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LPV control design by using LPVTools™ for an active anti-roll bar system of heavy vehicles

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Abstract

Vehicle rollover is a very serious problem for the safety of heavy vehicles, which can result in large financial and environmental consequences. In order to enhance the roll stability, most modern heavy vehicles are equipped with passive anti-roll bars. However they may be not sufficient to overcome critical situations, e.g during cornering or riding on uneven roads, at high speed. This paper is concerned with active anti-roll bar $H_{\infty}$/LPV control for single unit heavy vehicles. A Linear Parameter Varying (LPV) approach is proposed here in order to schedule the controller with the vehicle forward velocity (that is the varying parameter of the vehicle LPV model) and with the normalized load transfers at the two axles (that are considered into parameter dependent weighting functions for on-line performance adaptation to the rollover risk of heavy vehicles). The grid-based LPV approach is here used to synthesize the $H_{\infty}$/LPV controller through LPVTools™. The effectiveness of the $H_{\infty}$/LPV active anti-roll bar control is validated by using the TruckSim® simulation software with two different types of heavy vehicle: a bus and a truck, both fully loaded. The simulation results in the frequency and time domains show that the $H_{\infty}$/LPV active anti-roll bar control drastically improves the roll stability of the single unit heavy vehicle compared with the passive anti-roll bar case.

Keywords: Vehicle dynamics, Active anti-roll bar system, Linear Parameter Varying (LPV) control, $H_{\infty}$ control, Roll stability

1. Introduction

1.1. Background

Rollover of heavy vehicles is an important road safety problem world-wide. Although rollovers are relatively rare events, they are usually deadly accidents when they occur. According to the Federal National Highway Traffic Safety Administration (NHTSA), in the United States, there were 333,000 heavy vehicles involved in traffic crashes during 2012. There were 3,921 people killed in rollover crashes and 104,000 people injured (an increase of 18 percent from 2011). Rollover accidents are classified into four categories: preventable, potentially preventable, non-preventable and preventable unknown [1]. It is usually difficult for the driver to feel the rollover behaviour of a heavy vehicle. Investigations have shown that only a minority of rollover accidents could have been avoided with a warning device, potentially more with a skilled driver, but half of the rollover accidents were not preventable by driver action alone. The three major contributing factors to rollover accidents are side wind gusts, abrupt steering and braking manoeuvres by the driver. The main reason of rollover accidents in which heavy vehicles are involved is the roll stability loss when the tyre-road contact force on one of the side wheels becomes zero.

In order to prevent the rollover of vehicles, several schemes concerned with the possible active intervention onto the vehicle dynamics have been proposed as follows: active steering [2], active braking [3], active suspension [4],[5] and active anti-roll bars [6]. Among them, the active anti-roll bar system is the most common method used to improve the roll stability of heavy vehicles. Active anti-roll bars are usually made of a pair of hydraulic actuators. Lateral acceleration makes vehicles with conventional passive suspension tilt out of corners. The center of the sprung mass shifts outward of the ve-
hicle centerline, which creates a destabilizing moment that degrades roll stability. The lateral load response is reduced by active anti-roll bars that generate a stabilizing moment to counterbalance the overturning moment in such a way that the control torque leans the vehicle into the corners [7, 8]. One drawback of active anti-roll bars is that the maximum stabilizing moment is limited physically by the relative roll angle between the body and the axle [9].

Linear Parameter Varying (LPV) systems are an important class of systems, whose dynamics depend linearly on the state and input of the system, but could depend in a nonlinear way on scheduling parameters. The LPV paradigm considers that no a priori information about the scheduling parameter values is available, but that the parameter can be measured or estimated online [10], [11]. The interest in LPV systems is motivated by their use in gain-scheduling control techniques, and by the possibility to embed nonlinear systems into the LPV framework by covering nonlinearities within the scheduling parameters. Therefore the LPV framework enables, to some extent, the application of linear control methods to nonlinear systems, while providing rigorous statements on stability and performance of the closed-loop system.

1.2. Related works

In the literature, many works have been dedicated to design the active anti-roll bar system for single unit heavy vehicles. Many control strategies have been developed using the yaw-roll model such as:

- **Optimal control:** In [9], [12], [13] the authors used LQR method for the active anti-roll bar system of heavy vehicles. The control torques acting between the axle groups and the sprung mass are considered as the input control signal. The simulation results indicated that the normalized load transfers at all the axles reduced significantly, compared with the passive anti-roll bar system at 60 km/h.

- **$H_\infty$ and LPV control:** In [7], [14], [15] the LPV approach is applied for the active anti-roll bar system combined with active brake control on a single unit heavy vehicle. The forward velocity was considered as the varying parameter. The different actuator failures are identified by using a Fault Detection and Identification (FDI) filter. The $H_\infty$ control method was also applied to the yaw-roll model of a single unit heavy vehicle. It shows, using $\mu$ tool analysis, that the $H_\infty$ active anti-roll bar control is robust w.r.t. the forward velocity and sprung mass variations [16]. TruckSim® software is also used to evaluate the effectiveness of the active roll control system of heavy vehicles [17]. The simulations performed using TruckSim® indicated that a rollover threat detection system was further enhanced in combination with an active roll control system using active suspension mechanism.

In [18], [19] the authors proposed an integrated model with four electronic servo-valve hydraulic actuator models in a single unit heavy vehicle yaw-roll model. Then the LQR control method was applied to the active anti-roll bar, showing how it may solve a multi-objective problem considering the improvement of the roll stability, while taking into account the characteristics of the electronic servo-valve hydraulic actuators.

1.3. Paper contribution

Based on the integrated model presented in [19], this paper proposes a MIMO $H_\infty$/LPV controller, with three varying parameters (forward velocity and normalized load transfers at the two axles), designed using the grid-based LPV approach [20]. Hence, the main contributions of this paper can be summarized in the following points:

- Parameter dependant weighting functions are used to allow for performance adaptation to the rollover risk of heavy vehicles, characterized by the normalized load transfers at the two axles.
- The grid-based LPV approach [20] is used to synthesize the $H_\infty$/LPV controller depending on three varying parameters, which are the forward velocity and the normalized load transfers at the two axles. It is the first time that the grid-based LPV approach is applied for the heavy vehicle dynamic through LPVTMs™.
- The $H_\infty$/LPV active anti-roll bar control is validated using the TruckSim® simulation software with two different types of vehicle: a tour bus and a LCF (Low Cab Forward) truck, both fully loaded.
- The simulation results, in the frequency and time domains, show that the $H_\infty$/LPV active anti-roll bar control is a realistic solution which drastically improves the roll stability of a single unit heavy vehicle compared with the passive anti-roll bar.

