Q = S^{-1} \circ T$ admits a countable infinite family of eigenvalues and eigenvectors $(\omega_k, u_k)_{k \geq 1} \in \mathbb{C} \times H_D^1(\Omega; \mathbb{C}^2)$, with $|\omega_k| \leq |\omega_{k+1}|$, where each eigenvalue has finite multiplicity and $|\omega_k| \rightarrow \infty$.
2. The pairs $(\omega_k, u_k)_{k \geq 1}$ are the unique solutions of (12) in the following sense: if $(\omega, u) \in \mathbb{C} \times H_D^1(\Omega; \mathbb{C}^2)$ is any solution of (12), then there exists at least one $k \geq 1$ such that $\omega = \omega_k$ and u is a linear combination for all u_m for which $\omega_m = \omega$.

PROPOSITION 4 *Let $S(\omega)$ be a family of compact operators (for each ω fixed) and holomorphic with respect to $\omega \in \mathcal{D} \subset \mathbb{C}$ bounded. Define ω as a singular point if 1 is an eigenvalue of $S(\omega)$. Then either all $\omega \in \mathcal{D}$ are singular points or there are only a finite number of singular points in each compact subset of \mathcal{D} . In our case the result applies for (11) by taking $S(\omega) = \left(\frac{\mathcal{T}(\omega)}{\omega^2} + Id \right)^{-1}$.*

Optimization problem

Supposing that (10) admits a countable infinite family of solutions $(\omega_k, u_k)_{k \geq 1}$ in $\mathbb{C} \times H^1(\Omega; \mathbb{C})^d$, with the eigenfunctions, or modes, normalized by imposing that $\int_{\Omega} \rho |u_k|^2 \, dx = 1$, the objective function $\eta(\Omega)$ to be maximized is the modal loss factor (capacity to dissipate energy) of the structure for its first eigenvalue:

$$(14) \quad \eta(\Omega) = \frac{\Im(\omega_1^2)}{\Re(\omega_1^2)}.$$

Since the eigenvalues of (10) can not be naturally ordered in \mathbb{C} , ω_1 is computed as the closest $(\omega_k)_{k \geq 1}$ to the the smallest positive eigenvalue ω_1^0 solution of the real self-adjoint problem

$$(15) \quad -\operatorname{div}(A(0)e(u)) = \omega^2 \rho u \text{ in } \Omega.$$

3 Shape optimization

3.1 Shape sensitivity analysis

We briefly recall the definition and main results about shape derivation in dimension $d = 2$. Shape differentiation is a classical topic that goes back to Hadamard [3, 24]. Let the overall domain $\Omega \subset \mathbb{R}^2$ be fixed and bounded. Let $\Omega_1 \subset \Omega$ be a smooth open subset which is variable. Indeed, we consider variations of the type

$$(Id + \theta)(\Omega_1) := \{x + \theta(x) \text{ for } x \in \Omega_1\},$$

with $\theta \in W^{1,\infty}(\Omega; \mathbb{R}^2)$ such that $\|\theta\|_{W^{1,\infty}(\Omega; \mathbb{R}^2)} < 1$ and tangential on $\partial\Omega$ (i.e., $\theta \cdot n = 0$ on $\partial\Omega$; this last condition ensures that $\Omega = (Id + \theta)\Omega$). It is well known that, for sufficiently small θ , $(Id + \theta)$ is a diffeomorphism in Ω .

DEFINITION 5 The shape derivative of a function $J(\Omega_1)$ is defined as the Fréchet derivative in $W^{1,\infty}(\Omega; \mathbb{R}^2)$ at 0 of the application $\theta \rightarrow J((Id + \theta)\Omega_1)$, i.e.

$$J((Id + \theta)\Omega_1) = J(\Omega_1) + J'(\Omega_1)(\theta) + o(\theta),$$

where $\lim_{\theta \rightarrow 0} \frac{o(\theta)}{\|\theta\|_{W^{1,\infty}}} = 0$ and $J'(\Omega_1)$ is a continuous linear form on $W^{1,\infty}(\Omega; \mathbb{R}^2)$.