This paper is organised as follows: Section 2 details the LPV model of a single unit heavy vehicle with the forward velocity as scheduling parameter. Section 3 proposes the formulation of the $H_\infty$/LPV control problem. Section 4 illustrates the solution of the $H_\infty$/LPV control...
problem. Section 5 gives the simulation results analysis in the frequency and time domains where the LPV active anti-roll bar control is compared with the passive anti-roll bar case. Validation of the $H_\infty$/LPV active anti-roll bar control by using the TruckSim simulation software is analysed in section 6. Finally, some conclusions are drawn in section 7.

2. Vehicle modelling: an LPV approach

The integrated model of a single unit heavy vehicles was proposed in [19]. It includes four Electronic Servo-Valve Hydraulic (ESVH) actuators (two at the front and two at the rear axles) mounted in a linear yaw-roll model. In each ESVH actuator, the control signal is the electrical current $u$ opening the electronic servo-valve, the output is the force $F_{act}$ generated by the hydraulic cylinder.

2.1. The electronic servo-valve hydraulic actuator model [19]

![Diagram of the ESVH actuator](image)

Figure 1 illustrates the diagram of the considered ESVH actuator, made of an electronic servo-valve and a hydraulic cylinder. The spool valve displacement $X_v$ of the electronic servo-valve is controlled by the current $u$. The oil supply high pressure $P_t$ is always stored outside the electronic servo-valve and the spool valve displacement distributes high pressure oil into the two chambers of the hydraulic cylinder. The difference of pressures $\Delta P = P_1 - P_2$ inside the two chambers produces the output force $F_{act}$. The dynamical equations of the ESVH actuator are presented in equation (1) and the symbols are shown in Table 1.

\[
\begin{align*}
F_{act} &= A_P \Delta P \\
\frac{\partial \Delta P}{\partial t} + (K_p + C_p) \Delta P - K_v X_v + A_P \frac{\partial y_u}{\partial t} = 0 \\
\frac{\partial X_v}{\partial t} + \frac{1}{\tau} X_v - \frac{K_v}{\tau} u = 0
\end{align*}
\]

where $y_u$ is the displacement of the piston inside the hydraulic cylinder, $u$ the input current to the servo-valve.

![Figure 1: Diagram of the ESVH actuator](image)

2.2. Yaw-roll model of a single unit heavy vehicle [14]

The linear yaw-roll model is shown in the Figure 2, the differential equations of motion, i.e., the lateral dynamics, the yaw moment, the roll moment of the sprung mass, the roll moment of the unsprung masses at the two axles, are formalized in the equations (2):

\[
\begin{align*}
(mv &+ \delta) \phi' - m_v h \phi = F_{yf} + F_{yr} \\
-I_x \phi' + I_z \psi = F_{yf} r_f - F_{yr} r_r \\
(I_{xx} + m_v h^2) \phi' - I_c \psi &= m_v v h (\phi' + \psi) \\
+ m_v g h \phi - k_f (\phi - \phi_f) - b_f (\phi - \phi_f) \\
+ T_f - k_v (\phi - \phi_u) - b_r (\phi - \phi_u) + T_r \\
-r F_{yf} &= m_{urf} (r - h_{ur}) (\phi' + \psi) \\
+ m_{urf} g h u_r \phi_u - k_{urf} \phi_u \\
+ k_f (\phi - \phi_u) + b_r (\phi - \phi_u) + T_f \\
-r F_{yr} &= m_{urf} (r - h_{ur}) (\phi' + \psi) - m_{ur} g h u_r \phi_u \\
- k_{urf} \phi_u + k_r (\phi - \phi_u) + b_r (\phi - \phi_u) + T_r
\end{align*}
\]

where $T_f$ and $T_r$ are the torques generated by the active anti-roll bar system at the front and rear axles. The lateral tyre forces $F_{yf,r}$ in the direction of velocity at the wheel ground contact points are modelled by a linear stiffness as:

\[
\begin{align*}
F_{yf} &= \mu C_f \alpha_f \\
F_{yr} &= \mu C_r \alpha_r
\end{align*}
\]

with tyre side slip angles:

\[
\begin{align*}
\alpha_f &= -\beta + \delta_f - \frac{l_r \psi}{v} \\
\alpha_r &= -\beta + \frac{l_r \psi}{v}
\end{align*}
\]

The symbols of the yaw-roll model of a single unit heavy vehicle are detailed in Table 2.
The forces, as well as other characteristics of the ESVH actuators at each axle, do have the same magnitude and the opposite direction [19], therefore we can consider that the torque generated by the active anti-roll bar system at each axle is twice the torque generated by one electronic servo-valve hydraulic actuator. In this study, the characteristics of the ESVH actuators on the right at the front, and on the right at the rear axles will be used. The torque generated by the active anti-roll bar system at the front axle is determined by:

\[ T_f = 2l_{act}F_{act f} = 2l_{act}A_p\Delta P_f \]  

and the torque generated by the active anti-roll bar system at the rear axle is:

\[ T_r = 2l_{act}F_{act r} = 2l_{act}A_p\Delta P_r \]  

where \( \Delta P_f \) and \( \Delta P_r \) are respectively the difference of pressure of the hydraulic cylinder at the front and rear axles.

The dynamical equations of the ESVH actuators are written in the state-space representation form as follows:

\[ \dot{x} = A(\rho_1).x + B_1(\rho_1).w + B_2(\rho_1).u \]  

where \( \rho_1 \) is a varying parameter and the state vector:

\[ x = [\beta \ \psi \ \phi \ \phi_{uf} \ \phi_{ur} \ \Delta P_f \ \Delta P_r \ \Delta \psi_f \ \Delta \psi_r]^T \]

The exogenous disturbance (steering angle) is:

\[ w = [\delta_f]^T \]

and the control inputs (input currents):

\[ u = [u_f \ u_r]^T \]

The combination of equations (2) to (7) are the motion differential equations of the integrated model of a single unit heavy vehicle.

2.3. An LPV model of a single unit heavy vehicle

We can see in (2)-(4) that the yaw-roll model depends on the forward velocity \( v \) and on the inverse of the forward velocity \( \frac{1}{v} \). Moreover, when the vehicle is in motion, the forward velocity is one of the constantly changing parameters, and it depends on the driver and the motion condition of the vehicle. Here, the forward velocity \( v \) is chosen as a scheduling parameter.

Denoting \( \rho_1 = v \), the integrated model of a single unit heavy vehicle, merging the equations (2)-(7) is written in the state-space representation form as follows:

\[ \dot{x} = A(\rho_1).x + B_1(\rho_1).w + B_2(\rho_1).u \]  

where \( \rho_1 \) is a varying parameter and the state vector:

\[ x = [\beta \ \psi \ \phi \ \phi_{uf} \ \phi_{ur} \ \Delta P_f \ \Delta P_r \ \Delta \psi_f \ \Delta \psi_r]^T \]

The exogenous disturbance (steering angle) is:

\[ w = [\delta_f]^T \]

and the control inputs (input currents):

\[ u = [u_f \ u_r]^T \]

The model (8) is transformed into a Linear Parameter Varying (LPV) model, whose state-space entries depend continuously on a time varying parameter vector,
\( \rho_1(t) \). One characteristic of the LPV system is that it must be linear in the pair formed by the state vector \((x)\), and the control input vector \((u)\). The matrices \(A(\rho_1)\), \(B_1(\rho_1)\) and \(B_2(\rho_1)\) are generally nonlinear functions of the scheduling vector \(\rho_1\).

**Remark:** Let us note that \(A(\rho_1)\) is not affine in \(\rho_1\) since it includes \(\rho_1\) and \(\frac{1}{\rho_1}\). Therefore the classical polytopic approach cannot be used unless we consider two different parameters, which increases the conservatism.

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m)</td>
<td>Sprung mass</td>
</tr>
<tr>
<td>(m_{af,r})</td>
<td>Unsprung masses</td>
</tr>
<tr>
<td>(m)</td>
<td>The total vehicle mass</td>
</tr>
<tr>
<td>(v)</td>
<td>Forward velocity</td>
</tr>
<tr>
<td>(h)</td>
<td>Height of sprung mass from roll axis</td>
</tr>
<tr>
<td>(h_{af,r})</td>
<td>Height of unsprung mass from ground</td>
</tr>
<tr>
<td>(r)</td>
<td>Height of roll axis from ground</td>
</tr>
<tr>
<td>(a_\gamma)</td>
<td>Lateral acceleration</td>
</tr>
<tr>
<td>(\beta)</td>
<td>Side-slip angle at center of mass</td>
</tr>
<tr>
<td>(\psi)</td>
<td>Heading angle</td>
</tr>
<tr>
<td>(\psi)</td>
<td>Yaw rate</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>Side slide angle</td>
</tr>
<tr>
<td>(\phi)</td>
<td>Sprung mass roll angle</td>
</tr>
<tr>
<td>(\phi_{af,r})</td>
<td>Roll angle of unsprung masses</td>
</tr>
<tr>
<td>(\delta_f)</td>
<td>Steering angle</td>
</tr>
<tr>
<td>(u_f,r)</td>
<td>Control currents</td>
</tr>
<tr>
<td>(C_{f,r})</td>
<td>Tyre cornering stiffness on the axes</td>
</tr>
<tr>
<td>(k_{f,r})</td>
<td>Suspension roll stiffness on the axes</td>
</tr>
<tr>
<td>(b_{f,r})</td>
<td>Suspension roll damping on the axes</td>
</tr>
<tr>
<td>(k_{r,f})</td>
<td>Tyre roll stiffness on the axes</td>
</tr>
<tr>
<td>(l_{cx})</td>
<td>Roll moment of inertia of sprung mass</td>
</tr>
<tr>
<td>(l_{zz})</td>
<td>Yaw-roll inertial of sprung mass</td>
</tr>
<tr>
<td>(l_{zz})</td>
<td>Yaw moment of inertia of sprung mass</td>
</tr>
<tr>
<td>(l_{f},r)</td>
<td>Length from the CG to the axes</td>
</tr>
<tr>
<td>(l_{w})</td>
<td>Half of the vehicle width</td>
</tr>
<tr>
<td>(\mu)</td>
<td>Road adhesion coefficient</td>
</tr>
</tbody>
</table>

### 3. Formulation of the \(H_{\infty}/LPV\) control problem

#### 3.1. Performance criteria

To evaluate the rollover of vehicles, let us first consider the tyre force \((F_z)\) in the \(Z\) direction at the each wheel. The rollover occurs when \(F_z = 0\), and the wheel then starts to lift off the road.

However the value of the tyre force \((F_z)\) in the \(Z\) direction at the each wheel is not easy to measure or estimate.

For the yaw-roll model of the heavy vehicles in Figure 2, the normalized load transfer \(R_{zf}\) at the two axles is also used to evaluate the rollover, defined in [17], [23]. When the value of \(R_{zf}\) takes on the limit of \(\pm 1\), the wheel in the inner bend lifts off the road, and the rollover occurs.

\[
R_f = \frac{\Delta F_{zf}}{F_{zf}}, \quad R_r = \frac{\Delta F_{zr}}{F_{zr}} \tag{9}
\]

where \(F_{zf}\) is the total axle load at the front axle and \(F_{zr}\) at the rear axle. \(\Delta F_{zf}\) and \(\Delta F_{zr}\) are respectively the lateral load transfers at the front and rear axles, which can be given by:

\[
\Delta F_{zf} = \frac{k_{zf}\phi_{zf}}{l_{w}}, \quad \Delta F_{zr} = \frac{k_{zf}\phi_{zr}}{l_{w}} \tag{10}
\]

In the case of an obstacle avoidance in an emergency, the wheels at the rear axle lift off first for the truck, because the rollover of a vehicle is affected by the suspension stiffness to load ratio, which is greater at the rear axle than at the front one [6], [7]. However the other effect to be considered in the rollover of a vehicle is the distribution of the total load for the two axles. In the case of big buses, the engine is often mounted at the rear, so the wheels at the front axle usually lift off first. Therefore generally it is necessary to consider the rollover risk at the two axles of heavy vehicles.