3.2 Shape representation by the level-set method and multi-phase design

Level set method for topology optimization

The level-set method is a technique for capturing interfaces which are implicitly defined via the zero level-set of an auxiliary function. Over the last years, this method has been successfully applied to topology optimization problems. Define the working domain $\Omega \subset \mathbb{R}^d$ ($d = 2, 3$) bounded and the admissible shapes $\Omega_1 \subset \Omega$ and $\Omega \setminus \bar{\Omega}_1$. Then, the boundary of Ω_1 is described by means of a level set function ψ such that

$$(16) \quad \begin{cases} \psi(x) = 0 & \text{if } x \in \partial\Omega_1, \\ \psi(x) < 0 & \text{if } x \in \Omega \setminus \Omega_1, \\ \psi(x) > 0 & \text{if } x \in \Omega_1. \end{cases}$$

Under the action of a normal vector field $V(t, x)n(x)$, the shape Ω_1 evolves according to the Hamilton-Jacobi equation

$$(17) \quad \frac{\partial \psi}{\partial t}(t, x) + V(t, x)|\nabla \psi(t, x)| = 0,$$

$\forall t \in \mathbb{R}^+, \forall x \in \Omega$. The choice of the normal velocity V is based on the notion of shape derivative. Generally speaking, the shape derivative of a functional $J(\Omega_1)$ reads

$$(18) \quad J'(\Omega_1)(\theta) = \int_{\partial\Omega_1} \mathbb{T}\theta \cdot n \, ds,$$

where the integrand $\mathbb{T}(x)$ depends on the solutions of either (4)-(5) or (10). Note that in the case of multi-layered composite each ply shape Ω_i ($i = 1, \dots, N$) can move with its own

velocity. Since only the normal component of θ plays a role in (18), a descent direction θ for J satisfies

$$(19) \quad \theta = Vn \quad \text{and} \quad J'(\Omega_1)(\theta) = \int_{\partial\Omega_1} \mathbb{T}V ds \leq 0.$$

To ensure the decrease of J , the simplest choice is $V = -\mathbb{T}$. However, \mathbb{T} is a priori defined only on the interfaces Ω_1 while V must be defined in the entire domain Ω . If such an extension is not obvious or if we want to regularize the velocity fields, there is an alternative choice based on a different underlying scalar product (see e.g. [2]).

Multi-phase design

Given the above level-set framework, we consider multi-layered composites and viscoelastic damping treatments as multi-phase structures where the level set function ψ (or each ψ_i in the multi-layered case) represents the interface between two material phases occupying the sub-domains $\Omega_1 \subset \Omega$ and $\Omega \setminus \Omega_1$. As it was studied in [4], this interface can be sharp, in which case the elastic properties (denoted generically as A with A_0 and A_1 the respective values in each sub-domain) are defined piecewise as

$$(20) \quad A = A_0 + \chi(A_1 - A_0),$$

with $\chi = H(\psi)$ and H the Heaviside function, or smooth

$$(21) \quad A = A_0 + h(d_{\Omega_1})(A_1 - A_0),$$

where h is a smooth approximation of the Heaviside function and d_{Ω_1} the signed distance function associated to Ω_1

$$(22) \quad \begin{cases} d_{\Omega_1}(x) = 0 & \text{if } x \in \partial\Omega_1, \\ d_{\Omega_1}(x) = -\min_{x_I \in \partial\Omega_1} |x - x_I| & \text{if } x \in \Omega \setminus \Omega_1, \\ d_{\Omega_1}(x) = \min_{x_I \in \partial\Omega_1} |x - x_I| & \text{if } x \in \Omega_1. \end{cases}$$

For the ensuing analysis the latter multi-phase formulation is applied.

REMARK 6 In the case of viscoelastic structures, the global material density is defined as

$$\rho = \rho_0 + h(d_{\Omega_1})(\rho_1 - \rho_0),$$

where the densities ρ_0 and ρ_1 characterize $\Omega \setminus \Omega_1$ and Ω_1 respectively.

3.3 Shape derivatives

We only give the shape derivative formulas for the following criteria: Compliance, the inverse of the buckling load factor λ_1 (Section 2.1), and ω_1 (Section 2.2). For further details refer to [5], [4] and [10].