Since such performance indices are key parameters to evaluate the risk of rollover, they are considered here as scheduling parameters of the LPV control, in order to provide a stable and smooth control action when reaching critical situations. Let us note that a similar idea was used in [7], [14] considering the rear load transfer only in order to account for actuator fault and to switch on braking actuations when critical situations occur.

We then define \(\rho_2 = [R_f]\) and \(\rho_3 = [R_r]\).

#### 3.2. Performance specifications for the \(H_{\infty}/LPV\) control design

![Figure 3: The closed-loop interconnection structure of the LPV active anti-roll bar control.](image)

In this section the LPV control problem is presented for the active anti-roll bar system of heavy vehicles, using ESVH actuators. In Figure 3, the given \(H_{\infty}/LPV\)
control structure includes the nominal model $G(p_1)$, the controller $K(p_1, p_2, p_3)$, the performance output $z$, the control input $u$, the measured output $y$, the measurement noise $n$. The steering angle $\delta_f$ is the disturbance signal, set by the driver. The weighting functions $W_R, W_z(p_2, 3), W_n$ are presented thereafter, according to the considered performance objectives. The weighting functions matrix $W_c$ representing the performance output, is chosen as $W_c = \text{diag}[W_{cu}, W_{cr}]$. The purpose of the weighting functions is to keep the control inputs and normalized load transfers as small as possible over the desired frequency range up to $4\, \text{rad/s}$, which represents the limited bandwidth of the driver [7]. These weighting functions can be considered as penalty functions, that is, weights which should be large in the frequency range where small signals are desired and small where larger performance outputs can be tolerated. The weighting function $W_{cu}$ is chosen as $W_{cu} = \text{diag}[W_{cuf}, W_{cur}]$, corresponding to the input currents at the front and rear axles. The reason for keeping the control signals small is to avoid the actuator saturation, and the front and rear axles. The reason for keeping the control input small is to avoid the actuator saturation, and the front and rear axles. The reason for keeping the control input small is to avoid the actuator saturation, and the front and rear axles. The reason for keeping the control input small is to avoid the actuator saturation, and the front and rear axles. The reason for keeping the control input small is to avoid the actuator saturation, and the front and rear axles.

The weighting function $W_{cuf}$ chosen as $W_{cuf} = \text{diag}[W_{cuf f}, W_{cuf r}]$, corresponds to the input currents at the front and rear axles, and is selected as:

$$W_{cuf} = \frac{1}{0.4}; \quad W_{cu r} = \frac{1}{0.4} \quad (11)$$

The authors stress that the interest of parameter dependent weighting functions is to allow for performance adaptation to the rollover risk of heavy vehicles. Indeed, the ESVH actuators will be tuned according to the varying parameters in order to meet the desired performance. For example, as far as the normalized load transfer at the front is concerned, when the varying parameter $p_2 \to 1$, the gain of the weighting function $W_{cuf}$ is large, and therefore the normalized load transfer at the front will be penalized. In the same way, when $p_3$ is large, the normalized load transfer at the rear will be reduced.

The input scaling weight $W_R$ normalizes the steering angle to the maximum expected command. It is selected as $W_R = \pi/180$, which corresponds to a $1\, \text{deg}$ steering angle command.

The weighting function $W_n$ is selected as a diagonal matrix which accounts for sensor noise models in the control design. The noise weights are chosen as $0.01 (m/s^2)$ for the lateral acceleration and $0.01 (\text{deg/s})$ for the derivative of the roll angle $\phi$ [7].

3.3. The LPV generalized plant

According to Figure 3, the concatenation of the non-linear model [8] with the performance weighting functions has a partitioned representation in the following way:

$$\begin{bmatrix}
  \dot{x}(t) \\
  z(t) \\
  y(t)
\end{bmatrix} =
\begin{bmatrix}
  A(p) & B_1(p) & B_2(p) \\
  C_1(p) & D_{11}(p) & D_{12}(p) \\
  C_2(p) & D_{21}(p) & D_{22}(p)
\end{bmatrix}
\begin{bmatrix}
  x(t) \\
  w(t) \\
  u(t)
\end{bmatrix}$$

with the exogenous input $w(t) = [\delta_f, n]$, the control input $u(t) = [u_f, u_r]^T$, the measured output vector $y(t) = [a_i, \phi]^T$, and the performance output vector $z(t) = [u_f, u_r, R_f, R_r]^T$.

It is worth noting that, in the LPV model of the active anti-roll bar system [13], the varying parameters $p = [p_1, p_2, p_3]$ are known in real time. Indeed the parameter $p_1 = v$ is measured directly, while the parameters $R_f$ ($p_2$ and $p_3$) can be calculated by using the measured roll angle of the unsprung masses $\phi_{uf} [7]$.

3.4. $H_{\infty}$/LPV control problem

The control goal is to find a LPV controller $K(p)$ defined as:

$$\begin{bmatrix}
  \dot{x}_K(t) \\
  u(t)
\end{bmatrix} =
\begin{bmatrix}
  A_K(p) & B_K(p) \\
  C_K(p) & D_K(p)
\end{bmatrix}
\begin{bmatrix}
  x_K(t) \\
  y(t)
\end{bmatrix} \quad (14)$$

where $A_K(p), B_K(p), C_K(p), D_K(p)$ are continuous bounded matrix functions, which minimizes the induced $\ell_2$ norm of the closed-loop LPV system $\Sigma_{CL} = \text{LFT}(G, K)$, with zero initial conditions, i.e.:

$$\|\Sigma_{CL}(p)\|_{2 \to 2} = \sup_{\rho \in \rho_2} \sup_{w \in \mathcal{Z}_2} \frac{\|z(p)\|_2}{\|w\|_2} \quad (15)$$

The existence of a controller that solves the parameter dependent LPV $\gamma$-performance problem can be expressed as the feasibility of a set of linear matrix inequalities (LMIs), which can be solved numerically [7, 13, 24].

It is worth noting that:

- the above problem can be solved considering the parameter dependent stability of LPV systems, which is the generalization of the quadratic stability concept. Applying the parameter dependent stability concept, it is assumed that the derivative of parameters can also be measured in real time. This concept is less conservative than the quadratic stability [20, 25, 26].
• the possible controller dependence on \( \rho \) will be stated by the adopted solution in terms of the parameter-dependence, or not, of the Lyapunov matrix.

4. Solution of the \( H_\infty / LPV \) control problem

Let us consider the LPV generalized plant \( \{13\} \). First, recall that several methods have arisen for representing the parameter dependence in LPV models, and then for designing the LPV controllers, such as \( \{27\} \):

- **Linear Fractional Transformations (LFT) \( \{28\}, \{29\} \)**. The LFT models have state matrices that are rational functions of the parameter. Hence, their dependence on the parameter vector is modeled explicitly.

- **Polytopic solution**: A polytopic system is a convex combination of systems defined at each vertex of a polytope given by the bounds of the scheduling parameters \( \{30\}, \{31\}, \{32\} \). The synthesis of such a controller can be made in the framework of \( H_\infty / LPV \) based on the LMI solution for polytopic systems (the framework of quadratic stabilization).

This can be applied to LPV systems with an affine dependence on the parameters only.

- **Linearizations on a gridded domain (grid-based LPV) \( \{20\}, \{31\} \)**, which are obtained through Jacobian linearization at each grid point. Each linearization approximates the system’s dynamics in the vicinity of a particular grid point, and the grid of linearizations captures the system’s parameter dependence implicitly. Hence, linearization based LPV models do not require any special dependence on the parameter vector.

In this research, the authors are interested in the grid-based LPV approach for the active anti-roll bar system of heavy vehicles using the LPVTools™ toolbox \( \{33\} \). Indeed such an approach is interesting where the number of parameters increases since the polytopic approach may lead to very conservative results (due to the augmented size of the parameter set and the single Lyapunov function). It has been used successfully in several studies \( \{33\}, \{34\}, \{35\} \) and is now available in the LPVTools™ toolbox \( \{36\} \).

4.1. A solution to the LPV control design

Some brief recall on the synthesis of dynamic and feedback controllers for LPV presented here. More details can be found in the studies \( \{20\}, \{25\}, \{26\} \). The following theorem describes the LPV analysis problem when it is formulated in terms of the induced \( \mathcal{L}_2 \) norm of \( G(\rho) \) and the rate-bounds (\( \hat{V}, \gamma \)) of the parameter are taken into account \( \{17\} \).

**Theorem 1**: Given a compact set \( \mathcal{P} \subset \mathcal{R}^n \), the performance level \( \gamma \) and the LPV system \( \{13\} \), with restriction \( D_{11}(\rho = 0) \), the parameter-dependent \( \gamma \)-performance problem is solvable if there exist a continuously differentiable function \( X: \mathcal{R}^n \rightarrow \mathcal{R}^{m \times n} \), and \( Y: \mathcal{R}^n \rightarrow \mathcal{R}^{m \times n} \), such that for all \( \rho \in \mathcal{P} \), \( X(\rho) = X^T(\rho) > 0 \), \( Y(\rho) = Y^T(\rho) > 0 \) and

\[
\begin{bmatrix} 
\hat{A}(\rho)X(\rho) + X(\rho)\hat{A}^T(\rho) - \sum_{i=1}^n \hat{B}_i(\rho)B_i^T(\rho) & X(\rho)C_i^T(\rho) & \gamma^{-1}S_i(\rho) \\
C_i(\rho)X(\rho) & -I & 0 \\
Y^{-1}B_i(\rho) & 0 & -\nu \\
\end{bmatrix} < 0 
\]

Theorem 1 and its proof are found in \( \{20\}, \{25\}, \{26\} \). The constraints set by the LMIs in Theorem 1 are infinite dimensional, as is the solution space. The variables are \( X, Y : \mathcal{R}^n \rightarrow \mathcal{R}^{m \times n} \), which restricts the search to the span of a collection of known scalar basis functions.

Select scalar continuous differentiable basis functions \( \{g_i : \mathcal{R}^n \rightarrow \mathcal{R} \}_{i=1}^{N_1}, \{f_j : \mathcal{R}^n \rightarrow \mathcal{R} \}_{j=1}^{N_2} \), then the variables in Theorem 1 can be parametrized as:

\[
X(\rho) = \sum_{i=1}^{N_1} g_i(\rho)X_i, \quad Y(\rho) = \sum_{j=1}^{N_2} f_j(\rho)Y_j
\]

Currently, there is no analytical method to select the basis functions, namely \( g_i \) and \( f_j \). An intuitive rule for the basis function selection is to use those present in the open-loop state space data. In our case, several power series \( \{1, \rho^2\} \) of the scheduling parameters are chosen, based on the lowest closed-loop \( \mathcal{L}_2 \) norm achieved.

4.2. Grid-based LPV approach

The LPV system in the equation \( \{13\} \) is conceptually represented by a state-space system \( \mathcal{N}(\rho) \) that depends on a time varying parameter vector \( \rho \). A grid-based LPV model of this system is a collection of linearizations on a gridded domain of parameter values \( \{36\} \). For
general LPV systems, this conceptual representation requires storing the state-space system at an infinite number of points in the domain of \( \rho \). For each grid point \( \tilde{\rho}_k \) there is a corresponding LTI system \( (A(\tilde{\rho}_k), B(\tilde{\rho}_k), C(\tilde{\rho}_k), D(\tilde{\rho}_k)) \) which describes the dynamics of \( S(\tilde{\rho}_k) \) when \( \tilde{\rho}_k \) is held constant. All the linearized systems on the grid have identical inputs \( u \), outputs \( y \) and state vectors \( x \). Together they form a LPV system approximation of \( S(\rho) \) [55, 57]. Such a grid-based approach is here used to find an LPV controller [14] solving the Theorem 1.