Multi-layered composites

PROPOSITION 7 Let (λ_1, w_1) and u be the solutions of (4) and (5) respectively. Define the plane compliance as

$$\mathcal{E} = \int_{\Omega} Ae(u) : e(u) dx$$

and the adjoint state $p \in H_D^1(\Omega; \mathbb{R}^2)$ as the solution of

$$(23) \quad \int_{\Omega} Ae(p) : e(v) dx = \lambda_1 \int_{\Omega} B(v; w_1, w_1) dx$$

$\forall v \in H_D^1(\Omega; \mathbb{R}^2)$, where

$$B(v; w_1, w_1) = \int_{\Omega} (Ae(v) \cdot \nabla w_1) \cdot \nabla w_1 dx.$$

Moreover, assume λ_1 is a simple eigenvalue of problem (4) and let the buckling mode $w_1 \in H_D^2(\Omega)$ be normalized as $\int_{\Omega} B(u; w_1, w_1) dx = -1$. Then λ_1 is shape differentiable and

$$(24) \quad \lambda_1'(\Omega_1)(\theta) = \int_{\partial\Omega_1} \left(\llbracket D \rrbracket \nabla^2 w_1 : \nabla^2 w_1 + \lambda_1 (\llbracket A \rrbracket e(u) \cdot \nabla w_1) \cdot \nabla w_1 - \llbracket A \rrbracket e(u) : e(p) \right) \theta \cdot nds,$$

$$(25) \quad \mathcal{E}'(\Omega_1)(\theta) = \int_{\partial\Omega_1} \llbracket A \rrbracket e(u) : e(u) dx$$

where $\llbracket \cdot \rrbracket = \cdot_1 - \cdot_0$ denotes the jump through $\partial\Omega_1$.

Viscoelastic structures

PROPOSITION 8 Let (ω_1, u_1) the solutions of (10). Define $p_1 \in H_D^1(\Omega; \mathbb{C}^2)$ as the adjoint eigenvector solution of

$$(26) \quad \int_{\Omega} A^H(\omega_1)e(p) : e(\bar{v}) dx = \omega^2 \rho \int_{\Omega} p \cdot \bar{v} dx$$

$\forall v \in H_D^1(\Omega; \mathbb{C}^2)$, with $\omega^2 = \overline{\omega_1^2}$, $A^H = \bar{A}$ the conjugate transpose tensor of A and $\int_{\Omega} \rho |p_1|^2 = 1$. Moreover assume that ω_1 is a simple eigenvalue of (10). Then ω_1 is shape differentiable and

$$(27) \quad \omega_1'(\Omega_1)(\theta) = \frac{\int_{\partial\Omega_1} \theta \cdot n \left(\omega_1^2 \llbracket \rho \rrbracket u_1 \cdot \bar{p}_1 - \llbracket A \rrbracket (\omega_1) e(u_1) : e(\bar{p}_1) \right) ds}{\int_{\Omega} \left(2\omega_1 \rho u_1 \cdot \bar{p}_1 - \partial_{\omega} A(\omega_1) e(u_1) : e(\bar{p}_1) \right) dx}$$

where $\llbracket \cdot \rrbracket = \cdot_1 - \cdot_0$ denotes the jump through $\partial\Omega_1$.

4 Numerical results

4.1 Multi-layered composites

Test case: The goal of this test case is to design the lightest composite fuselage skin panel, subject to a shear load, as illustrated in Figure 3. Of particular interest is the study of the influence of the orthotropic plies oriented at 45° and -45° in the prevention of buckling. The objective function is the mass of structure constrained by different stiffness measures including

compliance and buckling load λ_1^{-1} . Due to the small curvature of the cylindrical panel section, an approximate plate model is used. The panel domain is a rectangle $\Omega = 2m \times 1m$, modeled as a multi-layered plate. The elastic properties of the main phase of each layer are described through one of the following tensors: $\mathcal{C}_{0^\circ}, \mathcal{C}_{90^\circ}, \mathcal{C}_{45^\circ}, \mathcal{C}_{-45^\circ}$ which correspond to rotations of the orthotropic material \mathcal{C} given by a carbon fiber/epoxy matrix.



Figure 3: Approximate flat model due to the small curvature of the cylindrical panel section.

Optimization algorithm: The optimization algorithm for solving (8) can be summarized as two nested loops:

1. An outer loop for the shape variable \mathcal{O} that solves (8) for a fixed stacking sequence ξ via a descent direction method, which is based on a shape sensitivity analysis coupled to a level set method described in Section 3.2.
2. An inner loop for the variable ξ where the constraint margin function $\mathcal{M}(\mathcal{O}) := \min_{\xi \in \mathcal{Y}_{ad}} G(\mathcal{O}, \xi)$ is evaluated by solving the integer programming problem via an outer approximation method [5].