4.3. The LPVToolbox\textsuperscript{TM} tool box

LPVToolbox\textsuperscript{TM} toolbox was developed by MUSYN, Inc. (G. Balas and the authors) but has been made freely available to the community. The toolbox can be available for download at:

www.aem.umn.edu/SeilerControl/software.shtml

The LPVToolbox\textsuperscript{TM} is a MATLAB toolbox for modeling and design of Linear Parameter-Varying (LPV) systems. The toolbox contains data structures to represent LPV systems in both the LFT and gridded (Jacobian-linearization) framework. The core of the toolbox is a collection of functions for model reduction, analysis, synthesis and simulation of LPV systems. The readers interested in the characteristics and capacities of the LPVToolbox\textsuperscript{TM} toolbox, can find more in [33, 35].

4.4. Grid-based LPV approach for the active anti-roll bar system

In this study, the grid-based LPV approach and LPVToolbox\textsuperscript{TM} are used to synthesize the \( H_\infty \)/LPV active anti-roll bar control for heavy vehicles. It requires a gridded parameter space for the three varying parameters \( \rho = [\rho_1, \rho_2, \rho_3] \). In the interconnection structure, the spacing of the grid points is selected on the basis of how well the \( H_\infty \) point designs perform for plants around the design point. The \( H_\infty \) controllers are synthesized for 10 grid points of the forward velocity in the range \( \rho_1 = v = [40\text{km/h}, 130\text{km/h}] \) and 5 grid points of the normalized load transfers at the two axles in a range \( \rho_2 = |R_f| = [0, 1], \rho_3 = |R_r| = [0, 1] \), respectively. In this work, we have chosen to design a controller that does not depend on the parameter derivatives (so the scalar basis functions [19] are constant). The following commands are used to make the grid points as well as the LPV controller synthesis by using LPVToolbox\textsuperscript{TM}:

\[
\text{rho1} = \text{pgrid('rho1',linspace(40/3.6,130/3.6,10))};
\]

\[
\text{rho2} = \text{pgrid('rho2',linspace(0,1,5))};
\]

\[
\text{rho3} = \text{pgrid('rho3',linspace(0,1,5))};
\]

and \([Klpv,normlpv] = \text{lpvsyn}(H_{\text{umeas},\text{ntcont}})\). The weighting functions for both the performance and robustness specifications are considered unique for the whole grid. The effect of the proposed \( H_\infty \)/LPV controller is shown both in the frequency and time domains. The parameter values of the ESVH actuators and of the yaw-roll model are those given in Table 3.

Table 3: Parameters of the yaw-roll model and ESVH actuator [7, 18].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_x )</td>
<td>12487 kg</td>
<td>( b_f )</td>
<td>( 100 \frac{N\text{m}}{\text{rad}} )</td>
</tr>
<tr>
<td>( m_{a,f} )</td>
<td>706 kg</td>
<td>( b_r )</td>
<td>( 100 \frac{N\text{m}}{\text{rad}} )</td>
</tr>
<tr>
<td>( m_{a,r} )</td>
<td>1000 kg</td>
<td>( k_f )</td>
<td>( 2060 \frac{kN \text{m}}{\text{rad}} )</td>
</tr>
<tr>
<td>( m )</td>
<td>14193 kg</td>
<td>( k_r )</td>
<td>( 3337 \frac{kN \text{m}}{\text{rad}} )</td>
</tr>
<tr>
<td>( I_{xx} )</td>
<td>24201 kgm(^2)</td>
<td>( k_f )</td>
<td>( 380 \frac{kN \text{m}}{\text{rad}} )</td>
</tr>
<tr>
<td>( I_{zz} )</td>
<td>4200 kgm(^2)</td>
<td>( k_r )</td>
<td>( 684 \frac{kN \text{m}}{\text{rad}} )</td>
</tr>
<tr>
<td>( r )</td>
<td>0.83 m</td>
<td>( C_f )</td>
<td>( 582 \frac{kN \text{m}}{\text{rad}} )</td>
</tr>
<tr>
<td>( h_{ui} )</td>
<td>0.53 m</td>
<td>( C_r )</td>
<td>( 783 \frac{kN \text{m}}{\text{rad}} )</td>
</tr>
<tr>
<td>( h )</td>
<td>1.15 m</td>
<td>( I_f )</td>
<td>1.95 m</td>
</tr>
<tr>
<td>( \beta_{\text{c}} )</td>
<td>6.89\times10^9 \frac{m}{s^2}</td>
<td>( I_r )</td>
<td>1.54 m</td>
</tr>
<tr>
<td>( \tau )</td>
<td>0.01 s</td>
<td>( A_p )</td>
<td>( 0.0123 m^2 )</td>
</tr>
<tr>
<td>( K_p )</td>
<td>( 4.2\times10^{-11} \frac{m^4}{s^2} )</td>
<td>( K_{x} )</td>
<td>( 2.5 \frac{m^2}{\text{rad}} )</td>
</tr>
<tr>
<td>( C_{lp} )</td>
<td>0</td>
<td>( C_{x} )</td>
<td>0</td>
</tr>
</tbody>
</table>

5.1. Analysis in the frequency domain

Various closed-loop transfer functions of the LPV active anti-roll bar system on heavy vehicles are shown in this section. The main objective of the active anti-roll bar system is to reduce the normalized load transfer at each axle \( (R_f) \). To evaluate the effectiveness of the proposed \( H_\infty \)/LPV active anti-roll bar controller, the two following cases will be considered: in the 1\textsuperscript{st} case, the varying parameters are \( \rho_1 = v = [40\text{km/h}, 130\text{km/h}] \), \( \rho_{2,3} = 0.5 \) and in the 2\textsuperscript{nd} case, the varying parameters are \( \rho_{2,3} = [0, 1] \), \( \rho_1 = v = 70 \text{ km/h} \).

5.1.1. The 1\textsuperscript{st} case: \( \rho_1 = v = [40\text{km/h}, 130\text{km/h}] \) (10 grid points)

We only consider the varying parameter \( \rho_1 = v = [40\text{km/h}, 130\text{km/h}] \) with 10 grid points, while the varying parameters \( \rho_{2,3} \) are kept constant at 0.5. Figures 4
shows respectively the transfer function of the normalized load transfers $\frac{\delta f}{\delta f}$ and the input currents $\frac{u_f}{\delta r}$ at the two axles, as well as the inverse of the weighting functions $\frac{1}{W_{df}}, \frac{1}{W_{dr}}, \frac{1}{W_{r}^o}, \frac{1}{W_{d}^o}$.

As shown in Figures 4, 5, and Table 4, the LPV active anti-roll bar system allows to reduce the normalized load transfers at the two axles compared with the passive anti-roll bar case in the frequency range up to more than 4 rad/s, which represents the limited bandwidth of the driver [9].

Figures 4, d show the transfer function gains of the input currents at the front $(\frac{u_f}{\delta r})$ and rear axles $(\frac{u_r}{\delta r})$, respectively. When the forward velocity increases, the controller input currents $(u_f, u_r)$ also increase. This indicates that the active anti-roll bar system requires more input current (i.e. energy) at higher forward velocity. Nonetheless, it remains in the allowed bound, which prevents from the actuator saturation.

5.1.2. The 2nd case: $\rho_{2,3} = [0, 1] \left( 5 \text{ grid points} \right)$

We consider the varying parameter $\rho_1 = v = 70 \text{ km/h}$, while the varying parameters $\rho_{2,3} = [0, 1]$ with 5 grid points for each parameter. Figure 5 shows the transfer function magnitude of the normalized load transfers at the two axles $\frac{R_{f,r}}{\delta r}$ when the varying parameters $\rho_{2,3}$ are at the lower and upper bounds ($\rho_2 = 0, \rho_2 = 1$ and $\rho_3 = 0, \rho_3 = 1$).

As shown in Figure 5, when the value of $\rho_{2,3}$ increase, the normalized load transfers at the two axles decrease in the frequency range up to more than 4 rad/s. And the reduction is about 19 dB between $\rho_{2,3} = 0$ and $\rho_{2,3} = 1$. The results above, indicate that the proposed $H_{\infty}$/LPV controller (with the parameter dependent weighting functions, including the normalized load transfers at the two axles) provides for performance adaptation to the rollover risk for heavy vehicles.
5.2. Analysis in the time domain

In this section, some simulation results in the time domain are shown for two different situations: the passive anti-roll bar and the LPV active anti-roll bar of the integrated model. The vehicle manoeuvre is a double lane change, which is a typical case to evaluate for the obstacle avoidance in an emergency. The manoeuvre has a 2.5 m path deviation over 100 m. The steering angle $\delta_f$ is shown in Figure 6a.

To validate the proposed $H_\infty$/LPV controller strategy for the active anti-roll bar system with three varying parameters $\rho_1 = v$, $\rho_2 = |R_f|$, $\rho_3 = |R_r|$, the following scenario is used:

- The initial forward velocity is 90 km/h, the vehicle runs on the dry road ($\mu = 1$). The total rolling resistance and aerodynamic resistance forces are ignored.
- When the obstacle is detected, the driver reduces the throttle and brakes to reduce the forward velocity of the vehicle. The total brake force increases from 0.5 s to 1.5 s and then the driver takes off the brake pedal, as shown in Figure 6b.
The differential equation for the forward velocity in case of the braking situation is determined as [24]:

$$mv = - \sum_{i=1}^{4} F_{bi}$$  \hspace{1cm} (20)

where $F_{bi}$ is the brake force at each wheel. Due to the brake force, the forward velocity reduces from 90 km/h to 76.5 km/h, as in Figure 5.

Figures 6, 7, 8 show the variation of varying parameters $\rho = [\rho_1, \rho_2, \rho_3]$. Figures 6, 8 show the normalized load transfers at the two axles. We can see that in the case of the passive anti-roll bar system, the rollover does occur indeed at the two axles, but in the case of the $H_\infty/LPV$ active anti-roll bar control, the maximum absolute value of the normalized load transfers at the two axles are respectively 0.55 and 0.46. This indicates that the $H_\infty/LPV$ active anti-roll bar control improves well the roll stability of heavy vehicles compared with the passive anti-roll bar case. The force of the actuators as well as the input current at the two axles are shown in Figure 8.

The simulation results both in frequency and time domains have proved the effectiveness of the $H_\infty/LPV$ active anti-roll bar controller synthesis, which considered the three varying parameters: the forward velocity and the normalized load transfers at the two axles, compared with the passive anti-roll bar case.

6. Validation of the $H_\infty/LPV$ active anti-roll bar control by using the TruckSim® simulation software

TruckSim® is a vehicle dynamics modelling software developed by Mechanical Simulation. It delivers accurately, detailed, and efficient methods for simulating the performance of multi-axle commercial vehicles such as $4 \times 2$ tractors, $6 \times 4$ tractors as well as box trucks, buses and trailers. From TruckSim® software, we can build vehicle models by defining many vehicle parameters which affect the dynamic behaviour of the model. To survey the proposed $H_\infty/LPV$ controller for the active anti-roll bar system by using the nonlinear vehicle model in TruckSim® software, the authors use the co-simulation between Matlab®/Simulink® and TruckSim® software. The diagram of the co-simulation is shown in Figure 7. The nonlinear vehicle model is determined from TruckSim® software, based on using the block S-function of Simulink. Meanwhile, the controller and the actuators are built directly in the Matlab®/Simulink® environment.

In the following validations, the authors tested the proposed $H_\infty/LPV$ controller for the active anti-roll bar system with two different types of vehicle: the tour bus and the LCF (Low Cab Forward) truck using the solid suspension system, both fully loaded. The engine is mounted at the rear of the vehicle for the tour bus and at the front for the LCF truck. To evaluate the rollover of vehicles, we consider the tyre force ($F_z$) in the Z direction at each wheel (rollover occurs when $F_z = 0$). The authors note that the steering angle in the following section is the angle of the steering wheel, which is directly controlled by the driver. In the co-simulation between Matlab®/Simulink® and TruckSim®, there are two solutions for the steering angle:

- First solution: the steering angle is made in the S-function of Simulink® and entered to TruckSim® through the S-function. With this solution, the trajectories of vehicle in the cases of the passive anti-roll bar and of the active anti-roll bar are often different, by the effect of the wheels lift off the road.
- Second solution: the steering angle is automatically changed to adapt with the vehicle trajectory. Here, the vehicle trajectories in the case of the passive anti-roll bar and of the active anti-roll bar will follow the target path, which fits the driver’s wishes.