Results

The optimal stacking sequence corresponds to the values: $[45^\circ, 90^\circ, -45^\circ, -45^\circ, 0^\circ, 0^\circ, 45^\circ, 0^\circ]$.

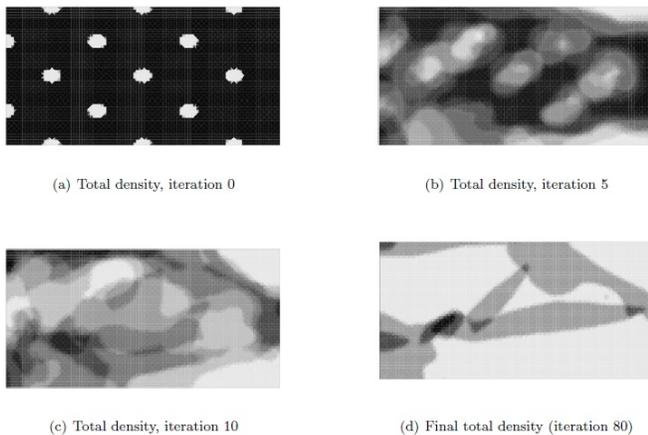


Figure 4: Evolution of the total material density of the composite laminate.

4.2 Viscoelastic structures

Test case: Let $\mathbb{D} = \Omega \times [0, \varepsilon] \subset \mathbb{R}^3$ be a plate in plane stress, with thickness $\varepsilon = 5mm$ and $\Omega = 2m \times 1m$, composed of the superposition of a fixed rectangular membrane of aluminum and an isotropic damping viscoelastic material 3M ISD112 [27]. We remark that the choice of material properties matches with the rational structure of $A(\omega)$ mentioned in Proposition 3. The cantilever Ω is fixed on the left at Γ_D and free on $\partial\Omega \setminus \Gamma_D$. The objective will be to maximize the loss factor η .

Solving the discrete non-linear eigenvalue problem: Denote as

$$(28) \quad \mathcal{T}_h(\omega) \cdot x = 0$$

the matrix representation of the FEM or discrete weak formulation of (10). Among the various methods to solve (28) (consult for instance [23]), we chose one of the simplest ones that consists in applying the Newton's method to the extended system:

$$(29) \quad F_z(\omega, x) = \begin{pmatrix} \mathcal{T}_h(\omega) \cdot x \\ \bar{z} \cdot x - 1 \end{pmatrix} = 0.$$

The second equation represents a normalization condition with $\|z\| = 1$ and $\bar{z} \cdot x^* \neq 0$, where x^* is the exact eigenvector of (28). We chose as initialization of the aforementioned algorithm (ω_1^0, u_1^0) solution of (15) and $z = u_1^0$.

Results

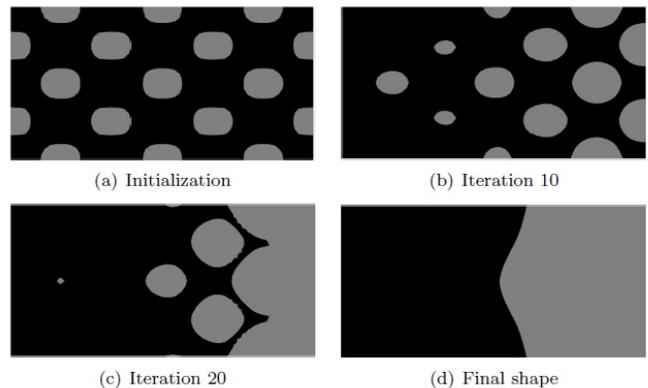


Figure 5: Evolution of the composite structure made of the superposition of aluminum (grey) and a viscoelastic material (black).

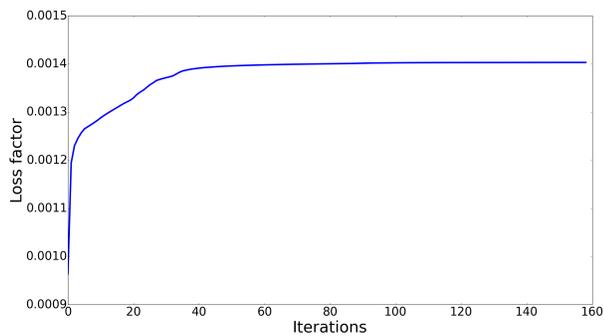


Figure 6: Evolution of the loss factor η during the optimization.

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