In the following validations, the second solution is used to define the steering angle.
6.1. Validation with the tour bus

A commercial passenger bus is probably one of the most popular people carrying vehicles in the world. Typically they are two axled vehicles (bus 2A) with a capacity of 45 passengers, as shown in Figure 8. In this validation, the total vehicle mass of the tour bus is 10620 kg, the double lane change is used to evaluate the roll stability of the tour bus when it runs at 100 km/h, as shown in Figure 9. This represents the situation when the driver wishes to overtake another vehicle. Figure 9 shows the time responses of the tour bus in the double lane change maneuver with the continued line for the $H_{\infty}/LPV$ active anti-roll bar and the dash line for the passive anti-roll bar.

Figures 9b, c, d show the time response of the roll angle of the sprung mass and the roll angle of the unsprung masses at the two axles, respectively. We can see that in the case of the $H_{\infty}/LPV$ active anti-roll bar controller, the roll angle of sprung and unsprung masses are significantly reduced compared to the passive anti-roll bar case (the reduction of the roll angle is about 6 deg for the sprung mass, 3 deg for the unsprung mass at the front axle, and 2 deg for the unsprung mass at the rear axle).
front and 4 deg for the unsprung mass at the rear axle). Figures 9, f, g, h show the time response of the tyre forces in the Z direction of all the wheels. We can see that in the case of the $H_w/LPV$ active anti-roll bar controller, all the tyre forces remain positive, which means that there is no wheel lift off the road. But in the case of the passive anti-roll bar, the left-front wheel lifts off at $2.6s \div 3.4s$ and $5.7s \div 6.5s$, the right-front wheel at $3.8s \div 5.3s$, the left-rear wheel at $5.8s \div 6.5s$, the right-rear wheel at $4s \div 5.5s$. These results show that the $H_w/LPV$ active anti-roll bar controller drastically improves the roll stability in the case of the tour bus fully loaded.

Figure 10 shows the trajectory of the tour bus in the double lane change maneuver. We would like that, in ideal conditions, the center of the vehicle mass follows the target path. But in fact, the trajectory of the center of the vehicle mass of the real vehicle can not satisfy that (due to the impact of the suspension system, the wheelbase and the wheels lift off from the road, etc). The trajectory of the vehicle in both cases of the $H_w/LPV$ active anti-roll bar and the passive anti-roll bar can only stick with the ideal target path as in Figure 10. Figure 9 shows the steering wheel angle controlled by the driver with the amplitude about 100 deg. To ensure the trajectory of the vehicle as in Figure 10 the driver generates the steering wheel angle difference between the $H_w/LPV$ active anti-roll bar case and the passive anti-roll bar case. We can easily see that the steering wheel angle in the case of the $H_w/LPV$ active anti-roll bar is smoother than in the case of the passive anti-roll bar.

Thus, in the $H_w/LPV$ active anti-roll bar case, the driver is less tired than in the passive anti-roll bar case.

6.2. Validation with the LCF truck

In this validation, the total vehicle mass of the LCF truck is 12549 kg, a sine wave steering manoeuvre is used to evaluate the roll stability of the LCF truck when it runs at 80 km/h, as shown in Figure 12b. Figure 12 shows the time response of the LCF truck. From figure 12b, the steering wheel angle $\delta_f$ is the sine wave with the amplitude of 90 deg and the repeating cycle time is 4s. In this validation, the driver applies the same steering wheel angles for both the $H_w/LPV$ active anti-roll bar case and the passive anti-roll bar case. Figures 12b, c, d show that in the case of the $H_w/LPV$ active anti-roll bar control, the roll angle of sprung and unsprung masses are significantly reduced compared to the case of the passive anti-roll bar, with the reduction of 10 deg for the roll angle of sprung and of 8 deg for the roll angle of the unsprung mass at the front axle, and 3 deg for the roll angle of the unsprung mass at the rear axle.

Figures 12b, f, g, h show the time response of the tyre forces in the Z direction of all the wheels. We can see that in the case of the $H_w/LPV$ active anti-roll bar controller, all the tyre forces remain positive, which means that there is no wheel lift off the road. But in the case of the passive anti-roll bar, all the wheels at the two axles lift off. These results show that the LPV active anti-roll bar control improves the roll stability in the case of the fully loaded LCF truck.

Figure 13 shows the trajectory of the LCF truck. We can see that even the forward velocity is held constant at 80 km/h but the LCF truck travels to point A in the case of the passive anti-roll bar, and to point B in the case of the $H_w/LPV$ active anti-roll bar. Since the wheels lift off the road in the case of the passive anti-roll bar, some performance characteristics of the vehicle are lost.

The simulation results for both the tour bus and LCF truck show a chaotic behavioural difference: the wheels at the front axle lift off the road before the wheels at
Figure 12: Time responses of the truck in the sine wave steering manoeuver.

the rear axle for the tour bus (see Figure 9, h). And for the LCF truck, it is the opposite (see Figure 12, h).

7. Conclusion

In this paper, the authors considered the nonlinear model of a single unit heavy vehicle as an LPV model, with the forward velocity as a scheduling parameter. The $H_\infty$/LPV active anti-roll bar controller is synthesized by using the grid-based LPV approach through LPVTools TM. Three varying parameters are considered to schedule the $H_\infty$/LPV controller: the forward veloc-
ity and the normalized load transfers at the two axles. The $H_{\infty}/LPV$ design is performed using parameter dependent weighting functions which allow adapting the performance to the risk of rollover in heavy vehicles. The simulation results in the frequency and time domains as well as the validation by using the TruckSim® software show that the $H_{\infty}/LPV$ active anti-roll bar controller drastically improves the roll stability of the single unit heavy vehicle compared with the passive anti-roll bar.

In the future, the effect of the oil passing through the electronic servo-valve on the closed-loop system by using a LPV approach for fault tolerant control design will be also an interesting area for further research.

References

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